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Theory of normal-incidence absorption for the intersubband transition in n-type indirect-gap semiconductor quantum wells

Wenlan Xu,* Y. Fu, and M. Willander
Department of Physics and Measurement Technology, Linköping University, S-581 83 Linköping, Sweden

S. C. Shen
National Laboratory for Infrared Physics, Shanghai Institute of Technical Physics, 200083 Shanghai, China
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We have studied systematically the normal-incidence-radiation absorption for the intersubband transition in the quantum well of n-type indirect-gap semiconductors. By a special choice of the coordinate system related to the sample growth direction, we have proposed a simple method to calculate the elements of the inverse effective-mass tensor. Using the concept of the invariance of the ellipsoidal constant-energy surface under coordinate transformations, the general expression for the absorption coefficient of the intersubband transition has been derived as a function of the sample growth direction. We have also investigated the maximal value of the normal-incidence-radiation absorption, the optimal growth direction for quantum-well detectors, and the comparison between normal- and parallel-incidence-radiation absorptions. These are of great utility for designing and optimizing the quantum-well infrared detector for normally incident radiations.

I. INTRODUCTION

Infrared detectors, based on the intersubband transitions in the n-type $GaAs/Ga_{1-x}Al_xAs$ quantum-well (QW) structure and other similar systems, have been developing rapidly.[1,2] The potential of the $GaAs/Ga_{1-x}Al_xAs$ system in the development of the infrared focus plane array (FPA) is great due to the fact that the system can be grown on a substrate of large area with a much higher uniformity than can the $Cd_xHg_{1-x}Te$ system. However, this type of QW infrared detector and its FPA have limited capability. The reason is that it can only absorb the component of radiation whose electric field is perpendicular to the layer plane of the QW. The absorption of the normal-incidence radiation is forbidden when the intersubband transitions are considered in this system. This makes the manufacture of the detector array very difficult and complicated, and the grating structure has to be made on the surface of the QW to improve the optical coupling and sensitivity.

Recently, QW structures based on indirect-gap semiconductors with anisotropic effective-mass tensors have been attracting much attention.[3–6] It is found that when the principal axes of the ellipsoid are tilted with respect to the QW growth direction, the effective-mass anisotropy of the conduction electrons in the ellipsoidal constant energy surfaces (valleys) can provide interactions and couplings between the parallel and perpendicular motions of the electrons. This makes the normal-incidence absorption possible based on the intersubband transitions in these systems.

When analyzing experimental data, the authors of Refs. 3–6 discussed the conditions of the normal-incidence radiation absorption based on the intersubband transitions of electrons in the QW structures, while in Ref. 7, a theoretical analysis has been given. However, these discussions and analyses were limited only in some special growth directions. The optimization of the QW growth direction and the limit of the absorption of the normal-incidence radiation based on the intersubband transitions have not been generally investigated yet.

In this paper, we present a general theory for the radiation absorption based on the intersubband transitions in n-type indirect-gap semiconductor QW's. In Sec. II we first formulate the matrix elements of the intersubband transition and the corresponding conditions for the absorption of the normal-incidence radiation in an n-type QW. In Sec. III we will introduce a simple method to calculate the elements of the inverse effective-mass tensor in the nonprincipal-axis coordinate system. By the concept of invariable quantities of ellipsoidal constant energy surface under the coordinate transformation in Sec. IV the relations among the elements of inverse effective-mass tensor are obtained so that the calculation of the absorption coefficients is greatly simplified. These relationships are actually our bases to investigate the limit and optimization of the normal-incidence-radiation absorption in Sec. IV. Also in Sec. IV, the absorptions of the parallel and normal-incidence radiations are to be studied and compared. The conditions of the optimal absorption and the limits of the normal-incidence absorption based on intersubband transitions in different material QW's are discussed in connection with the design of the QW detectors. A brief conclusion will be given in Sec. V.

II. MATRIX ELEMENT OF THE INTERSUBBAND TRANSITION AND THE CONDITION FOR NORMAL-INCIDENCE-RADIATION ABSORPTION

We set the growth direction of the QW as the z axis and the plane perpendicular to this direction as the xy

plane of the (x,y,z) coordinate system. In the effective-mass approximation, the Hamiltonian of the electrons in the QW is

$$\overleftrightarrow{H} = \frac{1}{2}\vec{P}\overleftrightarrow{W}\vec{P} + V(z) \ , \tag{1}$$

where $\vec{P}$ is the momentum operator, $\overleftrightarrow{W}$ is the 3×3 inverse effective-mass tensor taking into accounting the anisotropic effect of the band structure ($W_{ij} = W_{ji}$), and $V(z)$ is the potential energy. Since $\overleftrightarrow{H}$ is translationally symmetric in the xy plane, the wave function can be expressed as

$$\Psi_m(\vec{k}) = \phi_m(z)u(\vec{r})e^{i\vec{k}\cdot\vec{\rho}} \ , \tag{2}$$

where $u(\vec{r})$ is the Bloch function, which is approximated as the same for all m and $\vec{k}$ values, since we consider only the transitions among subbands. $\vec{k} = (k_x, k_y)$ and $\vec{\rho}$ are the wave vector and coordinate in the xy plane, respectively, and $\phi_m(z)$ is the envelope function. Because of the translational symmetry in the xy plane, $\vec{k}$ is a good quantum number. The quantum number of the energy level in the z direction is m. In the quantum well system, m is the subband index.

The kinetic energy of the electron, described by its vector potential $\vec{A}$, has to be replaced, in the presence of a radiation, by the expression

$$\frac{1}{2}\left(\vec{P} + \frac{e\vec{A}}{c}\right)\overleftrightarrow{W}\left(\vec{P} + \frac{e\vec{A}}{c}\right) \ , \tag{3}$$

where e is the unit charge and c the light velocity. Here the scalar potential has been taken as zero without loss of generality because of the arbitrariness in the gauge. The Lorentz condition and the choice of zero scalar potential imply $\nabla \cdot \vec{A} = 0$. Furthermore we can neglect nonlinear effects by disregarding the term in $\vec{A}^2$. Thus the interaction between the incident radiation and the electron is

$$\frac{e}{c}\vec{A}\overleftrightarrow{W}\vec{P} \ . \tag{4}$$

By Eq. (2), the matrix element of the transition between states $\Psi_m(\vec{k})$ and $\Psi_n(\vec{q})$ can be written as

$$\begin{aligned}\langle \Psi_m(\vec{k})|\vec{A}\overleftrightarrow{W}\vec{P}|\Psi_n(\vec{q})\rangle \\ = \left\langle u\phi_m e^{i\vec{k}\cdot\vec{\rho}}\left|\sum_{i,j} A_i W_{ij} P_j\right| u\phi_n e^{i\vec{q}\cdot\vec{\rho}}\right\rangle \ ,\end{aligned} \tag{5}$$

where $i,j = x,y,z$. Since $\phi_m(\vec{k})$ and $e^{i\vec{k}\cdot\vec{\rho}}$ are slowly varying functions over one unit cell, and $\vec{P}$ is a differential operator, the right-hand side of Eq. (5) can be expanded into two terms:

$$\begin{aligned}&\left\langle u\left|\sum_{i,j} A_i W_{ij} P_j\right| u\right\rangle \langle \phi_m e^{i\vec{k}\cdot\vec{\rho}}|\phi_n e^{i\vec{q}\cdot\vec{\rho}}\rangle \\ &+\langle u|u\rangle \left\langle \phi_m e^{i\vec{k}\cdot\vec{\rho}}\left|\sum_{i,j} A_i W_{ij} P_j\right|\phi_n e^{i\vec{q}\cdot\vec{\rho}}\right\rangle \ .\end{aligned} \tag{6}$$

In Eq. (6),

$$\langle \phi_m e^{i\vec{k}\cdot\vec{\rho}}|\phi_n e^{i\vec{q}\cdot\vec{\rho}}\rangle = \langle \phi_m|\phi_n\rangle\langle e^{i\vec{k}\cdot\vec{\rho}}|e^{i\vec{q}\cdot\vec{\rho}}\rangle$$

and vanishes for intersubband transitions due to the orthogonality of the envelope functions:

$$\langle \phi_m|\phi_n\rangle = 0 \quad (m \neq n) \ .$$

And because $\langle u|u\rangle = 1$,

$$\begin{aligned}&\langle \Psi_m(\vec{k})|\vec{A}\overleftrightarrow{W}\vec{P}|\Psi_n(\vec{q})\rangle \\ &= \left\langle \phi_m e^{i\vec{k}\cdot\vec{\rho}}\left|\sum_{i,j} A_i W_{ij} P_j\right|\phi_n e^{i\vec{q}\cdot\vec{\rho}}\right\rangle \\ &= {\sum_{i,j}}' A_i W_{ij}\langle \phi_m e^{i\vec{k}\cdot\vec{\rho}}|P_j|\phi_n e^{i\vec{q}\cdot\vec{\rho}}\rangle \\ &\quad + \sum_i A_i W_{iz}\langle \phi_m e^{i\vec{k}\cdot\vec{\rho}}|P_z|\phi_n e^{i\vec{q}\cdot\vec{\rho}}\rangle \ ,\end{aligned} \tag{7}$$

where the summation $\sum_{i,j}'$ excludes $j = z$. Since ϕ_n is only a function of z, the first term of the last equality in Eq. (7) is reduced to

$${\sum_{i,j}}' A_i W_{ij}\langle \phi_m|\phi_n\rangle\langle e^{i\vec{k}\cdot\vec{\rho}}|P_j|e^{i\vec{q}\cdot\vec{\rho}}\rangle,$$

which is zero when $m \neq n$. The second term of the last equality in Eq. (7)

$$\langle \phi_m e^{i\vec{k}\cdot\vec{\rho}}|P_z|\phi_n e^{i\vec{q}\cdot\vec{\rho}}\rangle = \langle \phi_m|P_z|\phi_n\rangle\langle e^{i\vec{k}\cdot\vec{\rho}}|e^{i\vec{q}\cdot\vec{\rho}}\rangle$$

because $e^{i\vec{q}\cdot\vec{\rho}}$ is only a function of $\vec{\rho} = (x,y)$. Since $\langle e^{i\vec{k}\cdot\vec{\rho}}|e^{i\vec{q}\cdot\vec{\rho}}\rangle = \delta_{\vec{k},\vec{q}}$, Eq. (7) finally becomes

$$\langle \Psi_m(\vec{k})|\vec{A}\overleftrightarrow{W}\vec{P}|\Psi_n(\vec{q})\rangle = \delta_{\vec{k},\vec{q}}\sum_i A_i W_{iz}\langle \phi_m|P_z|\phi_n\rangle \ . \tag{8}$$

Here we see that the intersubband transition occurs only while the momentum in the xy plane is conserved and the transition is between the envelop functions in the z direction. From Eq. (8), it is easy to see that a nonzero matrix element of intersubband transition for the normal-incidence radiation ($A_x, A_y \neq 0$ and $A_z = 0$) can be obtained only when W_{xz} or W_{yz} are not zero. This occurs when the QW growth direction does not coincide with any of the principal axes of the ellipsoid of the constant energy surface.

For electrons occupying an energy valley described by a spherical constant energy surface, e.g., the Γ electrons in $Al_xGa_{1-x}As$, $W_{ij} = 0$ if $i \neq j$ for any QW growth direction of, so that

$$\langle \Psi_m(\vec{k})|\vec{A}\overleftrightarrow{W}\vec{P}|\Psi_n(\vec{k})\rangle = A_z W_{zz}\langle \phi_m|P_z|\phi_n\rangle \ . \tag{9}$$

If $A_z = 0$, i.e., for a normal-incidence radiation, the transition matrix element is zero and thus the intersubband transitions are forbidden.

III. ELEMENTS OF THE INVERSE EFFECTIVE-MASS TENSOR

From the preceding section, we see that the elements of the inverse effective-mass tensor are of essential importance for the absorption of the normal-incidence radiation based on the intersubband transition of electrons in the QW structure. Therefore it is necessary to investigate the relations among these elements of the tensor and the growth direction of the QW.

For almost all the indirect-gap semiconductor materials, the conduction bands have either six symmetrical ellipsoidal constant energy surfaces (usually the X valley electrons) or eight symmetrical half ellipsoidal constant energy surfaces (usually the L valley electrons). We first discuss the X valley electrons. The major axes of the six ellipsoids are oriented along three crystal axes [100], [010], and [001], forming the principal coordinate system for the X valley electrons (X,Y,Z). The inverse effective mass tensor $\overleftrightarrow{W}_p$ is diagonal in this principal coordinate system. For example, the $\overleftrightarrow{W}_p$ matrix, for the two ellipsoids whose major axes are in the [001] direction, is in the form of

$$\overleftrightarrow{W}_p = \begin{pmatrix} w_t & 0 & 0 \\ 0 & w_t & 0 \\ 0 & 0 & w_l \end{pmatrix} , \tag{10}$$

where $w_t = 1/m_t$, $w_l = 1/m_l$, and m_l and m_t are the longitudinal and transverse effective masses, respectively.

When the growth direction of the quantum well $[l,m,n]$ does not coincide with any of the three principal axes, the inverse effective-mass tensor has nonzero off-diagonal elements in the new coordinate system (x,y,z), where the z direction is defined as $z=[l,m,n]$. This is the very reason why this type of quantum well can detect normal-incidence radiation, as we have already discussed in Sec. II. It is well known that the coordinate transformation matrix $\overleftrightarrow{T}$ between (X,Y,Z) and (x,y,z) is unitary, i.e., $\overleftrightarrow{T}^{-1} = \overleftrightarrow{T}^{\tau}$, where $\overleftrightarrow{T}^{-1}$ and $\overleftrightarrow{T}^{\tau}$ are the inversion and transpose of the matrix $\overleftrightarrow{T}$, respectively. The inverse effective-mass tensor $\overleftrightarrow{W}$ in the new coordinate system (x,y,z) becomes

$$\overleftrightarrow{W} = \overleftrightarrow{T}^{\tau}\overleftrightarrow{W}_p\overleftrightarrow{T} , \tag{11}$$

whose elements are

$$W_{ij} = \sum_k T_{ki}T_{kj}w_k , \tag{12}$$

where w_k is the diagonal element of $\overleftrightarrow{W}_p$, either w_t or w_l.

After defining the growth direction as the z axis, the choice of the x and y axes is arbitrary. However, we introduce here a special set of the x and y axes to simplify the mathematics, while at the same time do not lose its generality.[8] The y axis is chosen to be perpendicular to not only the already defined z axis, but also the major axis of the ellipsoid. The x axis is then uniquely determined by its orthogonality with both the y and z axes. In such a coordinate system, $W_{yy} = w_t$ and $W_{xy} = W_{yz} = 0$. The transformation matrix $\overleftrightarrow{T}$ between $\overleftrightarrow{W}_p$ and $\overleftrightarrow{W}$ is thus determined by the corresponding orthogonality property. For the ellipsoid whose major axis is in the [001] direction of the principal coordinate system,

$$\overleftrightarrow{T} = \begin{pmatrix} \dfrac{ln}{r\sqrt{l^2+m^2}} & \dfrac{-m}{\sqrt{l^2+m^2}} & \dfrac{l}{r} \\ \dfrac{mn}{r\sqrt{l^2+m^2}} & \dfrac{l}{\sqrt{l^2+m^2}} & \dfrac{m}{r} \\ \dfrac{-(l^2+m^2)}{r\sqrt{l^2+m^2}} & 0 & \dfrac{n}{r} \end{pmatrix} , \tag{13}$$

where $r^2 = l^2+m^2+n^2$. For other ellipsoids, the $\overleftrightarrow{T}$ matrices are obtained in the similar way.

For the L valley material, the calculation becomes complicated since the principal axes do not coincide with any of the [100], [010], and [001] directions. In order to discuss the coordinate transformation of the inverse effective-mass tensor, we must first transfer $\overleftrightarrow{W}_p$, expressed in the principal coordinate system (X,Y,Z), to the one in [100], [010], and [001] system and then to the system (x,y,z) determined by the growth direction. Therefore, Eq. (11) should be modified as

$$\overleftrightarrow{W} = \overleftrightarrow{T}^{\tau}\overleftrightarrow{B}\overleftrightarrow{W}_p\overleftrightarrow{B}^{\tau}\overleftrightarrow{T} , \tag{14}$$

whose elements are

$$W_{ij} = \sum_k T_{ki} \sum_{k'} B_{kk'}w_{k'} \sum_{k''} B_{k''k'}T_{k''j} . \tag{15}$$

For the ellipsoid whose major axis is in the [111] direction,

$$\overleftrightarrow{B} = \begin{pmatrix} \dfrac{1}{\sqrt{6}} & \dfrac{-1}{\sqrt{2}} & \dfrac{1}{\sqrt{3}} \\ \dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{3}} \\ \dfrac{-2}{\sqrt{6}} & 0 & \dfrac{1}{\sqrt{3}} \end{pmatrix} . \tag{16}$$

Similarly, the y axis in the new coordinate system (x,y,z) is set to be perpendicular to both the growth direction (the z axis in the new coordinate system) and the major axis of the ellipsoid. Again we have the relations $W_{yy} = w_t$ and $W_{xy} = W_{yz} = 0$. Thus

$$\overleftrightarrow{T} = \begin{pmatrix} \dfrac{(m+n)l-(m^2+n^2)}{rr_1} & \dfrac{n-m}{r_1} & \dfrac{l}{r} \\ \dfrac{(n+l)m-(n^2+l^2)}{rr_1} & \dfrac{l-n}{r_1} & \dfrac{m}{r} \\ \dfrac{(l+m)n-(l^2+m^2)}{rr_1} & \dfrac{m-l}{r_1} & \dfrac{n}{r} \end{pmatrix} , \tag{17}$$

where $r^2 = l^2+m^2+n^2$, as defined earlier, and $r_1^2 = (n-m)^2+(l-n)^2+(m-l)^2$.

W_{xz} and W_{zz} in the new coordinate system can be obtained by Eq. (15). However, we hereby present an alternative and simple method instead of the direct calculation of Eq. (15). Obviously W_{xz} and W_{zz} are linear functions of w_t and w_l [see Eq. (15)]. And it is known that if $w_t = w_l$, the inverse effective-mass tensor in the principal coordinate system is a numerical matrix (a unit matrix multiplied by a constant). Such a matrix is invariable under any unitary transformation. It is thus expected that for any w_t and w_l,

$$W_{zz} = (1-a)w_t + aw_l , \tag{18a}$$

$$W_{xz} = b(w_t - w_l) . \tag{18b}$$

In other words, in the expression of W_{zz} the sum of the coefficients of w_t and w_l equals one, while for W_{xz}, it is zero. Thus the calculation of W_{xz} and W_{zz} is extremely simplified. By Eq. (15), we can get a and b of Eq. (18) as

$$a = \left(\sum_k T_{kz} B_{kz}\right)^2 , \tag{19a}$$

$$b = -\left(\sum_k T_{kx} B_{kz}\right)\left(\sum_k T_{kz} B_{kz}\right) . \tag{19b}$$

Equations (18) and (19) are also valid for X valley by setting $\overleftrightarrow{B}$ as a unit matrix. Finally we have

$$W_{zz} = (1 - T_{zz}^2) w_t + T_{zz}^2 w_l , \tag{20a}$$

$$W_{xz} = -T_{zx} T_{zz} w_t + T_{zx} T_{zz} w_l . \tag{20b}$$

This method is of great convenience to determine the elements of the inverse effective mass tensor in the new coordinate system whose z axis is along the QW growth direction. However, the calculation of the matrix elements can be further simplified by a more detailed investigation. We will come back to this point later.

Now we would like to introduce the concept of the invariable quantities of the ellipsoidal energy surface in the coordinate transformations. These invariable quantities may greatly help us to understand the relations among different physical quantities.[9]

In the principal-axis coordinate system (X, Y, Z), the equation of the ellipsoidal constant energy surface of the QW system is written as

$$W_{XX}^2 k_X^2 + W_{YY}^2 k_Y^2 + W_{ZZ} k_Z^2 - E = 0 , \tag{21}$$

where k_i is the component of wave vector and E is the energy. However, in a general Cartesian coordinate system (x, y, z), the equation of the constant energy surface should be

$$W_{xx}^2 k_x^2 + W_{yy}^2 k_y^2 + W_{zz}^2 k_z^2 + 2W_{xy} k_x k_y + 2W_{yz} k_y k_z + 2W_{zx} k_z k_x - E = 0 . \tag{22}$$

There are four mathematically invariable quantities when transforming from the coordinate system (X, Y, Z) to (x, y, z). They are

$$\Gamma = \begin{vmatrix} W_{xx} & W_{xy} & W_{xz} & 0 \\ W_{xy} & W_{yy} & W_{yz} & 0 \\ W_{xz} & W_{yz} & W_{zz} & 0 \\ 0 & 0 & 0 & -E \end{vmatrix} = \begin{vmatrix} w_t & 0 & 0 & 0 \\ 0 & w_t & 0 & 0 \\ 0 & 0 & w_l & 0 \\ 0 & 0 & 0 & -E \end{vmatrix} = -E w_t^2 w_l , \tag{23a}$$

$$\Delta = \begin{vmatrix} W_{xx} & W_{xy} & W_{xz} \\ W_{xy} & W_{yy} & W_{yz} \\ W_{xz} & W_{yz} & W_{zz} \end{vmatrix} = \begin{vmatrix} w_t & 0 & 0 \\ 0 & w_t & 0 \\ 0 & 0 & w_l \end{vmatrix} = w_t^2 w_l , \tag{23b}$$

$$I = W_{xx} + W_{yy} + W_{zz} = 2w_t + w_l , \tag{23c}$$

$$J = W_{xx} W_{yy} + W_{yy} W_{zz} + W_{zz} W_{xx} - W_{xy}^2 - W_{yz}^2 - W_{zx}^2 = w_t^2 + 2 w_t w_l . \tag{23d}$$

In the above equations we have chosen $W_{XX} = W_{YY} = w_t$ and $W_{ZZ} = w_l$. Other choices of the $\overleftrightarrow{W}_p$ elements give the same results as shown by Eqs. (23).

Since $W_{yy} = w_t$ and $W_{xy} = W_{yz} = 0$ in the specially chosen coordinate system (x, y, z) as discussed in the beginning of this section, the above four invariable quantities are reduced to

$$W_{xx} W_{zz} - W_{xz}^2 = w_t w_l , \tag{24a}$$

$$W_{xx} + W_{zz} = w_t + w_l . \tag{24b}$$

Equations (24) are universal and thus are very useful to simplify the calculation of the elements of the inverse mass tensor and the discussion of the limits of the normal-incidence radiation absorption as well as its optimization. For example, from these two equations and Eq. (18a), we can write W_{xz} in a much simpler form

$$W_{xz}^2 = a(1-a)(w_t - w_l)^2 , \tag{25}$$

where a is defined by Eq. (19a). In other words, we have transformed the calculation of elements of the inverse effective-mass tensor into the calculation of one parameter a, which is evidently much simpler. In Table I, we list the expressions of the parameter a for X and the L valley electrons.

IV. LIMIT OF THE ABSORPTION COEFFICIENT AND THE OPTIMAL GROWTH DIRECTION OF A QW DETECTOR

From the infinite-barrier approximation and assuming that only the lowest subband is occupied so that the transition is between the ground and first excited states, the absorption coefficient α can be expressed as

$$\alpha(\hbar\omega) = \frac{S N_s}{\mu \omega L^3 A^2} \left| \sum_i A_i W_{iz} \right|^2 , \quad S = \frac{256 \hbar^2 e^2}{9 \Gamma c \epsilon_0} , \tag{26}$$

where N_s is the sheet density of carriers in the quantum well, $\Gamma/\hbar$ is the lifetime of the carrier in the first

TABLE I. The value of parameter a for X and L valleys.

X valley		L valley	
Major axis	ar^2	Major axis	$3ar^2$
[100]	l^2	[111]	$(l+m+n)^2$
[010]	m^2	[$\bar{1}$11]	$(-l+m+n)^2$
[001]	n^2	[1$\bar{1}$1]	$(l-m+n)^2$
		[11$\bar{1}$]	$(l+m-n)^2$

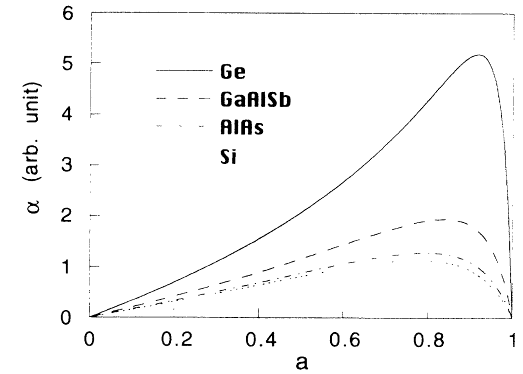

FIG. 1. The absorption coefficient of the normal incident radiation based on the intersubband transition as a function of the parameter a for Ge, $Ga_{1-x}Al_xSb$, AlAs, and Si well materials.

excited state, μ is the refraction index, L is the well width, and $\hbar\omega = 1.5\pi^2\hbar^2 W_{zz}/L^2$. Equation (26) is derived from our general formula of absorption coefficient as a function of intersubband transition (both the quantum well and superlattice systems). The relation between α and W_{ij} was discussed in Ref. 10, while the expression of α for quantum well detector was derived by Brown and Eglash.[11] It is recalled that for the normal-incidence radiation, $A_z = 0$, and in our selected coordinate system, $W_{xy} = 0$. For a nonpolarized incident radiation ($|A_x| = |A_y|$), from Eqs. (18a) and (25) and keeping $\hbar\omega$ constant, α can be expressed, apart from a constant, as

$$\alpha = W_{xz}^2/W_{zz}^{3/2} = \frac{a(1-a)(w_t - w_l)^2}{[(1-a)w_t + aw_l]^{3/2}} , \tag{27}$$

where $\mu \approx 3.5$ for all the materials under investigation. We have shown in Fig. 1 the absorption coefficient α as the function of the parameter a for four kinds of well materials which have been widely investigated. It is obtained from Eq. (27) that α reaches its maximal value $\alpha_{\max}$ when

$$a = \bar{a} = \frac{(3w_t + w_l) - \sqrt{(w_t + w_l)^2 + 12w_t w_l}}{2(w_t - w_l)} . \tag{28}$$

Using the formulas listed in Table I, we can calculate the a parameter for the different growth directions of the QW. By comparing the calculated a parameter with $\bar{a}$ of Eq. (28), the optimal growth direction for the absorption of the normal-incidence radiation can be obtained.

For X electrons in the energy valley with a [100] major axis and L electrons with a [111] major axis, we present the calculation results in Table II for the four different well materials. The value of $\sqrt{w_t}$ is also given, which will be useful for later discussions. It is clear that if the calculated optimal growth direction is $[l, m, n]$, the direction of $[m, l, n]$ for the [001] X valley and the directions obtained by the index rotation for the [111] L valley are also optimal because of the symmetry. The optimal growth directions for other valleys can be obtained in the similar way. It should be pointed out that in Table II only low-index optimal growth directions of the QW detector have been listed, where we have not taken into account the problems of the real material growth. In addition, the results are obtained for only a single valley whose major axis is listed in the table.

Table II and Fig. 1 have clearly demonstrated that among these four well materials studied, Ge is the best candidate for the largest absorption for the normal-incidence radiation based on the intersubband transitions. This conclusion agrees with the recent experimental results.[6] It is of interest to compare the absorption coefficients of indirect-gap systems with well known direct-gap systems, e.g., the GaAs well. Still keeping $\hbar\omega$ constant, it is easy to obtain from Eq. (26) that for a nonpolarized incident radiation $\alpha = 0.48$ at 45° incident illumination for the GaAs well where $w_t = w_l = 15.0$.

Now we would like to discuss the comparison between the absorption of the normal-incidence radiation and that of the parallel incident radiation for the same QW structure based on the intersubband transitions. For the parallel incident radiation and by Eq. (26), the absorption coefficient $\alpha_{\parallel}$ can be written as

$$\alpha_{\parallel} = W_{zz}^2/W_{zz}^{3/2} = \sqrt{W_{zz}} . \tag{29}$$

It is then more convenient to compare W_{xz} with W_{zz} instead of comparing Eq. (27) with Eq. (29). From Eqs. (18a) and (25), assuming $w_t = \beta w_l$ and omitting w_l, we have

$$W_{zz} = (1-a)\beta + a , \tag{30a}$$

$$W_{xz} = \sqrt{a(1-a)}(1-\beta) . \tag{30b}$$

Obviously, the parameter β is related to the anisotropic property of the effective-mass tensor of the well material ($0 < \beta \leq 1$). $\beta = 1$ means that the effective mass is isotropic so that $W_{xz} = 0$. The absorption coefficient of the intersubband transition for the normal-incidence radiation vanishes, similar to the Γ electrons in a $GaAs/Ga_{1-x}Al_xAs$ QW. The parameter a, defined in Eq. (19a), represents the orientation characteristics of the QW ($0 \leq a \leq 1$). Both $a = 0$ and $a = 1$ imply that the growth direction of the QW coincides with one of the principal axes so that $W_{xz} = 0$, and thus the absorption

TABLE II. The maximal value of the absorption coefficient and the optimal growth directions.

Material	Valley	Major axis	w_t[a]	w_l	$\bar{a}$	$\alpha_{\max}$	$\sqrt{w_t}$	Optimal [lmn]
AlAs	X	[001]	5.263	0.909	0.795	1.278	2.294	[102][113][203]
GaAlSb	L	[111]	6.304	0.766	0.839	1.941	2.511	[123][133][112]
Si	X	[001]	5.263	1.020	0.780	1.132	2.294	[102][113][203]
Ge	L	[111]	12.195	0.610	0.918	5.182	3.492	[122][112][133]

[a]The unit of the inverse effective-mass elements is $1/m_0$, where m_0 is the free electron mass.

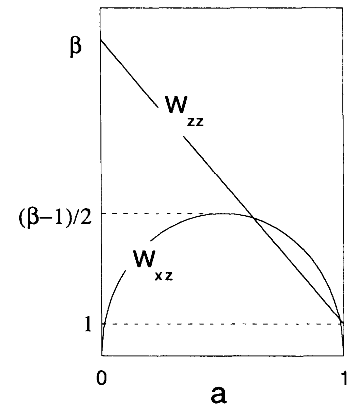

FIG. 2. Elements of inverse effective-mass tensor W_{zz} and W_{xz} (in unit of w_t) vs parameter a.

coefficient of the intersubband transition for the normal-incidence radiation vanishes, though the QW is made of the indirect gap semiconductor material. Figure 2 shows W_{zz} and W_{xz} as functions of parameter a. By a very simple calculation, it is found out that the straight line of W_{zz} and the curve of W_{xz} have two cross points only if

$$\beta > \beta_0 = 3 + \sqrt{8} = 5.828 \ .$$

In this case, when

$$\frac{3\beta - 1 - \sqrt{\beta^2 - 6\beta + 1}}{4(\beta - 1)} < a < \frac{3\beta - 1 + \sqrt{\beta^2 - 6\beta + 1}}{4(\beta - 1)} \ , \tag{31}$$

we will have $W_{xz} > W_{zz}$. In other words, the absorption of the normal-incidence radiation is larger than that of the parallel incident radiation. Thus, for the same QW's, the conditions for a larger absorption of the normal-incidence radiation are as follows: First, the anisotropy effect of the effective-mass tensor of the well material should be large enough so that the longitudinal mass m_l is at least 5.828 times larger than the transverse mass m_t. Second, the growth direction of the QW must satisfy Eq. (31). For example, for electrons in the $Ga_{1-x}Al_xSb$ L valley, $\beta = 8.229 > \beta_0$. If one chooses [2, 1, 3] or [2, 3, 3] as the growth direction $[l, m, n]$, the parameter a is 0.8571 or 0.9607, which satisfies Eq. (31). Thus the absorption of the normal-incidence radiation is larger than that of the parallel incident radiation in this QW. For a QW made of Ge, the same analysis can be done and a similar conclusion is obtained. However, for the Si QW, since $\beta = 5.16 < \beta_0$, the absorption of the normal-incidence radiation is always smaller than that of the parallel incident radiation for any growth direction.

Because the maximal value of the absorption coefficient of the parallel incident radiation based on the intersubband transitions is $\alpha_{\parallel\max} = \sqrt{w_t}$ (when the growth direction coincides with one principal axis), as listed in Table II, it is noticed that for a QW, the maximal value of the absorption coefficient of the parallel incident radiation is not always greater than that of the normal-incidence radiation.

V. SUMMARY

We have derived the conditions to absorb and detect the normal-incidence radiation based on the intersubband transitions in an n-type indirect-gap semiconductor quantum well. Quite different from previous publications, our investigation provides with a new method and we have established a simple and systematic theory to study the absorption of the normal-incidence radiation based on the intersubband transition in the n-type QW. The method is based on the special choice of the coordinate system associated with the growth direction of the QW, on the simple calculation of elements of the inverse effective-mass tensor, and on the application of invariable quantities of ellipsoidal constant energy surface under the transformation of the coordinate system.

We have derived a complete set of formulas for the absorption coefficient in terms of the sample growth direction $[l, m, n]$ and have obtained the optimal absorption conditions. Some optimal growth directions of the QW have been suggested for the maximal absorption of the normal-incidence radiation via the intersubband transition. The results obtained here are universal. They can be applied to any indirect-gap semiconductor materials and thus are of great help for the design of the highly efficient QW infrared detectors to interact directly with the normal-incidence radiation. The comparison of absorptions between the normal and the parallel incident radiation has unambiguously explained many related experimental results.[5]

It is easy to see that many conclusions about the selection of the materials and growth directions of the QW obtained in previous publications can be easily rederived based on our general theory. For example, the discussion about W_{zz}, W_{xz}, and the absorption coefficient given in Ref. 4 can be directly obtained from our Eq. (27). It should be noticed, however, that we have not considered the valley occupation in this paper and a further investigation along this line is in progress.

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* Permanent address: National Laboratory for Infrared Physics, Shanghai Institute of Technical Physics, 200083 Shanghai, China.

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