Landau theory of Fermi liquids and the integration-over-the-coupling-constant algorithm

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We have formulated a microscopic approach to the Landau theory of Fermi liquids. The method is based on the integration-over-the-coupling-constant algorithm and involves the use of suitable generalizations of Hubbard's many-body local field. Our investigation clearly demonstrates the crucial role played in the theory by the often neglected antiparallel spin correlations. A point we also emphasize is that in order to correctly employ this procedure it is necessary to make use of the expression for the charge susceptibility as appropriate to the case of an infinitesimally polarized system. As an exemplification, we present a fully self-consistent calculation of the effective mass and the anomalous g factor for the quasi-two-dimensional electron liquid occurring in Si inversion layers. It is shown that the present development solves some serious problems plaguing earlier theories.

I. INTRODUCTION

The Fermi liquid theory as developed by Landau,[1] Silin[2] and later by Nozières and Luttinger,[3,4] has been quite successful in providing a practical paradigm for the description of the physical properties of interacting Fermi systems when Luttinger theorem applies.[5] In particular, this theory has lead to several attempts aimed at a quantitative understanding of the transport properties of interacting electrons at metallic densities.

From an experimental viewpoint the quasi-two-dimensional (2D) electronic liquid (EL) realized in Si inversion layers has to date provided the most fruitful and interesting arena in which such theoretical framework can be put to work in practice. The most interesting feature of such systems is the possibility of being able to sweep, within the same sample, a reasonably wide range of electronic densities. As a consequence these systems are particularly suitable for the study of the many-body effects associated with the Coulomb interaction. In particular, Smith and Stiles[6] and Abstreiter *et al.*[7] have determined the density dependence of the cyclotron mass while Fang and Stiles[8] and Neugebauer *et al.*[9] have obtained the density dependence of the anomalous Landé factor g^* in such systems.

In his work Rice[10] presented a theory and a systematic analysis of the density dependence of a number of quasiparticle transport properties in a three-dimensional EL by obtaining an approximate microscopic expression for the Landau quasiparticle interaction function $f^{\sigma,\sigma'}_{\mathbf{k},\mathbf{p}}$. In Rice's approach, $f^{\sigma,\sigma'}_{\mathbf{k},\mathbf{p}}$ is obtained via a double functional derivative of an expression for the total energy of the EL first proposed by Hubbard[11,12] based on the integration-over-the-coupling-constant algorithm. Rice's paper carried out explicit calculations of transport properties by accounting for many-body effects beyond the simple random-phase approximation (RPA).[13,14] This was accomplished by accounting for parallel spin exchange and correlation corrections via the many-body local-field method introduced also by Hubbard.[12] Although it provided substantial contributions, Rice's paper contained an error that resulted in an incorrect treatment of the spin-antisymmetric channel.[15]

Rice's procedure was later implemented for a 2D EL by Ting, Lee, and Quinn (TLQ).[16] Although Rice's approach was corrected, this paper still included parallel spin exchange and correlation corrections only. As it turns out the results obtained by TLQ for the spin susceptibility (more precisely the Landé factor g^*) were rather unsatisfactory in that the predicted density dependence of g^* was opposite to that observed experimentally.[8,9,17] In the same paper TLQ showed that some degree of agreement with the expected and experimentally observed curve can be achieved if one renounces the Hubbard local-field correction and simply performs the calculation in the RPA. The corresponding RPA calculation for the case of a quasi-2D EL was later reported by the same authors.[18]

Recently a more sophisticated and extensive study of the microscopic many-body theory of an EL has been carried out by the present authors[19–21] as well as by Ng and Singwi[22] and Zhu and Overhauser.[23] In particular the work contained in Refs. 19–21 takes advantage of an approach that is not based on the integration-over-the-coupling-constant algorithm but rather on a more physically satisfactory self-consistent canonical transformation procedure. The results of these calculations are in reasonable agreement with the experimental findings. Since this work has been presented in a number of papers, it need not be discussed here.

In spite of recent developments, the apparent failure of the many-body theory of TLQ to account for the appropriate behavior of the spin response of the EL has to date surprisingly remained unexplained.

It is the purpose of the present paper to demonstrate how often neglected antiparallel spin correlations naturally arise from a theoretically correct procedure even within the integration-over-the-coupling-constant algo-

rithm in its present form. In this respect the present theory is based on a generalization of Rice's procedure in which new many-body local fields are used to account for some of the antiparallel spin correlations explicitly neglected in Refs. 10 and 16. More precisely the main new development with respect to the TLQ theory is the allowance of terms proportional to the antisymmetric many-body local field G_- (see below). Naively, terms of this sort could be thought to be absent from a description based on the integration-over-the-coupling-constant algorithm where only the knowledge of the charge response of the EL enters, a quantity which in the unpolarized state only depends on the spin-symmetric many-body local field G_+. As it turns out, however, what is actually necessary is the dependence of the charge response function *separately* upon infinitesimal variations of the occupation numbers of up- and down-spin states. This amounts to the knowledge of the charge response function of an *infinitesimally polarized* EL.[24] Naturally, this quantity contains both the fields G_+ and G_-.[20] As a consequence, although within the present simple approach G_- does not enter (as in fact it should) (Refs. 20 and 21) the expression for the quasiparticle energy, G_- does at least appear, and plays an important role in the quasiparticle interaction function $f^{\sigma,\sigma'}_{\mathbf{k},\mathbf{p}}$.

As an exemplification we also present an explicit parameter-free self-consistent calculation of the many-body effective mass m^* and of the Landé factor g^* for the case of the quasi-2D EL occurring in a [100]Si inversion layer. In view of the valley degeneracy which characterizes this system, our whole theory is worked out for the case of a multivalley EL. As we will show, allowance of the new corrections appears to satisfactorily resolve the problem plaguing the TLQ theory thereby shedding light on a rather puzzling theoretical question.

The paper is organized as follows. In Sec. II we present the correct procedure to be followed within the integration-over-the-coupling-constant algorithm. Within such a framework, we obtain the explicit expressions for both the quasiparticle energy and the quasiparticle interaction function. Corresponding expressions for the transport quantities of interest are also derived there. In Sec. III we present the above-mentioned explicit calculation of m^* and g^*. Lastly, in Sec. IV we discuss our results and compared them with previous work.

II. MICROSCOPIC APPROACH TO THE LANDAU THEORY OF FERMI LIQUIDS

Within the framework of the Landau theory, the quasiparticle energy $E^\sigma_{\mathbf{k}}$ and the quasiparticle interaction function $f^{\sigma,\sigma'}_{\mathbf{k},\mathbf{p}}$ can be obtained via suitable functional derivatives of the total energy E. In particular for a multivalley EL:

$$E^\sigma_{\mathbf{k}}=\frac{1}{\nu_v}\frac{\delta E}{\delta n^\sigma_{\mathbf{k}}} \tag{1}$$

and

$$f^{\sigma,\sigma'}_{\mathbf{k},\mathbf{p}}=\frac{1}{\nu_v}\frac{\delta^2 E}{\delta n^\sigma_{\mathbf{k}}\delta n^{\sigma'}_{\mathbf{p}}}\ , \tag{2}$$

where $\delta n^\sigma_{\mathbf{k}}$ corresponds to the change in the occupation number of spin-σ quasiparticles in one of the degenerate ν_v valleys.[25] Also to obtain $E^\sigma_{\mathbf{k}}$ and $f^{\sigma,\sigma'}_{\mathbf{k},\mathbf{p}}$ the functional derivatives are to be evaluated at $\delta n^\sigma_{\mathbf{k}}=0$. At this point a possible way to proceed is that suggested by Hubbard[11,12] and implemented by Rice[10] and TLQ.[16] One starts from the following exact expression for the total energy for the system based on the so-called integration-over-the-coupling-constant algorithm:[14]

$$E[n^\sigma_{\mathbf{k}}]=E_0+\int_0^1\frac{d\lambda}{\lambda}E_{\rm int}(\lambda)\ , \tag{3}$$

where E_0 is the kinetic energy of noninteracting electrons and the quantity $E_{\rm int}(\lambda)$ is defined as

$$E_{\rm int}(\lambda)=-\sum_{\mathbf{q}}\left\{\int_0^\infty\frac{d\omega}{2\pi}{\rm Im}[v_\lambda(q)\chi_{C,\lambda}(\mathbf{q},\omega)]+\frac{v_\lambda(q)N}{2}\right\}, \tag{4}$$

an expression that makes manifest the fact that the total energy of the EL is completely determined by the knowledge of the charge response χ_C for all values of the coupling constant from 0 to e^2. In the above equation, and in what follows, the subscript λ implies that the Fourier transform of the bare Coulomb interaction $v(q)$ is to be multiplied by the (real) scale factor λ. Now since it is necessary to be able to perform functional derivatives of $E[n^\sigma_{\mathbf{k}}]$ separately with respect to the occupation number of spin-up and spin-down electrons, in Eq. (4) the charge response χ_C should be taken to be appropriate to an EL with an infinitesimal polarization. For such a system an expression for χ_C has been derived in Ref. 20 and is given by

$$\chi_C(q,\omega)=\frac{\nu_v\chi_0^\uparrow(q,\omega)+\nu_v\chi_0^\downarrow(q,\omega)+4\nu_v^2v(q)G^v_-\chi_0^\uparrow(q,\omega)\chi_0^\downarrow(q,\omega)}{\mathcal{D}_v(q,\omega)}\ , \tag{5}$$

where the denominator $\mathcal{D}_v$ of this expression is defined by

$$\mathcal{D}_v=1-\nu_v v(q)(1-G^v_+-G^v_-)(\chi_0^\uparrow+\chi_0^\downarrow)-4\nu_v^2v(q)^2G^v_-(1-G^v_+)\chi_0^\uparrow\chi_0^\downarrow\ . \tag{6}$$

In Eqs. (5) and (6) χ_0^σ is a generalized Lindhard response function and can be defined in terms of the one-electron Green's function $g^\sigma(\mathbf{p},\omega)$ as follows:

$$\chi_0^\sigma(\mathbf{q},\omega)\equiv\sum_{\mathbf{p}}\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi i}g^\sigma(\mathbf{p},\epsilon)g^\sigma(\mathbf{p}+\mathbf{q},\epsilon+\omega)\ , \tag{7}$$

where

$$g^{\sigma}(\mathbf{p},\omega)\equiv\frac{n_{\mathbf{p}}^{\sigma}}{\omega-\epsilon_{\mathbf{p}}-i\eta}+\frac{1-n_{\mathbf{p}}^{\sigma}}{\omega-\epsilon_{\mathbf{p}}+i\eta}\ . \tag{8}$$

In the above equation, in order to be consistent with Niklasson's definition of the many-body local fields, the $n_{\mathbf{p}}^{\sigma}$ are exact occupation numbers.[26] Furthermore, the many-body local fields $G_{\pm}^{v}$ are, in general, functions of q and ω and are designed to account for the vertex corrections associated with charge- and spin-density fluctuations, respectively.[19,20] These functions are also defined so as to be appropriate to the case of an unpolarized multivalley system.[27]

For an unpolarized system Eq. (5) reduces to the following familiar form which can be treated as the defining equation for G_{+}^{v}:

$$\chi_C(q,\omega)\equiv\frac{\nu_v\chi_0(q,\omega)}{1-\nu_v v(q)(1-G_{+}^{v})\chi_0(q,\omega)}\ , \tag{9}$$

where $\chi_0=\chi_0^{\uparrow}+\chi_0^{\downarrow}$. G_{-}^{v} is in turn defined through the unpolarized spin response function as follows:

$$\chi_s(q,\omega)\equiv-\mu_B^2\frac{\nu_v\chi_0(q,\omega)}{1+\nu_v v(q)G_{-}^{v}\chi_0(q,\omega)}\ . \tag{10}$$

At this point the procedure involves taking the functional derivative of E with respect to $n_{\mathbf{k}}^{\sigma}$. Following Refs. 10 and 16 we do this by assuming that the only functional dependence of $E_{\rm int}(\lambda)$ of Eq. (4) on $n_{\mathbf{k}}^{\sigma}$ stems from the Lindhard functions χ_0^{σ}. To this purpose it is useful to note that

$$\frac{\delta\chi_0^{\sigma'}(\mathbf{q},\omega)}{\delta n_{\mathbf{p}}^{\sigma}}=\delta_{\sigma\sigma'}[g^{\sigma}(\mathbf{p}+\mathbf{q},\epsilon_{\mathbf{p}}^{\sigma}+\omega)+g^{\sigma}(\mathbf{p}-\mathbf{q},\epsilon_{\mathbf{p}}^{\sigma}-\omega)]\ . \tag{11}$$

Then, on using Eqs. (4)–(6) and (11), we obtain the following expression for the first functional derivative of $E_{\rm int}(\lambda)$:

$$\frac{\delta E_{\rm int}(\lambda)}{\delta n_{\mathbf{p}}^{\uparrow}}=-\nu_v\mathrm{Re}\sum_{\mathbf{q}}\int_0^{\infty}\frac{d\omega}{2\pi i}\frac{[v(q)+4\nu_v v(q)^2G_{-}^{v}\chi_0^{\downarrow}+4\nu_v^2v(q)^3(G_{-}^{v}\chi_0^{\downarrow})^2]_{\lambda}}{[\mathcal{D}_v]_{\lambda}^2}[g^{\uparrow}(\mathbf{p}+\mathbf{q},\epsilon_{\mathbf{p}}^{\uparrow}+\omega)+g^{\uparrow}(\mathbf{p}-\mathbf{q},\epsilon_{\mathbf{p}}^{\uparrow}-\omega)]_{\lambda}\ . \tag{12}$$

From this expression and Eqs. (3), (6), and (12), and by further assuming that $G_{\pm}^{v}$, χ_0^{σ}, and g^{σ} are all independent of λ, one arrives at the following simplified form for the quasiparticle energy in an unpolarized EL [Eq. (1)]:

$$E_{\mathbf{p}}=\epsilon_{\mathbf{p}}-\mathrm{Re}\sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi i}\frac{v(q)}{Q_{+}(q,\epsilon)}g(\mathbf{p}-\mathbf{q},\epsilon_{\mathbf{p}}-\epsilon)\ , \tag{13}$$

where

$$Q_{+}(q,\omega)=1-\nu_v v(q)(1-G_{+}^{v})\chi_0(q,\omega)\ , \tag{14}$$

and for simplicity the spin index has been left out in the expression for the quasiparticle energy. This expression coincides with that of both Refs. 10 and 16.

To obtain the Landau interaction function we take one more functional derivative of the quantity $E_{\rm int}(\lambda)$. Following the procedure outlined above we obtain for an unpolarized system

$$\frac{\delta^2E_{\rm int}(\lambda)}{\delta n_{\mathbf{p}}^{\uparrow}\delta n_{\mathbf{k}}^{\uparrow}}=-\mathrm{Re}\left[\frac{\nu_v v(|\mathbf{k}-\mathbf{p}|)}{Q_{+}(\mathbf{k}-\mathbf{p},\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{k}})^2}\right]_{\lambda}-\mathrm{Re}\sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi i}\left[\left\{\frac{2\nu_v^2v(q)^2(1-G_{+}^{v}+G_{-}^{v})}{Q-Q_{+}^3}-\frac{4\nu_v^2v(q)^2G_{-}^{v}}{Q_{-}Q_{+}^2}\right\}\tilde{g}\right]_{\lambda} \tag{15}$$

and

$$\frac{\delta^2E_{\rm int}(\lambda)}{\delta n_{\mathbf{p}}^{\uparrow}\delta n_{\mathbf{k}}^{\downarrow}}=-\mathrm{Re}\sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi i}\left\{\frac{2\nu_v^2v(q)^2(1-G_{+}^{v}+G_{-}^{v})}{Q_{-}Q_{+}^3}\tilde{g}\right\}_{\lambda}\ , \tag{16}$$

where we have defined

$$Q_{-}(q,\omega)=1+\nu_v v(q)G_{-}^{v}\chi_0(q,\omega)\ , \tag{17}$$

and

$$\tilde{g}\equiv[g(\mathbf{k}+\mathbf{q},\epsilon_{\mathbf{k}}+\epsilon)+g(\mathbf{k}-\mathbf{q},\epsilon_{\mathbf{k}}-\epsilon)]g(\mathbf{p}-\mathbf{q},\epsilon_{\mathbf{p}}-\epsilon)\ . \tag{18}$$

Finally making use of Eqs. (15) and (16), and upon carrying out the integration with respect to the coupling constant λ as prescribed in Eq. (3), the symmetric and the antisymmetric components of the Landau quasiparticle interaction function are obtained from Eq. (2) and can be expressed as

$$f_s\equiv\frac{f_{\mathbf{k},\mathbf{p}}^{\uparrow,\uparrow}+f_{\mathbf{k},\mathbf{p}}^{\uparrow,\downarrow}}{2}=-\frac{v(|\mathbf{k}-\mathbf{p}|)}{2Q_{+}(\mathbf{k}-\mathbf{p},0)}-\nu_v\mathrm{Re}\sum_{\mathbf{q}}\int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi i}\frac{v(q)^2(1-G_{+}^{v})}{Q_{+}^2}\tilde{g} \tag{19}$$

and

$$f_a \equiv \frac{f^{\uparrow,\uparrow}_{\mathbf{k},\mathbf{p}} - f^{\uparrow,\downarrow}_{\mathbf{k},\mathbf{p}}}{2} = -\frac{v(|\mathbf{k}-\mathbf{p}|)}{2Q_+(\mathbf{k}-\mathbf{p},0)} + 2\nu_v \mathrm{Re}\sum_{\mathbf{q}} \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i} v(q)^2 G^v_- F\tilde{g} \ , \tag{20}$$

where

$$F(q,\omega) = \frac{1}{[Q_- - Q_+]^2}\left[\frac{(Q_- - Q_+)}{Q_+} + \ln\left|\frac{Q_+}{Q_-}\right|\right] . \tag{21}$$

In Eqs. (19) and (20), the first term on the right-hand side is the contribution from the screened exchange while the remaining term is a higher-order correlation contribution.

At this point it is possible to obtain useful explicit expressions for the transport quantities of interest. For instance, the quasiparticle effective mass is obtained in this framework from[16]

$$\frac{m^*}{m} = 1 + \frac{m^*}{\pi}\int_0^{2\pi} \frac{d\phi}{2\pi} f_s(\phi)\cos\phi \ , \tag{22}$$

where the notation is such that ϕ is the angle between the vectors $\mathbf{k}$ and $\mathbf{p}$ which, in the spirit of the Landau theory, have been assumed to lie on the Fermi surface. As it turns out, in order to carry out explicit calculations, we find that it is easier to use the alternative definition of m^* given by[28]

$$\frac{1}{m^*} \equiv \frac{1}{p_F}\left[\frac{\partial E_{\mathbf{p}}}{\partial p}\right]_{p_F} . \tag{23}$$

A quantity of particular interest in the present work is the Landé factor g^*. Within Fermi-liquid theory g^* can be expressed in terms of m^* and the spin-antisymmetric Landau interaction function f_a of Eq. (20) by performing the quadrature via a contour integration along the circumference of the first and the third quadrants of the complex frequency plane. We obtain

$$\frac{g}{g^*} = 1 + \frac{m^*}{\pi}\int_0^{2\pi}\frac{d\phi}{2\pi} f_a(\phi) = 1 - \frac{m^*}{(2\pi)^2}\int_0^{2\pi} d\phi \frac{v(|\mathbf{k}-\mathbf{p}|)}{Q_+(\mathbf{k}-\mathbf{p},0)} + \frac{2\nu_v m^* m}{\pi^3}\int_0^{\infty} dz \int_0^{\infty} du\, v(q)^2 G^v_-(q,i\omega) \times F(q,i\omega) P_+(z,u) \ , \tag{24}$$

where

$$P_+(z,u) = \frac{[(z^2-u^2-1)^2+(2zu)^2]^{1/2}+(z^2-u^2-1)}{[(z^2-u^2-1)^2+(2zu)^2]} \ , \tag{25}$$

and we have introduced the variables $z = q/2p_F$, and $u = \omega m/qp_F$.

III. SELF-CONSISTENT DETERMINATION OF m^* AND g^* IN INVERSION LAYERS

In order to attempt a realistic quantitative calculation of the quasiparticles transport properties in a quasi-2D EL a number of modifications of the standard homogeneous EL theory must be implemented. A complete discussion of this problem can be found in Refs. 18 and 21. It will suffice here to state that one must account for the specific geometry of a metal-insulator-semiconductor system, its various dielectric constants, the crystal axis orientation, the corresponding subband structure with its associated particular value for the electron band mass, and the valley degeneracy. The actual numerical values used in the present calculation are those appropriate to a [100] silicon inversion layer and are given in the caption of Fig. 1. The subband structure also leads to a modification of the bare Coulomb interaction which will depend on the subband occupation.

In order to actually calculate m^* and g^* for various electronic densities, appropriate expressions for the many-body local fields $G^v_{\pm}(\mathbf{q},\omega)$ must be employed. The procedure we have used coincides with that described in Ref. 21 and is based on a self-consistent determination of $G^v_{\pm}$, for which a static model expression is assumed.[30] More specifically we take for $G^v_{\pm}$ the following simple form:

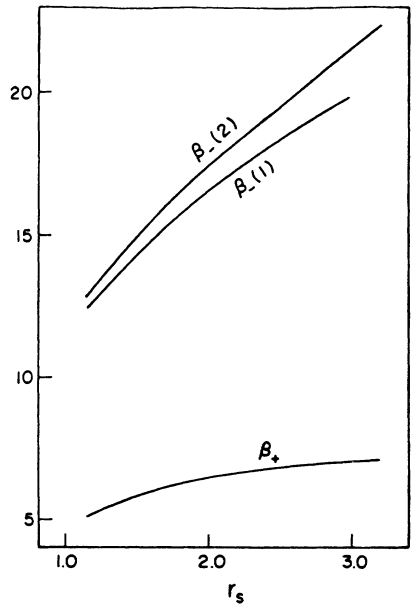

FIG. 1. Calculated values of the functions β_+ and β_-, defined in Eq. (26), vs r_s. The curves labeled $\beta_-(1)$ and $\beta_-(2)$ correspond to the theories in Refs. 21 and this paper, respectively. The following values have been used for the parameters in our calculations: 0.19 for the band-mass ratio, 2 for the valley degeneracy, 3.8 and 11.8, respectively, for the oxide and semiconductor dielectric constants, 5330 Å for the thickness of the oxide layer, and 32.5 Å for the average electronic distance from the oxide-semiconductor interface. For comparison $r_s = 2$ corresponds to a density of 1.7×10^{12} cm^{-2}.

$$G_{\pm}^{v}(\mathbf{q})=\frac{L[(q^2+p_F^2)^{1/2}]}{L(q)}\frac{G_{\pm}^{v}(\infty)q}{\{q^2+[\beta_{\pm}G_{\pm}^{v}(\infty)p_F]^2\}^{1/2}}\ , \tag{26}$$

where the form factor $L(q)$ is given in Refs. 18 and 21 and accounts for the modification of the bare Coulomb interaction in the inversion layer. Furthermore, in the above expression $G_{\pm}^{v}(\infty)$ are the values of the valley many-body local fields for large values of q and ω and are determined[29] by the value at the origin of the static pair correlation function $g(0)$.[14] The corresponding exact expressions are given by[20]

$$G_{+}^{v}(\infty)=[1+(v_v^{-1}-2)g(0)] \tag{27}$$

and

$$G_{-}^{v}(\infty)=v_v^{-1}g(0)\ . \tag{28}$$

Finally, $\beta_{\pm}$ are suitable functions of the electronic density which ensure the correct limiting behavior of $G_{\pm}$ for small q and $\omega=0$.

At this point the calculation proceeds as follows. First the values of both β_{+} and $g(0)$ are determined as a function of the density from existing calculations of the total energy of a 2D EL. In particular we have extracted these functions from the results of Jonson whose work was based on the popular theory of Singwi *et al.*[31] Then, the effective mass m^* can be determined as a function of the density from Eqs. (13), (23), and (26) since, in the present formulation, the only unknown quantity β_{-} does not enter its expression. Once m^* is known we use a trial value for β_{-} and calculate a preliminary value for g^* from Eqs. (24) and (25) at each density. At this point by means of the relationship

$$\frac{\chi_S(q\to 0,0)}{\chi_S^0}=\frac{m^*}{m}\frac{g^*}{g}\ , \tag{29}$$

involving g^*, m^*, the free-electron Pauli spin susceptibility χ_S^0, and the static long-wavelength spin susceptibility, we deduce a new function β_{-} by making use of Eqs. (26) and (10). These new values for β_{-} are then used as input for a new iteration of the whole procedure. The calculation is terminated when self-consistency is finally achieved for the values of β_{-} (or equivalently g^*) at each density. As a consequence the present calculation is free of arbitrary parameters.

In Fig. 1, we present the self-consistent values of $\beta_{\pm}$ as a function of the density parameter r_s (the average electronic distance expressed in effective Bohr radii). In this figure, as well as in the remaining ones, the curves labeled (1) correspond to the results of our full theory of Ref. 21, while the curves labeled (2) are the results of the present theory.

In Fig. 2 we display our results for the quasiparticle effective mass m^* as a function of the areal density n. There the empty circles are the experimental data from Ref. 6. The dashed line corresponds to the RPA calculation where G_{+} and G_{-} are arbitrarily set to zero.

Finally, in Fig 3 we have plotted our results for the enhanced Landé factor g^*. There, along with the results

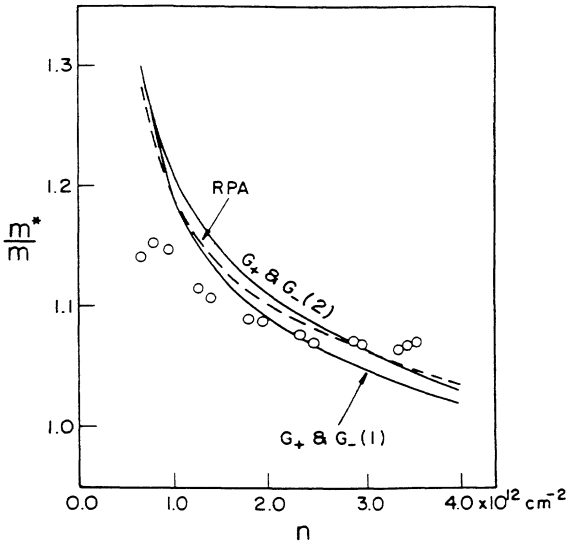

FIG. 2. Plot of the ratio m^*/m vs the electronic areal density n. The values of the parameters are the same as in Fig. 1. The solid curves labeled G_{+} & $G_{-}(1)$ and G_{+} & $G_{-}(2)$ correspond to our full theory of Ref. 21 and this paper, respectively. The dashed line corresponds to the RPA calculation, i.e., $G_{\pm}=0$. The circles are the experimental data taken from Ref. 6.

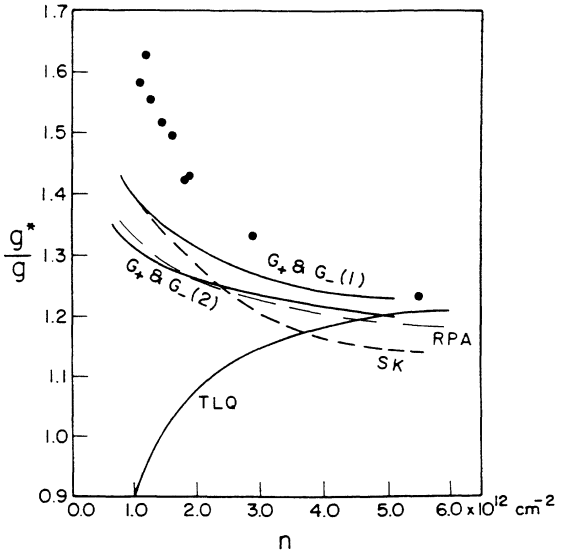

FIG. 3. Plot of g^*/g vs the electronic areal density n. The values of the parameters are the same as in Fig. 1. The solid curves labeled G_{+} & $G_{-}(1)$ and G_{+} & $G_{-}(2)$ correspond to our full theory of Ref. 21 and this paper, respectively. The curve labeled TLQ is the prediction of Ref. 16 which is appropriate for a 2D EL. The dashed line corresponds to the RPA calculation, i.e., $G_{\pm}=0$. The full circles are the original data from Ref. 8, while the dashed curve labeled SK corresponds to the same data after the rescaling by Suzuki and Kawamoto (Ref. 32).

of the new theory and that of Ref. 21 we have presented the RPA curve as well as the theoretical prediction of TLQ for a 2D EL that we have actually numerically reproduced on our own. For comparison we have also displayed the original data from Ref. 8 as well as their rescaled version according to Suzuki and Kawamoto from Ref. 32. Although our calculation is for a quasi-2D EL, the improvement of the present theory [curve G_+ and G_- (2)] over that of TLQ is quite obvious.

IV. DISCUSSION AND CONCLUSIONS

We have developed a microscopic approach to the Landau theory of Fermi liquids based on the integration-over-the-coupling-constant algorithm and a generalization of the Hubbard theory of many-body local fields. The method is the same as that previously employed by Rice[10] and TLQ.[16] Our contribution consists in accounting on equal footing for both parallel and antiparallel spin correlations. The latter were neglected in previous theories.

Our main results are represented by the explicit formulas for the Landau quasiparticle interaction function $f^{\sigma,\sigma'}_{\mathbf{k},\mathbf{p}}$ provided in Eqs. (19) and (20). Of interest is also our result for the Landé factor g^* which is proportional to the Wilson ratio. An explicit expression for such a quantity is provided in Eq. (24). At variance with previous work[16] our expression for f_a, the spin-antisymmetric component of $f^{\sigma,\sigma'}_{\mathbf{k},\mathbf{p}}$, does contain both G_+ and G_- as separate quantities describing different physical processes. For the sake of clarity we remind the reader that in a multivalley EL these two quantities are related to the single-valley spin-dependent many-body local fields as follows:[33]

$$G^v_\pm \equiv \frac{1}{\nu_v}\left[G^{\uparrow\uparrow}+(\nu_v \pm \nu_v - 1)G_c^{\uparrow\downarrow}\right], \tag{30}$$

where in turn $G^{\uparrow\uparrow}=G^{\uparrow\uparrow}_{x,\text{intra}}+G^{\uparrow\uparrow}_{c,\text{intra}}$. Here the subscripts x and c refer to exchange and correlation effects, while intra refers to intravalley processes. While G^v_+ is designed to mimic the vertex corrections associated with charge fluctuations, G^v_- accounts for spin-fluctuations-induced vertex corrections.

The Hubbard approximation[12] used by Rice and TLQ corresponds to the case in which only the processes associated with $G^{\uparrow\uparrow}$ are kept, since Hubbard's original idea was to account for exchange effects. Clearly in this case we have $G^v_+=G^v_-=G^{\uparrow\uparrow}/\nu_v$. Under this (in general incorrect) assumption our results (once they are expressed for the case of a 2D EL) reduce to that of TLQ. In this sense some contributions from spin fluctuations are in a crude way included in the TLQ theory. On the other hand, the results of the analysis presented in Sec. III of the present paper clearly display in vivid fashion the importance of the proper and concomitant inclusion of the contributions stemming from both G^v_+ and G^v_-. From a quantitative viewpoint, in particular, the formulation of TLQ suffers from the problem that, in general, even within a static model G^v_+ is larger than G^v_-. A direct inspection of the expression for g^*, given by Eq. (24), shows that assuming for these two quantities the same value, leads to an overestimation of the higher-order dynamical correlation contributions with respect to the screened exchange term. This problem is compounded by the fact that these terms have opposite sign. The result is a sizable underestimation of the renormalization effect which can be seen to get worse as the electronic density decreases.

A number of technical remarks concerning the present approach are here in order. In implementing the type of microscopic approach to the Landau theory employed in this work, it is crucial to be able to keep track separately of the occupation numbers of quasiparticles with opposite spin projections. The problem with Rice's theory[10] stems from not having properly recognized this point. In our opinion the shortcomings of his analysis originate from this oversight rather than being inherent to his method as a whole, as suggested in Ref. 34. In the present formulation the proper treatment is ensured by the use of the expression for the charge response of the EL in an infinitesimally polarized state. From a formal viewpoint the results of Ref. 10 correspond to the arbitrary choice of $G^v_-=0$.

Another interesting point to be noted is that, in the regime of interest, the functional derivative and the integration over the coupling constant can be freely taken in any order. This is consistent with the assumption of validity of the Landau quasiparticle concept.

We discuss next the limitations of the present formulation of the EL theory based on the integration-over-the-coupling-constant algorithm. The main shortcoming of the approach is the fact that the local field G^v_- only appears in f_a. Also puzzling is the fact that the quasiparticle energy given in Eq. (13) only contains G^v_+ and, therefore, leaves out most of the contributions associated with spin fluctuations. That such contributions should appear in the quasiparticle energy is not only expected on general grounds, but has also been recently established by a number of different theoretical approaches to the theory of EL.[33,19–35,36] In this respect a more physically sound theory of the quasiparticle properties in an EL has been presented in Ref. 21. There, expressions for the Landau quasiparticle interaction functions and the Landé factor g^* have been derived. For comparison the numerical results of such a more complete theory applied to the case of an inversion layer of the type studied in Sec. III have been provided in Figs. 1–3.

We think that problems in the present formulation of the EL theory based on the integration-over-the-coupling-constant algorithm are likely to originate from a number of uncontrolled approximations which are customarily made in the present context with no proper justification. As mentioned in Sec. II the main approximations used are the following: (1) In taking the functional derivative of $E_{\text{int}}(\lambda)$ [Eq. (4)] the functional dependence of the many-body local fields $G^v_\pm$ on (separately) $n^\uparrow_{\mathbf{p}}$ and $n^\downarrow_{\mathbf{p}}$ was neglected. (2) In performing the integration over the coupling constant λ in Eq. (3) the dependence on λ contained in G^v_+, χ^σ_0, and g^σ was also neglected. It is our opinion that the first of these two approximations is the most suspect. Accordingly, we believe that meaningful progress from the present status of the integration-over-the-coupling-constant algorithm can be achieved

through a systematic study of the many-body local fields in a polarized electron liquid.

Finally, we conclude by noting that a meaningful comparison between alternative many-body theories was achieved here only by studying the behavior of the Wilson ratio g^*/g. As it turns out a similar analysis focused on the many-body enhancement of the effective mass m^*/m is less effective since, for the EL, this quantity displays, in general, a similar and rather uninspiring density dependence within virtually any reasonable approximation.

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