Correlated state of double layers of electron fluids

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When the interlayer separation is not large, the ground state of a double layer of two-dimensional electron fluids in a large magnetic field is determined by correlations arising from the interlayer Coulomb repulsion. We suggest that, as a consequence, the independent translational invariance of the electrons in each layer is lost, independent of the filling fraction of the Landau levels. The gap measured recently in tunneling across the bilayer is explained and experiments are proposed to test our description.

With a device fabricated from GaAs-$Ga_{1-x}Al_xAs$ heterojunctions, Eisenstein, Pfeiffer, and West[1] have recently measured the tunneling conductance across a bilayer of ultrapure two-dimensional fluids in large perpendicular magnetic fields. At very low temperatures, they find a well-defined gap Δ in the conductance as a function of bias voltage. The gap depends only weakly on the magnetic field and there is a zero-bias conductance which as a function of temperature is exponentially activated with a characteristic energy scale of the order of Δ. The in-plane conductivity is similar to that found in single-layer devices. This behavior of the tunneling conductance is not associated with the quantized Hall effect, although weak features are seen in Δ at those values of magnetic field where the quantized Hall effect is prominent.

We propose that the presence of a well-defined gap in the tunneling conductance is a reflection of the existence of a correlated electron state across the bilayer in a situation in which the interlayer Coulomb interaction is important while quantum-mechanical tunneling is negligible. This state breaks the independent translational invariance of the electrons in each layer in order to take advantage of electron-electron interactions across the bilayer. In a simple calculation, the correlated state occurs when the bilyaer separation is large enough that the tunneling matrix element is less than the intralayer Coulomb energy. This condition is $t_\perp \ll e^2/\kappa r_s$, where κ is the dielectric constant, r_s is the interelectron separation within each layer, and $t_\perp$ is the tunneling rate between the layers. Under these circumstances, each layer is a "Mott insulator" with respect to delocalization between the layers.

In the experimental setup,[1] the interlayer separation d is of the order of the intralayer particle separation r_s and the filling factors are not near specific fractions. Under these circumstances, we believe it is essential to include the interlayer Coulomb correlations on the same footing as the intralayer ones. Approaches which depend only on single-layer correlations[1-6] may be relevant when the interlayer spacing is large enough or when the system is near a specific fractional filling, as in the discussion of Ref. 2. We also distinguish the experimental parameters from the case in which the interlayer spacing is very small, so that there is substantial interlayer overlap and the proper electron states are the symmetric and antisymmetric combinations from the two wells as in the experiment of Boebinger and co-workers.[7]

At the experimental[1] filling factors, Hartree-Fock calculations for a single layer favor crystallization. For $\nu \gtrsim \frac{1}{5}$, this state is destroyed by quantum fluctuations and a fluid state is realized. When two layers are in proximity, the electron repulsion across the layers favors correlated "molecular" charge distributions in which an excess electron density in one layer is near a diminished density in the other, so that each layer no longer has a translationally invariant electron distribution. The molecular charge fluctuations lower the interlayer Coulomb correlation energy at the expense of the intralayer energy. When $t_\perp$ is small, this tendency is only opposed by the intralayer repulsion but it is a substantial effect for $d \simeq r_s$. With a Monte Carlo treatment of a lattice model (no kinetic energy) for each of the two-dimensional (2D) electron layers, Efros and Pikus,[8] in work of which ours is independent, have computed a low-temperature low-bias tunneling gap which they interpret as due to correlations of the sort we discuss here.

In order to obtain a rough estimate of the regime in which such a state might arise, we use a variational wave function similar to the one employed[9] to describe a quantum Hall liquid with both spin polarizations. It has also been used[10] to discuss the correlations in bilayers when $\nu \simeq \frac{1}{4}$ or $\frac{1}{2}$.

For a variational wave function of the bilayer, we write[9]

$$\Psi_{mmn} = \mathcal{A}\,\psi_L^{(m)}(\{u_i\})\,\psi_R^{(m)}(\{v_j\})\,\psi_{LR}^{(n)}(\{u_i - v_j\})\,, \qquad (1)$$

where the u_i and v_j are the (complex) coordinates of the electrons in the left and right layers and $\mathcal{A}$ is the antisymmetrization operator, which does not mix the electrons in the separate layers. This approximation is expected to be valid as long as the tunneling is weak, as in the case we discuss. Aside from their Gaussian envelopes

$\exp[-u^2/4l^2]$, with l the magnetic length, $l=\sqrt{ch/eH}$, the $\psi_{L,R}^{(m)}$ are polynomials of order $mN(N-1)/2$ (the layers have equal density), and ψ_{LR}^{n} is a polynomial of order nN^2, where N is the number of electrons in each layer. At high magnetic fields we are justified in restricting Ψ to contain only wave functions of the first Landau level; under this circumstance, n is an integer (which we shall take equal to 1 in order to have the most favorable case). The interparticle separation r_s in each layer may be found from the highest power in the wave function of one of the electron coordinates. For Ψ_{mmn} it is $r_s=l\sqrt{m+n}$. Then the filling factor for each layer is[9] $\nu=1/(m+n)$.

Since there is effectively no kinetic energy in the problem, the ground-state energy is given by

$$E=E_L+E_R+E_{LR}=\frac{(\psi_L\psi_{LR},V_L\psi_L\psi_{LR})}{(\psi_L\psi_{LR},\psi_L\psi_{LR})}+\frac{(\psi_R\psi_{LR},V_R\psi_R\psi_{LR})}{(\psi_R\psi_{LR},\psi_R\psi_{LR})}+\frac{(\Psi_{mmn},V_{LR}\Psi_{mmn})}{(\Psi_{mmn},\Psi_{mmn})}\ , \tag{2}$$

where the V's are the Coulomb interactions. We discuss these integrals as follows: In the expression for E_L, we may perform the integration over all the right coordinates $\{v_j\}$. This gives an expression of the form

$$E_L=\int(\Pi_i d^2u_i)\chi_L(\{u_i\})V_L\chi_L(\{u_i\})\ , \tag{3}$$

where χ_L^2 is a polynomial in the $\{u_i\}$ and we have assumed all wave functions normalized. χ_L^2 is given by

$$\chi_L^2(\{u_i\})=\int(\Pi_j d^2v_j)\psi_{LR}^2\ . \tag{4}$$

It is easy to see that the highest power of any u_i in the polynomial $\chi_L(\{u_i\})$ is $N(m+n)$. Therefore the effective interparticle spacing in the left-layer wave function $\chi_L(\{u_i\})$ is $r_s=l\sqrt{m+n}$. If the ground state were translationally invariant in each layer, then we would expect E_L and E_R to be functions of r_s only, independent of d.

The interlayer correlation energy is a competition between two effects. When two layers are in proximity with no charge redistribution, $E_{LR}=0$ as the electron repulsions are canceled by the stationary positive countercharges. However, a correlated molecular charge readjustment will lead to an *attractive* contribution to the electrostatic energy. Let x be a configurational coordinate which measures the charge displacements. Then the interlayer Coulomb interaction will lower the energy by an amount depending on a combination which, schematically, has the form $\sqrt{d^2+(x+r_s)^2}$. For small x/d, this gives a negative contribution to the energy which is *linear* in x. Opposing this effect is the increase of intralayer Coulomb energy due to the distortion, which is *quadratic* in x since the undistorted state is at a minimum for the intralayer energy. Therefore, the lowest energy will always be realized for a correlated distorted state. We write

$$E_L+E_R+E_{LR}=f(r_s)-(e^2/r_s)g(d/r_s)\ , \tag{5}$$

where $g(d/r_s)$ measures the correlation energy and $g(\infty)=0$.

Let us compare this situation with that of two uncorrelated layers with wave function

$$\Psi_0=\psi_L^{(m_0)}\psi_R^{(m_0)}\ , \tag{6}$$

in which the two layers each have translationally invariant charge distributions of density $n_0=1/(l^2m_0)$, that is, particle separation $r_s^0=l\sqrt{m_0}$. Thus, $\psi_L^{(m_0)}$ is the ground-state wave function of a single layer of density n_0 in the applied magnetic field. This is just the density of the positive background which is unchanged when interlayer correlations are introduced. Thus, the correlated wave function with $r_s=l\sqrt{m+n}$ must have the same density $n_0=1/[l^2(m+n)]$ (to cancel the positive background) as the uncorrelated one, so that $m_0=m+n$. Therefore, the energy for the uncorrelated state of Eq. (6) is simply the $f(r_s)$ of Eq. (5). Assuming that E_L, E_R are smooth, we argue that the correction to the energy [the second term in Eq. (5)] due the correlated charge redistribution is negative and that therefore the correlated state is always favored over the uncorrelated one. The argument fails near quantized Hall fractional fillings, where the energy as a function of changes in the pair distribution function in each layer might have a cusp. Indeed, in the actual experiments, deviations from a weak nomotonic magnetic-field dependence are seen at fields corresponding to filling fractions $\nu=1$, $\frac{2}{3}$, and $\frac{3}{5}$.

We have made a rough estimate of the interlayer correlation energy by calculating the change in energy of crystalline layer by introducing a charge at a perpendicular distance d from the center of a square plaquette of a 2D crystal and allowing the plaquette to readjust its nearest-neighbor separation, keeping the rest of the crystal fixed. The result is shown in Fig. 1, where the correlation energy $g(d/r_s)$ (in units of $e^2/\kappa r_s$) is plotted. This shows the behavior of the correlation energy as a function of layer spacing at fixed density.

It is clear that, with states like Eq. (2), the usual method of calculating the tunneling current as a convolu-

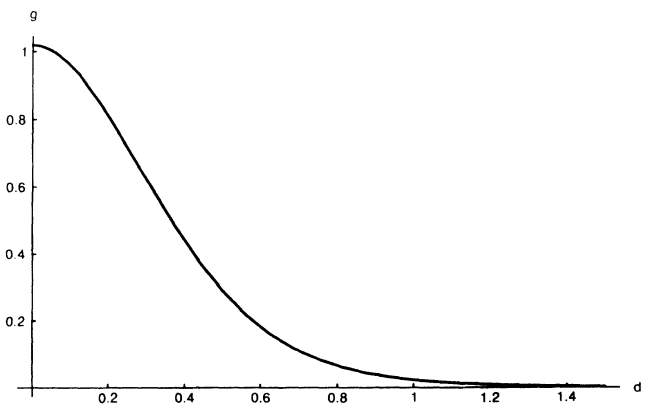

FIG. 1. Correlation energy in units of $e^2/\kappa r_s$ as a function of interlayer separation in units of r_s.

tion of the spectral functions on the two sides is invalid. We argue that the correlation energy leads to a gap in the tunneling characteristic, as observed.[1] An electron tunneling between the layers contributes to the current at the leads only if it is excited out of the correlated ground state. This requires an energy of order of the correlation energy $(e^2/\kappa r_s)g(d/r_s)$, which will be the order of magnitude of the gap Δ in the tunneling spectrum. In effect, this gap expresses the stiffness of the distorted state with broken translational invariance.

The observed tunneling gap is of order 2 meV compared to an $e^2/\kappa r_s$ of 22 meV, thus a measured $g(d/r_s)$ of order 0.1. Our calculation then gives $d/r_s \simeq 0.8$. In the actual experiment, the width of each layer is 200 Å, the barrier width is 175 Å, and r_s is 250 Å. Thus the effective d is somewhat larger than r_s. On the other hand, the layers are not strictly two dimensional at these length scales, so that the intralayer Coulomb effects are softer than those of our rough calculation of $g(d/r_s)$, which is in any case an underestimate since further-neighbor (Madelung) correlations are neglected.

The observed weak magnetic-field dependence of Δ may be understood qualitatively by the observation that an increase in magnetic field shrinks the electron orbitals, which enhances the correlation energy. Therefore, we expect a slight increase of Δ with increasing magnetic field.[11]

The conductance of the bilayer for electric field parallel to the layers should be normal, with a somewhat larger effective mass due to the interlayer correlations. Recently, observations of the interlayer Coulomb interactions have been reported for bilayers via the so-called Coulomb-drag effect.[12,13] These experiments, carried out in weak or zero field, have been accounted for by interlayer Coulomb scattering in the 2D electron fluids;[14,15] thus, they should have a characteristic T^2 temperature dependence. In a similar geometry, but at high magnetic field, we can suggest an experiment to test the lack of independent translational invariance of the ground state we have described: Apply the electric field along the plane in only one of the two layers and measure the current in both of the layers. They should be comparable in magnitude and have identical temperature dependence. A variant is to construct a bilayer where one layer has electron carriers and the other has hole carriers, as in Ref. 13. Then the currents in the two layers should be in opposite directions[13] for an applied electric field in only one layer. This effect arises solely from the molecular interlayer Coulomb correlations and is distinct from that already observed at low magnetic fields,[12,13] where the drag is temperature dependent. In our case, the leading effect is temperature independent.

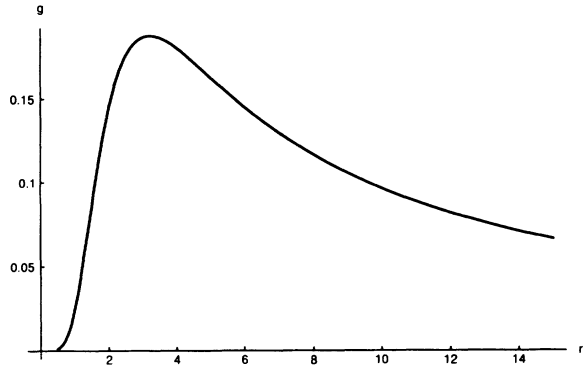

FIG. 2. Correlation energy in units of $e^2/\kappa d$ as a function of interelectron separation r_s in units of d.

In our picture, the competition between intra- and interlayer effects produces a consequence which may be experimentally accessible. For fixed interlayer spacing d the correlation energy increases in magnitude for small r_s, has a maximum which in our very rough calculation is in the neighborhood of $r_s \simeq 3d$, and then decreases for larger r_s. Thus, we predict that the tunneling gap will increase with density at high density but decrease with density at low density. The behavior is shown in Fig. 2, where the correlation energy is plotted as a function of r_s/d. Complementary to Fig. 1, this shows the behavior of the correlation energy (in units of $e^2/\kappa d$) as a function of r_s (i.e., density) for fixed layer separation.

In conclusion, it is our view that the experimental results of Eisenstein, Pfeiffer, and West[1] depend strictly on interlayer correlations and cannot be explained by single-layer correlations. Tunneling into single layers should reveal single-layer effects and so far there is no single-layer evidence[11] for a true gap as seen in Ref. 1.

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Phys. Rev. Lett. **64**, 681 (1990) is widely quoted as showing a magnetic-field-induced gap for tunneling into a single layer. While these authors do find some decrease of the tunneling conductance as a perpendicular magnetic field is turned on, the temperature dependence is rather weak. This is unlike the activated temperature dependence of a true gap, as seen in Ref. 1.

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