Resonant tunneling in the presence of a two-level fluctuator: Average transparency

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(Received 15 June 1993; revised manuscript received 18 January 1994)

We consider electrons tunneling through a double-barrier resonant-tunneling structure (DBRTS) and interacting with a defect which has internal degrees of freedom. Usually such a defect has two (or more) metastable configurations and can switch between them due to its interaction with a thermal bath. Interaction between the tunneling electron and the dynamic defect creates a *noisy environment* surrounding the DBRTS, and leads to fluctuations in time of the resonant level. Such fluctuations result in inelastic tunneling and low-frequency noise. This paper is focused on the problem of inelastic resonant tunneling. We have calculated the average transparency for various relations between the switching rate of the dynamic defect, the escape rate of the electron from the resonant level in the well, the coupling strength between the electron and the dynamic defect, and the temperature. The results derived here are entirely different from those found in phonon-assisted resonant tunneling, because phonons obey the Bose statistics but a two-level fluctuator behaves as an effective spin.

I. INTRODUCTION

Since the work of Tsu and Esaki,[1] the resonant tunneling through a double-barrier resonant-tunneling structure (DBRTS) has been investigated extensively. In the last few years much effort has been devoted to the study of the effect on resonant tunneling of the electron-phonon interaction, the electron-electron interaction, an external magnetic field, or a high-frequency ac electric field. All these works are based on the assumption that the DBRTS is perfect and does not contain defects which have internal degrees of freedom. Such defects are different from the static imperfection of interfaces.

Defects with internal degrees of freedom have been detected in the surrounding of even very high quality point contacts.[2–5] Usually a defect has two (or more) metastable configurations and can switch between them due to its interaction with a thermal bath. As a result, the so-called random telegraph noise appears.[2–5] The role of defects with internal degrees of freedom in the surroundings of point contacts was extensively discussed in the literature (see for review Ref. 6 and the references therein). Recently new features of electron transport due to Kondo effect in scattering by two-level impurities and electron-electron interaction were considered.[7–9]

It is quite reasonable to believe that such fluctuating defects, or the so-called *elementary fluctuators* (EF's) exist also in the surroundings of a DBRTS. Since EF's behave as dynamic degrees of freedom, their effect on resonant tunneling is an interesting and important phenomenon. In general, when the quasibound state in the well is coupled to an EF, the resonant level ε_0 will fluctuate in time, leading to both an inelastic tunneling and a low-frequency noise in the conductance. The fluctuations are induced by hops of the EF between its states. These hops, in turn, are due to the coupling of the EF to a thermal bath.

In this paper we study the inelastic resonant tunneling due to the interaction between the tunneling electron and a two-level EF. The aim is to calculate the time-average transparency $T(\varepsilon)$ of the DBRTS for the electron with the energy ε which is directly related to the Ohmic conductance σ $[\sigma = (e^2/2\pi\hbar)T(\varepsilon_F)]$ where ε_F is the Fermi level. The low-frequency noise originated from such an interaction will be treated elsewhere.

We use a simple model that the two-level EF is coupled to a phonon thermal bath and to the quasibound state in the well, but not to any state in either the emitter or the collector.[10] The model is described in Sec. II and is schematically illustrated in Fig. 1. According to the model, a *hop* of EF between its states leads to switching of the electron level inside the well between the values $\varepsilon_0 \pm J$, where J is the electron-EF coupling constant. As a result, our problem is fundamentally different from the phonon-assisted resonant tunneling[11,12] because phonons obey the Bose statistics, but the two-level EF's behave as effective dynamical spins. It is also different from the problem of direct electron interaction with a two-level impurity.[7–9] It should be mentioned that dynamical defects with internal degrees of freedom are intrinsic to all disordered materials, and their properties have been extensively analyzed both experimentally and theoretically (for a review see Ref. 6). In the present work we

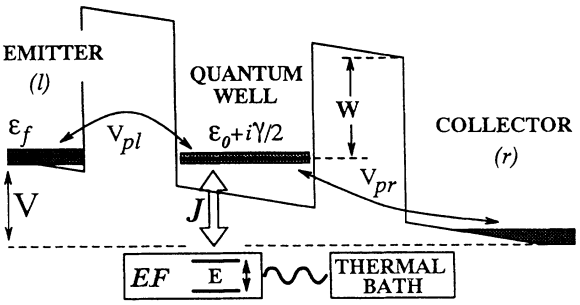

FIG. 1. A schematic illustration of the model system for a double-barrier resonant-tunneling structure in a *noisy environment.* ε_F is the Fermi energy, V is the bias voltage, V_{pl} (V_{pr}) is the tunneling matrix element between the emitter (collector) and the well, E is the level split of the EF. The energy $\varepsilon_0 \pm J$ corresponds to upper (lower) state of the EF. Consequently, J is the coupling constant between the EF and the resonant level inside the well.

will use the theoretical approach which was developed earlier.[6,13,14] We will outline the main steps of this analytical method in Sec. II, keeping in mind that the details of mathematical derivations have been published in Refs. 6, 13, 14, and the references therein. According to the relative values of relevant time scales in the system, the EF will be classified as *slow* or *fast.* The analysis for a slow EF is simpler, and so we will treat this case in Sec. III in order to illustrate a clear physical picture before we study a fast EF in Sec. IV. In a large DBRTS there may exist many EF's and the shape of the transparency spectrum will be sensitive to their spatial distribution. This problem will be treated in Sec. V. In Sec. VI essential conclusions will be discussed. To analyze the temperature dependence of the average transparency, one needs to use the random process with different hopping times from upper and lower states. The properties of such a process[15,16] are discussed in the Appendix.

II. MODEL AND ANALYTICAL METHOD

With the effective-mass approximation and assuming perfect interfaces, the simplest model for a DBRTS is one dimensional (1D). This 1D model coupled to an EF is schematically shown in Fig. 1. In terms of the notations in Fig. 1, when the DBRTS is decoupled from the environment, the electron Hamiltonian for the DBRTS is

$$H_e = \sum_{p,\nu} \varepsilon_{p,\nu} c^\dagger_{p,\nu} c_{p,\nu} + \varepsilon_0 c^\dagger_0 c_0 + \sum_{p,\nu} \{V_{p,\nu} c^\dagger_0 c_{p,\nu} + \text{H.c.}\}\,, \tag{1}$$

where p is the momentum of the electron, $\nu = l$ (or r) refers to emitter (or collector), and subscript 0 refers to the state within the well. In the present work we will set $\hbar = 1$ and so the energy is measured in frequency.

Corresponding to the 1D model shown in Fig. 1, besides the electron Hamiltonian H_e, the total Hamiltonian $H = H_e + H_{\text{EF}} + H_{\text{int}} + H_{\text{EF-ph}} + H_{\text{ph}}$ also contains the EF Hamiltonian

$$H_{\text{EF}} = \frac{E}{2}\sigma_z\,, \tag{2}$$

which interacts with the electron in the well via

$$H_{\text{int}} = J c^\dagger_0 c_0 \sigma_z\,, \tag{3}$$

and with the thermal bath H_{ph} via

$$H_{\text{EF-ph}} = \frac{1}{2}\sum_{k,q} M_q \sigma_x b^\dagger_{k+q/2} b_{k-q/2}\,. \tag{4}$$

In the above equations, σ_x and σ_z are Pauli matrices, E is the separation between the two levels of the EF, and b_k (or $b^\dagger_k$) is the phonon annihilation (or creation) operator. Due to the coupling H_{int}, as EF changes its state through one-phonon transitions, the electronic resonant level in the well is $\varepsilon_0 + J$ when the EF occupies the *upper state,* and is $\varepsilon_0 - J$ when the EF occupies the *lower state.* As we have mentioned in the Introduction, the main function of the EF is to transfer the high-frequency fluctuations of the phonon bath into the low-frequency fluctuations of the quasibound level in the well.

The transparency of a DBRTS for an electron with incoming energy ε and outgoing energy ε' has been derived[11] as

$$T(\varepsilon,\varepsilon') = \gamma_l(\varepsilon)\gamma_r(\varepsilon') \int \frac{d\eta\, ds\, d\tau}{2\pi} \mathcal{G}(\tau,s,\eta) \times e^{i(\varepsilon\tau - \varepsilon'\tau + \varepsilon'\eta - \varepsilon s)}\,. \tag{5}$$

γ_l (or γ_r) is the escape probability of an electron from the well to the emitter (or collector), and can be calculated from the matrix elements V_{pl} and V_{pr}. The so-called transmission Green's function is defined as

$$\mathcal{G}(\tau,s,\eta) = \Theta(s)\Theta(\eta)\langle c_0(\tau - s) c^\dagger_0(\tau) c_0(\eta) c^\dagger_0(0)\rangle_q\,, \tag{6}$$

where $\Theta(s)$ is the Heaviside function, $c_0(t)$ and $c^\dagger_0(t)$ are Heisenberg operators, and angular brackets mean quantum and thermal average with respect to the total Hamiltonian including the environment[11]

$$\langle\cdots\rangle_q \equiv \text{Tr}\,[\exp(-H/k_B T)\cdots]\,.$$

$\mathcal{G}(\tau,s,\eta)$ is in fact a two-particle Green's function with special arrangement of times. Equation (5) is derived under the assumption that an electron interacts with the environment only when it occupies the quasibound state in the well.

To avoid ambiguity, we use the term *noninteracting system* for the electronic part when the DBRTS is decoupled from the environment (namely, $J = 0$ in H_{int}). For a noninteracting system, we have

$$\mathcal{G}^0(\tau,s,\eta) = G^0_R(\eta)[G^0_R(s)]^*\,, \tag{7}$$

where

$$G_R^0(s) = -\Theta(t)\langle c(s)c^\dagger(0)\rangle_{q0} \tag{8}$$

is the bare retarded Green's function, the average $\langle\,\rangle_{q0}$ being calculated with the Hamiltonian H_e for the noninteracting system. In the wide band approximation[11] where the escape rates γ_l and γ_r are assumed to be energy independent, (8) becomes

$$G_R^0(s) = -\Theta(s)\exp(-i\varepsilon_0 s - \gamma s)\,, \tag{9}$$

where $\gamma = (\gamma_l + \gamma_r)/2$. In this case it is easy to obtain the transparency as

$$T^0(\varepsilon,\varepsilon') = \frac{\gamma_l\gamma_r}{(\varepsilon-\varepsilon_0)^2+\gamma^2}\delta(\varepsilon-\varepsilon')\,. \tag{10}$$

We are interested in the total transparency $T(\varepsilon) = \int d\varepsilon' T(\varepsilon,\varepsilon')$ of an electron with incoming energy ε, as well as in its form factor $S(\varepsilon) \equiv T(\varepsilon)/\int d\varepsilon T(\varepsilon)$. For a noninteracting system, they are simply

$$T^0(\varepsilon) = \frac{\gamma_l(\varepsilon)\gamma_r(\varepsilon)}{(\varepsilon-\varepsilon_0)^2+[\gamma(\varepsilon)]^2} \tag{11}$$

and

$$S^0(\varepsilon) = \frac{1}{\pi}\frac{\gamma}{(\varepsilon-\varepsilon_0)^2+\gamma^2}\,, \tag{12}$$

which is a Lorentzian.

Let us return to the complete coupled system illustrated in Fig. 1. Corresponding to $T(\varepsilon,\varepsilon')$, the reflection probability $R(\varepsilon,\varepsilon')$ can be written as[11]

$$R(\varepsilon,\varepsilon') = \frac{\gamma_l(\varepsilon')}{\gamma_r(\varepsilon')}T(\varepsilon,\varepsilon') + \delta(\varepsilon-\varepsilon') \times\{1+2\gamma_l(\varepsilon)\,\mathrm{Im}\,[G_R(\varepsilon)]\}\,, \tag{13}$$

where

$$G_R(\varepsilon) = \int dt\, e^{i\varepsilon t}[-i\Theta(t)\langle c_0(t)c_0^\dagger(0)\rangle_q] \tag{14}$$

is the retarded one-particle Green's function. The total reflection probability of an electron with incoming energy ε is simply $R(\varepsilon) = \int d\varepsilon' R(\varepsilon,\varepsilon')$. Integrating over the energy ε' in (13), we have

$$\int d\varepsilon' \frac{\gamma(\varepsilon')}{\gamma_r(\varepsilon')}T(\varepsilon,\varepsilon') + 2\gamma_l(\varepsilon)\,\mathrm{Im}\,[G_R(\varepsilon)] = T(\varepsilon) + R(\varepsilon) - 1\,. \tag{15}$$

Consequently, in the wide band approximation, the relation

$$T(\varepsilon) = -\frac{2\gamma_l\gamma_r}{\gamma}\,\mathrm{Im}\,[G_R(\varepsilon)] \tag{16}$$

is equivalent to the conservation law $T(\varepsilon) + R(\varepsilon) = 1$. Since approximations will be used in our calculation of tunneling transparency, we have to make sure that the so-obtained results will obey the above conservation law. This is indeed the case as shown in the later part of this paper by checking the relation (16).

As mentioned in the Introduction, though the effect of EF's on resonant tunneling through a DBRTS was not investigated before, a powerful method of *stochastic differential equations*[15,17–20] (SDE) has already been developed to study various phenomena related to dynamical defects. This approach was introduced much earlier by Anderson[21] and Kubo[22,23] to solve the problem in connection to nuclear magnetic resonance, and was extensively used recently to analyze the spectral diffusion in glasses.[24–26] Let W be the energy separation between the quasibound level ε_0 in the well and the top of the lower barrier as indicated in Fig. 1. There exists a proof[27,28] which, when applied to our problem, shows that the SDE approach can reproduce all essential results derived from the Green's functions method up to the second order in $(J/W)^2$. The SDE approach is then an adiabatic approximation, which takes into account the influence of dynamical fluctuations on the phase of the wave function, but neglects the interlevel transition caused by the switching of the EF between its two states. While a comprehensive discussion of the validity of the SDE approach can be found elsewhere,[28] it is certainly valid for our system shown in Fig. 1 because the ratio J/W is small.

Since the mathematical structure of the SDE approach has been given in detail in several places,[15,17–22,24–28] here we will only outline the SDE approach in terms of physical pictures. We are interested in the change of the resonant level ε_0 in the well due to its interaction with the EF. Within the framework of the SDE approach, such fluctuations of ε_0 are described by the replacement $\varepsilon_0 \to \varepsilon_0 + J\xi(t)$, where $\xi(t)$ is a stationary random process which describes the switching of the EF between the values $+1$ and -1. Consequently, the transmission Green's function (6) becomes a random function of time, and the transparency (5) needs to be averaged over this random process. The final result depends on the statistics of the random process $\xi(t)$, which is determined by the interaction $H_{\text{EF-ph}}$ between the EF and the thermal bath.

The random process $\xi(t)$ is closely connected to the switching rates between the states of an EF. For the present work, we consider an EF of two levels with the switching rate Γ_u for the upper level and Γ_d for the lower one. At a given temperature T, they are related as $\Gamma_u/\Gamma_d = \exp(E/k_BT)$. As we have mentioned earlier, in this paper we are interested in the high temperature region such that the coupling between the EF and the electrons in the emitter and the collector can be ignored. However, even in this high temperature region there are two distinguishable cases $k_BT \gg E$ and $k_BT \le E$. From now on we use the terminology *higher temperature* for $k_BT \gg E$ and *lower temperature* for $k_BT \le E$.

Besides the random process $\xi(t)$, the transparency spectrum also depends on the interplay of the three physical parameters: the total switching rate $\Gamma \equiv \Gamma_u + \Gamma_d$ of the EF, the total escape rate $\gamma \equiv (\gamma_l + \gamma_r)/2$ of the electron from the resonant level, and the coupling constant J between the electron and the EF. If $\gamma \gg \Gamma$, we have a slow fluctuator and the calculation of the transparency spectrum is simple for both higher and lower tempera-

ture. We will present this calculation in Sec. III.

However, for a fast fluctuator with $\Gamma \geq \gamma$, the random process for lower temperature is very different from the random process for higher temperature. We will first analyze the simpler case of higher temperature. If $k_BT \gg E$, then $\Gamma_u \simeq \Gamma_d \simeq \Gamma/2$ and the simplest $\xi(t)$ is the so-called dichotomic, or the random telegraph process, which has been extensively used to study the nuclear magnetic resonance.[16,21–23] For this process, the variable $\xi(t)$ switches between -1 and $+1$ randomly in time with the characteristic rate $\Gamma/2$. The static probabilities for the states with $\xi = +1$ and $\xi = -1$ are equal. The conditional probability $Q(\xi_1, t_1|\xi_0, t_0)$ to find the value $\xi(t_1) = \xi_1$ at time t_1 under the condition that at time t_0 the value of the variable was $\xi(t_0) = \xi_0$ is given as[16]

$$Q(\xi_1, t_1|\xi_0, t_0) = \frac{1}{2}[1 + e^{-\Gamma|t_1 - t_0|}]\delta_{\xi_1,\xi_0} + \frac{1}{2}[1 - e^{-\Gamma|t_1 - t_0|}]\delta_{\xi_1,-\xi_0}\,. \tag{17}$$

If we define $\langle\,\rangle_\xi$ as a *statistical* average over the random process $\xi(t)$, then from this conditional probability, it follows that $\langle\xi(t)\rangle_\xi = 0$ and the correlation function is $\langle\xi(t)\xi(t')\rangle_\xi = \exp(-\Gamma|t - t'|)$. Thus the spectrum of the random telegraph process is a Lorentzian with characteristic width Γ. We will return to this case in Sec. IV A.

At lower temperature $k_BT \leq E$, the random process $\xi(t)$ can deviate drastically from the random telegraph process because the hopping times from the upper state of EF and from its lower state are considerably different (see Sec. IV B).

III. A SLOW FLUCTUATOR

In this case with $\gamma \gg \Gamma$, in a period of lifetime $1/\gamma$ the electron experiences no change of the environment. Therefore we simply calculate the transparency for a given value of energy $\varepsilon + J\xi$. Then with a given probability distribution $P_1(\xi)$ for the variable $\xi \equiv \xi(t)$, we readily obtain the average transparency spectrum

$$\langle T(\varepsilon)\rangle_\xi = \int d\xi\, P_1(\xi) T^0(\varepsilon + J\xi)\,. \tag{18}$$

For a two-level EF, the probability distribution is[29]

$$P_1(\xi) = p_u\delta(\xi - 1) + p_d\delta(\xi + 1)\,,$$

where

$$p_u = \frac{\exp(-E/2k_BT)}{2\cosh(E/2k_BT)}\,, \quad p_d = 1 - p_u = \frac{\exp(E/2k_BT)}{2\cosh(E/2k_BT)}\,.$$

Hence we obtain the average transparency

$$\langle T(\varepsilon)\rangle_\xi = p_u T^0(\varepsilon + J) + p_d T^0(\varepsilon - J)\,. \tag{19}$$

In general, the average transparency exhibits a double-peak structure, which is well resolved if $J \geq \gamma$. The above results are valid for both the higher and the lower temperature.

IV. A FAST FLUCTUATOR

The situation becomes much more complicated when $\Gamma \geq \gamma$. In our problem the ratio J/W is small, where W is the energy separation shown in Fig. 1. Then, for $\Gamma \geq \gamma$, the only influence of EF is the phase breaking and therefore we can use the one-level approximation.[24,28] Within this approximation the phase destruction is taken into account with the replacement of $\exp(\pm i\varepsilon_0 t)$ by

$$\exp\left\{\pm i\varepsilon_0 t \pm iJ\int_0^t \xi(t')dt'\right\}.$$

As a result, the transmission Green's function (6) is obtained as

$$\mathcal{G}(\tau, s, \eta) = \exp(-\gamma s - \gamma\eta + i\varepsilon_0 s - i\varepsilon_0\eta) \times\Theta(s)\Theta(\eta)\Phi(\tau, s, \eta)\,, \tag{20}$$

where

$$\Phi(\tau, s, \eta) \equiv \left\langle \exp\left[iJ\int_0^\infty \vartheta_{\tau,s,\eta}(t')\xi(t')\,dt'\right]\right\rangle_\xi \tag{21}$$

and

$$\vartheta_{\tau,s,\eta}(t) = \Theta(t - \tau + s) - \Theta(t - \tau) - \Theta(t) + \Theta(t - \eta)\,. \tag{22}$$

By comparing these expressions with (7) and (9), it is clear that the effect of the EF is represented by the function $\Phi(\tau, s, \eta)$. As a result, we have reduced the quantum mechanical problem of the transmission Green's function calculation to the statistical average over the random process $\xi(t)$.

The $\vartheta(t)$ function is discontinued at the times 0, $\tau - s$, η and τ. We shall use the notations t_i for these times, and arrange $t_1 < t_2 < t_3 < t_4$. For our analysis it is convenient to set the zero reference energy at ε_0 and to measure all times in units $1/J$. If we introduce the auxiliary function

$$\Psi(\tau, s, \eta, t) \equiv \left\langle \exp\left(i\int_{t_1}^t \vartheta_{\tau,s,\eta}(t')\xi(t')\,dt'\right)\right\rangle_\xi\,, \tag{23}$$

then $\Phi(\tau, s, \eta)$ is just $\Psi(\tau, s, \eta, t_4)$, because $\vartheta_{\tau,s,\eta}(t) = 0$ for $t < t_1$ and for $t > t_4$. In the following we will calculate the Green's function (20) via this auxiliary function for both $E \ll k_BT$ and $E \geq k_BT$. For the former, $\xi(t)$ is a random telegraph process, and for the latter, one must take into account the difference of the hopping rates from the EF's states.

A. $S(\varepsilon)$ for $E \ll k_BT$

For a random telegraph process, from the definition (23) of $\Psi(\tau, s, \eta, t)$ and its correlation properties,[18,24] it satisfies the differential equation

$$\frac{d^2\Psi}{dt^2} + \left(\Gamma + \frac{1}{\vartheta}\frac{d\vartheta}{dt}\right)\frac{d\Psi}{dt} + \vartheta^2\Psi = 0, \tag{24}$$

with the initial conditions $\Psi(\tau,s,\eta,t_1) = 1$ and $\dot{\Psi}(\tau,s,\eta,t_1) = 0$. Using the coupling (4) between the EF and the thermal bath, the switching rate Γ can be derived as[13]

$$\Gamma = \Gamma_0 \coth(E/2k_BT) , \tag{25}$$

where $\Gamma_0 = E^3/E_c^2$, and E_c is a characteristic energy depending on the coupling constant M_q. The typical value of E_c/k_B is 20–30 K.[13] The shape of $\vartheta(t)$ given by (22) consists of two nonoverlapping pulses with the amplitudes ± 1, and hence ϑ^2 is 1 in the pulse regions and 0 in the gap regions. In both the pulse region and the gap region, the term $\dot{\vartheta}/\vartheta$ in (24) vanishes. In the gap region the solution of (24) is $\Psi(\tau,s,\eta,t) = b_1 + b_2 e^{-\Gamma t}$, and in the pulse region it is $\Psi(\tau,s,\eta,t) = a_1 e^{-s_1 t} + a_2 e^{-s_2 t}$ with

$$s_1 = \frac{\Gamma}{2} + \sqrt{\left(\frac{\Gamma}{2}\right)^2 - 1}, \quad s_2 = \frac{\Gamma}{2} - \sqrt{\left(\frac{\Gamma}{2}\right)^2 - 1} . \tag{26}$$

The coefficients a_1, a_2, b_1, and b_2 are determined from the continuity conditions of $\Psi(\tau,s,\eta,t)$ and $\dot{\Psi}(\tau,s,\eta,t)$.

The complete solution of $\Psi(\tau,s,\eta,t)$ will be derived with the transfer matrix approach. Let us introduce the vectors

$$\vec{a} \equiv \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \vec{b} \equiv \begin{pmatrix} b_1 \\ b_2 \end{pmatrix},$$

$$\vec{\Psi}(\tau,s,\eta,t) \equiv \begin{pmatrix} \Psi(\tau,s,\eta,t) \\ \dot{\Psi}(\tau,s,\eta,t) \end{pmatrix} . \tag{27}$$

In the pulse region, say $t_1 < t < t_2$, we also define two matrices

$$\hat{l} \equiv \begin{pmatrix} 1 & 1 \\ -s_1 & -s_2 \end{pmatrix}, \quad \hat{d}(t) \equiv \begin{pmatrix} e^{-s_1 t} & 0 \\ 0 & e^{-s_2 t} \end{pmatrix} . \tag{28}$$

Since $\vec{a} = \hat{l}^{-1}\hat{d}^{-1}(t_1)\vec{\Psi}(\tau,s,\eta,t_1)$ and

$$\vec{\Psi}(\tau,s,\eta,t_1) = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

we have for $t_1 < t < t_2$

$$\vec{\Psi}(t) = \hat{l}\hat{d}(\hat{t})\hat{d}^{-1}(t_1)\hat{l}^{-1}\vec{\Psi}(t_1) . \tag{29}$$

Similarly, for the gap region we introduce two matrices

$$\hat{m} \equiv \begin{pmatrix} 1 & 1 \\ 0 & -\Gamma \end{pmatrix}, \quad \hat{q}(t) \equiv \begin{pmatrix} 1 & 0 \\ 0 & e^{-\Gamma t} \end{pmatrix} . \tag{30}$$

Then, the complete function $\vec{\Psi}$ can be expressed as

$$\vec{\Psi}(t_1,t_2,t_3,t_4) = \hat{L}(t_4)\hat{M}^{-1}(t_3)\hat{M}(t_2)\hat{L}^{-1}(t_1)\vec{\Psi}(t_1) , \tag{31}$$

where

$$\hat{L}(t) \equiv \hat{l}\hat{d}(t), \quad \hat{M}(t) \equiv \hat{q}^{-1}(t)\hat{m}^{-1}\hat{L}(t) . \tag{32}$$

The Green's function will then be obtained from $\vec{\Psi}(t_1,t_2,t_3,t_4)$ by integrating over the proper time interval. Depending on the relative values of τ, s, and η, the four times t_1, t_2, t_3, and t_4 are assigned accordingly. As an example, for the situation $s > \eta$, various assignments are listed in Table I. The case $s < \eta$ can be treated in a similar way. Now we are ready to calculate the form factor $S(\varepsilon) \equiv T(\varepsilon)/\int d\varepsilon T(\varepsilon)$. The integrated transparency $T(\varepsilon) = \int d\varepsilon' T(\varepsilon,\varepsilon')$ is much simplified if the escape rate γ is energy independent (as within the wide band approximation[11]). In this case, from (5) the integral $\int d\varepsilon' T(\varepsilon,\varepsilon')$ yields a factor $\delta(\eta - \tau)$. Consequently, (22) reduces to $\vartheta(t) = \Theta(t-\tau+s) - \Theta(t)$ and the solution of (24) is $\Psi = \hat{l}\hat{d}(t_2 - t_1)\hat{l}^{-1}$. A straightforward calculation leads to the form factor

$$S(\varepsilon) = \frac{f(\varepsilon) + f(-\varepsilon)}{2\pi(s_1 - s_2)} , \tag{33}$$

where

$$f(\varepsilon) \equiv \frac{s_1}{s_2 + \gamma + i\varepsilon} - \frac{s_2}{s_1 + \gamma + i\varepsilon} . \tag{34}$$

With this approach we can also calculate the retarded Green's function, and then using the expression (16) to obtain the integrated transparency $T(\varepsilon)$, from which the form factor $S(\varepsilon)$ is again derived. The so-derived $S(\varepsilon)$ has exactly the same form as (33). Consequently, our approximation indeed satisfies the conservation law $T(\varepsilon) + R(\varepsilon) = 1$.

To analyze these results let us restore the correct dimensions for the energies and write

$$s_1 = \frac{1}{2}(\Gamma + \sqrt{\Gamma^2 - 4J^2}), \quad s_2 = \frac{1}{2}(\Gamma - \sqrt{\Gamma^2 - 4J^2}) .$$

For $\Gamma > 2J$, both s_1 and s_2 are real. Therefore $S(\varepsilon)$ has one peak centered at $\varepsilon = \varepsilon_0 \equiv 0$ (we have set ε_0 as our zero reference energy), resulting from the overlap of the *positive* Lorentzian with characteristic width s_2 and the *negative* one with the width s_1. Thus $S(\varepsilon)$ cannot be represented as a single Lorentzian. In the limit $\Gamma \gg J$ and under the condition $J^2 \gg \gamma\Gamma$, the height of the peak is

TABLE I. Arrangement of the times τ, s, and η for $s > \eta$.

Condition	t_1	t_2	t_3	t_4
$\tau > \eta$	0	$\tau - s$	η	τ
$s < \tau < \eta$	0	$\tau - s$	τ	η
$0 < \tau < s$	$\tau - s$	0	τ	η
$\tau < 0$	$\tau - s$	τ	0	η

$$S(0)=\frac{1}{\pi}\frac{\Gamma+\gamma}{(s_2+\gamma)(s_1+\gamma)}\simeq\frac{1}{\pi}\frac{\Gamma}{J^2}. \tag{35}$$

This result is similar to the dynamical line narrowing in the paramagnetic and the nuclear magnetic resonance.[21]

If $\Gamma<2J$, s_1 and s_2 are complex and the structure of $S(\varepsilon)$ consists of two peaks positioned at $\varepsilon=\pm J$, with equal peak height $2/\pi(\Gamma+2\gamma)$ and equal peak width $\Gamma+\gamma$. For $E=0.03k_BT$ and $\gamma=0.1J$, the evolution of $S(\varepsilon)$ from the single-peak structure to the double-peak structure is illustrated in Fig. 2, following the variation of Γ_0 which appears in (25). We see in Fig. 2 that $S(\varepsilon)$ splits into two sharp peaks as Γ_0/J decreases from $\Gamma_0/J=0.1$ (curve 1) through $\Gamma_0/J=0.01$ (curve 2) to $\Gamma_0/J=0.001$ (curve 3).

We will close this analysis with a remark that the present results differ entirely from the emergence of a phonon replica in the phonon-assisted resonant tunneling through a DBRTS.[11,12,30] The origin of this difference is that a two-level fluctuator behaves as a spin rather than obeys the Bose statistics.

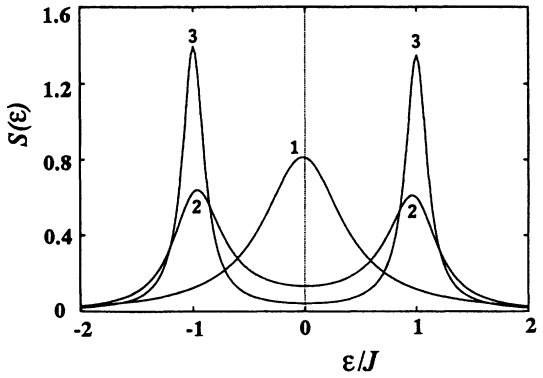

FIG. 2. The average spectrum form factor $S(\varepsilon)$ in the presence of a fast fluctuator with $\gamma/J=0.1$ at higher temperature $E/k_BT=0.03$. The value of Γ_0/J is 0.1 for curve 1, 0.01 for curve 2, and 0.001 for curve 3. The zero reference energy for ε is ε_0.

B. $S(\varepsilon)$ for $E\geq k_BT$

When the interlevel splitting E of the EF is comparable with k_BT, the switching rate Γ_u of the upper level differs much from the switching rate Γ_d of the lower level. To solve for the function $\Psi(t)$ in (23), it is convenient[15,20,16] to express it as $\Psi(t)\equiv\langle x(t)+1\rangle_\xi$, where the random function $x(t)$ satisfies the stochastic differential equation

$$\frac{dx(t)}{dt}=i\vartheta(t)\xi(t)x(t)+i\vartheta(t)\xi(t), \tag{36}$$

with the initial condition $x(t_1)=0$. To calculate the average $\langle x(t)\rangle_\xi$ of the solution of SDE (36), one can employ the general procedure outlined in Refs. 15, 17, 19, and 20. The part of the calculations which is relevant to our present problem is given explicitly in the Appendix. Using this method we readily obtain the Laplace transform $\Psi_L(s)$ of $\Psi(t)$ as

$$\Psi_L(s)=\frac{\langle g(s,\xi)\rangle_s+\left[\langle\Gamma(\xi)g(s,\xi)\rangle_s^2-\langle\Gamma^2(\xi)g(s,\xi)\rangle_s\langle g(s,\xi)\rangle_s\right]/\langle\Gamma(\xi)\rangle_s}{1-\langle\Gamma^2(\xi)g(s,\xi)\rangle_s/\langle\Gamma(\xi)\rangle_s}, \tag{37}$$

where $g(s,\xi)\equiv[s+\Gamma(\xi)+i\xi]^{-1}$ with $\Gamma(+1)\equiv\Gamma_u$ and $\Gamma(-1)\equiv\Gamma_d$. The static average $\langle h(\xi)\rangle_s$ is defined as $\langle h(\xi)\rangle_s\equiv p_uh(+1)+p_dh(-1)$, where p_u (or p_d) is the static probability for the state $\xi=+1$ (or $\xi=-1$) of the random variable $\xi(t)$.

Let us define $\mathcal{N}$ and n as, respectively, the Planck and the Fermi function of argument E/k_BT. We can then easily derive the expressions $\Gamma_d=\Gamma_0\mathcal{N}$, $\Gamma_u=\Gamma_0(\mathcal{N}+1)$, $p_u=n$, and $p_d=1-n$, with the relation $p_u\Gamma_u=p_d\Gamma_d$. Using the above definition of average to obtain $\langle\Gamma(\xi)\rangle_s=\Gamma\cosh^{-2}(E/2k_BT)$, from (37) we get the final form

$$\Psi_L(s)=\frac{1}{s_1-s_2}\left[\frac{s_1-\Gamma(1-d^2)-id}{s+s_1}-\frac{s_2-\Gamma(1-d^2)-id}{s+s_2}\right], \tag{38}$$

where $d\equiv p_d-p_u=\tanh(E/2k_BT)$, and with energies in units of J,

$$s_1=\frac{1}{2}(\Gamma-\sqrt{\Gamma^2+i4d\Gamma-4}),$$
$$s_2=\frac{1}{2}(\Gamma+\sqrt{\Gamma^2+i4d\Gamma-4}). \tag{39}$$

As in the case $E\ll k_BT$, knowing $\Psi_L(s)$, from (20)–(23) we can derive the form factor $S(\varepsilon)$. When performing this calculation, one must keep in mind that for $t<0$, certain quantities should be replaced by their complex conjugates. The final result can be expressed in a very compact form

$$S(\varepsilon)=\frac{1}{\pi}\mathrm{Re}\,\Psi_L(\gamma-i\varepsilon). \tag{40}$$

Under the condition $\Gamma_u=\Gamma_d$, $d=0$ and then the above equation reduces to (35) for $\varepsilon=\varepsilon_0\equiv0$. In the limit $J\gg\Gamma$, the $S(\varepsilon)$ given by (40) consists of two well-resolved peaks, one centers at $\varepsilon=+J$ with weight p_u, and the other centers at $\varepsilon=-J$ with weight p_d. Figure 3 demonstrates the evolution of $S(\varepsilon)$ when Γ/J and/or E/k_BT vary. If we fix the value $E/k_BT=0.3$,

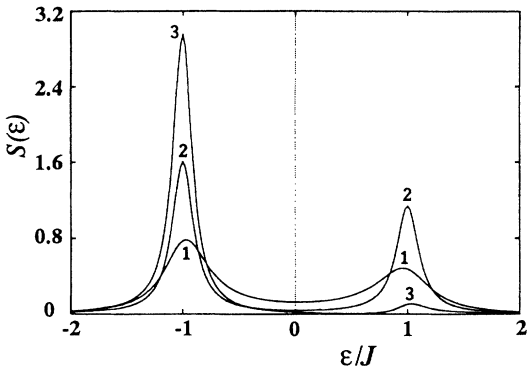

FIG. 3. The average spectrum form factor $S(\varepsilon)$ in the presence of a fast fluctuator at lower temperature. The values of $(E/k_BT;\Gamma_0/J)$ are (0.3;0.1) for curve 1, (0.3;0.01) for curve 2, and (3.0;0.1) for curve 3. The zero reference energy for ε is ε_0.

$S(\varepsilon)$ changes from curve 1 for $\Gamma_0/J = 0.1$ to curve 2 for $\Gamma_0/J = 0.01$. On the other hand, if we fix the value $\Gamma_0/J = 0.1$, the variation of $S(\varepsilon)$ is more dramatic from curve 2 for $E/k_BT = 0.3$ to curve 3 for $E/k_BT = 3.0$. Comparing Fig. 3 and Fig. 2, we see that at lower temperature the double-peak structure of $S(\varepsilon)$ is more pronounced.

As in the case $E \ll k_BT$, one can easily prove the current conservation law $T(\varepsilon) + R(\varepsilon) = 1$ by direct calculation of the imaginary part of the average Green's function. Actually, the property responsible for the current conservation is the possibility to express the function $\vartheta(t)$ defined by Eq. (22) as a system of two *nonoverlapping* pulses. As a result, our model is consistent with the relation (16) in general case.

V. MANY FLUCTUATORS

If a DBRTS is large enough to exhibit high $1/f$ noise, it is important to consider the effect of many EF's on the resonant level in the quantum well. Let us assume N noninteracting EF's, and modify the relevant parts of the Hamiltonian (2) and (3) as

$$H_{\rm EF} = \frac{1}{2}\sum_{i=1}^{N} E_i \sigma_z^{(i)} \tag{41}$$

and

$$H_{\rm int} = \sum_{i=1}^{N} J_i c_0^\dagger c_0 \sigma_z^{(i)} . \tag{42}$$

Since the EF's are assumed to be independent of each other, the associated random processes $\xi^{(i)}$ are also independent of each other. In reality the distribution of the coupling strength J_i can be quite sharp, while the distribution of E_i and the distribution of the corresponding switching rate Γ_i are broad. We will again treat separately the two cases distinguished by the switching rates.

A. Slow switching

Let us consider the situation that $\gamma \gg \Gamma_i$ for all i. We start from the situation where the distribution of J_i is within a narrow range. To obtain all main results, we will simplify the mathematical treatment by approximating all $J_i \simeq J$. Let us at first ignore the difference between Γ_u and Γ_d for all the EF's, namely, we assume random telegraph process for all random processes $\xi^{(i)}$.

The essential problem here is to calculate the probability distribution of the resonant level. This is most conveniently done by introducing the generating functional

$$K_N(x) = \prod_{i=1}^{N} \langle e^{-ix\xi^{(i)}} \rangle_{\xi^{(i)}} . \tag{43}$$

For $N = 1$ and for $p_d = p_u$, it is easy to obtain[29] $K_1(x) = \cos x$. For $N \gg 1$ we can use the Holtsmark method[31] to perform the average in (43). In this way we get[32]

$$K_N(x) = \exp\left[-N\int_0^\infty d\Gamma\, \mathcal{P}(\Gamma)[1 - K_1(x)]\right] , \tag{44}$$

where $\mathcal{P}(\Gamma)$ is the distribution function of the switching rate Γ with $\int_0^\infty d\Gamma \mathcal{P}(\Gamma) = 1$. For our problem under the approximation that all $J_i \simeq J$, $K_1(x)$ is simply Γ independent.

The average transparency spectrum can be calculated with (18), provided a correct probability $P_1^{(N)}(\xi)$ is used instead of $P_1(\xi)$. This probability $P_1^{(N)}(\xi)$ is just a Fourier component of $K_N(x)$, and is a Gaussian distribution

$$P_1^{(N)}(\xi) = (1/\sqrt{2\pi N}) \sum_k \delta(\xi - k) \exp(-k^2/2N) . \tag{45}$$

Substituting this distribution into (18) and integrating over k, we get for $N \gg 1$

$$S(\varepsilon) = \frac{1}{2\pi}\int dt\, e^{-\gamma|t| - NJ^2t^2/2} \cos(\varepsilon - \varepsilon_0)t. \tag{46}$$

It is clear that the resonant peak has Lorentzian *wings* in the region $|\varepsilon - \varepsilon_0| \gg J\sqrt{N}$, but a Gaussian central part with characteristic width $J\sqrt{N}$. The typical transparency spectrum is shown in Fig. 4 for $\frac{J}{\gamma}\sqrt{2N} = 2.0$.

To investigate the temperature dependence of the shift of the resonant level in this limit case, one should use the explicit temperature-dependent expressions of p_u and p_d. Then, for $N = 1$ we obtain from (43)

$$K_1(x) = \cos x - i(p_d - p_u)\sin x.$$

Substituting this expression into (44), we found that at lower temperature the only effect of $p_d \neq p_u$ is, in all relevant equations, to replace the actual number N of EF's by an effective number

$$N_{\rm eff}(T) \equiv \int dJ dE d\Gamma \frac{\mathcal{P}(J,E,\Gamma)}{\cosh^2(E/2k_BT)} . \tag{47}$$

The physical meaning of $N_{\rm eff}(T)$ is the number of EF's

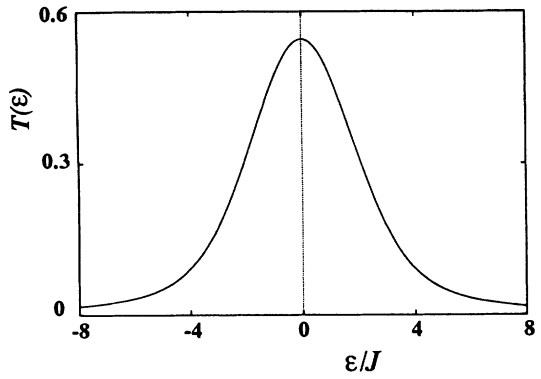

FIG. 4. The average transparency spectrum $T(\varepsilon)$ in the presence of N slow fluctuators with $J\sqrt{2N}/\gamma = 2.0$ at higher temperature. The zero reference energy for ε is ε_0.

that can switch states at the given temperature T. If the probability function $\mathcal{P}$ does not depend on E, then we have simply $N_{\text{eff}}(T) \propto T$. It is important to point out that because of $p_d \neq p_u$, there is a net average shift of the resonant level due to the switches of EF's. This shift consists of a temperature-independent part, which can be absorbed in the renormalization of ε_0, and a temperature-dependent part

$$\delta\varepsilon_0(T) \equiv \frac{2J}{N_{\text{eff}}(T)} \int dE d\Gamma \mathcal{P}(E,\Gamma) n(E/k_BT) . \quad (48)$$

A finite $\delta\varepsilon_0(T)$ will cause a temperature dependence of the resonant-tunneling current through a DBRTS.

B. Fast switching

The case of fast switching requires much tedious calculation. Here we will outline the general procedure of analysis. Our previous equation (21) is obtained for a single EF. We first use this equation to calculate the $\Phi_i(\tau, s, \eta)$ for the ith EF. Then we define the function Φ_N as the product of all these $\Phi_i(\tau, s, \eta)$'s for the N EF's. Using the Holtsmark procedure we arrive at the following expression for Φ_N:

$$\Phi_N(\tau, s, \eta) = \exp\left[- \int dJ dE d\Gamma \frac{\mathcal{P}(J,E,\Gamma)}{\cosh^2(E/2k_BT)} \times [1 - \Psi(t_1, t_2, t_3, \tilde{t})] \right] , \quad (49)$$

where the auxiliary function $\Psi(t_1, t_2, t_3, \tilde{t})$ is given by (31). This expression is similar to (44) for the case of slow switching.

We are interested in the integrated transparency $T(\varepsilon)$. Since the escape rate γ is energy independent within the wide band approximation,[11] from (5) the integral $\int d\varepsilon' T(\varepsilon, \varepsilon')$ yields a factor $\delta(\eta - \tau)$. Making use of this simplification and (40), $\Psi(t_1, t_2, t_3, \tilde{t})$ reduces to the form

$$\Psi(t_1, t_2, t_3, \tilde{t}) = \frac{1}{\pi(s_1 - s_2)} \int_{-\infty}^{\infty} d\omega \cos(\omega\tilde{t}) \times \left[\frac{s_2}{\omega - is_1} - \frac{s_1}{\omega - is_2} \right] = 1 - \exp\left(-\frac{1}{2}\Gamma\tilde{t} \right) \times \left[\cosh I\tilde{t} + \frac{\Gamma}{2I} \sinh I\tilde{t} \right] , \quad (50)$$

where $I = \sqrt{(\Gamma/2)^2 + id\Gamma - 1}$. Again, calculating the imaginary part of the retarded Green's function by a procedure similar to the one described above and comparing the result with (49) one can check the current conservation law in the case of many EF's (the calculations are straightforward but rather cumbersome).

The $\tilde{t}$ dependence of $\Psi(t_1, t_2, t_3, \tilde{t})$ is determined by the distribution function of J_i and Γ_i. We remind ourselves that in (40) the energies are in units of J. With the quantity $d \equiv p_d - p_u$ included in the renormalized energy shift, for $J\tilde{t} \ll 1$ we can easily derive the result

$$\Psi(t_1, t_2, t_3, \tilde{t}) \approx 1 - J^2\tilde{t}^2/2.$$

Consequently, if the distribution function $\mathcal{P}(J, E, \Gamma)$ decreases rapidly with the increasing J such that a cutoff value J_c (as well as cutoff values $\Gamma_{\max}$ and $\Gamma_{\min}$ for the Γ distribution) can be introduced, $T(\varepsilon)$ will consist of a Gaussian kernel with characteristic width $J_c\sqrt{N_{\text{eff}}(T)}$ and Lorentzian *wings* in the regions $|\varepsilon - \varepsilon_0| \gg J_c\sqrt{N_{\text{eff}}(T)}$. This spectrum has similar structure as that for the slow switching case, where $N_{\text{eff}}(T)$ plays the role of the actual number N of elementary fluctuators.

C. Crossover region

We have seen that for fast fluctuators, the crossover in the spectrum $T(\varepsilon)$ from the Gaussian shape to the Lorentzian shape is determined by the *maximal* coupling J_c. Therefore, if the distribution of J is smooth and exhibits no cutoff behavior, the spectrum will be entirely different. In particular, this is the case when the coupling between the resonant level and the EF's is due to a dipole force which decays in space as A/R^3. In such a situation, if there is no geometrical cutoff for the nearest-neighbor distance, the distribution function is characterized by

$$\mathcal{P}(J, E, T) \propto \int d^3R \delta(J - AR^{-3}) \propto J^{-2} .$$

On the other hand, the quantity $1 - \Psi(t_1, t_2, t_3, \tilde{t})$ in the right-hand side of (49) is proportional to $J^2\tilde{t}^2$ only if $J\tilde{t} < 1$, but is oscillating in the region $J\tilde{t} > 1$. Consequently, the integral of J in the exponent of (49) is estimated as

$$\int_0^{\tilde{t}^{-1}} dJ \frac{1}{J^2} J^2\tilde{t}^2 \propto \tilde{t} .$$

With this estimation, we can show that the transparency

spectrum is a Lorentzian with the width proportional to $|A|$. This behavior is similar to that discussed by Anderson[33] in connection with the problem of ESR spectra.[34]

We can analyze the problem in more detail with the $\mathcal{P}(J,E,\Gamma)$ of the dipole interaction for which the distributions of J and Γ are broad. We will use the form[6]

$$\mathcal{P}(J,E,\Gamma)=\frac{P_0\langle|A|\rangle_{or}}{J^2}\,\frac{1}{\Gamma}\Theta(\Gamma_{\max}-\Gamma)\Theta(\Gamma-\Gamma_{\min})\,, \tag{51}$$

where $\Gamma_{\max}=\Gamma_0\coth(E/2k_BT)$ and $\Gamma_{\min}=\Gamma_l\coth(E/2k_BT)$, with Γ_l being some minimal value of the switching rate. In the above equation, P_0 is the density of states for EF's, and $\langle\,\rangle_{or}$ means an average over the direction of dipole. The factor $1/\Gamma$ reflects the exponentially broad distribution of switching rates. Using the procedure given in Refs. 25, 33, and 34, one can prove that for such distribution and for $\Gamma_0\ll\nu_d\simeq P_0\langle|A|\rangle_{or}k_BT$, the function $\Phi(\tau,s,\eta)$ in (49) has the simple form

$$\Phi(\tau,s,\eta)=\exp(-|\tilde{t}|\nu_d)\,. \tag{52}$$

Therefore it is straightforward to show that the transparency spectrum $T(\varepsilon)$ is a Lorentzian with the characteristic width $\gamma+\nu_d$. The crossover from the Gaussian spectrum to the Lorentzian one takes place when the distance between the DBRTS and its nearest EF becomes less than $(P_0k_BT)^{-1/3}$.

Thus the shape of the transparency spectrum is sensitive to both the spatial arrangement of the EF's, and the interaction mechanism.

VI. DISCUSSION AND CONCLUSION

The influence of an EF on the resonant tunneling depends strongly on the relation between the switching rate Γ, the resonant level shift J due to the change of EF state, and the electron escape rate γ. If $\Gamma\ll J$ and $\Gamma\ll\gamma$, electrons can tunnel resonantly through two static levels $\varepsilon_0\pm J$ and result in a split of the transparency spectrum. The amount of split is proportional to J, and the width is proportional to the larger one of Γ and γ. Hence a two-peak structure is well resolved if J is larger than both γ and Γ. On the other hand, if $\Gamma\gg J\gg\gamma$, between switching the electron does not have enough time to form two separate levels $\varepsilon_0\pm J$. In this case the transparency spectrum is a broad peak, the width of which is the larger value of J^2/Γ and γ.

Since Γ increases with temperature, the temperature dependence of the width may change from one parameter regime to the other. The exact form of the temperature dependence is determined by the mechanism of EF transition. In the case of quantum tunneling, $\Gamma\propto T^3$ if the EF interacts with phonons, and $\Gamma\propto T$ if the EF interacts with electrons. In the case of activation, $\Gamma\propto\exp(-E_a/k_BT)$. Therefore a certain crossover temperature should exist at which the temperature dependence of Γ changes drastically. To estimate this crossover temperature as well as the activation energy one needs a microscopic model of EF. So far it is not clear what kinds of localized modes interact with tunneling electrons. It could be disorder-induced soft atomic vibrations which are responsible for many dynamical properties of amorphous media. There is a relatively good model for this kind of modes,[6] which allows one to estimate roughly the crossover temperature (about 10–20 K) and the activation energy (about 100–300 meV). Another possible mechanism is the hopping of localized electrons in the doped region between donor states, which can create fluctuating fields inside the quantum well.

In general it is very difficult to estimate the rate Γ and the shift J. It is known that in any disordered material there exists an exponentially wide range of relaxation rates. Thus, if the defect is originated from an amorphous region, one can find practically any value of Γ. The quantity J is determined by the microscopic structure of the defect and by its spatial position. In any case, to observe EF one should study small DBRTS with thick barriers so the escape rate γ is small.

If the size of the DBRTS is big, many EF's contribute to the spectrum of resonant tunneling. The typical shape and width of the spectrum have been shown to be dependent on the spatial distribution of EF's and on the interaction mechanism. If the distribution of J exhibits a cutoff feature, the shape is a Gaussian of width $J\sqrt{N_{\rm eff}}$, where $N_{\rm eff}$ is the effective number of EF's which can change states at a given temperature. This quantity depends on both the quality of the DBRTS and the microscopic nature of the EF. If the spatial distribution of EF's is uniform and the interaction obeys the dipole law AR^{-3}, the spectrum is a Lorentzian with width of the order $\langle|A|\rangle_{or}$.

As a conclusion, we want to emphasize that an attempt to work out a theory of resonant tunneling in the presence of soft dynamical modes has been made. The model used allows one to analyze qualitatively the current-voltage curve of a DBRTS. To get more quantitative information a microscopic model for defects as well as new experiments are required. It would be extremely useful to perform experiments on structures of different sizes and at different temperatures to observe the crossovers from tunneling to activation and the crossover from a single EF to many EF's. Based on such information a realistic model can be constructed for quantitative studies. The implication of the phenomena investigated in this paper to the low-frequency noise will be considered in a separate paper.

ACKNOWLEDGMENTS

We are grateful to M. Jonson, R. Shekhter, and V. Shumeiko for helpful discussion. The present work was supported by the Norwegian Research Council for Science and the Humanities (NAVF) Grant No. RT/SDU 424.90/009.

APPENDIX: ANALYSIS OF THE STOCHASTIC PROCESSES WITH DIFFERENT HOPPING RATES

Here we outline the scheme of calculation which is very similar to the one discussed in Ref. 20. Assume that a

random quantity $\xi(t)$ has a discrete set of values $\{c_k\}$. The rate of hopping $\Gamma(\xi)$ from the state $\xi(t)$ depends on the value of $\xi(t)$. Note that $\Gamma(\xi)$ is defined as the *total rate* for tunneling out of state $\xi(t)$ to *any* other possible states. The transition probability density to hop from the state ξ' to the state ξ during the infinitesimally small time interval dt can be expressed as[15,20]

$$Q(\xi, dt|\xi', 0) = [1 - \Gamma(\xi')\,dt]\delta(\xi - \xi') + \Gamma(\xi')\frac{\Gamma(\xi)p(\xi)}{\langle\Gamma\rangle_s}\,dt\,, \tag{A1}$$

where $p(\xi)$ is a steady-state distribution of the random quantity ξ. If ξ is a discrete quantity having the values $\{c_k\}$ with static probabilities $\{p_k\}$ then $p(\xi) = \sum_k p_k\delta(\xi - c_k)$. The static average $\langle F\rangle_s$ of a deterministic function $F(t,\xi)$ is defined as $\langle F\rangle_s = \int F(\xi)\,p(\xi)\,d\xi$. The quantity $\Gamma(\xi)p(\xi)/\langle\Gamma\rangle_s$ has the meaning of the probability of the value $\xi(t) = \xi$ after at least one jump.

One can easily derive the linear equation for the conditional probability $Q(\xi, t|\xi', 0)$,

$$\frac{\partial Q}{\partial t} = \hat{S}Q\,, \tag{A2}$$

where

$$\hat{S}Q \equiv -\Gamma(\xi)Q + \frac{\Gamma(\xi)}{\langle\Gamma\rangle_s}p(\xi)\int\Gamma(\xi_1)\,Q(\xi_1, t|\xi', 0)\,d\xi_1 \tag{A3}$$

has the meaning of a master equation. Knowing the $\hat{S}$ operator it is possible to derive an equation[19] for an arbitrary average of the type $U(t) \equiv \langle F(t,\xi)A_t[\xi(\tau)]\rangle_\xi$, where $F(t,\xi)$ is an arbitrary deterministic function of its variables, while $A_t[\xi(\tau)]$ is an arbitrary function of t and a functional of the process $\xi(\tau)$ for $\tau < t$. This functional is a *retarded functional.* The equation to be derived has the form

$$\frac{dU(t)}{dt} = \frac{\partial U(t)}{\partial t} + \langle[\hat{S}^+F(t,\xi)]A_t[\xi(\tau)]\rangle_\xi\,. \tag{A4}$$

Here $\hat{S}^+$ is the operator conjugate to $\hat{S}$, which operates on a function $f(t,\xi)$ as

$$\hat{S}^+f(t,\xi) \equiv -\Gamma(\xi)f(t,\xi) + \frac{\Gamma(\xi)\langle\Gamma f\rangle_s}{\langle\Gamma\rangle_s}\,. \tag{A5}$$

Indeed,

$$\int d\xi\, f(\xi, t|\xi', 0)\hat{S}Q \equiv \int d\xi\, Q(\xi, t|\xi', 0)\hat{S}^+f$$

in agreement with the definition of a conjugated operator. Combining (A5) and (A2) one obtains the probability density as[20]

$$\frac{dU(t)}{dt} = \frac{\partial U}{\partial t} - \langle\Gamma F A_t\rangle_\xi + \frac{\langle\Gamma F\rangle_s}{\langle\Gamma\rangle_s}\langle\Gamma A_t\rangle_\xi\,. \tag{A6}$$

We outline the derivation of (A4) as follows.[19,20] In the definition of the statistical average

$$\langle FA_t\rangle = \int F(t,\xi)Q(\xi, t|\{\xi_i\},\{t_i\})A_t(\{\xi_i\},\{t_i\})$$

$$\times p(\{\xi_i\},\{t_i\})\,d\xi\prod_i d\xi_i\,, \tag{A7}$$

$\{\xi_i\}$ is the set of values of the random quantity ξ, and $\{t_i\} < t$ is the set of times. The quantity $Q(\xi, t|\{\xi_i\},\{t_i\})$ is the multievent conditional probability to find the value ξ at time t under the condition $\xi(t_i) = \xi_i$, while $p(\{\xi_i\},\{t_i\})$ is the multievent distribution function. Consequently the product $Q(\xi, t|\{\xi_i\},\{t_i\})p(\{\xi_i\},\{t_i\})$ is the joint probability to find $\xi(t) = \xi$, $\xi(t_i) = \xi_i$. Taking the time derivative, we get

$$\frac{d}{dt}\langle FA_t\rangle_\xi = \frac{\partial}{\partial t}\langle FA_t\rangle_\xi + B\,, \tag{A8}$$

where

$$B = \int F(t,\xi)A_t\,\frac{\partial Q(\xi, t|\{\xi_i\},\{t_i\})}{\partial t}$$

$$\times p(\{\xi_i\},\{t_i\})\,d\xi\prod_i d\xi_i\,. \tag{A9}$$

Using the master equation (A2) one obtains

$$\begin{aligned} B &= \int FA_t\,[\hat{S}Q]p(\{\xi_i\},\{t_i\})\,d\xi\prod_i d\xi_i \\ &= \int[\hat{S}^+F(t,\xi)]A_tQ\,p(\{\xi_i\},\{t_i\})\,d\xi\prod_i d\xi_i \\ &= \langle[\hat{S}^+F(t,\xi)]A_t\rangle_\xi\,. \end{aligned} \tag{A10}$$

Now we apply this formalism to the calculation of an average over the random process in question of a set of stochastic differential equations. Consider a set

$$\dot{\vec{x}} = \hat{D}(\xi)\vec{x} + \vec{f}\,, \tag{A11}$$

where the dynamical variable $\vec{x}$ is an n-dimensional vector $[x_1(t), x_2(t), \ldots, x_n(t)]$, and $\vec{f}$ is an n-dimensional random force which can depend statistically on the random process $\xi(\tau)$ for $\tau < t$. The $n \times n$ matrix $\hat{D}$ depends also on $\xi(t)$.

We will first average (A11) over the statistics of the process $\xi(t)$, and then average it over the statistics of the random variable $\vec{f}$. The second average will be defined as $\langle\,\rangle_f$. The first average leads to

$$\langle\dot{\vec{x}}\rangle_\xi = \langle\hat{D}(\xi)\vec{x}\rangle_\xi + \langle\vec{f}\rangle_\xi\,. \tag{A12}$$

It is important that the solution $\vec{x}(t)$ is indeed a *retarded functional* of the process ξ. Then, one can apply the (A6) to decompose the correlation in the average $\langle\hat{D}\,[\xi(t)]\,\vec{x}(t)\rangle$. Inserting $F = \hat{D}$ and $A_t = \vec{x}$ into (A6), we get

$$\frac{d}{dt}\langle\hat{D}\vec{x}\rangle_\xi = -\langle\Gamma\hat{D}\vec{x}\rangle_\xi + \frac{\langle\Gamma\hat{D}\rangle_s}{\langle\Gamma\rangle_s}\langle\Gamma\vec{x}\rangle_\xi + \langle\hat{D}\dot{\vec{x}}\rangle_\xi$$
$$= -\langle\Gamma\hat{D}\vec{x}\rangle_\xi + \frac{\langle\Gamma\hat{D}\rangle_s}{\langle\Gamma\rangle_s}\langle\Gamma\vec{x}\rangle_\xi + \langle\hat{D}^2\vec{x}\rangle_\xi + \langle\hat{D}\vec{f}\rangle_\xi . \tag{A13}$$

As a result, the average $\langle\hat{D}\vec{x}\rangle_\xi$ is expressed in terms of the averages $\langle\Gamma\vec{x}\rangle_\xi$, $\langle\Gamma\hat{D}\vec{x}\rangle_\xi$, and $\langle\hat{D}^2\vec{x}\rangle_\xi$. This procedure can be continued. Introducing the notations

$$\vec{x}_{km} \equiv \langle\Gamma^m\hat{D}^k\vec{x}\rangle_\xi, \vec{f}_{km} \equiv \langle\Gamma^m\hat{D}^k\vec{f}\rangle_\xi, \tag{A14}$$
$$\hat{D}_{km} \equiv \frac{\langle\Gamma^{m+1}\hat{D}^k\rangle_s}{\langle\Gamma\rangle_s},$$

we get the chain of equations

$$\dot{\vec{x}}_{km} = \hat{D}_{km}\vec{x}_{01} + \vec{f}_{km} + \vec{x}_{k+1,m} - \vec{x}_{k,m+1}, \tag{A15}$$

with the initial condition $\vec{x}(0) = 0$. One can show that for all k and m, we also have $\vec{x}_{km}(0) = 0$.

Now let us introduce the inverse operator $\hat{I}$ as

$$\hat{I}g = \int_0^t g(t_1)\,dt_1 ,$$

and iterate the term $(\vec{x}_{k+1,m}\vec{x}_{k,m+1})$ on the right-hand side of (A15). At fixed values c_k of the variable $\xi(t)$, such iteration generates a geometrical progression. As a result, one obtains[20]

$$\vec{x}_{km} = \frac{1}{\langle\Gamma\rangle}\left\langle\frac{\Gamma^{m+1}\hat{D}^k}{\frac{d}{dt}+\Gamma-\hat{D}}\right\rangle_s \vec{x}_{01} + \left\langle\frac{\Gamma^m\hat{D}^k}{\frac{d}{dt}+\Gamma-\hat{D}}\vec{f}\right\rangle_s . \tag{A16}$$

Finally, we arrive at the following set of equations for the coupled quantities $\langle\vec{x}\rangle$ and $\vec{x}_{01}$:

$$\langle\vec{x}\rangle_\xi = \frac{1}{\langle\Gamma\rangle_s}\left\langle\frac{\Gamma}{\frac{d}{dt}+\Gamma-\hat{D}}\right\rangle_s \vec{x}_{01} + \left\langle\frac{1}{\frac{d}{dt}+\Gamma-\hat{D}}\vec{f}\right\rangle_s , \tag{A17}$$

$$\vec{x}_{01} = \frac{1}{\langle\Gamma\rangle_s}\left\langle\frac{\Gamma^2}{\frac{d}{dt}+\Gamma-\hat{D}}\right\rangle_s \vec{x}_{01} + \left\langle\frac{\Gamma}{\frac{d}{dt}+\Gamma-\hat{D}}\vec{f}\right\rangle_s . \tag{A18}$$

One can see that the "force" $\vec{f}$ enters the equations linearly. Consequently the average over its statistics does not change the results and one can include it in the definition of $\langle\,\rangle_s$.

It is convenient to make the Laplace transform over the variable t. Finally, we have

$$\vec{X}(s) = \frac{1}{\langle\Gamma\rangle}\left\langle\frac{\Gamma}{s+\Gamma-\hat{D}}\right\rangle_s \vec{X}_{01} + \left\langle\frac{1}{s+\Gamma-\hat{D}}\vec{F}(s)\right\rangle_s , \tag{A19}$$

$$\vec{X}_{01} = \frac{1}{\langle\Gamma\rangle}\left\langle\frac{\Gamma^2}{s+\Gamma-\hat{D}}\right\rangle_s \vec{X}_{01}(s) + \left\langle\frac{\Gamma}{s+\Gamma-\hat{D}}\vec{F}(s)\right\rangle_s \tag{A20}$$

for the Laplace transforms $\vec{X}(s)$ and $\vec{X}_{01}(s)$ of the variables $\langle\vec{x}\rangle$ and $\vec{x}_{01}$, respectively. The average over the statistics of f can be included in $\langle\,\rangle_s$ as

$$\langle\cdots\vec{F}(s)\rangle_s = \int\int\cdots\vec{f}P(s,\vec{f}|\xi)\,p(\xi)\,d\xi\,d\vec{f}. \tag{A21}$$

The relation $\Psi(t) \equiv \langle x(t)+1\rangle_\xi$ introduced at the beginning of Sec. IV B corresponds to the case of one-component vector $\vec{x}$ and $D = f = i\vartheta(t)\xi(t)$. With the help of the outlined procedure the expression (37) is readily obtained.

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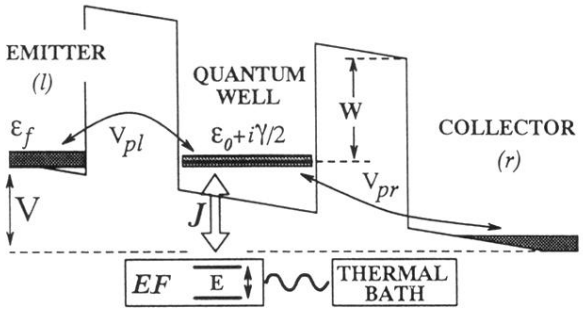

FIG. 1. A schematic illustration of the model system for a double-barrier resonant-tunneling structure in a *noisy environment*. ε_F is the Fermi energy, V is the bias voltage, V_{pl} (V_{pr}) is the tunneling matrix element between the emitter (collector) and the well, E is the level split of the EF. The energy $\varepsilon_0 \pm J$ corresponds to upper (lower) state of the EF. Consequently, J is the coupling constant between the EF and the resonant level inside the well.