Persistent currents and conductance of a metal loop connected to electron reservoirs

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(Received 17 August 1993; revised manuscript received 10 January 1994)

We have calculated the persistent current and the conductance of a normal metal loop connected to two electron reservoirs in the presence of magnetic flux. The geometry considered here facilitates simultaneous measurement of the persistent current and the conductance. We show that, in general, the magnitude of the persistent current in a loop depends on the direction of the direct current flow from one reservoir to another, a feature that can be experimentally verified.

I. INTRODUCTION

The electronic and magnetic properties of mesoscopic systems have recently received much attention in the light of several experimental observations.[1–4] In mesoscopic systems typically of nm sizes, the effective distance the electrons travel between inelastic collisions at low temperatures (typically millikelvin) can exceed the sample dimension. In such a situation the electron maintains the coherence of the single particle wave function across the entire sample. Mesoscopic systems thus can be modeled as phase coherent elastic scatterers. For systems of size larger than the inelastic mean free path, inelastic collisions disrupt the phase coherence of electron wave evolution. This effectively breaks the system into different uncorrelated regions. These phase breaking scattering processes can be included via averaging arguments. In these mesoscopic systems, quantum interference phenomena[1–5] observed and predicted include the normal-state Aharanov-Bohm resistance oscillations and persistent currents in metallic loops pierced by a magnetic field, universal conductance fluctuations, sample specific non-self-averaging fluctuations in conductance as the magnetic field or chemical potential is varied, nonlocal current voltage relations, violations of Onsager relationships, the Coulomb blockade effect in microtunnel junctions, and several other effects. The guiding theme for mesoscopic systems is quantum coherence along the whole sample.

Persistent currents in mesoscopic normal metal rings have recently received much attention in the light of the experimental observations of these phenomena.[6–8] There have been several theoretical attempts to explain the discrepancy between the measured current amplitude and the results based on the noninteracting electron models.[1,9–14] Prior to the experimental observations, Büttiker, Imry, and Landauer in their pioneering work suggested the existence of persistent current in an ordered one-dimensional ring threaded by a magnetic flux.[15] The persistent current has an amplitude of ev_f/L (where v_f is the Fermi velocity and L is the circumference of the ring) and is periodic in magnetic flux. General quantum mechanical principles require that the wave functions, eigenvalues, and hence all observables be periodic with a flux ϕ threaded by the loop with a period $\phi_0, \phi_0 = hc/e$ being the elementary flux quantum. This current is an equilibrium property of the ring and is given by the flux derivative of the total energy of the ring. These currents can also be attributed to the sensitivity of the eigenstates to the boundary conditions along the ring (the magnetic field tunes the boundary condition). The magnetic field destroys the time reversal symmetry and as a consequence the degeneracy of the states, carrying current clockwise and anticlockwise, is lifted. Depending on the position of the Fermi level, uncompensated current flows in either of the directions. For an ideal isolated ring without impurities and at zero temperature the nature of the persistent current depends on the total number N of the electrons and the persistent current exhibits a saw-tooth-type behavior as a function of the magnetic flux ϕ. For N even, the jump discontinuities occur from the value $-(2ev_f/L)$ to $(2ev_f/L)$ at $\phi=0$, $\pm\phi_0, \pm 2\phi_0$, etc., and at $\phi = \pm\phi_0/2, \pm 3\phi_0/2$, etc. for N odd. Studies have been extended to include multichannel rings, disorder, spin-orbit coupling,[16] and electron-electron interaction effects.[1,9–16] For the multichannel quasi-one-dimensional ring, the average amplitude decreases as a function of strength of the disorder. In the presence of strong disorder (i.e., when the localization length of electronic eigenfunctions is smaller than the ring size L) the persistent current decreases exponentially with L. The root mean square amplitude of the current is calculated to be of the same order as the amplitude of the current. In the case of weak disorder, i.e., when the elastic mean free path of an electron is less than L, the persistent current decreases algebraically (i.e., as $1/L$). In this regime the main effect of the disorder is to open a gap at each crossing point of energy levels, thus reducing the slopes of the curves $E(\phi)$. For the multichannel systems there is no correlation on the average between different channels, in the absence of disorder. Consequently the total current is $\sqrt{m}$ times the one channel current, where m is the number of channels. However, the result differs in the diffusive regime due to the compensation between currents in different channels. Inelastic scatterings do not destroy the effect. At finite temperature T, for L less than the phase coherence length L_ϕ, the main effect arises due to the mixing of contributions of the levels in an energy interval $k_B T$. This mechanism reduces the current, since adjacent levels give opposite

0163-1829/94/49(19)/13685(6)/$06.00

contributions to the current. In the case $L > L_\phi$, the current vanishes exponentially with L/L_ϕ. The typical magnitude of the persistent current at $T=0$ for L between 1 and 3 μm and for a Fermi wave vector k_f between 10^{10} m^{-1} (metallic ring) and 10^8 m^{-1} (semiconductor ring) varies between 1 and 5 nA.[1]

Most of the theoretical treatments to date have been done on isolated systems. In an isolated system the number of electrons is fixed and the statistical mechanical treatment must be based on the canonical ensemble. Persistent currents in open conductors have found little attention in the literature.[17–21] It was Büttiker who first gave a treatment[17] of a small normal metal loop coupled to an electron reservoir (open system). The reservoir acts as a source and sink for electrons and is characterized by a well defined chemical potential μ, and by definition there is no phase relationship between the absorbed and emitted electrons. Thus the reservoir acts as an inelastic scatterer and as a source of energy dissipation. Since the reservoir keeps the chemical potential in the loop fixed, the statistical mechanical description for this system corresponds to a different ensemble, namely the grand canonical ensemble. This implies that the open and closed loop systems belong to different statistical treatments. The exact description of the system is important, as the dependence of the current on flux has a different behavior if the chemical potential is held fixed or if the number of electrons is fixed to an odd or even number.[16] In our recent treatment[21] we have extended Büttiker's discussion to a case wherein electrons from the reservoir enter and leave the ring in a subbarrier regime characterized by evanescent modes throughout the circumference of the loop. In such a situation the persistent current arises due to two nonclassical effects, namely, the Aharonov-Bohm effect and quantum tunneling. The dependence of the current on the length of the ring is similar to that arising due to states localized by a static disorder.

In our present treatment we consider a one-dimensional metal loop of length L coupled to two electron reservoirs as shown in Fig. 1. This idealization to one-dimension corresponds experimentally to a network of high-mobility quantum wires with narrow width such that only the lower subband is filled. Our calculations are for noninteracting system of electrons. In such a geometry the Aharonov-Bohm effect manifests itself not only in a transport phenomenon such as two terminal conductance but also in a persistent current. The left and the right reservoirs are characterized by chemical potentials μ_1 and μ_2, respectively. We consider here a free electron network and we have introduced a δ-function impurity at a length l_2 to the right of the junction J_1 (shown as $\times$ in Fig. 1). If $\mu_1 > \mu_2$ the net current flows from the left to the right (along $R_1 J_1 R_2$) and vice versa if $\mu_1 < \mu_2$. At the junction J_2 an ideal wire of length l_1 is connected to the metallic loop. Except at the δ-function impurity of strength V the potential is taken to be zero (free electron network). The scattering of the electronic wave function occurs at the junctions J_1,J_2 and at the impurity site. The metallic loop is isolated from the direct current flow. However, in general it is not essential. Such a geometry facilitates measuring the persistent current in a loop and the conductance of an entire network simultaneously. In our model we have complete spatial separation between elastic processes in the loop and the inelastic processes in the reservoirs. These inelastic processes in the reservoir are essential to obtain a finite conductance. Now consider a situation wherein steady flux of electrons with an energy E is injected from the reservoir 1. These electrons moving to the right are first scattered at the junction J_1 and subsequently at J_2 and I (along with multiple reflections at J_1,J_2, and I). The electrons emitted by the reservoir 2 are first scattered at I and subsequently at J_1 and J_2. Consequently for these two different cases the electron wave functions will have different complex amplitudes at the junction J_2. This effectively corresponds to a different boundary condition at the junction point J_2. As already stated, the persistent current in a metallic loop is sensitive to the boundary condition, and hence we observe that the magntiude of the persistent current depends on the direction of the current flow. Obviously the conductance of an entire network (calculated via the quantum transmission coefficient) does not depend on the direction of the current flow. This suggests that there is no simple scaling relation between the persistent currents and the conductance of the entire network. In Sec. II we present the theoretical treatment, Sec. III is devoted to results and conclusions.

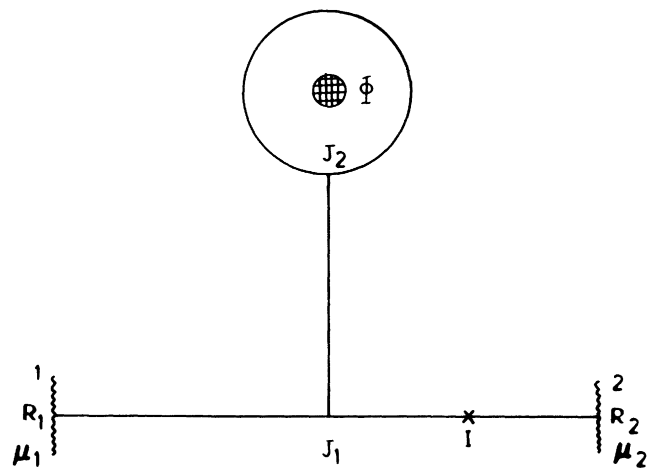

FIG. 1. An open metallic loop coupled to two electron reservoirs via an ideal conductor.

II. THEORETICAL TREATMENT

In this section we derive an expression for the persistent current and the transmission coefficient by solving a scattering problem. Except for the point I (where we have introduced a δ function potential) the electronic potential is assumed to be identically zero throughout the sample. We do not assume any particular form for the scattering matrix for junctions J_1 and J_2, but rather we derive them from the first principles using the quantum waveguide theory on networks.[22] Since the two reservoirs are mutually phase incoherent, we have to solve the problem separately for the electrons emitted from the left and the right reservoirs. First we consider the case wherein electrons are emitted from the left reservoir. The reservoirs emit carriers with the Fermi distribution $f(E)=(\exp[(E-\mu_1)/k_B T]+1)^{-1}$. This results in a current flowing from the left to the right. The appropriate wave function in the absence of magnetic field in the ideal lead and in the region $R_1 J_1$ is given by

$$\psi_1(x_1)=e^{ikx_1}+Re^{-ikx_1} . \tag{1}$$

This wave function represents the carriers injected from the reservoir 1 and reflected towards the same reservoir. Here k is a wave vector and the energy of the injected particle is given by $E=\hbar^2k^2/2m$. Throughout our calculations, we have set units of h, e, and m to be unity. Wave functions in other regions can be written down explicitly as

$$\psi_2(x_1)=Ee^{ikx_1}+Fe^{-ikx_1} , \tag{2}$$

$$\psi_3(x_1)=te^{ikx_1} , \tag{3}$$

$$\psi_4(x_2)=Ae^{ikx_2}+Be^{-ikx_2} , \tag{4}$$

$$\psi_5(x_3)=Ce^{ikx_3}+De^{-ikx_3} , \tag{5}$$

where Eqs. (2)–(5) are for the regions $J_1 I, IR_2, J_1 J_2$ and for the loop, respectively. The coordinates for the regions $R_1 J_1 IR_2, J_1 J_2$, and the loop are x_1, x_2, and x_3, respectively. We will assume the origin of the coordinates x_1 and x_2 to be at the junction point J_1 and for x_3 to be at the junction point J_2. At J_2, x_2 takes a value l_2 and x_3 takes values 0 and L, the circumference of the loop. We have to use the Griffith boundary conditions[23] at the junctions. These boundary conditions are due to the single-valuedness of wave functions and conservation of the current (Kirchoff law). For example, at the junction J_1 we have[22]

$$\psi_1(0)=\psi_2(0)=\psi_4(0) \tag{6}$$

and

$$\sum d\psi_i/dx_i=0 . \tag{7}$$

Here all the derivatives are either outward or inward from the junction. Using the Griffith boundary conditions one can easily verify that the junction scattering matrix is given by

$$S=\begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 2/3 & -1/3 & 2/3 \\ 2/3 & 2/3 & -1/3 \end{bmatrix} .$$

The above scattering matrix is very specific to the free electron junction. If we assume an additional scatterer at the junction (say, by assuming potential barriers at the junction) we will get many other choices for the scattering matrix, which are unitary and symmetric. For simplicity, in the present analysis we have not considered other cases. The boundary conditions at the point I due to the δ-function potential of strength V are given by

$$\psi_2(l_2)=\psi_3(l_2) , \tag{8}$$

$$\frac{d\psi_3(l_2)}{dx}-\frac{d\psi_2(l_2)}{dx}=2V\psi_3(l_2) . \tag{9}$$

In the presence of a magnetic field in the loop we can choose a gauge for the vector potential in which the field does not appear explicitly in the Hamiltonian. The magnetic field manifests itself only in the boundary condition. The boundary conditions (6) and (7) for junction J_2 do not change, however the electrons propagating from the junction point J_2 and back to the same point along the ring pick up an additional phase $\alpha=2\pi\phi/\phi_0$ for a clockwise and a phase $-\alpha$ for an anticlockwise motion.[22] Here ϕ and ϕ_0 are the magnetic flux and the flux quantum (hc/e), respectively. Using all the boundary conditions mentioned above and using Eqs. (1)–(5) we get

$$1+R=A+B=E+F , \tag{10}$$

$$1-R-A+B-E+F=0 , \tag{11}$$

$$Ae^{ikl_1}+Be^{-ikl_1}=C+De^{-i\alpha}=Ce^{i\alpha+ikL}+De^{-ikL} , \tag{12}$$

$$Ae^{ikl_1}-Be^{-ikl_1}-C+Ce^{ikL+i\alpha}-De^{-ikL}+De^{-i\alpha}=0 , \tag{13}$$

$$Ee^{ikl_2}+Fe^{-ikl_2}=te^{ikl_2} , \tag{14}$$

$$ikte^{ikl_2}-ikEe^{ikl_2}+ikFe^{-ikl_2}=2Vte^{ikl_2} . \tag{15}$$

Here L is the length of the loop and l_1, l_2 are the lengths of the segments $J_1 J_2$ and $J_1 I$, respectively. Using Eqs. (10)–(15) we can solve for the coefficients C, D, and t. These solutions have been obtained analytically using mathematica.

III. RESULTS AND DISCUSSIONS

The persistent current in the loop in the energy interval dE around E is given by[17]

$$dj_{LR}=k(|C|^2-|D|^2) , \tag{16}$$

$$dj_{LR}/k=\frac{16\sin(\alpha)\sin(kl)[-4V^2-2k^2+4V^2\cos(2kl)-4Vk\sin(2kl)]}{\Omega} , \tag{17}$$

where

$$\begin{aligned}\Omega=\{&114V^2+87k^2-(21V^2+12k^2)\cos[2kl]-(66V^2+27k^2)\cos[4kl]-27V^2\cos[6kl]\\&-(32V^2+24k^2)\cos[2kl]\cos[2\alpha]+16Vk\sin[2kl]\cos[2\alpha]\\&-24V^2\cos[4kl]\cos[2\alpha]-(176V^2+136k^2)\cos[kl]\cos[\alpha]\\&+(104V^2+72k^2)\cos[3kl]\cos[\alpha]+72V^2\cos[5kl]\cos[\alpha]\\&+(56V^2+40k^2)\cos[2\alpha]+47Vk\sin[2kl]+48Vk\sin[4kl]\\&+27Vk\sin[6kl]+24V\sin[4kl]\cos[2\alpha]-48Vk\sin[kl]\cos[\alpha]\\&-56Vk\sin[3kl]\cos[\alpha]-72Vk\sin[5kl]\cos[\alpha]\}\ .\end{aligned} \tag{17a}$$

For simplicity we have taken $l_1=l_2=L=l$, the equations are too complicated to reproduce here otherwise. As expected the current varies cyclically with the flux, where the period is given by ϕ_0 and is antisymmetric in the flux ϕ. It also has components of higher harmonics. The current also oscillates between the positive and negative values as a function of the energy.

The expression for the transmission probability $T=tt^*$ is given by

$$T=\frac{8k^2[4\sin(kl)\cos(\alpha)-3\sin(2kl)]^2}{\Omega}\ , \tag{18}$$

where Ω is given in Eq. (17a). The quantum mechanical transmission probability is related to the two probe conductance G of the network[24] by the Landauer formula $G=(e^2/\hbar)T$, or the dimensionless conductance g is given by $G/(e^2/\hbar)$. The Landauer formula expresses the conductance in terms of scattering properties at the Fermi energy. The conductance also oscillates with a period ϕ_0 and is symmetric in the flux ϕ.

We have also set up a problem wherein electrons enter the lead from the right reservoir (this results in a direct current flow from the right to the left). Following the earlier procedure, the persistent current in this case is given by

$$dj_{RR}/k=\frac{-32k^2\sin(\alpha)\sin(kl)}{\Omega}\ , \tag{19}$$

with Ω as in Eq. (17a). The expression for T remains unchanged [Eq. (18)]. One can easily notice from Eqs. (17) and (19) that the magnitude of the persistent current carried by an electron with energy E depends on the direction of the direct current flow. Only in the special case, where we set the strength of the δ-function potential $V=0$, do we get identical persistent current (independent of the direction of the direct current flow). This is because we restore the symmetry between the left and the right with respect to the loop. The magnitude of the persistent current vanishes for $\phi=0$ as it should. At temperature $T=0$ the total persistent current is obtained by adding all contributions from levels with energies less than the chemical potential. Hence, if $\mu_1>\mu_2$, we have the total persistent current $J_1=\int_0^{\mu_2}n(E)(dj_{LR}+dj_{RR})dE+\int_{\mu_2}^{\mu_1}n(E)dj_{LR}dE$, and for $\mu_1<\mu_2$ we have the total current $J_2=\int_0^{\mu_1}n(E)(dj_{LR}+dj_{RR})dE+\int_{\mu_1}^{\mu_2}n(E)dj_{RR}dE$. Here $n(E)$ is the density of states in one dimension.[17] Thus by keeping $|\mu_1-\mu_2|$ fixed (i.e., fixed applied voltage) we get a different persistent current depending on the direction of current flow. The same argument can be extended to the finite temperatures by including the Fermi functions.

In Figs. 2 and 3 we have plotted the dimensionless persistent currents dj/k and the dimensionless conductance g as a function of kl, for a fixed value of magnetic flux to flux quantum ratios $\alpha=2\pi\phi/\phi_0=0.7$ and $Vl=10.0$, respectively. In Figs. 4 and 5 we have plotted the persistent currents and conductance, respectively, for fixed $kl=7.0$ and $Vl=10.0$ as a function of α. Persistent currents and

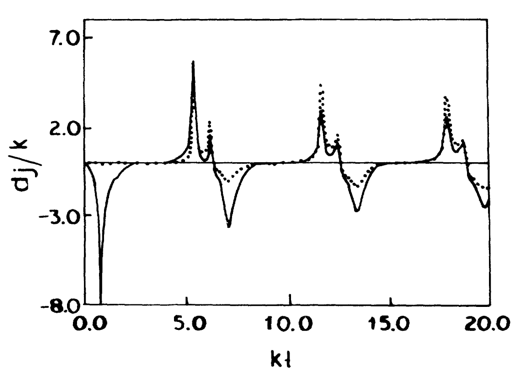

FIG. 2. Persistent current versus kl for a fixed value of flux $\alpha=0.7$ and $Vl=10.0$. The dashed curve represents dj_{RR}/k and the solid curve represents dj_{LR}/k.

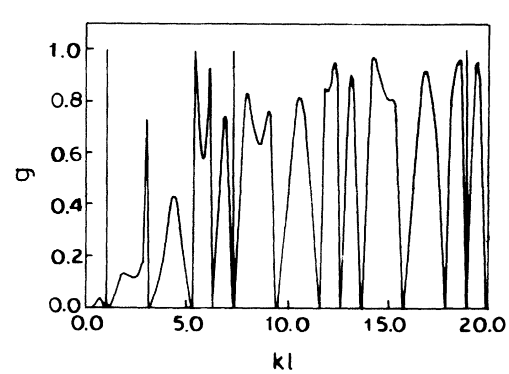

FIG. 3. Conductance g versus kl for a fixed value of flux $\alpha=0.7$ and $Vl=10.0$.

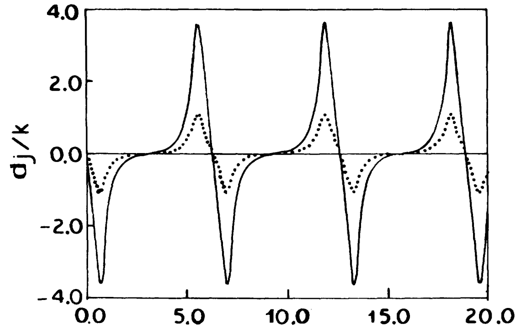

FIG. 4. Persistent current versus flux α for a fixed value of $kl=7.0$ and $Vl=10.0$. The dashed curve represents dj_{RR}/k and the solid curve represents dj_{LR}/k.

the conductance are flux periodic. The electrical conductance exhibits resonances as a function of kl, i.e., the transmission probability T exhibits a peak transmission ($T=1$) for certain values of kl. This occurs whenever the incident electron energy coincides with one of the eigenenergies of the ring attached to an additional stub J_1J_2. The deviations from the values of the exact energy states of the closed ring follow from the fact that the coupling to the reservoir via J_1 and J_2 causes additional scatterings (or perturbations) and shifts the energy levels. It is the flux dependence of these resonances which gives rise to a strong oscillatory behavior in g (for details, see Refs. 25 and 26). The Aharonov-Bohm oscillations in magnetoconductance (or resistance) have been observed experimentally.[4] The actually observed magnetoresistance exhibited irregular oscillations as a function of the magnetic field. These reproducible oscillations vary from sample to sample and are not time dependent and are also called magnetofingerprints. The irregular behavior of these oscillations is associated with the multichannel case in conjunction with the disorder.

One can also notice from Figs. 2 and 4 the difference between the values of the persistent current carried by electron emitted by the left reservoir (solid line) and emitted by the right reservoir (dashed lines). This shows clearly that persistent currents in a metal loop connected to two reservoirs depend on the direction of direct current flow from one reservoir to the other. Electrons emitted by the reservoir enter the loop via junctions J_1 and J_2. These electrons in the loop will eventually reach the reservoirs via junctions after some time delay. Thus, coupling of the loop to the reservoirs gives rise to the finite lifetime broadening of the electron states in the loop. Consequently the persistent current shows a broadened feature as a function of kl compared to that for an isolated ring. In Fig. 6 we have plotted the persistent currents dj_{LR} and dj_{RR} as a function of dimensionless impurity potential Vl, for a fixed value of $kl=7.0$ and $\alpha=0.7$. The magnitude of dj_{RR} decreases monotonically to zero as Vl goes to ∞. This follows from the fact that, in this limit, electrons emitted from the right reservoir do not enter the loop. The absolute magnitude of dj_{LR} increases monotonically to an asymptotic value. This asymptotic value corresponds to the geometry truncated at the point I and effectively the metallic loop is connected to a single reservoir. When the metallic loop is connected to two reservoirs the electrons emitted by a single reservoir partially enter the loop and partially get transmitted directly to the other reservoir, whereas for a metallic loop connected to a single reservoir all the electrons emitted by it will enter and leave the loop. This manifests itself as an increase in persistent current for a loop connected to a single reservoir as compared to that of a loop connected to multiple reservoirs.

In conclusion, we have shown that the magnitude of the persistent current in a normal metal loop connected to two reservoirs depends on the direction of the direct current flow, which should be an experimentally verifiable feature. There is no simple scaling relation between the persistent currents and the conductance of the entire network. This follows from the fact that, unlike the persistent currents, the conductance does not depend on the direction of the direct current flow. However, for a closed ring there exists a relation between the persistent current carried by an eigenstate and the conductance (transmission amplitude) of the loop.[9] Here the transmission amplitude for a ring is to be calculated by cutting a ring at any point and connecting the two end points to an

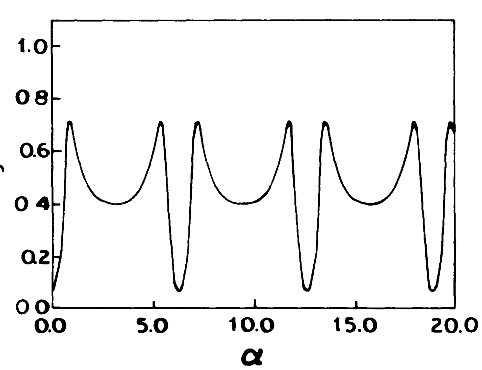

FIG. 5. Conductance g versus flux α for a fixed value of $kl=7.0$ and $Vl=10.0$.

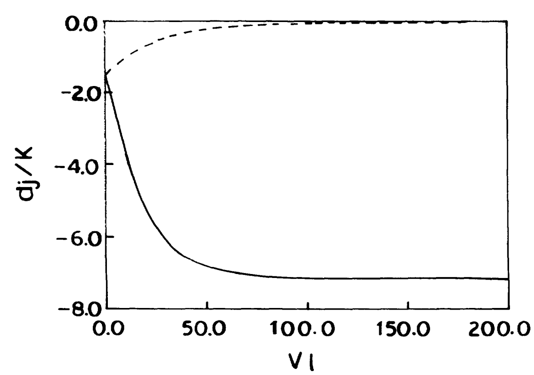

FIG. 6. Persistent current versus impurity potential Vl for a fixed value of $\alpha=0.7$ and $kl=7.0$. The dashed curve represents dj_{RR}/k and the solid curve represents dj_{LR}/k.

ideal wire. Such a unique relation does not exist for the open system considered here. The difference between the magnitudes of the persistent currents (on the direction of the current flow) can be made significant by adjusting the impurity potential. This can be achieved experimentally by having a gate in one of the leads connected to the reservoirs and by appropriately varying the gate voltage. Such an experiment can also be useful for separating the persistent currents from all other parasital currents (or signals) associated with measurements.[27]

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[27] Benoit *et al.* (see Ref. 1) have suggested an interesting possibility to separate the persistent currents from other parasital signals, namely to change the number of electrons in an isolated ring by applying light. The effect of light is to trap some electrons, thus changing the total number of electrons.