Phonon amplification by absorption of an intense laser field in a quantum well of polar material

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A theory of the amplification of the polar-optical-phonon population in a quantum well of polar material under an intense laser field is presented by taking into account the discontinuity of the electronic effective mass crossing the interface of the quantum-well materials. We have found that for the confined-polar-optical phonons, the number of phonons will grow with time if the laser field strength E_d is greater than a threshold value E_{dL}^c and the phonon wave vector $q<2k_{\|}$. For the interfacial-polar-optical (IPO) phonons, the conditions of the amplification of the phonon population are $E_d>E_{dsi}^c$ ($i=1,2$) and $q<2k_{\|}$ in the well region and $q<\min(2k_{\|},k_{2l}+k_{2l'})$ in the barrier regions. We have also found that the amplification of the IPO-phonon population is easier in material i than in material j if the electronic effective mass $m_i^*<m_j^*$. Under the conditions of the small phonon wave vector q and large quantum-well potential V_0, the number of the $(\alpha-\mathbf{q})$ IPO phonons is not amplified under an intense laser field and the amplification of the $(\alpha+\mathbf{q})$ IPO-phonon population is mainly determined by the amplification of the $(\alpha+\mathbf{q})$ IPO-phonon population inside the well region. The change regulation of the rates of the phonon excitations with the quantum-well width are studied.

I. INTRODUCTION

Phonon amplification by absorption of laser field energy is a subject extensively studied in bulk semiconductors.[1-7] The main reasons for which the subject is studied extensively are its application to intraband absorption experiments and its significance as an independent investigation.[8] The main results of the research are that in an intense-field limit only multiphoton absorption processes are significant, and the phonon population grows with time under some conditions.

With the development of modern experimental technology, the fabrication of quantum wells and superlattices is realizable. Naturally, phonon amplification by the absorption of laser field energy in such confined structures should show the characterization of the electron-photon-phonon interaction in such low-dimensional structures. Recently, Sakai and Nunes studied acoustic-phonon and optical-phonon amplification by the absorption of a laser field in a semiconductor superlattice.[8] Feng and Chen studied the amplification of the interfacial-optical-phonon population in a heterostructure system under an intense laser field.[9] But neither of these papers considers the discontinuity of the electronic effective mass crossing the interface of the materials. In this paper, taking into account the discontinuity of the electronic effective mass crossing the interfaces of the quantum-well materials, we will discuss the problem of the amplification of the quantum-well optical-phonon population under an intense laser field.

Our paper is organized as follow. In Sec. II, we describe the electronic wave function under a laser field and the electron-optical-phonon interaction Hamiltonian. In Sec. III, we calculate the rates of optical-phonon excitations in a quantum well of polar materials. In Sec. IV, we discuss the conditions and regulations of the phonon amplification. The conclusions of the paper are given in Sec. V.

II. THE STATE OF THE ELECTRON AND THE ELECTRON-PHONON INTERACTION HAMILTONIAN

The quantum well we consider, which consists of different polar materials such as GaAs/AlAs, is shown in Fig. 1. The numbers 1, 2, and 3 in Fig. 1 express different polar materials. Materials 2 and 3 are identical. Let there be an electromagnetic plane wave propagating normal to the interfaces of the quantum well and penetrating well into the sample. In order to simplify some mathematical treatment, we suppose that the wavelength of the electromagnetic plane wave is far greater than both the mean free path of the electrons and the width of the quantum well, so that the spatial dependence of the laser wave can be neglected (dipole approximation).

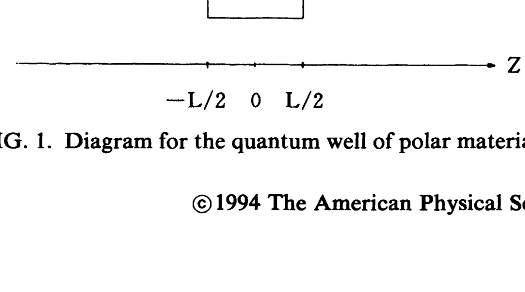

FIG. 1. Diagram for the quantum well of polar material.

Here we calculate the electron wave function. Accordingly, the Schrödinger equation for an electron moving in a quantum well shown in Fig. 1 under a laser field is

$$i\hbar\frac{\partial\Psi(\gamma,z,t)}{\partial t}=\frac{1}{2m^*}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}(t)\right]^2\Psi(\gamma,z,t)+V(z)\Psi(\gamma,z,t)\ , \tag{1}$$

where the vector potential $\mathbf{A}(t)$ can be expressed as

$$\mathbf{A}(t)=\mathbf{e}_{\parallel}A_0\cos(\omega t)\ , \tag{2}$$

and $V(z)$ is the potential of the quantum well,

$$V(z)=\begin{cases}0, & z<-\frac{L}{2}\ \text{or}\ z>\frac{L}{2}\\ V_0, & |z|\le\frac{L}{2}\ .\end{cases} \tag{3}$$

$\mathbf{e}_{\parallel}$ is the unit vector parallel to the interface, m^* is the electronic effective mass. It is easy to obtain the solution of Eq. (1):

$$|\mathbf{k}_{\parallel},l\rangle=\Psi_{\mathbf{k}_{\parallel}l}(\gamma,z,t)=\frac{1}{\sqrt{s}}\phi_l(z)\exp\left[i\mathbf{k}_{\parallel}\cdot\gamma-\frac{i}{\hbar}\int_0^t E_l dt\right]\times\begin{cases}\exp\left\{-\frac{i}{2m_1^*\hbar}\int_0^t\left[\hbar\mathbf{k}_{\parallel}-\frac{e}{c}\mathbf{A}(t')\right]^2dt'\right\}, & |z|\le\frac{L}{2}\\ \exp\left\{-\frac{i}{2m_2^*\hbar}\int_0^t\left[\hbar\mathbf{k}_{\parallel}-\frac{e}{c}\mathbf{A}(t')\right]^2dt'\right\}, & z\le\frac{L}{2}\ \text{or}\ z>\frac{L}{2}\ ,\end{cases} \tag{4}$$

where $\phi_l(z)$ is the electronic wave function moving along the z axis, and can be expressed in the following form. For the even-parity state,

$$\phi_l(z)=\left[\frac{2}{L}\right]^{1/2}B_{sl}\times\begin{cases}C_{sl}\exp(k_{2l}z), & z\le\frac{L}{2}\\ \cos(k_{1l}z), & |z|\le\frac{L}{2}\\ C_{sl}\exp(-k_{2l}z), & z>\frac{L}{2}\ ,\end{cases} \tag{5a}$$

with

$$B_{sl}=\left[1+\frac{\left[\cos\frac{k_{1l}L}{2}\right]^2}{k_{2l}L}\right]^{1/2}, \tag{5b}$$

$$C_{sl}=\cos\left[\frac{k_{1l}L}{2}\right]\exp(+k_{2l}L/2)\ , \tag{5c}$$

where the energy of the lth subband is determined by

$$\tan\left[\frac{k_{1l}L}{2}\right]=\frac{k_{2l}m_1^*}{k_{1l}m_2^*}\ . \tag{5d}$$

For the odd-parity state,

$$\phi_l(z)=\left[\frac{2}{L}\right]^{1/2}B_{Al}\times\begin{cases}C_{Al}\exp(k_{2l}z), & z\le\frac{L}{2}\\ \sin(k_{1l}z), & |z|\le\frac{L}{2}\\ -C_{Al}\exp(-k_{2l}z), & z>\frac{L}{2}\ ,\end{cases} \tag{6a}$$

with

$$B_{Al}=\left[1+\frac{\left[\sin\left[\frac{k_{1l}L}{2}\right]\right]^2}{k_{2l}L}\right]^{1/2}, \tag{6b}$$

$$C_{Al}=-\sin\left[\frac{k_{1l}L}{2}\right]\exp(k_{2l}L/2)\ , \tag{6c}$$

where the energy of the lth subband is determined by

$$\tan\left[\frac{k_{1l}L}{2}\right]=-\frac{k_{1l}m_2^*}{k_{2l}m_1^*}\ . \tag{6d}$$

Within Eqs. (5) and (6), $k_{1l}^2=(2m_1^*/\hbar^2)E_l$ and $k_{2l}^2=(2m_2^*/\hbar^2)(V_0-E_l)$.

The polar-optical-phonon models of the quantum well of polar materials have been studied extensively in the past decade. The main results of this research have been proof of the existence of confined bulklike phonon modes and interface phonon modes. At the present, the Huang-Zhu model[10] for the confined-polar-optical (CPO) phonon of the quantum well can be regarded as the best among all the models of CPO phonons.[11] By using the Huang-Zhu model[10] for CPO phonons, the Fourier component of the electron–CPO-phonon interaction Hamiltonian in a quantum well of polar materials is given by

$$V_{n\alpha}^c(\gamma,z,\mathbf{q})=\lambda V^{-1/2}t_n(q)\Phi_{n\alpha}(q)e^{i\mathbf{q}\cdot\gamma}\ , \tag{7a}$$

with

$$\lambda^2=4\pi e^2\hbar\omega_{\mathrm{TO}}(\varepsilon_\infty^{-1}-\varepsilon_0^{-1})\ , \tag{7b}$$

$$\Phi_{n\pm}=\begin{cases}\sin\left[\frac{n\,\mu_n z}{L}\right]+\frac{C_n z}{L}, & n=3,5,7,\ldots \\ \cos\left[\frac{n\pi}{L}z\right]-(-1)^{n/2}, & n=2,4,6,\ldots,\end{cases} \tag{7c}$$

$$t_n(q)=(2I_n)^{-1/2}, \tag{7d}$$

and

$$I_n=\frac{1}{L}\int_{-L/2}^{L/2}dz\left[q^2\Phi_n^2+\left[\frac{d\Phi_n}{dz}\right]^2\right], \tag{7e}$$

where $\mathbf{q}$ is the phonon wave vector parallel to an interface, and $\mathbf{r}$ is the two-dimensional localization vector parallel to the interfaces. The constant in Eq. (7c) is successive solutions of the equation

$$\tan\left[\frac{\mu_n\pi}{2}\right]=\frac{\mu_n\pi}{2}$$

with the smallest one (μ_1) being omitted, and C_n in Eq. (7c) being given by

$$C_n=-2\sin\left[\frac{\mu_n\pi}{2}\right].$$

Several authors have discussed the interfacial-polar-optical (IPO) phonon of the quantum well.[12,13] In this paper, we will make use of the electron–IPO-phonon interaction Hamiltonian given by Lassing.[13] According to Ref. 13, the Fourier component of the Hamiltonian is given by the following:

$$V_{\alpha\mu}(\mathbf{r},z,\mathbf{q})=\left[\frac{2\pi e^2 f_{\alpha\mu}}{\hbar S q\,\omega_{\alpha\mu}}\right]^{1/2} e^{i\mathbf{q}\cdot\gamma-qL/2}(e^{qz}+e^{-qz}) \times(1\pm\gamma)^{-1/2}. \tag{8a}$$

The index $\alpha=a,s$ denotes the parity with respect to the z axis; the index $\mu=\pm$ distinguishes between the solutions of given parity. The quantities that enter Eq. (8a) are defined as follows:

$$\gamma=\exp(-qL), \tag{8b}$$

$$f_{\alpha\mu}=\left|\frac{\hbar^2(\omega_{\alpha\mu}^2-\omega_{\mathrm{TO1}}^2)(\omega_{\alpha\mu}^2-\omega_{\mathrm{TO2}}^2)}{(\omega_{\alpha+}^2-\omega_{\alpha-}^2)(\varepsilon_{1\alpha}+\varepsilon_{2\alpha})}\right|, \tag{8c}$$

$$\varepsilon_{1s}=\varepsilon_{1\infty}(1-\gamma),\quad \varepsilon_{2s}=\varepsilon_{2\infty}(1+\gamma), \tag{8d}$$

$$\varepsilon_{1a}=\varepsilon_{1\infty}(1+\gamma),\quad \varepsilon_{2a}=\varepsilon_{2\infty}(1-\gamma), \tag{8e}$$

$$(\hbar\omega_{\alpha\pm})^2=\frac{P_\alpha\pm[P_\alpha-(\varepsilon_{1\alpha}+\varepsilon_{2\alpha})(\varepsilon_{1\alpha}\omega_{\mathrm{LO1}}^2\omega_{\mathrm{TO2}}^2+\varepsilon_{2\alpha}\omega_{\mathrm{LO2}}^2\omega_{\mathrm{TO1}}^2)]^{1/2}}{\varepsilon_{1\alpha}+\varepsilon_{2\alpha}}, \tag{8f}$$

and

$$P_\alpha=[\varepsilon_{1\alpha}(\omega_{\mathrm{TO2}}+\omega_{\mathrm{LO1}}^2)+\varepsilon_{2\alpha}(\omega_{\mathrm{TO1}}^2+\omega_{\mathrm{LO2}}^2)]/2,\quad \alpha=s,a \tag{8g}$$

where $\varepsilon_{i\infty}$ $(i=1,2)$ is the high-frequency dielectric constant, ω_{T01}, ω_{T02} and ω_{LO1}, ω_{LO2} are the bulk TO-phonon frequencies and bulk LO-phonon frequencies, respectively, assumed here to be dispersionless. Index 1 refers to the well material, and index 2 to the barrier material.

III. RATES OF THE CPO- AND IPO-PHONON EXCITATIONS

In what follows, we deduce first the rates of the CPO-phonon excitations. We express the number of CPO phonons whose quantum number are $(n\alpha\mathbf{q})$ as $N_{n\alpha\mathbf{q}}$. The index α denotes the symmetry of the CPO phonons. Considering the processes of the phonon absorption and emission at the same time, the kinetic equation may be written as

$$\frac{dN_{n\alpha\mathbf{q}}}{dt}=\sum_{\mathbf{k}_\parallel l'l}((N_{n\alpha\mathbf{q}}+1)f[E_1(\mathbf{k}_\parallel+\mathbf{q},l')]\{1-f[E_1(\mathbf{k}_\parallel,l)]\}-N_{n\alpha\mathbf{q}}\{1-f[E_1(\mathbf{k}_\parallel+\mathbf{q},l')]\}f[E_1(\mathbf{k}_\parallel,l)])T_{n\alpha\mathbf{q}}(\mathbf{k}_\parallel l',l),$$

where $f[E_1(\mathbf{k}_\parallel,l)]$ is the Fermi distribution function, $T_{n\alpha\mathbf{q}}(\mathbf{k}_\parallel,l',l)$ is the transition probability per unit time for the electronic transitions from an initial state $|\mathbf{k}_\parallel,l\rangle$ to a final state $|\mathbf{k}_\parallel',l'\rangle$ in the l'th subband. By making use of Eqs. (4) and (7), we can easily obtain the expression for $T_{n\alpha\mathbf{q}}(\mathbf{k}_\parallel,l',l)$. The calculation of $T_{n\alpha\mathbf{q}}(\mathbf{k}_\parallel,l',l)$ is given in Appendix A.

As usual,[1–9] the phonon population satisfies the condition $N_{n\alpha\mathbf{q}}\gg 1$. By using this condition, Eq. (9) may be rewritten as

$$\frac{dN_{n\alpha\mathbf{q}}}{dt}=\gamma_{n\alpha\mathbf{q}}N_{n\alpha\mathbf{q}}, \tag{10}$$

where $\gamma_{n\alpha\mathbf{q}}$ is the rate of change of the population of the $(n\alpha\mathbf{q})$ phonon state, and

$$\gamma_{n\alpha q}=\sum_{\mathbf{k}_{\parallel}l'l}\{f[E_1(\mathbf{k}_{\parallel}+\mathbf{q},l')]-f[E_1(\mathbf{k}_{\parallel},l)]\}T_{n\alpha q}\ . \tag{11}$$

The semiconductor quantum well will be assumed to be in the region of low temperature so that we may simplify the carrier distribution function by using the step function at the Fermi energy E_F defined by

$$f[E(\mathbf{k}_{\parallel},l)]=\begin{cases}1 & \text{if } E(\mathbf{k}_{\parallel},l)\le E_F\\ 0 & \text{if } E(\mathbf{k}_{\parallel},l)> E_F\ .\end{cases}$$

By making use of the intense laser field condition, namely $\Lambda_L \gg \hbar\omega$, and assuming that laser-photon-absorption processes dominate,[8,9] we can obtain the final expression for the rate of change of the CPO-phonon population $\gamma_{n\alpha q}$, namely

$$\gamma_{n\alpha q}=\frac{\lambda^2}{16\pi\hbar L}\left[\frac{m_1^*t_n(q)}{\hbar^2 q}\right]^2\sum_{ll'}|F_{n\alpha}(l',l)|^2(B_lB_{l'})^2K_L(l',l)\ , \tag{12}$$

with

$$\begin{aligned}K_L(l',l)=&\left[\frac{2\hbar^2q^2}{m_1^*}(E_F-E_l-\hbar\omega_{\text{LO}}+\Lambda_L)-\left[\frac{\hbar^2q^2}{2m_1^*}+E_{l'}-E_l-\hbar\omega_{\text{LO}}+\Lambda_L\right]^2\right]^{1/2}\\ &-\left[\frac{2\hbar^2q^2}{m_1^*}(E_F-E_l)-\left[\frac{\hbar^2q^2}{2m_1^*}+E_{l'}-E_l-\hbar\omega_{\text{LO}}+\Lambda_L\right]^2\right]^{1/2},\end{aligned} \tag{13}$$

where

$$\Lambda_L=\frac{e\hbar}{\omega m_1^*}\mathbf{q}\cdot\mathbf{E}_d\ . \tag{14}$$

The expression for $F_{n\alpha}(l',l)$ in Eq. (12) can be found in Appendix A.

The mechanism for the phonon amplification, that is the reasons for which the phonon population increases, are the intrasubband and intersubband absorption of light by charge carriers in quantum wells accompanied by the emission or absorption of phonons in order for the electrons to gain the necessary momentum for the transition. For an interface system consisting of different materials, because of the difference of the electronic effective mass in the different regions of the system, the total energy of the electron is different in different regions of the system even though the energy of the electron moving along the z axis is the same. This result shows that the amplification of the phonon population via the absorption of the laser energy will be different in different material regions of the system. In that case, the kinetic equation is written to account for this fact of effective-mass difference corresponding to different material regions. Hence, for the interface system shown in Fig. 1, the kinetic equation for the IPO phonon should be written as

$$\frac{d\,{}^iN_{\alpha\mu q}}{dt}=\sum_{\mathbf{k}_{\parallel}l'l}(({}^iN_{\alpha\mu q}+1)f[E_i(\mathbf{k}_{\parallel}+\mathbf{q},l')]\{1-f[E_i(\mathbf{k}_{\parallel},l)]\}-{}^iN_{\alpha\mu q}\{1-f[E_i(\mathbf{k}_{\parallel}+\mathbf{q},l')]\}f[E_i(\mathbf{k}_{\parallel},l)])T^i_{\alpha\mu q}(\mathbf{k}_{\parallel},l',l)$$

$$(i=1,2,3)\ , \tag{15}$$

where ${}^iN_{\alpha\mu q}$ $(i=1,2,3)$ is the number of IPO phonons of parity α and frequency $\omega_{\alpha\mu}$ in the ith region, and $T^i_{\alpha\mu q}(\mathbf{k}_{\parallel},l',l)$ is the transition probability per unit time in the ith region for the electronic transition caused by the scattering of the $(\alpha\mu\mathbf{q})$ IPO phonons from an initial state $|\mathbf{k}_{\parallel},l\rangle$ to final state $|\mathbf{k}'_{\parallel},l'\rangle$ in the l'th subband. The expression for $T^i_{\alpha\mu q}(\mathbf{k}_{\parallel},l',l)$ is given in Appendix B. The total number of the $(\alpha\mu\mathbf{q})$ IPO-phonon modes is $N_{\alpha\mu q}={}^1N_{\alpha\mu q}+{}^2N_{\alpha\mu q}+{}^3N_{\alpha\mu q}$. By making use of the condition ${}^iN_{\alpha\mu q}\gg 1$, Eq. (15) is simplified as

$$\frac{d\,{}^iN_{\alpha\mu q}}{dt}={}^i\gamma_{\alpha\mu q}\,{}^iN_{\alpha\mu q},\quad i=1,2,3\ , \tag{16}$$

with

$${}^i\gamma_{\alpha\mu q}=\sum_{\mathbf{k}_{\parallel}l'l}\{f[E_i(\mathbf{k}_{\parallel}+\mathbf{q},l')]-f[E_i(\mathbf{k}_{\parallel},l)]\}T^i_{\alpha\mu q}(\mathbf{k}_{\parallel},l',l)\ ,\quad i=1,2,3\ . \tag{17}$$

If we make use of the same approximations and assumptions for the IPO phonons as for the CPO phonons mentioned above, we can deduce the rates of change of the IPO-phonon population in each material regions of the system as the following:

$$
\begin{aligned}
{}^{3}\gamma_{\alpha\mu\mathbf{q}} &= {}^{2}\gamma_{\alpha\mu\mathbf{q}} \\
&= 2\left[\frac{m_2^* e}{\hbar^3 qL}\right]^2 \frac{f_{\alpha\mu}}{q\omega_{\alpha\mu}} \frac{e^{-qL}}{1\pm\gamma} \sum_{l'l} |B_l B_{l'} C_l C_{l'}|^2 |F^2_{\alpha\mu}(l',l)|^2 K_{s2}(l',l) , \qquad (18)
\end{aligned}
$$

$$
{}^{1}\gamma_{\alpha\mu\mathbf{q}} = 4\left[\frac{m_1^* e}{\hbar^2 qL}\right]^2 \frac{f_{\alpha\mu}}{q\omega_{\alpha\mu}} \frac{e^{-qL}}{1\pm\gamma} \sum_{l'l} (B_l B_{l'})^2 |F^1_{\alpha\mu}(l',l)|^2 K_{s1}(l',l) , \qquad (19)
$$

with

$$
\begin{aligned}
K_{si} = &\left[\frac{2\hbar^2 q^2}{m_i^*}(E_F - E_l - \hbar\omega_{\alpha\mu} + \Lambda_{si}) - \left[\frac{\hbar^2 q^2}{2m_i^*} + E_{l'} - E_l - \hbar\omega_{\alpha\mu} + \Lambda_{si}\right]^2\right]^{1/2} \\
&- \left[\frac{2\hbar^2 q^2}{m_1^*}(E_F - E_l) - \left[\frac{\hbar^2 q^2}{2m_i^*} + E_{l'} - E_l - \hbar\omega_{\alpha\mu} + \Lambda_{si}\right]^2\right]^{1/2} , \qquad (20)
\end{aligned}
$$

where

$$
\Lambda_{si} = \frac{e\hbar}{m_i^* \omega} \mathbf{q}\cdot\mathbf{E}_d , \quad i = 1,2 .
$$

The expression for $F^i_{\alpha\mu}(l',l)$ $(i=1,2)$ in Eqs. (18) and (19) is given in Appendix B.

IV. DISCUSSION

The mechanism for the phonon amplification mentioned above shows that the phonon amplification should satisfy energy and momentum conditions so that the electron can transit from an initial state $|\mathbf{k}_{\parallel}, l\rangle$ to all the possible final states $|\mathbf{k}_{\parallel}, l'\rangle$ in the l'th subband. In what follows we will deduce these conditions.

Equations (10) and (16) show us that if the rates of change of the phonon population are greater than zero, the phonon population will grow with time, whereas for rates less than zero there is a damping of the phonon population. Accordingly, the sign of the rates of change of the phonon population and the sign of the term in the square root in Eqs. (13) and (20) determine the conditions under which the phonons can be amplified. Analyzing Eqs. (12) and (18)–(20), we can obtain the conditions which the laser field must satisfy for the phonon amplification. The conditions are

$$
\Lambda_L > \hbar\omega_{\mathrm{LO}} \qquad (21)
$$

for the CPO phonons, and

$$
\Lambda_{si} > \hbar\omega_{\alpha\mu} , \quad i = 1,2 \qquad (22)
$$

for the IPO phonons. On the other hand, since we have used the low-temperature condition in the calculation above, the energy provided to the electron-phonon system by the laser field must be greater than Fermi energy of the electrons, so that the electron can transit from the ground state to a higher-energy state. This means that

$$
\Lambda_L, \Lambda_{si} > E_F , \quad i = 1,2 . \qquad (23)
$$

Combining Eqs. (21)–(23), we have

$$
\Lambda_L > \max(E_F, \hbar\omega_{\mathrm{LO}}) \qquad (24)
$$

for the CPO phonons, and

$$
\Lambda_{si} > \max(E_F, \hbar\omega_{\alpha\mu}) , \quad i = 1,2 \qquad (25)
$$

for the IPO phonons, respectively. Feng and Chen[9] have obtained a relation for a single heterostructure similar to Eq. (25).[9] Equations (24) and (25) simply mean that if the drift velocity of the electron $v = eE_d/(m^*\omega)$, as imposed by an intense laser field, exceeds the phonon-phase velocity v_{ph} $(=\omega_{\mathrm{LO}}/q$ for CPO phonons or $\omega_{\alpha\mu}/q$ for IPO phonons) for $\hbar\omega_{\mathrm{LO}}$ (or $\hbar\omega_{\alpha\mu}) < E_F$, or exceeds $(k_F/(2q))v_F$ for $\hbar\omega_{\mathrm{LO}}$ (or $\hbar\omega_{\alpha\mu}) < E_F$, a deformation potential for multiphonon excitations can be generated in the well (or in the interfaces).

For a given optical-phonon mode, the conditions $\Lambda_L = \max(E_F, \hbar\omega_{\mathrm{LO}})$ and $\Lambda_{si} = \max(E_F, \hbar\omega_{\alpha\mu})$ $(i=1,2)$ define the critical field strength E^c_{dL} for the IPO phonons and E^c_{dsi} for the IPO phonons, respectively. These critical strengths are

$$
E^c_{dL} = \frac{m_1^* \omega}{eq\hbar} \max(E_F, \hbar\omega_{\mathrm{LO}}) \qquad (26)
$$

and

$$
E^c_{dsi} = \frac{m_i^* \omega}{eq\hbar} \max(E_F, \hbar\omega_{\alpha\mu}) \quad (i = 1,2) . \qquad (27)
$$

Equations (26) and (27) express that (a) in material 1, the amplification of the IPO-phonon population is easier to attain than that of the CPO-phonon population if $\hbar\omega_{\mathrm{LO}} > \hbar\omega_{\alpha\mu} > E_F$; (b) the CPO and IPO phonons will be excited at the same time if $\hbar\omega_{\mathrm{LO}} < E$ and $\hbar\omega_{\alpha\mu}(E)$; and (c) the amplification of IPO phonons is attained more easily in the well than in the barriers.

As a numerical example of the value of E^c_{α}, we consider a GaAs/AlAs quantum well. The material constants for GaAs are taken to be $m_1^* = 0.067 m_0$ and $\hbar\omega_{\mathrm{LO}} = 36.25$ meV; for AlAs, $m_2^* = 0.15 m_0$. The Fermi energy is related to the carrier concentration, and taken here to be 0.05 eV. For an intense CO_2 laser, we take the wavelength to be $\lambda = 10.6$ μm. From Eqs. (26) and (27),

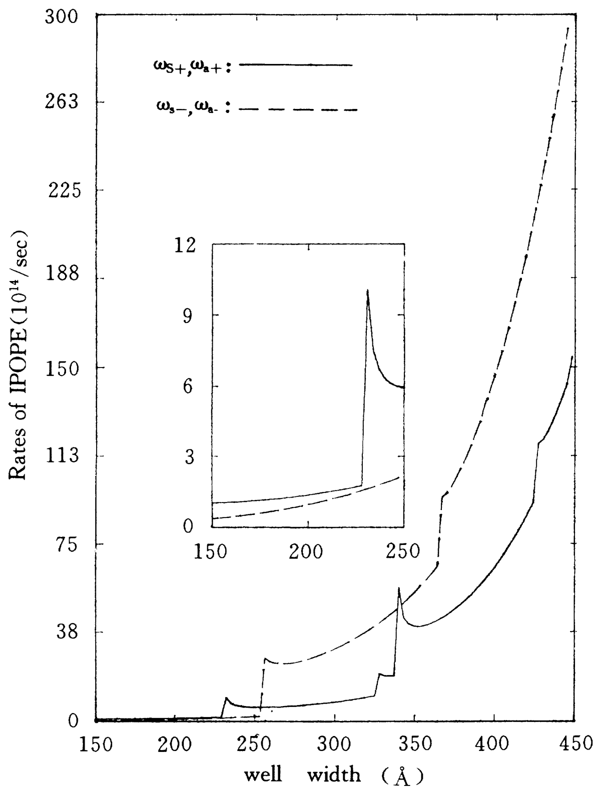

FIG. 2. Rates of the interfacial-polar-optical phonon excitation (IPOPE) shown as a function of the quantum-well width L. The parameters used were $q=2.11\times10^8$ (m^{-1}) and $E_d=2.317\times10^7$ (V/m). The rate curves for the excitations of the phonon with different parity but same values of μ are overlapped.

the critical laser field strength can be obtained as $E_{dL}^c=E_{ds1}^c=1.9488\times10^7$ (V/m) and $E_{ds2}^c=4.3637\times10^7$ (V/m) for $q=2.11\times10^8$ (m^{-1}). By making use of the expression for the critical laser intensity $I_c=C(E_d^c)^2/(8\pi)$, we can easily find that I_c is of order 10^8 W/cm, which is well within present experimental capabilities. These data on the critical field strength show that if we express the critical laser intensity which can excite the IPO phonon in the barriers as I_{s2}^c, and those in the well as I_{s1}^c, then $I_{s2}^c\sim 5I_{s1}^c$. When the laser intensity I we used in the absorption experiment satisfies $I_{s1}^c<I<I_{s2}^c$, the kinetic equation for the IPO phonon population can be written as

$$\frac{dN_{\alpha\mu\mathbf{q}}}{dt}={}^1\gamma_{\alpha\mu\mathbf{q}}{}^1N_{\alpha\mu\mathbf{q}}\ .$$

We will only discuss this case in Fig. 2.

It should be noted that Eqs. (24) and (25) are not the only necessary conditions for the phonon amplification. The momentum conditions must also be satisfied. We have obtained the momentum conditions for the phonon amplification at the same time we obtained Eqs. (12), (18), and (19). The conditions are

$$q<2k_{\parallel} \tag{28}$$

for the CPO phonons,

$$q<2k_{\parallel} \tag{29}$$

for the IPO phonons in the well region, and

$$q<\min(2k_{\parallel},k_{2l'}+k_{2l}) \tag{30}$$

for the IPO phonons in the barriers. The momenta $k_{\parallel}$ in the above equations satisfy the following relations:

$$\left[\frac{2m_1^*}{\hbar^2}(E_F-E_l)\right]^{1/2}<k_{\parallel}<\left[\frac{2m_1^*}{\hbar^2}(E_F-E_l-\hbar\omega_{\mathrm{LO}}+\Lambda_L)\right]^{1/2}$$

for the CPO phonons, and

$$\left[\frac{2m_i^*}{\hbar^2}(E_F-E_l)\right]^{1/2}<k_{\parallel}<\left[\frac{2m_i^*}{\hbar^2}(E_F-E_l-\hbar\omega_{\mathrm{Lo}}+\Lambda_L)\right]^{1/2},\quad i=1,2$$

for the IPO phonons.

The terms in the square roots impose the following restriction to the initial and final energies of the electron for the phonons to be amplified:

$$E_{l'}<\left[\frac{2\hbar^2q^2}{m_i^*}(E_F-E_l)\right]^{1/2}-\left[\frac{\hbar^2q^2}{2m_i^*}-E_l-\hbar\omega_q+\Lambda_i\right] \tag{31}$$

and

$$E_l<E_F\ . \tag{32}$$

The term m_i^* ($i=1,2$) in Eq. (31) equals m_1^* when we discuss the amplification of the phonon population in region 1, and m_2^* in regions 2 and 3. $\hbar\omega_q$ equals $\hbar\omega_{\mathrm{LO}}$ for CPO phonons and $\hbar\omega_{\alpha\mu}$ for IPO phonons.

Proceeding further we consider the amplification of the IPO population in the following analysis. From Eqs. (16)–(19), we write the kinetic equation of the total population in the following form:

$$\frac{dN_{\alpha\mu\mathbf{q}}}{dt}={}^2\gamma_{\alpha\mu\mathbf{q}}({}^2N_{\alpha\mu\mathbf{q}}+{}^3N_{\alpha\mu\mathbf{q}})+{}^1\gamma_{\alpha\mu\mathbf{q}}{}^1N_{\alpha\mu\mathbf{q}}\ . \tag{33}$$

Under conditions $q \ll 1$ and $qL \ll 1$, we see from Eqs. (B10)–(B18) that ${}^1\gamma_{\alpha-\mathbf{q}} = {}^2\gamma_{\alpha-\mathbf{q}} = 0$, and hence

$$\frac{dN_{\alpha-\mathbf{q}}}{dt} = 0 \tag{34}$$

for the IPO phonons of frequency $\omega_{\alpha-}$ for a given parity, that is, the population of the IPO phonons whose frequencies are $\omega_{\alpha-}$ is not amplified under an intense laser field.

Let us analyze the magnitude of the quantities in the sum of Eqs. (18) and (19). Because the difference of the electronic effective mass in a quantum-well system, such as GaAs/AlAs, is not very large, we have $K_{s1} \sim K_{s2}$. On the other hand, considering $k_{2l} \gg k_{1l}$ under the condition of large V_0 (quantum-well potential), we have from Eqs. (B10) and (B18), for small q,

$$|C_{l'}C_l F^2_{\alpha+}(l',l)|^2 \sim \frac{4}{(k_{2l}+k_{2l'})^2} ,$$

$$|F^1_{\alpha+}(l',l)|^2 \sim \left| \frac{\sin \frac{k_{1l'}-k_{1l}}{2} L}{k_{1l'}-k_{1l}} + \frac{\sin \frac{k_{1l'}+k_{1l}}{2} L}{k_{1l'}+k_{1l}} \right|^2 .$$

Obviously, $|C_{l'}C_l F^2_{\alpha+}(l',l)|^2$ is a small quantity as compared with $|F^1_{\alpha+}(l',l)|^2$ for all the possible l' and l. Taking into account the results mentioned above, we have the following result:

$${}^1\gamma_{\alpha+\mathbf{q}} \gg {}^2\gamma_{\alpha+\mathbf{q}} .$$

Hence, Eq. (33) can be simplified as

$$\frac{dN_{\alpha+\mathbf{q}}}{dt} = {}^1\gamma_{\alpha+\mathbf{q}} {}^1N_{\alpha+\mathbf{q}} \tag{35}$$

for the IPO phonons of frequency $\omega_{\alpha+\mathbf{q}}$ for a given parity. Equation (35) can be understood physically by considering that the electrons are located mainly in the quantum well under the condition of large V_0.

We now discuss the relationship between the rates of change of the phonon population and the quantum-well width under a defined q and a defined laser field strength. In order to make the problem definitive, we consider a GaAs/AlAs quantum well. The material constants are all taken from Refs. 15 and 16. Let $q = 2.11 \times 10^8$ (m^{-1}) and $E_d = 2.317 \times 10^7$ (V/m). The change regulation of the rates of the phonon excitons with the quantum-well width are shown in Fig. 2 for the IPO phonons, and in Fig. 3 for CPO phonons. Figure 2 shows, first, that the rates increase with the increase of the well width, which is the result of an increase in the number of levels (below the Fermi energy) with an increase of the well width, following which more electrons can transit to higher levels; second, the rates are not related to the parity of the IPO phonons but to the frequency of the IPO phonons; and third, the rate of the $\omega_{\alpha+}$ IPO-phonon amplification is greater than that of the $\omega_{\alpha-}$ phonon amplification when $L < 251$ Å, but less than that of the $\omega_{\alpha-}$ phonon amplification when $L > 251$ Å. Figure 3 shows the amplification curves of the $n = 2$, 3, 4, and 5 mode phonon population. From Fig. 3 we can see that the rate for

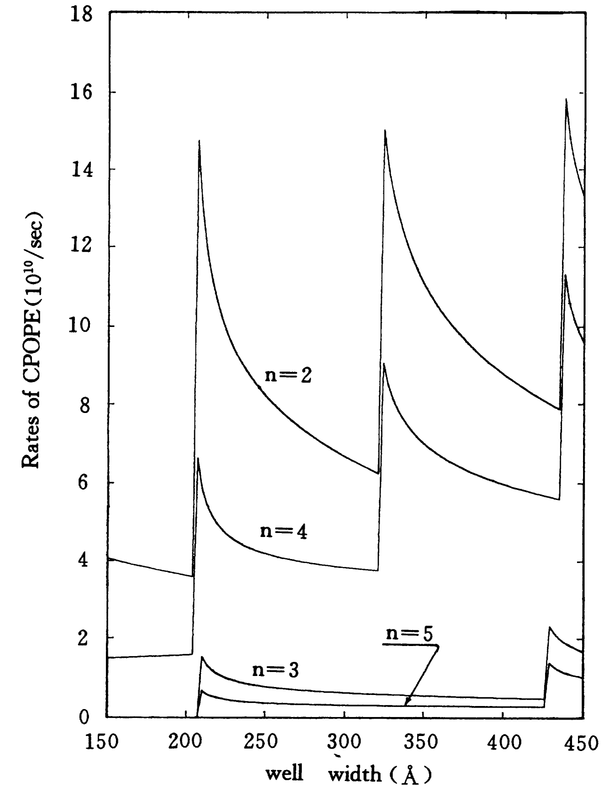

FIG. 3. Rates of the confined-polar-optical phonon excitation (CPOPE) shown as a function of the quantum-well width L. The parameters used were $q = 2.11 \times 10^8$ (m^{-1}) and $E_d = 2.317 \times 10^7$ (V/m).

the $n = 2$ mode phonons is greater than those of the other modes, and that the rates of the even-parity phonons are greater than those of the odd-parity phonons. With the increase of the quantum-well width, the height of the resonant peaks in Fig. 3 increases and the intervals between two adjacent peaks decrease. The increased height of the resonant peaks for the amplification curve of the $n = 4$ mode phonon population is seen to be greater than that of the amplification curve of the $n = 2$ mode phonon population, where the number of resonant peaks for the phonon with a given parity is not related to the value of the phonon mode number n. Finally, comparing Figs. 2 and 3, we see that the rates of interfacial-phonon excitation are much greater than those of confined-phonon excitation.

The results mentioned above can be observed by a pulsed-intense-laser Raman experiment under low-temperature conditions. At low temperature ($T < 10$ K), the number of optical phonons is infinitesimal and, thus, the observed signal originates from a nonthermal phonon population, that is, the amplified phonon population. In order to observe the results mentioned above, several requirements for the samples used in the experiment should be satisfied. First, the in-plane momenta of the phonon should satisfy Eqs. (28)–(30). This requirement can be achieved by using samples whose (110) cleavage planes are polished at certain angles. Second, one of the sides of the quantum-well system into which the laser beam is to be injected should be masked, and then the masked layer of the well or the barrier regions of the quantum-well system be removed by an electron beam lithography facility according to the requirement of the experiment.

V. CONCLUSION

In this paper, taking into account the discontinuity of the electronic effective mass crossing the interfaces of the quantum well, we have discussed the amplification of CPO- and IPO-phonon populations in a quantum-well system under an intense laser field. Our results show that (a) for the CPO phonon, the number of phonons will grow if the laser field strength $E_d > E_{dL}^c$ and the phonon wave vector $q < 2k_\parallel$; (b) for IPO phonons, the phonon population will grow if the laser field strength $E_d > E_{dsi}^c$ $(i=1,2)$ and the phonon wave vector $q < 2k_\parallel$ in material 1 and $q < \min(2k_\parallel, k_{2l'}+k_{2l})$ in material 2 or 3; (c) because $m_1^* < m_2^*$ in a quantum well, $E_{ds1}^c < E_{ds2}^c$ and hence the amplification of the IPO-phonon population is easier in the well than in the barriers; and (d) under the conditions of large V_0 and small phonon wave vector q, the number of the $(\alpha - \mathbf{q})$ IPO phonons is not amplified under an intense field, and the amplification of the $(\alpha+\mathbf{q})$ IPO-phonon population is determined mainly by the amplification of the $(\alpha+\mathbf{q})$ phonon population inside the well. Numerical calculations for the rates of change of the phonon amplification show that (i) the rates for the IPO-phonon excitations increase with the increase of the well width L; (ii) there are some resonant peaks in the amplification curves of the CPO-phonon population; (iii) the height of the peaks increases with the increase of the well width L; and (iv) the number of the peaks for the phonons having defined parities is the same and is not related to the mode number n of the phonons.

It should be noted that a net amplification of the optical-phonon population requires that the phonon growth rate be greater than the linear losses $\eta(q)$ due to processes other than phonon emission or absorption by electrons.[14] By the time an instability is obtained when condition $\gamma_{\mathbf{q}} > \eta(q)$ is satisfied, the phonon population grows with time and saturates due to nonlinear relaxation mechanisms which should be effective in stabilizing these amplified optical phonons.[1,17] This ultimately limits the phonon population. Since, to my knowledge, there are no suitable experimental data to determine the order of the linear losses $\eta(q)$ in the quantum-well system we have used in this paper, and the numerical calculation of $\eta(q)$ is a very tedious work,[14] we cannot here compare $\gamma_{\mathbf{q}}$ with $\eta(q)$ to obtain an order-of-magnitude estimate of the laser threshold intensity for an actual optical-phonon amplification. We will calculate $\eta(q)$ and compare with $\gamma_{\mathbf{q}}$ in details in a following paper.

APPENDIX A

In this appendix, treating the electron–CPO-phonon interaction as the perturbation, we deduce the transition matrix elements and transition probability of the electron acted upon by CPO phonons. The transition matrix element between the initial state $|\mathbf{k}_\parallel, l\rangle$ and the final state $|\mathbf{k}_\parallel', l'\rangle$ is

$$A_{n\alpha}(\mathbf{k}_\parallel', l'; \mathbf{k}_\parallel, l) = \frac{1}{i\hbar}\int d\gamma\, dz\, dt \langle \mathbf{k}_\parallel' l' | V_{n\alpha}^c(\gamma, z, \mathbf{q}) | \mathbf{k}_\parallel, l\rangle \ . \tag{A1}$$

By substituting Eqs. (4) and (7) into (A1), and using the formula for the Bessel-function expansion,

$$\exp(-ix \sin\omega t) = \sum_{\nu=-\infty}^{\infty} J_\nu(x)\exp(-i\nu\omega t) \ , \tag{A2}$$

where $J_\nu(x)$ is a Bessel function of order ν. In fact, since $\{\exp(-i\nu\omega t)\}$ is an orthogonal-complete function set, Eq. (A2) can also be regarded as a Fourier expansion of the function $\exp[-ix\sin(\omega t)]$. Thus index ν can be regarded as the times of the frequency of the laser field. The integration of Eq. (A1) can be performed easily, to yield

$$A_{n\alpha}(\mathbf{k}_\parallel', l'; \mathbf{k}_\parallel, l) = \sum_{\nu=-\infty}^{\infty} A_{n\alpha\nu}(\mathbf{k}_\parallel', l'; k_\parallel, l) \ , \tag{A3}$$

with

$$A_{n\alpha\nu}(\mathbf{k}_\parallel', l'; \mathbf{k}_\parallel, l) = \frac{(2\pi)^2\lambda}{i\hbar S V^{1/2}} t_n(q)\delta(\mathbf{k}_\parallel' - \mathbf{k}_\parallel - \mathbf{q}) \times \widetilde{F}_{n\alpha}(l', l) J_\nu\left[\frac{\Lambda_L}{\hbar\omega}\right] R_\nu(l', l) \ , \tag{A4}$$

where

$$\widetilde{F}_{n\alpha}(l', l) = \int_{-L/2}^{L/2} dz\, \Phi_{n\alpha}(z)\varphi_{l'}^*(z)\varphi_l(z) \ , \tag{A5}$$

$$R_\nu(l', l) = \frac{e^{i[E(\mathbf{k}_\parallel', l') - E(\mathbf{k}_\parallel, l) - \hbar\omega_{\mathrm{LO}} - \nu\hbar\omega]\tau/\hbar} - 1}{[E(k_\parallel', l') - E(\mathbf{k}_\parallel, l) - \hbar\omega_{\mathrm{LO}} - \nu\hbar\omega]\frac{i}{\hbar}} \ . \tag{A6}$$

It is easy to prove that

$$|R_\nu(l', l)|^2 \sim 2\hbar\pi\tau\delta[E_1(\mathbf{k}_\parallel', l') - E_1(\mathbf{k}_\parallel, l) - \hbar\omega_{\mathrm{LO}} - \nu\hbar\omega] \ , \tag{A7}$$

where

$$E_1(\mathbf{k}_\parallel, l) = \frac{\hbar^2 k_\parallel^2}{2m_1^*} + E_l \ . \tag{A8}$$

Substituting Eq. (5) or (6) into (A5), we have the following result:

$$\widetilde{F}_{n\alpha}(l', l) = \tfrac{1}{2} B_{l'} B_l F_{n\alpha}(l', l) \ . \tag{A9}$$

The function $F_{n\alpha}(l', l)$ is concerned with the parity of the initial and final states of the electron. When the initial state of the electron is an even-parity state,

$$F_{n+} = \frac{4C_n}{L^2}\left[\frac{\sin\frac{k_{1l}+k_{1l'}}{2}L}{(k_{1l}+k_{1l'})^2} + \frac{\sin\frac{k_{1l'}-k_{1l}}{2}L}{(k_{1l'}-k_{1l})^2}\right] \ , \tag{A10}$$

$$F_{n-} = 0 \tag{A11}$$

for an odd-parity final state, and

$$F_{n+}=0\ , \tag{A12}$$

$$F_{n-}=-2(-1)^{n/2}\delta_{l'l} \tag{A13}$$

for an even-parity final state. When the initial state of the electron is an odd-parity state,

$$F_{n+}=\frac{4C_n}{L^2}\left[\frac{\sin\frac{k_{1l'}+k_{1l}}{2}L}{(k_{1l}+k_{1l'})^2}-\frac{\sin\frac{k_{1l'}-k_{1l}}{2}L}{(k_{1l'}-k_{1l})^2}\right]\ , \tag{A14}$$

$$F_{n-}=0 \tag{A15}$$

for an even-parity final state, and

$$F_{n+}=0 \tag{A16}$$

$$F_{n-}=-2(-1)^{n/2}\delta_{l'l} \tag{A17}$$

for an odd-parity final state.

The transition probability per unit time $T_{n\alpha}(\mathbf{k}_\parallel,l',l)$ for the electron transition from an initial state to all possible final states in the l'th subband due to electron scattering by CPO phonons can be written as

$$T_{n\alpha}(\mathbf{k}_\parallel,l',l)=\sum_\nu \frac{S}{(2\pi)^2}\int d\mathbf{k}'_\parallel |A_{n\alpha\nu}(\mathbf{k}'_\parallel,l';\mathbf{k}_\parallel,l)|^2\ . \tag{A18}$$

Since we have assumed that the laser field strength is large enough, that is $\Lambda/\hbar\omega\gg 1$, the following approximation can be employed;[5,9]

$$\sum_\nu \left[J_\nu\left[\frac{\Lambda}{\hbar\omega}\right]\right]^2\delta(E-\nu\hbar\omega)=\tfrac{1}{2}[\delta(E-\Lambda)+\delta(E+\Lambda)]\ . \tag{A19}$$

Assuming that $\Lambda>E_F$ (Fermi energy) and that laser-photon-absorption processes dominate, we can neglect the contribution of the first δ function in Eq. (A19). Putting Eq. (A4) into Eq. (A18), using Eqs. (A7) and (A19), we have

$$\begin{aligned}T_{n\alpha}(\mathbf{k}_\parallel,l',l)=&\frac{\pi\lambda^2}{4\hbar V}(t_n(q))^2(B_{l'}B_l)^2|F_{n\alpha}(l',l)|^2\\ &\times\delta[E_1(\mathbf{k}_\parallel+\mathbf{q},l')-E_1(\mathbf{k}_\parallel,l)\\ &-\hbar\omega_{\mathrm{LO}}+\Lambda_L]\ ,\end{aligned} \tag{A20}$$

with

$$\Lambda_L=\frac{\hbar e\mathbf{q}\cdot\mathbf{E}_d}{m_1^*\omega}\ . \tag{A21}$$

APPENDIX B

Treating the electron–IPO-phonon interaction as the perturbation, we can deduce the transition matrix elements and the electronic transition probability from an initial state to all possible final states in the l'th subband due to electron–IPO-phonon scattering. The transition matrix elements between the initial state $|\mathbf{k}_\parallel,l\rangle$ and final state $|\mathbf{k}'_\parallel,l'\rangle$ in the different material regions are

$$A^i_{\alpha\mu}(\mathbf{k}'_\parallel,l';\mathbf{k}_\parallel,l)=\frac{1}{j\hbar}\int d\mathbf{r}\,dz\,dt\langle\mathbf{k}'_\parallel,l'|V^I_{\alpha\mu}(\gamma,z,\mathbf{q})|\mathbf{k}_\parallel,l\rangle\ , \tag{B1}$$

where the integral on the z coordinate is in the ith material region. Substituting Eqs. (4) and (8) into (B1), we have

$$A^i_{\alpha\mu}(\mathbf{k}'_\parallel,l';\mathbf{k}_\parallel,l)=\sum_{\nu=-\infty}^{\infty}A^i_{\alpha\mu\nu}(\mathbf{k}'_\parallel,l';\mathbf{k}_\parallel,l)\quad(i=1,2,3)\ , \tag{B2}$$

with

$$\begin{aligned}A^i_{\alpha\mu\nu}(\mathbf{k}'_\parallel,l';\mathbf{k}_\parallel,l)=&\frac{(2\pi)^2}{j\hbar S}\left[\frac{2\pi e^2 f_{\alpha\mu}}{\hbar Sq\omega_{\alpha\mu}}\right]^{1/2}\frac{e^{-qL/2}}{(1\pm\gamma)^{1/2}}\\ &\times\tilde{F}^i_{\alpha\mu}(l',l)J_\nu\left[\frac{\Lambda_i}{\hbar\omega}\right]R^i_\nu(\mathbf{k}'_\parallel,l';\mathbf{k}_\parallel,l)\ ,\end{aligned} \tag{B3}$$

where

$$R^i_\nu(\mathbf{k}'_\parallel,l';\mathbf{k}_\parallel,l)=\frac{e^{j[E_i(\mathbf{k}'_\parallel,l')-E_i(\mathbf{k}_\parallel,l)-\hbar\omega_{\alpha\mu}-\nu\hbar\omega]\tau/\hbar}-1}{[E_i(\mathbf{k}'_\parallel,l')-E_i(\mathbf{k}_\parallel,l)-\hbar\omega_{\alpha\mu}-\nu\hbar\omega]\frac{j}{\hbar}}\quad(i=1,2,3)\ , \tag{B4}$$

$$\Lambda_i=\frac{e\hbar\mathbf{q}\cdot\mathbf{E}_d}{m_i^*\omega}\quad(i=1,2,3)\ , \tag{B5}$$

$$\tilde{F}^3_{\alpha\mu}(l',l)=\int_{-\infty}^{-L/2}dz\,\varphi^*_{l'}(z)\varphi_l(z)[e^{qz}\pm e^{-qz}]\ , \tag{B6}$$

$$\tilde{F}^2_{\alpha\mu}(l',l)=\int_{L/2}^{\infty}dz\,\varphi^*_{l'}(z)\varphi_l(z)[e^{qz}\pm e^{-qz}]\ , \tag{B7}$$

$$\tilde{F}^1_{\alpha\mu}(l',l)=\int_{-L/2}^{L/2}dz\,\varphi^*_{l'}(z)\varphi_l(z)[e^{qz}\pm e^{-qz}]\ . \tag{B8}$$

It is easy to prove with Eqs. (5) and (6) that

$$\tilde{F}^i_{\alpha\mu}(l',l)=\frac{2}{L}B_{l'}B_l\begin{cases}2F^1_{\alpha\mu}(l',l), & i=1\\ C_{l'}C_lF^i_{\alpha\mu}(l',l), & i=2,3\ ,\end{cases} \tag{B9}$$

$$\begin{aligned}F^2_{\alpha\mu}(l',1)=\pm F^3_{\alpha\mu}(l',l)&=\frac{e^{-(k_{2l'}+k_{2l}+q)L/2}}{k_{2l'}+k_{2l}+q}\\ &\pm\frac{e^{-(k_{2l'}+k_{2l}-q)L/2}}{k_{2l'}+k_{2l}-q}\quad(q<k_{2l}+k_{2l'})\ .\end{aligned} \tag{B10}$$

The expressions of the $F^1_{\alpha\mu}(l',l)$ are concerned with the symmetry of the electron in the final and initial states, respectively. When the initial state of the electron is an even-parity state,

$$F^1_{\alpha+}(l',l)=\cosh\frac{qL}{2}\left[\frac{(k_{1l'}-k_{1l})\sin\frac{k_{1l'}-k_{1l}}{2}L}{(k_{1l'}-k_{1l})^2+q^2}+\frac{(k_{1l'}+k_{1l})\sin\frac{k_{1l'}+k_{1l}}{2}L}{(k_{1l'}+k_{1l})^2+q^2}\right]$$
$$+\sinh\frac{qL}{2}\left[\frac{\cos\frac{k_{1l}+k_{1l'}}{2}L}{(k_{1l'}+k_{1l})^2+q^2}+\frac{\cos\frac{k_{1l'}-k_{1l}}{2}L}{(k_{1l'}-k_{1l})^2+q^2}\right]q \tag{B11}$$

$$F^1_{\alpha-}(l',l)=0 \tag{B12}$$

for an even-parity final state, and

$$F^1_{\alpha+}(l',l)=0\ , \tag{B13}$$

$$F^1_{\alpha-}(l',l)=\cosh\frac{qL}{2}\left[\frac{\sin\frac{k_{1l'}+k_{1l}}{2}L}{(k_{1l'}+k_{1l})^2+q^2}+\frac{\sin\frac{k_{1l'}-k_{1l}}{2}L}{(k_{1l'}-k_{1l})^2+q^2}\right]q$$
$$-\sinh\frac{qL}{2}\left[\frac{(k_{1l'}+k_{1l})\cos\frac{k_{1l'}+k_{1l}}{2}L}{(k_{1l'}+k_{1l})^2+q^2}+\frac{(k_{1l'}-k_{1l})\cos\frac{k_{1l'}-l_{1l}}{2}L}{(k_{1l'}-k_{1l})^2+q^2}\right] \tag{B14}$$

for an odd-parity final state. When the initial state of the electron is an odd-parity state,

$$F^1_{\alpha-}(l',l)=\cosh\frac{qL}{2}\left[\frac{\sin\frac{k_{1l'}+k_{1l}}{2}L}{(k_{1l'}+k_{1l})^2+q^2}-\frac{\sin\frac{k_{1l'}-k_{1l}}{2}L}{(k_{1l'}-k_{1l})^2+q^2}\right]q$$
$$-\sinh\frac{qL}{2}\left[\frac{(k_{1l'}+k_{1l})\cos\frac{k_{1l'}+k_{1l}}{2}L}{(k_{1l'}+k_{1l})^2+q^2}+\frac{(k_{1l'}-k_{1l})\cos\frac{k_{1l'}-k_{1l}}{2}L}{(k_{1l'}-k_{1l})^2+q^2}\right] \tag{B15}$$

$$F^1_{\alpha+}(l',l)=0 \tag{B16}$$

for an even-parity final state, and

$$F^1_{\alpha+}(l',l)=\cosh\frac{qL}{2}\left[\frac{(k_{1l'}-k_{1l})\sin\frac{k_{1l'}-k_{1l}}{2}L}{(k_{1l'}-k_{1l})^2+q^2}-\frac{(k_{1l'}+k_{1l})\sin\frac{k_{1l'}+k_{1l}}{2}L}{(k_{1l'}+k_{1l})^2+q^2}\right]$$
$$+\sinh\frac{qL}{2}\left[\frac{\cos\frac{k_{1l'}-k_{1l}}{2}L}{(k_{1l'}-k_{1l})^2+q^2}-\frac{\cos\frac{k_{1l'}+k_{1l}}{2}L}{(k_{1l'}+k_{1l})^2+q^2}\right]q \tag{B17}$$

$$F^1_{\alpha-}(l',l)=0 \tag{B18}$$

for an odd-parity final state.

Making use of the definition of the δ function, we have

$$|R^i_\nu(\mathbf{k}'_\parallel,l';\mathbf{k}_\parallel,l)|^2\sim 2\pi\hbar\tau\delta[E_i(\mathbf{k}'_\parallel,l')-E_i(\mathbf{k}_\parallel,l)-\hbar\omega_{\alpha\mu}-\nu\hbar\omega],\quad i=1,2,3\ . \tag{B19}$$

The transition probability per unit time, $T^i_{\alpha\mu\mathbf{q}}(\mathbf{k}_\parallel,l',l)$ $(i=1,2,3)$, for the electron transition from an initial state to all possible final states in the l'th subband due to the electron–IPO-phonon scattering can be written as

$$T^i_{\alpha\mu\mathbf{q}}(\mathbf{k}_\parallel,l',l)=\sum_\nu\frac{S}{(2\pi)^2}\int d\mathbf{k}'_\parallel|A^i_{\alpha\mu\nu}(\mathbf{k}'_\parallel,l';\mathbf{k}_\parallel,l)|^2\ . \tag{B20}$$

Putting Eq. (B3) into (B16), making use of Eqs. (B19) and (A19) and the intense laser field condition $\Lambda_i/\hbar\omega\gg 1$, assuming that $\Lambda_i > E_F$ and the laser-photon-absorption processes dominate, we have

$$T^2_{\alpha\mu q}(\mathbf{k}_\parallel,l',l)=T^3_{\alpha\mu q}(\mathbf{k}_\parallel,l',l)$$
$$=\frac{4\pi}{\hbar L^2}\left[\frac{2\pi e^2 f_{\alpha\mu}}{\hbar S q\omega_{\alpha\mu}}\right]\frac{e^{-qL}}{1\pm\gamma}|B_l B_{l'} C_l C_{l'}|^2|F^2_{\alpha\mu}(l',l)|^2\delta[E_2(\mathbf{k}_\parallel+\mathbf{q},l')-E_2(\mathbf{k}_\parallel,l)-\hbar\omega_{\alpha\mu}+\Lambda_2] , \tag{B21}$$

$$T^1_{\alpha\mu q}(\mathbf{k}_\parallel,l',l)=\frac{16\pi}{\hbar L^2}\left[\frac{2\pi e^2 f_{\alpha\mu}}{\hbar S q\omega_{\alpha\mu}}\right]\frac{e^{-qL}}{1\pm\gamma}|B_l B_{l'}|^2|F^1_{\alpha\mu}(l',l)|^2\delta[E_1(\mathbf{k}_\parallel+\mathbf{q},l')-E_1(\mathbf{k}_\parallel,l)-\hbar\omega_{\alpha\mu}+\Lambda_1] , \tag{B22}$$

where

$$E_i(\mathbf{k}_\parallel,l)=\frac{\hbar^2 k_\parallel^2}{2m_i^*}+E_l,\quad i=1,2 .$$

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