Spin-orbit coupling in one-dimensional conducting rings

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Spin-orbit coupling of electron motion in a one-dimensional conducting ring is examined with a confining potential to keep the electron path exactly in a circle. It is found that a non-symmetrical potential is essential for the onset of spin-orbit coupling. This result may be useful for understanding some mesoscopic phenomena, e.g., persistent currents.

The recent experimental observation of persistent currents in mesoscopic disordered conducting rings[1] has attracted much interest. The spin-orbit coupling of the electron motion in a one-dimensional (1D) ring was examined and was considered to give an important contribution to the persistent currents.[2] The interaction between the electron spin and its orbital motion has been known for a long time. For an electron moving in the central-potential field, this interaction can be derived from the Dirac electron equation and can be written in the well-known form $\mathbf{S}\cdot\mathbf{L}$.[3] For an electron moving in a one-dimensional mesoscopic ring, the form of its spin-orbit coupling was discussed only recently by Aronov and Lyanda-Geller.[4]

In the present paper, we introduce a more realistic model of a 1D ring by adding a confining potential such that the exact 1D effects are phenomenologically taken into account. This investigation of spin-orbit coupling may be useful for understanding the persistent currents and other mesoscopic phenomena in 1D rings.

Let us propose a confining potential

$$U(r,z)=\frac{U_0}{4}(e^{az}+e^{-bz})(e^{a'(r-r_0)}+e^{-b'(r-r_0)}) . \tag{1}$$

The electrons moving in a 1D ring of radius r_0 would be described by the confining potential with parameters a, b, a', $b'\to\infty$. Here we employ cydrindrical coordinates (r,θ,z). The electrons would be confined to move on a planar ring with radius $r=r_0$ and $z=0$. If $a=b$ and $a'=b'$, the confinement will be symmetric about the $z=0$ plane. If $a\neq b$ but $a-b=\text{const}$, the confining potential will not be symmetric about the $z=0$ plane. For $a'\neq b'$ but $a'-b'=\text{const}$, the confining potential will be asymmetric about $r=r_0$.

Dirac's equation for an electron in the confining potential U and electromagnetic vector potential $\mathbf{A}$ is

$$\left[c\boldsymbol{\alpha}\cdot\left[\mathbf{p}+\frac{e}{c}\mathbf{A}\right]+\beta mc+U\right]\psi=E\psi , \tag{2}$$

where U is the confining potential given by (1). After repeating the well-known calculations,[5] we can obtain the Zeeman-splitting energy and spin-orbit coupling energy in the presence of the confining potential. The Hamiltonian can be written

$$H'=\frac{e\hbar}{2mc}\sigma\cdot(\nabla\times\mathbf{A}) -\frac{\hbar}{4m^2c^2}\sigma\cdot\left[\left[\mathbf{p}+\frac{e}{c}\mathbf{A}\right]\times\nabla U\right] . \tag{3}$$

The second term in (3) is the spin-orbit coupling. Consider the 1D ring in the limit case; we have

$$\begin{aligned}\nabla U&=\hat{\mathbf{r}}\frac{\partial u}{\partial r}\bigg|_{z=0,r=r_0}+\hat{\mathbf{z}}\frac{\partial U}{\partial Z}\bigg|_{z=0,r=r_0}\\&=(a'-b')U_0\hat{\mathbf{r}}+(a-b)U_0\hat{\mathbf{z}}\\&=\alpha_1\hat{\mathbf{r}}+\alpha_2\hat{\mathbf{z}} ,\end{aligned} \tag{4}$$

where $\hat{\mathbf{r}}$ and $\hat{\mathbf{z}}$ are unit vectors along radial and axial directions, respectively, and $\alpha_1=(a'-b')U_0$, $\alpha_2=(a-b)U_0$. From (3) and (4) we get

$$\begin{aligned}H'=&\frac{e\hbar}{2mc}\sigma\cdot\mathbf{B}-\hbar\omega_1(\sigma_x\cos\theta+\sigma_y\sin\theta)\left[-i\frac{d}{d\theta}-\frac{\phi}{\phi_0}\right]\\&+\frac{i\hbar\omega_1}{2}(\sigma_x\sin\theta-\sigma_y\cos\theta)+\hbar\omega_2\sigma_z\left[-i\frac{d}{d\theta}-\frac{\phi}{\phi_0}\right] .\end{aligned} \tag{5}$$

Here $\hbar\omega_1=\alpha_1\hbar^2/(4m^2c^2)$ and $\hbar\omega_2=\alpha_2\hbar^2/(4m^2c^2)$. For the sake of convenience, we use the same notations of Ref. 4. In Eq. (2) of Ref. 4 the lower indices x and y in $[P-(e/c)\mathbf{A}]$ should be interchanged. Our expression (5) differs from the spin-orbit interaction Hamiltonian in Ref. 4 by an excess of the last two terms:

$$\hbar\omega_2\sigma_z\left[-i\frac{d}{d\theta}-\frac{\phi}{\phi_0}\right]+\frac{i\hbar\omega_1}{2}(\sigma_x\sin\theta-\sigma_y\cos\theta) . \tag{6}$$

This difference is important. First, it can easily be checked that omission of the second term in (6) will lead to a non-Hermitian Hamiltonian. Adding the electron kinetic-energy term, our Hermitian Hamiltonian would be

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$$H=\hbar\omega\left[-i\frac{d}{d\theta}-\frac{\phi}{\phi_0}\right]^2+\frac{e\hbar}{2mc}\sigma\cdot\mathbf{B}$$
$$+\hbar\omega_2\sigma_z\left[-i\frac{d}{d\theta}-\frac{\phi}{\phi_0}\right]$$
$$-\hbar\omega_1(-\sigma_x\cos\theta+\sigma_y\sin\theta)\left[-i\frac{d}{d\theta}-\frac{\phi}{\phi_0}\right]$$
$$+\frac{i\hbar\omega_1}{2}(\sigma_x\sin\theta-\sigma_y\cos\theta)\ . \qquad (7)$$

Second, the third term on the right-hand side of (7) represents the interaction of the z-direction spin with orbital motion, which can be combined with the first term to give

$$\hbar\omega\left[-i\frac{d}{d\theta}-\frac{\phi}{\phi_0}+\frac{\omega_1}{2\omega}\sigma_z\right]^2+\text{const}\ .$$

So an effective flux, the sign of which depends on the eigenvalue of σ_z, should be added into the Aharonov-Bohn magnetic flux. Further, the external confining potential affects the spin-orbit coupling of electrons in the ring. For $a\neq b$, $a-b=$const and $a'\neq b'$, $a'-b'=$const, i.e., the confining potential is nonsymmetric both about the $z=0$ plane and $r=r_0$, then there is spin-orbit coupling. On the contrary, if the confining potential is symmetric both about $z=0$ and $r=r_0$ there is no spin-orbit coupling. Nonsymmetry about the $z=0$ plane contributes coupling motion with σ_z. Nonsymmetry about $r=r_0$ contributes coupling of electron orbital motion with σ_x and σ_y. Moreover, nonsymmetry about $r=r_0$ contributes the spin flip of electrons along orbital motion.

So, in a completely symmetric confining potential in a 1D ring, the electron spin is not coupled to its orbital motion. The nonsymmetric confining potential will give the different coupling terms which have different effects on electron motion.

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