Universality in the resistivity-temperature relationship for decagonal quasicrystals

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The temperature T dependence of the resistivity of decagonal single quasicrystals available at present has been carefully examined from 77 to 340 K. All the samples show a precisely linear T dependence of resistivity along the periodic direction, while in the quasicrystalline plane a quadratic term was added to fit the data. The linear T dependence in the large temperature range implies a very low transport Debye temperature, and can be easily understood by the nearly filled band model used to explain Hall-effect anisotropy. The T dependence of the resistivity in the quasicrystalline plane can be satisfactorily explained by the generalized Faber-Ziman theory, proving an enhanced electron-phonon interaction and an enhanced phonon-phonon coupling in the quasicrystalline plane.

The discovery of quasicrystals has raised great interest in how the new symmetry should affect the electronic properties of the materials. Recent results on the stable icosahedral quasicrystals^{1,2} as well as on decagonal single quasicrystals³ strongly suggest the interaction of the Fermi surface with a quasi-Brillouin-zone boundary, leading to a substantial decrease of the density of states at the Fermi surface. However, this kind of instability, known as the Hume-Rothery law, is predominately determined by the local structural symmetry, and has nothing to do with the long-range quasicrystalline order. One expects that the latter should put its fingerprint on the properties of phonons and, hence, the electron-phonon interaction which can be felt by electrons in their transport process if the electron-phonon scattering is not completely covered by defect scattering. Especially, the success in growing decagonal single quasicrystals provides an opportunity to compare the transport differences between the periodic and quasicrystalline directions. Indeed, great anisotropy has been observed in resistivity,⁴⁻⁶ thermopower,⁴ Hall effect,^{3,7} and thermal conductivity.⁸ Here we present our careful measurements of the T dependence of the resistivity on the decagonal single quasicrystals available at present. The results can consistently be explained by the generalized Faber-Ziman theory,⁹ proving an enhanced electron-phonon interaction in the quasicrystalline plane.

Four kinds of decagonal quasicrystals, Al-Si-Cu-Co, Al-Ni-Co, Al-Cu-Co No. 1, and Al-Cu-Co No. 2, were measured in our experiments. The details of the growth of Al-Si-Cu-Co and Al-Cu-Co No. 1 single quasicrystals were reported in Ref. 10. The Al-Ni-Co single quasicrystals were grown by a method similar to that reported in Ref. 11 and the Al-Cu-Co No. 2 single quasicrystals by the directional solidification method.¹² Their chemical compositions have been measured in detail.³ The composition of Al-Cu-Co No. 2 is different from that of Al-Cu-Co No. 1. The quasicrystals of all these materials were well formed columnar prisms with tenfold rotational symmetry and proved to be single domained over the whole sample which was usually a few tenths of a mm in diameter and several mm in length. The x-ray-diffraction

studies on all these materials showed diffraction peaks only slightly broader than those for good crystals, indicating the good quality of the quasicrystals. Convergent beam-diffraction patterns along the columnar axis displays Kikuchi lines with a regular intensity distribution revealing the tenfold rotational symmetry and no appreciable distortion.

The key problem in getting reliable results of the amplitude as well as the T dependence of resistivity is to avoid the mixing of the behaviors in different directions. By using needle-shape samples, it is not a serious problem to measure the resistivity along tenfold axis ρ^c , for which we have used the usual four-leads configuration. Every needle-shape sample has a length of $500-800 \ \mu m$ which is more than twelve times of the dimension of its cross section. The measurements of the resistivity in the quasicrystalline plane, ρ^{qc} , were made by the van der Pauw method,¹³ taking great care to ensure the samples' main planes perpendicular to the tenfold axis. The quasicrystals for measuring ρ^{qc} were cut and polished to thin slices of 2–10 μ m in thickness and 100–200 μ m in length as measured by scanning electron microscopy. For every sample the final check of the right alignment was the resistivity isotropy, i.e., when one of the current electrodes was interchanged with the voltage electrode at the opposite corner of the sample, one should get exactly the same T dependence. The electrodes were made by spotwelding Pt wires of 20 μ m in diameter and the resistivity was measured by the ac method with an EG&G Model 5210 lock-in amplifier. Altogether seven samples were measured for ρ^c and eight samples were measured for ρ^{qc} from 77 to 350 K. ρ^c for all the measured samples follows a linear T dependence (Fig. 1). The relative deviations of the measured data from the linear fit lines are all within the measured accuracy of $\pm 0.1\%$. The linear T dependence of ρ^c was observed down to ~40 K for Al-Si-Cu-Co (Ref. 4) and Al-Ni-Co (Ref. 14) below which ρ^c became T independent. The lack of a rapid drop of ρ^c , the large temperature range of linear dependence, together with the large α^c ($\alpha^c = d\rho^c/dT$), imply a very low transport Debye temperature Θ_T . Using the Grüneisen-

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FIG. 1. The normalized resistivity along the periodic direction. The solid curves are measured data, and the dashed curves are the linear fit curves (A + BT): (a) Al-Cu-Co No. 1, (b) Al-Ni-Co, (c) Al-Si-Cu-Co, (d) Al-Cu-Co No. 2.

Bloch equation¹⁵

$$\rho = \rho_0 + (\rho_{\Theta} - \rho_0) 4.226 \left[\frac{T}{\Theta_T} \right]^5 \\ \times \int_0^{\Theta_T / T} \frac{x^5 dx}{(e^x - 1)(1 - e^{-x})} , \qquad (1)$$

where ρ_0 is the residual resistivity, and ρ_{Θ} the resistivity at Θ_T , we can quantitatively estimate the transport Debye temperatures for the four kinds of decagonal quasicrystals. We find that every measured curve in Fig. 1 could be fitted by the Grüneisen-Bloch equation within $\pm 0.1\%$ when Θ_T in Eq. (1) is less than 32 K for Al-Ni-Co, 22 K for Al-Cu-Co No. 1, 56 K for Al-Cu-Co No. 2, and 40 K for Al-Si-Cu-Co, in strong contrast with the estimated dynamical Debye temperatures Θ_D .¹⁶

Figure 2 presents the results for ρ^{qc} . Satisfactory



FIG. 2. The normalized resistivity in the quasicrystalline plane. The solid curves are measured data, and the dashed curves are their quadratic polynomial fit curves $(A+BT+CT^2)$: (a) Al-Cu-Co No. 1, (b) Al-Ni-Co, (c) Al-Si-Cu-Co, (d) Al-Cu-Co No. 2.

agreement with the experimental data is obtained when a quadratic term is added to the linear term. We notice that the quadratic term is always negative while the coefficients of the linear term can change sign from sample to sample. A negative T^2 behavior was often found in metalline glasses and has been accounted for by the generalized Faber-Ziman theory.⁹ As we mentioned above, for decagonal quasicrystals the transport Debye temperature may be very low and the inelastic single-phonon scattering may be pretty strong. We argue that our data can satisfactorily fit the generalized Faber-Ziman theory in the high-T limit when both the inelastic single-phonon scattering and the multiphonon scattering are taken into account. The multiphonon scattering in the form of a Debye-Waller factor¹⁷ can be expanded at high T,

$$e^{-2W} \approx 1 - 2W \approx 1 - \beta T . \tag{2}$$

The resistivity is expressed as

$$\rho = (1 - \beta T)(\rho_0 + \alpha T) = \rho_0 + (\alpha - \rho_0 \beta)T - \alpha \beta T^2 , \qquad (3)$$

where ρ_0 is the residual resistivity, α the parameter describing electron-phonon interaction, and β the parameter describing phonon-phonon coupling. There is no adjustable parameter in Eq. (3), and we can unambiguously determine ρ_0 , α , and β for every sample. Equation (3) shows that we should always have a negative quadratic term. But the linear term can be either positive or negative, depending on the competition of inelastic singlephonon scattering with the multiphonon scattering.

An interesting result of our measurements is that the parameter β in Eq. (3) is nearly a constant in the quasicrystalline plane ($\beta^{qc} \approx 6 \times 10^{-4} \text{ K}^{-1}$). This can be clearly seen from Fig. 3 where β is plotted as a function of α . In the figure we also show the data taken from Refs. 5 and 6 as well as the data for metallic glasses,¹⁸ all of which can be fitted with Eq. (3). The phonon-phonon



FIG. 3. The phonon-phonon coupling parameter (β) plotted as a function of the electron-phonon interaction parameter (α) in the quasicrystalline plane. •, \blacktriangle , and \blacksquare denote the data for Al-Ni-Co, \bigcirc and \square for Al-Cu-Co, and \bigcirc for Al-Si-Cu-Co. •, \bigcirc , and \bigcirc are our data, \blacktriangle taken from Ref. 5, and \blacksquare and \square taken from Ref. 6. As a contrast, the data for Ag-Cu-Ge metallic glass taken from Ref. 18 are also included in the figure (*).

	$ ho_0^c$ ($\mu \Omega { m cm}$)	$ ho_0^{ m gc}$ ($\mu\Omega$ cm)	$ ho_0^{gc}/ ho_0^c$	$lpha^c$ ($\mu\Omega \mathrm{cm/K}$)	$lpha^{ m qc}$ ($\mu\Omega m cm/K$)	α^{qc}/α^{c}	$egin{array}{c} eta^{ m qc} \ (10^{-4}/ m K) \end{array}$	$ \beta^{c} $ (10 ⁻⁴ /K)
AlNiCo	27	164	6.1	0.030	0.13	4.3	4.89	< 0.21
AlSiCuCo	50	429	8.6	0.047	0.12	2.6	5.87	< 0.26
AlCuCo No. 1	53	245	4.6	0.100	0.19	1.9	4.93	< 0.14
AlCuCo No. 2	39	405	10.4	0.025	0.18	7.0	6.07	< 0.34

TABLE I. The three parameters in Eq. (3), ρ_0 , α , and β , for the four kinds of single quasicrystals. Notice the great differences between directions.

coupling parameter for the quasicrystalline plane, β^{qc} , is independent of the electron-phonon interaction parameter, α^{qc} .

To describe the behavior along the crystalline axis by Eq. (3), we conclude that the absolute value of the phonon-phonon coupling parameter β^c is always less than 4×10^{-5} K⁻¹ for all the measured quasicrystals (Table I). From the table we see that the three parameters, ρ_0 , α , and β in the quasicrystalline plane are always much larger than that along the crystalline axis. The large α and β may imply an enhanced electron-phonon interaction and an enhanced phonon-phonon coupling in the quasicrystalline plane, respectively.

The enhanced electron-phonon interaction and phonon-phonon coupling can also be seen in Fig. 3 which shows that both the electron-phonon interaction parameter α and the phonon-phonon coupling parameter β in the quasicrystalline plane of decagonal quasicrystals are larger than that in metallic glasses. The enhancement of electron-phonon interaction can be clearly demonstrated by the contribution of phonon scattering to the resistivity normalized to each current carrier which is equivalent to the reciprocal of mobility, $\mu_{ph}^{-1} = \alpha T/R_H$, where R_H is the Hall coefficient, ³ the ideal resistivity per carrier, $(\mu_{ph}e)^{-1}$, is plotted in Fig. 4 against T. Obviously, for all



FIG. 4. The specific resistivity (see text) $(\mu_{ph}e)^{-1}$ as a function of *T*. Notice that $(\mu_{ph}e)^{-1}$ in the quasicrystalline plane is always higher than that along the crystalline axis. •, **A**, **I**, and **★** denote the data for Al-Ni-Co, Al-Si-Cu-Co, Al-Cu-Co No. 1, and Al-Cu-Co No. 2 quasicrystals in the quasicrystalline plane, while \circ , \triangle , \Box , and **★** denote the data for Al-Ni-Co, Al-Si-Cu-Co, Al-Si-Cu-Co, Al-Cu-Co No. 1, and Al-Cu-Co No. 1, and Al-Cu-co No. 2 quasicrystals in the crystalline axis, respectively.

the Al-Ni-Co, Al-Cu-Co, and Al-Si-Cu-Co quasicrystals, $(\mu_{\rm ph}e)^{-1}$ in the quasicrystalline plane is substantially larger than that along the tenfold axis, proving an enhancement of electron-phonon interaction in the quasicrystalline plane. The electron-phonon enhancement effect was also shown by thermopower measurements.⁴

The above description is phenomenological in the sense that they have not been touched with the specific structure of the decagonal quasicrystals. In the following, we will try to take into account the specific structure for explaining the low transport Debye temperature and the multiphonon scattering process.

The essence of the deviation of Θ_T from Θ_D is that while all of the phonon modes contribute to Θ_D , not all of them are equally effective in the electron-phonon scattering process. The latter process greatly depends on the structures of the Fermi surface in addition to the phonon spectrum. Since the phonon energies are much lower than the Fermi energy, the electron-phonon scattering occurs only with the states prior and after the scattering confined within a thin shell of the Fermi surface and observing the momentum conservation law

$$\mathbf{q} = \mathbf{k}_2 - \mathbf{k}_1, \quad (|\mathbf{k}_1| \approx |\mathbf{k}_2| \approx |\mathbf{k}_F|), \quad (4)$$

where k_1 and k_2 are the momenta of the initial and the final state of the electron, respectively, **q** the phonon momentum, and \mathbf{k}_F the Fermi wave vector. Very low Θ_T means that mainly low-frequency phonons are involved in the process.

In Ref. 3, we presented an electron structure model successfully explaining the Hall-effect anisotropy in decagonal quasicrystals. In the nearly filled quasi-Brillouin zone the Fermi surface of electrons can be equated to a Fermi surface of holes with very low Fermi energy, similar to the treatment in semiconductors. In this case, only low-frequency phonons couple with carriers and give substantial contribution to the resistance, leading to the low transport Debye temperatures estimated above. An alternative treatment is that, since the Fermi surface is very close to the quasi-Brillouin boundary, the Umklapp process, with low-frequency phonons taking part in, may be much more important than for ordinary metals. This is supported by the fact that the ratio Θ_T / Θ_D for alkali metals is close to or larger than unity but Θ_T / Θ_D for alkaline-earth metals is markedly smaller.¹⁹

In quasicrystals, the phonon momentum q is not a good quantum number. In other words, phonons are strongly coupled with each other in the material. If an electron is scattered by a phonon, it would be also scat-

tered by those phonons coupled with this phonon. This provides an explanation why the multiphonon scattering is found in the quasicrystalline plane and not found along the crystalline axis of the decagonal quasicrystals.

In conclusion, the careful measurements of the T dependence of resistivity in decagonal single quasicrystals show some universalities of these decagonal quasicrystals. The precisely linear temperature dependence in the crystalline axis strongly suggests a very low transport Debye temperature. The measured curves in the quasicrystalline plane can be satisfactorily explained by the generalized

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Faber-Ziman theory, which means multiphonon scattering plays an important role in the quasicrystalline plane. The data also show that the phonon-phonon coupling in the multiphonon scattering process does not depend on the electron-phonon interaction, and there is an enhancement of electron-phonon interaction in the quasicrystalline plane compared to that along the periodic direction.

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