

## Effective action of a (2+1)-dimensional system of nonrelativistic fermions in the presence of a uniform magnetic field: Dissipation effects

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The effective action of nonrelativistic fermions in 2+1 dimensions is analyzed at finite temperature and chemical potential in the presence of a uniform magnetic field perpendicular to the plane. The method used is a generalization of the derivative expansion technique. The induced Chern-Simons term is computed and shown to exhibit Hall quantization. Effects of dissipation due to collisions are also analyzed.

In many situations in condensed-matter physics one deals with systems of fermions in an electromagnetic field, which has both dynamical and background parts. The fractional quantum Hall effect<sup>1</sup> is one of these interesting systems, where the electrons are submitted to a high magnetic field perpendicular to the plane of confinement and interacting through a strong Coulomb interaction. In this particular system, correlation effects give rise to the Hall plateaus at fractional values of the filling factor and are expected to cause Wigner crystallization at lower densities.<sup>2</sup> Another interesting system, believed to be relevant to high- $T_c$  superconductors,<sup>3</sup> is the anyon gas in 2+1 dimensions.<sup>4</sup> The particles in this system can be viewed as electrons carrying flux tubes; this leads to a statistical magnetic field proportional to the anyon density.

A useful tool in studying the low-energy excitations in such systems is the effective action formalism of quantum field theory. This is simply defined as the sum of all connected one-particle irreducible vacuum diagrams in the presence of any background field.<sup>5,6</sup> It has been successfully applied in the classic case of BCS superconductivity,<sup>7,8</sup> where all the properties of the superconductor including the gap field, free energy, and so on, can be derived.

Formally the effective action is achieved by integrating out the fermions, which results in an expression involving a fermionic determinant. One then has to concoct a method of evaluating this determinant. The derivative expansion technique<sup>9</sup> is one such method. It has been extremely useful in the calculation of anomaly induced vortices in 3+1 dimensions,<sup>10</sup> proving the bosonization in 1+1 dimensions,<sup>11</sup> and showing the origin of Chern-Simons term in 2+1 dimensions.<sup>12</sup> Recently, combined with the large- $N$  expansion technique, this method has been applied successfully to a (2+1)-dimensional system of charged fermions interacting through a four-Fermi term and exhibiting superconductivity via the Kosterlitz-Thouless mechanism.<sup>13</sup> However, so far, only plane-wave states have been used as the underlying single-particle states in this technique. In this paper, we propose a generalization of this technique to the case where the single-particle states are localized, such as Lan-

dau states in the presence of a uniform magnetic field.

To illustrate the technique, we apply it to a system of nonrelativistic fermions in 2+1 dimensions subjected to a uniform magnetic field perpendicular to the plane and interacting with a fluctuating gauge field whose dynamics can be described by the usual Maxwell term, or eventually a Chern-Simons term. This system, in connection with anyonic superconductivity, has been recently analyzed in Refs. 14 and 15 where a different approach based on the inhomogeneity derivative expansion technique has been used. Unlike that one, our technique gives the higher derivative terms in a straightforward way and in the form of some known polynomials.

The calculation will be carried out at finite temperature and finite chemical potential. To obtain information on the influence of the electron gas on the dynamics of the photon, it is useful to integrate over the electron degree of freedom, obtaining the effective action for the gauge field. Here we show that in this process a Chern-Simons term emerges with a coefficient depending on the magnetic field and the temperature, in agreement with the result derived in Refs. 14 and 15. Finally, the full gauge field propagator at finite temperature is constructed and is shown to be transverse in accordance with gauge invariance.

The action at finite temperature describing the fermions coupled to the gauge field is written as

$$S = \int_0^\beta d\tau \int d^2x \psi^\dagger [ip_\tau + \mu - \epsilon(-i\nabla - e\mathbf{A} - e\mathbf{a}) + iea_\tau] \psi + S_{\text{kin}}[a], \quad (1)$$

where  $\mathbf{A}$  is the background field and the last term is the kinetic term of the fluctuating gauge field  $\mathbf{a}$ .  $\mu$  is the chemical potential of the system, and  $\beta = 1/T$  where  $T$  is the absolute temperature of the system. We are assuming a parabolic dispersion for the fermions:  $\epsilon(\mathbf{k}) = \mathbf{k}^2/2m$ .

The effective action  $S_{\text{eff}}$  of this system is a functional of  $a_\mu$  and is expressed as a path integral over the fermionic variables

$$\int D\psi^\dagger D\psi e^{S} = e^{-S_{\text{eff}}}, \quad (2)$$

where

$$S_{\text{eff}} = -\text{Tr} \ln \{ ip_\tau + \mu - \epsilon(-i\nabla - e\mathbf{A} - e\mathbf{a}) + ie\mathbf{a}_\tau \} + S_{\text{kin}}[a]. \quad (3)$$

Varying twice with respect to the gauge field  $\mathbf{a}_\mu$ , we get the different components of the polarization tensor  $\Pi_{\mu\nu}$ .

To illustrate the technique, we present the derivation of one of these components,  $\Pi_{ij}$ , in some detail. The other components are computed in a similar way. For  $\Pi_{ij}$ ,  $a_\tau$  can be discarded since no derivative with respect to it is taken.

First one expands the logarithm inside the trace as follows:

$$S_{\text{eff}} = -\text{Tr} \ln(ip_\tau + \mu - \hat{H}_0) - \text{Tr} \left[ \frac{1}{ip_\tau + \mu - \hat{H}_0} \left[ \frac{e}{2m} (\boldsymbol{\Pi} \cdot \mathbf{a} + \mathbf{a} \cdot \boldsymbol{\Pi}) - \frac{e^2}{2m} \mathbf{a}^2 \right] \right] + \frac{1}{2} \left[ \frac{e}{2m} \right]^2 \text{Tr} \left[ \frac{1}{ip_\tau + \mu - \hat{H}_0} (\boldsymbol{\Pi} \cdot \mathbf{a} + \mathbf{a} \cdot \boldsymbol{\Pi}) \frac{1}{ip_\tau + \mu - \hat{H}_0} (\boldsymbol{\Pi} \cdot \mathbf{a} + \mathbf{a} \cdot \boldsymbol{\Pi}) \right] + \dots \quad (4)$$

Now by varying twice with respect to  $a$  and setting  $a = 0$ , we get

$$\begin{aligned} \frac{\delta^2 S_{\text{eff}}}{\delta a^i(x) \delta a^j(y)} &= \frac{e^2}{m} \text{Tr} \left[ \frac{1}{ip_\tau + \mu - \hat{H}_0} \delta^3(\hat{R} - x) \delta^3(\hat{R} - y) \right] \delta^{ij} \\ &+ \left[ \frac{e}{2m} \right]^2 \text{Tr} \frac{1}{ip_\tau + \mu - \hat{H}_0} [\hat{\Pi}^i \delta^3(\hat{R} - x) + \delta^3(\hat{R} - x) \hat{\Pi}^i] \\ &\times \frac{1}{ip_\tau + \mu - \hat{H}_0} [\hat{\Pi}^j \delta^3(\hat{R} - y) + \delta^3(\hat{R} - y) \hat{\Pi}^j]. \end{aligned} \quad (5)$$

In these equations  $\hat{H}_0$  is the unperturbed Hamiltonian giving rise, in this case, to Landau states  $|\zeta\rangle \equiv |l, X\rangle$ .<sup>16</sup>  $\hat{\Pi}^i$  is the momentum operator in the presence of the gauge field  $\mathbf{A}$  given by  $p^i - eA^i$  and  $\hat{R}$  stands for the operators  $(\hat{x}, \hat{y})$ . The defining commutation relations between these operators are<sup>16</sup>

$$\hat{H}_0 = (1/2m) \hat{\Pi}^2 \quad \text{with} \quad [\hat{\Pi}_x, \hat{\Pi}_y] = -i/l_B^2. \quad (6)$$

The energy associated with the states  $|l, X\rangle$  is given by  $E_\zeta = (l + \frac{1}{2})\omega_c$  where  $\omega_c = eB/m$  is the cyclotron frequency and each state has a degeneracy  $m\omega_c/2\pi \equiv 1/2\pi l_B^2$ . The guiding center coordinates are defined by

$$\hat{X} = \hat{x} - l_B^2 \hat{\Pi}_y, \quad \hat{Y} = \hat{y} + l_B^2 \hat{\Pi}_x, \quad (7a)$$

with the commutation relations

$$[\hat{X}, \hat{\Pi}_x] = [\hat{Y}, \hat{\Pi}_y] = 0, \quad [\hat{X}, \hat{Y}] = il_B^2. \quad (7b)$$

In the basis  $|l, X\rangle$  we have

$$\hat{X}|X\rangle = X|X\rangle, \quad \hat{Y}|X\rangle = -il_B^2(\partial/\partial X)|X\rangle. \quad (7c)$$

And finally,

$$[\hat{x}, \hat{p}_\tau] = i. \quad (8)$$

To evaluate the trace in Eq. (5), we first use a spectral representation of the  $\delta$  function:

$$\delta^3(\hat{R} - x) = \int [d^3q/(2\pi)^3] e^{iq \cdot (\hat{R} - x)}, \quad (9)$$

and by making use of the commutation relation (8) to move  $\hat{x}$  to the left, we obtain in Fourier space

$$\Pi^{ij}(\mathbf{q}, \omega) = \frac{e^2}{m} \frac{1}{\beta} \sum_n \sum_\zeta \frac{1}{i\omega_n + \mu - E_\zeta} + \frac{e^2}{4m^2} \frac{1}{\beta} \sum_n \sum_{\zeta\zeta'} \frac{\langle \zeta | (\hat{\Pi}^i e^{iq \cdot \hat{x}} + e^{iq \cdot \hat{x}} \hat{\Pi}^i) | \zeta' \rangle \langle \zeta' | (\hat{\Pi}^j e^{-iq \cdot \hat{x}} + e^{-iq \cdot \hat{x}} \hat{\Pi}^j) | \zeta \rangle}{(i\omega_n + \mu - E_\zeta)(i\omega_n - i\omega + \mu - E_{\zeta'})}, \quad (10)$$

where we have a sum over the so-called Matsubara frequencies  $\omega_n = 2\pi(n + \frac{1}{2})/\beta$  and a sum over Landau states.  $\omega$  is an imaginary Bose frequency. The first sum can be done easily by contour integration (see the Appendix); the result is

$$\Pi^{ij}(\mathbf{q}, \omega) = \frac{e^2}{m} \sum_\zeta f_\zeta \delta^{ij} + \frac{e^2}{4m^2} \sum_{\zeta\zeta'} \frac{f_\zeta - f_{\zeta'}}{E_\zeta - E_{\zeta'} - i\omega} \langle \zeta | (\hat{\Pi}^i e^{iq \cdot \hat{x}} + e^{iq \cdot \hat{x}} \hat{\Pi}^i) | \zeta' \rangle \langle \zeta' | (\hat{\Pi}^j e^{-iq \cdot \hat{x}} + e^{-iq \cdot \hat{x}} \hat{\Pi}^j) | \zeta \rangle. \quad (11)$$

where  $f_\xi$  is the familiar Fermi-Dirac distribution function

$$f_\xi = \frac{1}{1 + e^{\beta(E_\xi - \mu)}} .$$

Using the same procedure, we obtain the other components of the polarization tensor; these are given by

$$\Pi^{i\tau} = \frac{ie^2}{2m} \sum_{\xi\xi'} \frac{f_\xi - f_{\xi'}}{E_\xi - E_{\xi'} - i\omega} \langle \xi | (\hat{\Pi}^i e^{i\mathbf{q}\cdot\hat{\mathbf{r}}} + e^{i\mathbf{q}\cdot\hat{\mathbf{r}}} \hat{\Pi}^i) | \xi' \rangle \langle \xi' | e^{-i\mathbf{q}\cdot\hat{\mathbf{r}}} | \xi \rangle \quad (12)$$

and

$$\Pi^{\tau\tau} = -e^2 \sum_{\xi\xi'} \frac{f_\xi - f_{\xi'}}{E_\xi - E_{\xi'} - i\omega} |\langle \xi | e^{i\mathbf{q}\cdot\hat{\mathbf{r}}} | \xi' \rangle|^2 . \quad (13)$$

It is worth mentioning that this last equation is related to the density-density correlation function and that its analysis at finite frequency  $\omega$  gives information about so-called magnetoplasmon collective excitations. A detailed investigation of these excitations along with other results will be reported elsewhere.<sup>17</sup>

Having these components, we can now construct the transverse polarization tensor  $\Pi^{\mu\nu}$ . Due to rotational invariance and current conservation,<sup>18,19</sup> it is expressed in terms of four tensors,

$$\Pi^{\mu\nu} = \Pi_t T^{\mu\nu} + \Pi_l L^{\mu\nu} + \Pi_0 E^{\mu\nu} + \Pi_p P^{\mu\nu} , \quad (14)$$

which are defined as

$$\begin{aligned} T_{\mu\nu} &= \delta_\mu^i \delta_\nu^j (\mathbf{q}^2 \delta_{ij} - q_i q_j) , \\ L_{\mu\nu} &= -q_\mu q_\nu + q^2 g_{\mu\nu} - T_{\mu\nu} , \\ E_{\mu\nu} &= \varepsilon_{\mu\nu\lambda} q^\lambda , \\ P_{\mu\nu} &= (\delta_\mu^i \delta_\nu^0 + \delta_\mu^0 \delta_\nu^i) \varepsilon_{ki} q^k q^j \\ &\quad + \delta_\mu^i \delta_\nu^j (\varepsilon_{ik} q^k + \varepsilon_{jk} q^i) q^k q^0 . \end{aligned} \quad (15)$$

These tensors correspond, respectively, to the  $B^2$ ,  $E^2$ , Chern-Simons, and  $B \nabla \mathbf{E}$  terms in the effective action. Although the last term is irrelevant in the infrared limit, it is included here because the algebra formed by the first three tensors does not close without it. The coefficients  $\Pi_t$ ,  $\Pi_l$ ,  $\Pi_0$ ,  $\Pi_p$  are analytical functions of  $\mathbf{q}$  and  $\omega$ ; they are expressed as

$$\begin{aligned} \Pi_l &= -\frac{1}{q^2} \Pi^{00} , \\ \Pi_t &= \frac{1}{q^2} \left[ \Pi^{ii} + 2 \frac{q_0^2}{q^2} \Pi^{00} \right] , \\ \Pi_0 &= \frac{q_\lambda \varepsilon^{\mu\nu\lambda}}{q^2} \Pi_{\mu\nu} , \\ \Pi^{i0} &= q^i q^0 \Pi_l + \varepsilon^{ij} q^j (\Pi_0 + q^2 \Pi_p) . \end{aligned} \quad (16)$$

In computing the different components of the polarization tensor, one makes use of the following relations:<sup>20,21</sup>

$$\begin{aligned} |\langle \xi | e^{i\mathbf{q}\cdot\hat{\mathbf{r}}} | \xi' \rangle|^2 &= |\langle l, X | e^{i\mathbf{q}\cdot\hat{\mathbf{r}}} | l', X' \rangle|^2 \\ &= |J_{ll'}(u)|^2 \delta_{X, X' + l_B^2 q} , \end{aligned} \quad (17a)$$

where  $u = l_B^2 (q_x^2 + q_y^2) / 2$ ,

$$|J_{ll'}(u)|^2 = (l! / l'!) e^{-u} u^{l'-l} [L_{l'}^{l'-l}(u)]^2 , \quad (17b)$$

and  $L_{l'}^{l'-l}$  is an associated Laguerre polynomial for  $l' \geq l$  (for  $l' \leq l$ , interchange  $l$  and  $l'$  in this formula). It is also helpful to use the following relation, which is easy to establish,

$$\{ \hat{\Pi}^k, e^{i l_B^2 \varepsilon_{ij} q^i \hat{\Pi}^j} \} = \varepsilon^{nk} \frac{2}{i l_B^2} \frac{\partial}{\partial q^n} e^{i l_B^2 \varepsilon_{ij} q^i \hat{\Pi}^j} , \quad (18)$$

here the curly braces denote the anticommutator.

In principle, using these relations, it is straightforward, although tedious, to evaluate exactly the different components of the polarization tensor. However, we shall give here only the long-wavelength limit of these coefficients. At finite temperature, it turns out that  $\Pi_l$  has a pole when  $q \rightarrow 0$  and  $\omega = 0$  indicating that the limits  $(q, \omega) \rightarrow 0$  and  $T \rightarrow 0$  do not commute.<sup>22</sup> Consequently the effective action has a nonlocal electric-electric field term. This is reminiscent of the anomalous skin effect in metals where the static transverse conductivity shows the same singular behavior  $\sigma(q, 0) = C/q$ .<sup>22</sup> In the long-wavelength limit we obtain

$$\begin{aligned} \Pi_l &= \frac{e^2 m \omega_c}{2\pi} \frac{1}{q^2} \left[ \sum_{l=0}^{\infty} \frac{\partial f}{\partial E_l} \right] - \frac{e^2}{2\pi \omega_c} \left[ \sum_{l=0}^{\infty} f_l \right] \\ &\quad - \frac{e^2}{2\pi} \left[ \sum_{l=0}^{\infty} \frac{\partial f}{\partial E_l} (l + \frac{1}{2}) \right] , \end{aligned} \quad (19a)$$

$$\begin{aligned} \Pi_t &= \frac{e^2}{\pi m} \left[ \sum_{l=0}^{\infty} (l + \frac{1}{2}) f_l \right] \\ &\quad + \frac{e^2 \omega_c}{2\pi m} \left[ \sum_{l=0}^{\infty} \frac{\partial f}{\partial E_l} (l + \frac{1}{2})^2 \right] , \end{aligned} \quad (19b)$$

$$\Pi_0 = \frac{e^2}{2\pi} \left[ \sum_{l=0}^{\infty} f_l \right] + \frac{e^2 \omega_c}{2\pi} \left[ \sum_{l=0}^{\infty} \frac{\partial f}{\partial E_l} (l + \frac{1}{2}) \right] , \quad (19c)$$

$$\Pi_p = -\frac{e^2}{\pi m \omega_c} \left[ \sum_{l=0}^{\infty} (l + \frac{1}{2}) f_l \right] . \quad (19d)$$

To gain more insight, we now consider the zero-temperature limit when the Fermi level is pinned in a gap between two Landau levels

$$E_N < \epsilon_F < E_{N+1} .$$

In this case, we make the replacement

$$\sum_{l=0}^{\infty} f_l \rightarrow (N+1)$$

and

$$\sum_{l=0}^{\infty} (l + \frac{1}{2}) f_l \rightarrow \frac{(N+1)^2}{2} , \quad N=0, 1, \dots .$$

The coefficient of the Chern-Simons term is thus

$$\Pi_0 = \frac{e^2}{2\pi} (N + 1). \tag{20}$$

This coefficient is exactly the Hall conductivity  $\sigma_{yx}$ ,<sup>23</sup> and hence Eq. (20) expresses the famous integral quantum Hall effect discovered by von Klitzing.<sup>24</sup>

Next, we consider the effect of dissipation due to collisions with impurities. This is known to be of considerable importance for the two-dimensional gas, especially in connection with transport properties. This effect is accounted for by means of a relaxation time  $\tau$ , which is usually obtained by computing the imaginary part of the fermions' self-energy in the presence of a scattering potential. However, one has to do the calculation in a self-consistent way because of polarization effects. Furthermore, this relaxation time, in general, depends on each Landau level. Hereafter, we assume a single finite relaxa-

tion, and compute its effect on the polarization tensor. The fermion propagator is then modified to<sup>7,25</sup>

$$\frac{1}{i\omega_n + \mu - E_\xi - i \frac{\text{sgn}(\omega_n)}{2\tau}}, \tag{21}$$

which can also be written as

$$\int_{-\infty}^{+\infty} dk_0 \sigma(k_0 + \mu - E_\xi) \frac{1}{i\omega_n - k_0}, \tag{22a}$$

where use has been made of the spectral function

$$\sigma(k_0) = \frac{1}{2\pi\tau} \frac{1}{k_0^2 + \frac{1}{4\tau^2}}. \tag{22b}$$

Consequently, in Eqs. (11)–(13) one has then to make the following substitution

$$\sum_{\xi\xi'} \frac{f_\xi - f_{\xi'}}{E_\xi - E_{\xi'} - i\omega} \rightarrow \sum_{\xi\xi'} \int_{-\infty}^{+\infty} dk_0 \int_{-\infty}^{+\infty} dk'_0 \sigma(k_0 - E_\xi) \sigma(k'_0 - E_{\xi'}) \frac{f(k_0) - f(k'_0)}{k_0 - k'_0 - i\omega}. \tag{23}$$

We find it here more interesting to determine the effect of a finite lifetime on the Chern-Simons coefficient, since the quantization shown in Eq. (20) has a crucial consequence on two-dimensional electron gas. Using the substitution (23) in the expression of  $\Pi_0$  given in (16), we find

$$\Pi_0(\omega) = \frac{e^2 \omega_c^2}{2\pi} \sum_{l=0}^{\infty} (l+1) \int_{-\infty}^{+\infty} dk_0 \int_{-\infty}^{+\infty} dk'_0 \sigma(k_0 - E_l) \sigma(k'_0 - E_{l+1}) \frac{f(k_0) - f(k'_0)}{(k_0 - k'_0)^2 + \omega^2}. \tag{24}$$

Next, we perform the  $k'_0$  integration in the first term and the  $k_0$  integration in the second term of this equation, giving

$$\begin{aligned} \Pi_0(\omega) = \frac{e^2 \omega_c^2}{2\pi} \sum_{l=0}^{\infty} \int_{-\infty}^{+\infty} dk_0 f(k_0) \sigma(k_0 - E_l) \\ \times \{ (l+1) g_\omega(k_0 - E_{l+1}) \\ - l g_\omega(k_0 - E_{l-1}) \}, \end{aligned} \tag{25a}$$

where we have introduced the function

$$\begin{aligned} g_\omega(x) &= \int_{-\infty}^{+\infty} dk_0 \frac{\sigma(k_0)}{(k_0 + x)^2 + \omega^2} \\ &= \left[ 1 + \frac{1}{\omega\tau} \right] \frac{1}{x^2 + (\omega + 1/2\tau)^2}. \end{aligned} \tag{25b}$$

Now in a strong magnetic field we have  $\omega_c \tau \gg 1$  and Eq. (25a) reduces to

$$\Pi_0(\omega) = \frac{e^2 \rho}{m} \omega_c g_\omega(\omega_c), \tag{26}$$

where  $\rho$  is the particle density given by

$$\rho = \frac{m \omega_c}{2\pi} \sum_{l=0}^{\infty} \int_{-\infty}^{+\infty} dk_0 f(k_0) \sigma(k_0 - E_l),$$

and the static Chern-Simons coefficient is

$$\Pi_0(0) = \frac{e^2 \rho}{m} \frac{\omega_c \tau^2}{\tau^2 \omega_c^2 + \frac{1}{4}}. \tag{27}$$

In the absence of dissipation ( $\tau \rightarrow \infty$ ), this expression reduces to that given in Eq. (19c). For the sake of comparison, we give here the same induced coefficient in the case of massive Dirac fermions with dissipation effects when the magnetic field is absent.<sup>26</sup> In this case, we obtain

$$\Pi_0 = \frac{1}{2} \frac{\partial}{\partial m} (\rho/m). \tag{28a}$$

For relativistic fermions, the particles density is given by

$$\rho = \frac{2m}{\beta} \sum_n \int \frac{d^2 p}{(2\pi)^2} \frac{1}{[\omega_n + \text{sgn}(\omega_n)/2\tau - i\mu]^2 + \mathbf{p}^2 + m^2}. \tag{28b}$$

From this, we get at  $T = 0$

$$\begin{aligned} \Pi_0 = \frac{1}{4\pi^2} \frac{|m|}{m} \{ \arctan 2\tau(|m| + \mu) \\ + \arctan 2\tau(|m| - \mu) \}. \end{aligned} \tag{28c}$$

In the absence of dissipation, this equation reproduces the by now familiar Chern-Simons term  $\Pi_0 = \text{sgn}(m)/4\pi$ .

In concluding we would like first to emphasize that this method is useful for any background field for which we know the exact eigenstates. In particular Eqs. (10)–(13) are exactly the same when the background field has a constant magnetic field as well as a constant electric field not too strong to prevent the destruction of Landau levels quantization<sup>27</sup> and the particles creation, the only change to be made is in the eigenstates' energy which is now shifted by the electric field and consequently the degeneracy is lifted. Another advantage of this technique is the exact form, in terms of Laguerre polynomials, obtained for the higher derivative terms [or in Fourier space for any  $(\mathbf{q}, \omega)$ ]. As noted in the manuscript, this is of particular interest for the magnetoplasmon collective excitations, when one is concerned with higher values of  $\mathbf{q}$ . Application of this technique to the case of massive and massless Dirac fermions in the presence of a uniform magnetic field is currently under progress and will be reported elsewhere.

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#### APPENDIX

In this appendix, we evaluate the Matsubara sum appearing in Eqs. (10)–(13). We adopt the standard technique of contour integration. The summations are of the form

$$S = -\frac{1}{\beta} \sum_{n=-\infty}^{n=+\infty} g(i\omega_n), \quad (\text{A1})$$

where  $\omega_n = (2\pi/\beta)(n + \frac{1}{2})$ . We shall do an integral of the form

$$I = \lim_{R \rightarrow \infty} \int \frac{dz}{2\pi i} f(z)g(z), \quad (\text{A2})$$

where the contour is a large circle of radius  $R$  in the limit  $R \rightarrow \infty$ . The function  $f(z)$  is chosen to generate poles at the points  $i\omega_n$ . The function which does this is

$$f(z) = \frac{1}{1 + e^{\beta z}},$$

the residue at its poles is  $(-1/\beta)$ . In the limit  $R \rightarrow \infty$ , the integral vanishes,  $I=0$  so that the summation is given by the residues

$$S = \sum \text{residue of } f(z)g(z) \text{ at the poles of } g(z). \quad (\text{A3})$$

Applying this procedure we derive the following formula:

$$\frac{1}{\beta} \sum_n \frac{1}{i\omega_n + \mu - E_\xi} = f_\xi \quad (\text{A4})$$

and

$$\frac{1}{\beta} \sum_n \frac{1}{(i\omega_n + \mu - E_\xi)^2} = \frac{\partial f_\xi}{\partial E_\xi}, \quad (\text{A5})$$

and finally

$$\begin{aligned} & \frac{1}{\beta} \sum_n \sum_{\xi\xi'} \frac{F_{\xi\xi'}(q)}{(i\omega_n + \mu - E_\xi)(i\omega_n + \mu - E_{\xi'} - i\omega)} \\ &= \sum_\xi \frac{\partial f_\xi}{\partial E_\xi} F_{\xi\xi}(q) \delta_{\omega=0} + \sum_{\xi \neq \xi'} \frac{f_\xi - f_{\xi'}}{E_\xi - E_{\xi'} - i\omega} F_{\xi\xi'}(q), \end{aligned} \quad (\text{A6})$$

where we have used the identity

$$f(E_\xi - i\omega) = f(E_{\xi'}),$$

because  $\omega$ , being a Bose frequency, is given by  $2\pi s/\beta$  with  $s$  as an integer.

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