Superconductivity in a high magnetic field: Excitation spectrum and tunneling properties

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The quasiparticle excitation spectrum of a type-II superconductor placed in a high magnetic field near $H_{c2}(T)$ is shown to be gapless. The gap turns to zero at the points in the magnetic Brillouin zone that are in correspondence with the vortex lattice in real space. When the field decreases below a certain critical value, branch crossings occur and gaps start opening up at the Fermi surface. The strong dispersion around the gapless points leads to an algebraic temperature dependence in the thermodynamic functions and the algebraic voltage dependence in the tunneling conductance between the microscope tip and superconductor in a scanning tunneling microscope experiment. The crossover from the localized midgap states at the core of an isolated vortex to the coherent quasiparticle band at high fields is reflected in a qualitative change of differential conductance.

I. MOTIVATION

The properties of a superconductor placed in an external magnetic field have been a subject of considerable interest for a long time. Particularly significant in this context is the mixed phase of a type-II superconductor in which the external field can coexist with superconductivity in the form of a quantized flux lattice. Discovery of high-temperature superconductors (HTS's), exhibiting strongly type-II behavior, has only fueled further intense studies of such systems. The familiar Abrikosov-Gorkov (AG) microscopic theory based on the semiclassical approximation[1] for a magnetic field yields a very good description of most conventional type-II materials. This theory completely neglects Landau level quantization of electronic energies in a magnetic field, which is justified for a range of fields and temperatures such that $\hbar\omega_c \ll k_BT$ in a clean system ($\hbar\omega_c = eH/mc$). For large impurity concentrations this condition translates to $\omega_c \ll 2\pi\tau^{-1}$, where τ is the scattering lifetime. Under these conditions electrons occupy a huge number of closely separated Landau levels so that either temperature or impurity scattering completely erases the significance of the quantized energy levels in a magnetic field. Recently Tešanović *et al.* examined the opposite limit to the one described by AG theory and discovered that the inclusion of Landau levels leads to reentrant behavior at high fields[2] where the superconductivity is enhanced by a magnetic field. Numerous other interesting effects are predicted as manifestations of the Landau-level structure in superconductors.[2] This behavior is mostly pronounced in the low-carrier-density systems where the high-field limit can be achieved by application of experimentally available fields. It is well known that HTS's are *inherently* strongly type-II systems, i.e., their behavior in a magnetic field is not the consequence of doping the materials by impurities. Instead, they are actually quite clean systems with the strong type-II behavior being due to their low carrier densities. There is a sharply defined Landau level structure in such a superconductor and it should be included in any complete study of HTS's in a magnetic field. We expect that the effects of the Landau-level quantization will be most pronounced at low temperatures and high fields, near the semiclassical $H_{c2}(T)$.

Particularly interesting in this context is the problem of the quasiparticle excitation spectrum in the mixed state of the superconductor. Recent scanning tunneling microscope (STM) experiments[3] have revealed the local distribution of quasiparticle states in the vortex core. Several theoretical works have followed[4–6] explaining experimental results of Hess and co-workers, based on the solution of the Bogoliubov-deGennes (BdG) equations for the quasiparticle excitation spectrum in the isolated vortex case. This situation is obtained when the external magnetic field is rather low (typically $10^{-2} - 10^{-3}$ T) so that vortices are well separated. It is natural to inquire what would be the result of such an STM experiment at higher fields (> 1 T). Such an experiment would probe the electronic structure of the vortex lattice since the STM probe would be able to scan more than just a single vortex. A clean HTS sample would be an excellent candidate for such an experiment: at low temperature and high fields the mean free path of the electrons in these systems will become much longer than the separation of the vortices. Therefore, the quasiparticle excitations will propagate *coherently* through many unit cells of the vortex lattice. This coherent propagation will lead to qualitatively new features in the STM pattern.

In Sec. II of this paper we present the solution of BdG equations for the quasiparticle excitation spectrum of the vortex lattice. We show that the type-II superconductor in a high magnetic field has a gapless excitation spectrum with the strong dispersion around gapless points at the Fermi surface. This high-field gapless behavior, first discussed by Dukan *et al.*,[2] is dictated by topological considerations and is shown here to persist even relatively far below the $H_{c2}(T)$ line. We also discuss the mechanism of the gap opening in lower fields. In Sec. III we discuss the behavior of the thermodynamic properties and the density of states for such a gapless superconductor. In Sec. IV we show that the S-N-S tunneling conductance and the STM conductance of the superconductor in a high magnetic field have an algebraic voltage dependence.

II. THE QUASIPARTICLE EXCITATION SPECTRUM

We consider a three-dimensional (3D) weakly interacting electronic system in a magnetic field with the model interaction $V(\mathbf{r}_1,\mathbf{r}_2) = -V\delta(\mathbf{r}_1-\mathbf{r}_2)$, arising from the electron-phonon and electron-electron pairing mechanism. We assume V is only weakly dependent on magnetic field. The Hartree-Fock Hamiltonian for such a system is

$$H_{\rm HF} = \frac{1}{2m}\sum_\alpha \int \Psi^\dagger_\alpha(\mathbf{r})\left(-i\hbar\nabla + \frac{e}{c}\mathbf{A}\right)^2 \Psi_\alpha(\mathbf{r})d^3r + \int \Delta(\mathbf{r})\Psi^\dagger_\uparrow(\mathbf{r})\Psi^\dagger_\downarrow(\mathbf{r})d^3r + \text{H.c.} \ , \quad (1)$$

where $\Delta(\mathbf{r})$ is the superconducting order parameter given by the self-consistent equation

$$\Delta(\mathbf{r}) = V\langle\Psi_\uparrow(\mathbf{r})\Psi_\downarrow(\mathbf{r})\rangle. \quad (2)$$

We take the order parameter to be uniform along the field direction. In the mean field (MF) approximation $\Delta(\mathbf{r})$ forms the Abrikosov lattice of vortices that in the Landau gauge $\mathbf{A} = H(-y,0,0)$ has the form[7]

$$\Delta_0(\mathbf{r}) = \Delta\sum_n \exp\left(\pi\frac{b_x}{a}n^2\right) \times \exp\left[i2\pi nx/a - (y/l_H + \pi n l_H/a)^2\right] \quad (3)$$

where $l_H = \sqrt{\hbar c/eH}$ is the magnetic length and Δ is the amplitude. The vortex lattice is characterized by unit vectors $\mathbf{a} = (a,0,0,)$ and $\mathbf{b} = (b_x,b_y,0)$ ($b_x = 0$, $b_y = a$ for quadratic lattice and $b_x = \frac{1}{2}a$, $b_y = \frac{\sqrt{3}}{2}a$ for triangular lattice). The flux through the unit cell is given by the expression $ab_y = \pi l_H^2$. The above form of the order parameter is entirely contained in the lowest Landau level of the Cooper charge $2e$ and represents the excellent approximation as long as the region of interest is close to the $H_{c2}(T)$ line in a phase diagram. In lower fields, $H \approx H_{c2}(T)/(2j+1)$, $\Delta(\mathbf{r})$ contains contributions from higher Landau levels ($j \geq 1$) that can be easily obtained by the action of the operator[8]

$$\Pi^\dagger(\mathbf{r}) = \frac{l_H}{2}\left(-i\frac{\partial}{\partial x} - \frac{\partial}{\partial y} + 2\frac{y}{l_H^2}\right) \quad (4)$$

j times on $\Delta_0(\mathbf{r})$ yielding

$$\Delta_j(\mathbf{r}) = \Delta_j\sum_n \exp\left(i\pi\frac{b_x}{a}n^2\right) \times \exp\left[i2\pi n\frac{x}{a} - \left(\frac{y}{l_H} + \frac{\pi n}{a}l_H\right)^2\right] \times H_j\left[\sqrt{2}\left(\frac{y}{l_H} + \frac{\pi n}{a}l_H\right)\right]. \quad (5)$$

$H_j(x)$ is the Hermite polynomial of the order j. Amplitudes Δ_j can be generated from the self-consistent equation (2). Because of the symmetry, only $\Delta_{j=4k}(\mathbf{r})$ for the quadratic and $\Delta_{j=6k}(\mathbf{r})$ for the triangular lattice will contribute to the general form of the order parameter $\Delta(\mathbf{r}) = \sum_j \Delta_j(\mathbf{r})$. Only these functions have the same position and vorticity of zeros to produce the order parameter of the correct symmetry. Nevertheless, we can show that the higher level contributions ($k \neq 0$) are not crucial for our problem. They introduce extra wiggles in the spatial dependence of the order parameter far from the vortex positions and do not bring any new essential features in the form of the excitation spectrum.

In order to diagonalize the Hamiltonian (1), we use the magnetic sublattice representation[9] with basis functions characterized by quasimomentum $\mathbf{q}$ perpendicular to the direction of the field. Since the electronic charge is half as large as the Cooper pair charge, we choose the electronic unit cell spanned by vectors $2\mathbf{a}$ and $\mathbf{b}$ in order to enclose one full electronic flux in the unit cell. Then, the Magnetic Brillouin Zone (MBZ) is defined by vectors $\mathbf{a}^* = (b_y/l_H^2, -b_x/l_H^2)$ and $\mathbf{b}^* = (0, 2a/l_H^2)$. In this basis we can write the BdG transformations[10] as

$$\Psi_\uparrow(\mathbf{r}) = \sum_{k_z,\mathbf{q},n}\left[u_{k_z,\mathbf{q},n}c_{\uparrow k_z,\mathbf{q},n} - v^\dagger_{-k_z,-\mathbf{q},n}c^\dagger_{\downarrow -k_z,-\mathbf{q},n}\right] \times\phi_{k_z,\mathbf{q},n}(\mathbf{r}),$$
$$\Psi_\downarrow(\mathbf{r}) = \sum_{k_z,\mathbf{q},n}\left[u_{k_z,\mathbf{q},n}c_{\downarrow k_z,\mathbf{q},n} + v^\dagger_{-k_z,-\mathbf{q},n}c^\dagger_{\uparrow -k_z,-\mathbf{q},n}\right] \times\phi_{k_z,\mathbf{q},n}(\mathbf{r}) \quad (6)$$

where $\phi_{k_z,\mathbf{q},n}(\mathbf{r})$ are the eigenfunctions of the magnetic translation group (MTG) in the Landau gauge belonging to the nth Landau level:

$$\phi_{k_z,\mathbf{q},n}(\mathbf{r}) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}l_H}}\sqrt{\frac{b_y}{L_xL_yL_z}}\exp(ik_z\zeta)\sum_m \exp\left(i\frac{\pi b_x}{2a}m^2 - imq_yb_y\right) \times\exp\left\{i\left(q_x + \frac{\pi m}{a}\right)x - 1/2\left[\frac{y}{l_H} + \left(q_x + \frac{\pi m}{a}\right)l_H\right]^2\right\}H_n\left[\frac{y}{l_H} + \left(q_x + \frac{\pi m}{a}\right)l_H\right]. \quad (7)$$

ζ is the spatial coordinate and k_z is the momentum along the field direction. Product $L_xL_yL_z$ is the volume of the system.

The Cooper pairs are formed from the electrons having opposite crystalline momenta $\mathbf{q}$ and spins within the same Landau level (diagonal pairing) and from the electrons belonging to the Landau levels separated by $\hbar\omega_c$ or more (off-diagonal pairing). In sufficiently high magnetic fields so that $\Delta \ll \hbar\omega_c$ we can use the quantum limit approximation (QLA) (see Dukan *et al.* in Ref. 2) which takes into account only diagonal pairing and ignores off-diagonal pairing completely. This is justified only if an additional condition is fulfilled, i.e., if the number of occupied Landau levels n_c is less than $\approx E_F/T_{c0}$, where E_F is the Fermi energy and T_{c0} is the zero-field transition temperature. This is a situation that can readily be achieved in low-carrier-density systems by application of fields in the 10–30 T range. In lower fields, where n_c is large, off-diagonal terms have to be included as well: their number grows as n_c^2, while the number of diagonal terms grows as n_c and for sufficiently large $n_c \gg E_F/T_{c0}$ these terms will eventually come to dominate. For the moment, we ignore Zeeman splitting but we will show below how our results can be generalized to the case when the Zeeman effect is included.

Taking the order parameter in the form $\Delta(\mathbf{r}) = \sum_j \Delta_j(\mathbf{r})$ and after performing the BdG transformations (6) we get the following set of equations:

$$E^N_{k_z,\mathbf{q}} u^N_{k_z,\mathbf{q},n} = \left(\frac{\hbar^2 k_z^2}{2m} + \hbar\omega_c(n+1/2) - \mu\right) u^N_{k_z,\mathbf{q},n} + \sum_m \Delta_{nm}(\mathbf{q}) v^N_{k_z,\mathbf{q},m},$$

$$-E^N_{k_z,\mathbf{q}} v^N_{k_z,\mathbf{q},n} = \left(\frac{\hbar^2 k_z^2}{2m} + \hbar\omega_c(n+1/2) - \mu\right) v^N_{k_z,\mathbf{q},n} - \sum_m \Delta_{mn}(\mathbf{q})^* u^N_{k_z,\mathbf{q},m} \qquad (8)$$

where $\Delta_{nm}(\mathbf{q}) = \sum_j \Delta^j_{n,m}(\mathbf{q})$ is the matrix element of $\Delta(\mathbf{r}) = \sum_j \Delta_j(\mathbf{r})$ between electronic states $(k_z, \mathbf{q}, n)$ and $(-k_z, -\mathbf{q}, m)$ and is given by

$$\Delta^j_{nm}(\mathbf{q}) = \frac{\Delta_j}{\sqrt{2}} \frac{(-1)^m}{2^{n+m}\sqrt{n!m!}} \frac{(n+m)!}{(n+m-j)!} \times \sum_k \exp\left(i\pi \frac{b_x}{a} k^2 + 2ikq_y b_y - (q_x + \pi k/a)^2 l_H^2\right) H_{n+m-j}\left[\sqrt{2}\left(q_x + \frac{\pi k}{a}\right) l_H\right]. \qquad (9)$$

$\Delta^j_{nm}(\mathbf{q})$ constitutes a magnetic lattice in $\mathbf{q}$-space (i.e., a lattice which is invariant under MTG transformations in $\mathbf{q}$-space) and belongs to the $(n+m-j)$th Landau level of charge $e/2$ in this space. In the same way as in real space, the operator (4) can be constructed in $\mathbf{q}$-space:

$$\Pi^\dagger(\mathbf{q}) = \frac{1}{2l_H}\left(-i\frac{\partial}{\partial q_y} - \frac{\partial}{\partial q_x} + 2q_x l_H^2\right) \qquad (10)$$

in order to obtain all $\Delta^j_{nm}(\mathbf{q})$ from $\Delta^{j=0}_{00}(\mathbf{q})$ belonging to the lowest level in this space. The behavior of $\Delta^j_{nm}(\mathbf{q})$ is very important for the excitation spectrum and should be investigated in detail. The set of $\Delta^j_{nm}(\mathbf{q})$ can be classified by the position and order of its zeros: All $\Delta^j_{nm}(\mathbf{q})$ with $(n+m-j+4k)$ for quadratic and $(n+m-j+6k)$ for triangular lattices (k is an integer) have the same set of zeros with the similar dispersion around each zero. They differ considerably only far from these points. Therefore, the contributions $\Delta_{j=4k}(\mathbf{r})$ $[\Delta_{j=6k}(\mathbf{r})]$ with $k \neq 0$ for quadratic (triangular) lattices to $\Delta(\mathbf{r}) = \sum_j \Delta_j(\mathbf{r})$ have similar matrix elements around singular points to the matrix elements $\Delta^{j=0}_{nm}(\mathbf{q})$ obtained for the order parameter entirely in the lowest Landau level. Their inclusion does not bring any new qualitative feature in the form of the excitation spectrum and we proceed by taking only $j = 0$ contributions to $\Delta(\mathbf{r})$. In high magnetic fields ($\Delta \ll \hbar\omega_c$) we follow the QLA and ignore the off-diagonal pairing. Then, BdG equations (8) can be solved analytically yielding the quasiparticle excitation spectrum of the form

$$E_{k_z,\mathbf{q},n} = \pm\sqrt{\left[\frac{\hbar^2 k_z^2}{2m} + \hbar\omega_c(n+1/2) - \mu\right]^2 + |\Delta_{nn}(\mathbf{q})|^2}. \qquad (11)$$

There are n_c gapless branches in the above spectrum. The gap $\Delta_{nn}(\mathbf{q})$ turns to zero on the Fermi surface at the set of points $\{q_j = q_{yj} + iq_{xj}\}$ in the MBZ which are in direct correspondence with the position of the vortices $\{z_i = x_i + iy_i\}$, e.g., $q_j l_H = z_i/l_H$. While these zeros are of the first order, we have found that $n = 1+2k$ branches for quadratic and $n = 2+3k$ branches for triangular lattices have in addition zeros of the second and third order, respectively. Configuration of the zeros in the MBZ and corresponding vorticities are such as to preserve exactly one positive vorticity per unit cell of the order parameter in real space. Figure 1 shows the $E_{k_F,\mathbf{q},0}$ branch in the spectrum (11) of the triangular vortex lattice.

Lowering the magnetic field (but still in the region of $\Delta \leq \hbar\omega_c$), the number of occupied Landau levels grows and the off-diagonal coupling becomes important (see the discussion above). The Cooper pairs are formed from the electrons in states $(k_z, \mathbf{q}, n)$ and $(-k_z, -\mathbf{q}, n \pm m)$ where $m \ll \Omega_D/\hbar\omega_c$ (Ω_D is a Debye frequency). Inclusion of the off-diagonal matrix elements $\Delta_{nm}(\mathbf{q})$ makes solving the BdG equations (8) a cumbersome problem of diagonalizing the $2(n_c+M) \times 2(n_c+M)$ matrix that can be done numerically. Initially, it seems that the inclusion of the off-diagonal matrix elements $\Delta_{nm}(\mathbf{q})$ destroys the gapless behavior of the excitation spectrum (11): Off-diagonal matrix elements $\Delta_{nn+k}(\mathbf{q})$, where k is an odd integer, have zeros on the lattice dual to the lattice formed by zeros of diagonal $\Delta_{nn}(\mathbf{q})$ and there are no points in the MBZ where zeros of all matrix elements coincide. However, our numerical results show that there are always n_c gapless branches in the excitation spectrum with the position of zeros in the (q_x, q_y) plane exactly the same as those found within QLA. The role of off-diagonal matrix elements is to shift the value of Fermi momentum k_{Fn} at which the gapless behavior occurs in QLA to the new value k'_{Fn} estimated from the condition

$$\frac{\hbar^2 k'^2_{Fn}}{2m} - \frac{\hbar^2 k^2_{Fn}}{2m} \approx \left(|\Delta_{nn-k}(\mathbf{q}_j)|^2 - |\Delta_{nn+k}(\mathbf{q}_j)|^2\right)/\hbar\omega_c, \qquad (12)$$

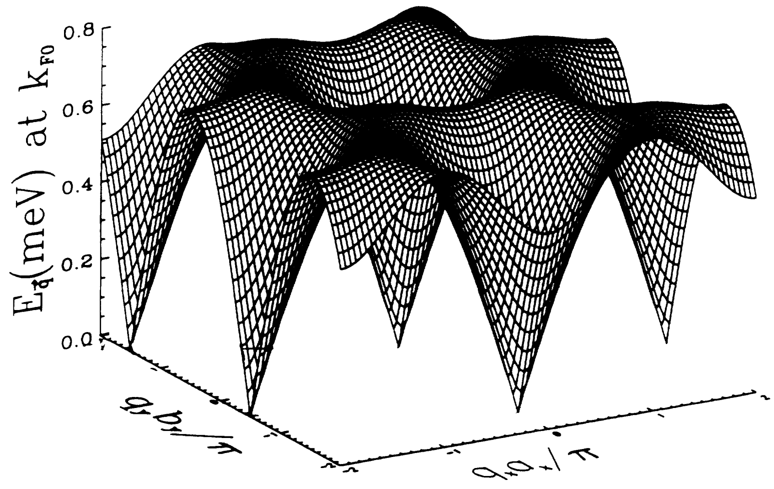

FIG. 1. $E_{k_F,\mathbf{q},0}$ branch in the spectrum (11) of the triangular vortex lattice when $\Delta/\hbar\omega_c = 1/20$.

where n is the label of the Landau level. The shift (12) corresponds to the change in self-energies of the normal electronic Green functions by the inclusion of the off-diagonal pairing. Figure 2 shows one of the gapless branches in the excitation spectrum that occurs at the Fermi momentum k'_{F3} determined from condition (12) as $\hbar^2 k'^2_{F3}/2m \approx 0.85(\Delta^2/\hbar\omega_c)$ for the case when all the Landau levels separated by $\hbar\omega_c$ and $2\hbar\omega_c$ are coupled. The gapless feature of the excitation spectrum is a direct consequence of the behavior of the order parameter in real space: Accommodating the magnetic field in the interior of the superconductor, the order parameter assumes a nonuniform periodic configuration with exactly one flux quantum per unit cell. This topology is directly reflected in the energy spectrum and results in the gapless behavior of the quasiparticle excitation spectrum. This result will hold as long as the region of interest in the phase diagram is far enough from the $H_{c1}(T)$ line so that the magnetic field inside the superconductor is approximately uniform and as long as the temperature is low enough for the quasiparticles to propagate coherently over many unit cells of the vortex lattice.

If the magnetic field is lowered even further so that $\Delta \geq \hbar\omega_c$, mixing of Landau levels becomes very strong and the Landau level structure is not very well defined anymore. Monitoring the shift in k_{Fn} as a function of $\Delta/\hbar\omega_c$, we find that when $\Delta/\hbar\omega_c$ reaches the critical value estimated from the relation

$$\left(\frac{\Delta}{\hbar\omega_c}\right)_{\text{critical}} \approx \frac{1}{2f_{nn+1}} \qquad (13)$$

nth and $(n+1)$th gapless branches cross each other. Increasing $\Delta/\hbar\omega_c$ (corresponds to decrease in magnetic field) above the critical value (13) opens the gap at the Fermi surface in this branch. The "form factor" f_{nn+1} ($n = 0, 2, 4, \ldots$ for the quadratic and $n = 0, 3, 6, \ldots$ for the triangular lattice) behaves as $f_{nn+1} \sim \frac{1}{2^n n}$ for n large, so that the critical value of $\Delta/\hbar\omega_c$ increases as more and more Landau levels cross the Fermi surface. The above branch crossing is similar to the behavior described in Ref. 11, where the influence of an ordinary external periodic potential on a 2D electron system in a magnetic field was investigated.

In the discussion so far, we have ignored the Zeeman term $-g\mu_B\hat{\sigma}\cdot\mathbf{H}(\mathbf{r})$ which is justified for lot of materials that have very small effective g factors. Now, we show how our results can be generalize for the case when $g \neq 0$. The interesting situation happens when $g \approx 2$, e.g., the

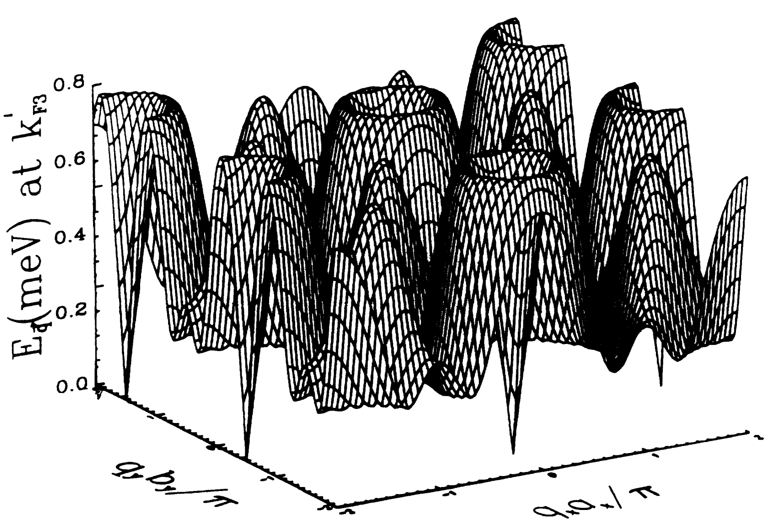

FIG. 2. $E(k'_{F3}, \mathbf{q})$ branch in the quasiparticle excitation spectrum of the triangular vortex lattice when $\Delta/\hbar\omega_c = 1/5$ (off-diagonal pairing is included).

Zeeman splitting is closed to the cyclotron splitting making the nth spin-up Landau level nearly degenerate with the $(n+1)$th spin-down one. In this case the lattice of zeros found in the $g=0$ case makes a transition to the dual lattice described above for the off-diagonal matrix elements $\Delta_{nm}(\mathbf{q})$ with $n+m$ odd. Then, when $g \approx 4$ the dual lattice transforms back to the original lattice, and so on. When $0 < g < 2$ one should pair electrons with the momenta along the field axis k_z and $-k_z + q_{zn}$ for all possible q_{zn} required to offset the Zeeman splitting. In this case, one should consider a nonuniform (along the field direction) order parameter of the form $\Delta(x,y)\exp(\sum_n q_{zn}\zeta)$, where $\Delta(x,y)$ is given by (3). The diagonalization of the BdG equations (8) is a very complicated task now due to the determination of the vectors $\mathbf{q}_{zn}$. Nevertheless, we still find the gapless points in the excitation spectrum for the case where only few Landau levels are occupied.

III. THERMODYNAMIC PROPERTIES AND DENSITY OF STATES

The strong dispersion around gapless points at the Fermi surface leads to unusual temperature behavior of the thermodynamic functions at low temperatures. Our calculations show that the heat capacity at low temperatures behaves as

$$c_V \approx \frac{k_B}{(2\pi l_H)^2}\left(\frac{\Delta}{\hbar v_F}\right)\frac{N_g}{a_n^2}\left[\left(\frac{k_B T}{\Delta}\right)^3 + a_n^{\alpha-1}\left(\frac{k_B T}{\Delta}\right)^{\alpha}\right] \tag{14}$$

where $\alpha = 2$ for the quadratic and $\alpha = 5/3$ for the triangular lattice. N_g is the number of the gapless branches at some value of $\Delta/\hbar\omega_c$ (see the discussion above). The $(k_BT/\Delta)^\alpha$ behavior in (14) is due to the presence of second- (third-) order zeros for the quadratic (triangular) lattice in some of the branches of the energy spectra, while the $(k_BT/\Delta)^3$ behavior is a consequence of linear dispersion around first-order zeros in all of the gapless branches. Coefficient a_n in (14) that measures slope around zero, depends strongly on the chosen lattice symmetry but also on the strength of the magnetic field. In lower fields more Landau levels cross the Fermi surface and mix together making the slope around zero steeper (e.g., increasing a_n, compare Fig. 1 and Fig. 2). Furthermore, it was mentioned before that in lower fields the contribution to the order parameter from higher Landau levels should be included as well. This will introduce wiggles in the region between zeros in $\mathbf{r}$-space, making the slope around zero in the energy spectra even steeper. Also, as the value of $(\frac{\Delta}{\hbar\omega_c})$ increases the number of gapless branches N_g decreases, making the algebraic temperature dependence in (14) weaker.

It is interesting to see how the superconducting density of states changes from the standard BCS form due to the presence of zeros at the Fermi surface in the quasiparticle excitation spectra. Our calculations show that the low-energy density of states per gapless branch of the spectrum has a behavior

$$N_s(E) \approx N_f(0)\left[\frac{1}{a_n}\left(\frac{E}{\Delta}\right)^2 + \frac{1}{a_n^\alpha}\left(\frac{E}{\Delta}\right)^\alpha\right] \tag{15}$$

where $\alpha = 1$ for the quadratic and $\alpha = 2/3$ for the triangular lattice. $N_f(0)$ is the density of states of the free 3D system of electrons in a magnetic field at the Fermi level. The term $(\frac{E}{\Delta})^2$ in (15) comes from first-order zeros while $(\frac{E}{\Delta})^\alpha$ is due to second- (third-) order zeros in the gapless branches of the excitation spectrum of the quadratic (triangular) vortex lattice.

IV. THE TUNNELING PROPERTIES

In this section we present a theoretical study of various tunneling characteristics of a superconductor in a high magnetic field exhibiting the gapless behavior described above. We present the results for the triangular vortex lattice that is known to be the lowest energy state in the mixed phase. First, we consider a simple problem of tunneling between two superconductors separated by a thin insulating layer. The geometry is such that the tunneling occurs primarily along the vortex lines. The tunneling problem was studied by Bardeen[12] and Cohen *et al.*[13] in the approximation of the semiconductor band model where the chemical potentials of two superconductors differ by the applied voltage, i.e. $eV = \mu_L - \mu_R$. The potential drop eV occurs in the insulating region between superconductors, which is typically a metal oxide. Cohen *et al.* introduced the concept of a tunneling Hamiltonian:

$$H_T = \sum_{[k],[p]} T_{kp} c_k^\dagger c_p + \text{H.c.}\,, \tag{16}$$

where c_k (c_p) are set of operators describing electrons in a left (right) superconductor. The tunneling matrix element T_{kp} transfers particles through the insulating layer and is not spin dependent. The tunneling in the superconductors will take place over a very narrow span of energies around the Fermi level, therefore it is adequate to treat the transfer rate T_{kp} as constant T_0 evaluated at $[k_F]$ and $[p_F]$. This is a proper formalism for the voltages in our problem ($eV \ll \Delta \approx 1$ meV).

Following Ref. 14, tunneling current through the insulating layer between the superconductors is given by

$$I(V,T) = 2e\sum_{[k],[p]} |T_{kp}|^2 \int_{-\infty}^{+\infty} \frac{d\varepsilon}{2\pi} A_R(k,\varepsilon)A_L(p,\varepsilon+eV) \times[n_F(\varepsilon) - n_F(\varepsilon+eV)] \tag{17}$$

where $A_R(k,\varepsilon)$ and $A_L(p,\varepsilon+eV)$ are the spectral functions of the right and left superconductor, respectively. $[k] = (\mathbf{k},k_z,n)$ ($[p] = (\mathbf{p},p_z,m)$) is a set of quantum numbers representing the quasiparticle states in the left (right) superconductor and $n_F(\varepsilon)$ is a Fermi distribution. We anticipate that the most significant contribution to the tunneling current for small voltages ($eV \ll \Delta$) will come from the states around the gapless points in the quasiparticle excitation spectrum. In the vicinity of these special points $(\mathbf{q}_i, k_F)$ at the Fermi surface the energy spectrum (11) can be approximated as

$$E(\mathbf{q},k_z) \approx \sqrt{\left(\frac{E_F}{k_F}k_z\right)^2 + \Delta^2 a_\alpha^2 (q_x^2 l^2 + q_y^2 l^2)^\alpha} \tag{18}$$

where α is the order of the zero, while the coefficient a_α

measures slope around zero. E_F is the Fermi level and k_F is the Fermi momentum. We are interested in the tunneling along the vortex lines, i.e., along the direction of magnetic field. For such a tunneling, in-plane quasimomenta are conserved up to the reciprocal lattice vector in the first MBZ. The quasiparticle from the left with quasimomentum $\mathbf{q} \approx \mathbf{q}_i$ can tunnel in four equivalent states in the right superconductor with the same contribution to the tunneling current. After some algebra expression (17) reduces to

$$I_{||}(V,T) \approx \frac{16\pi e}{\hbar} N_{gR} N_{gL} |T_0|^2 \sum_{\mathbf{q}} \sum_{k_z, q_z > 0} [n_F(E_{\mathbf{q}k_z}) - n_F(E_{\mathbf{q}q_z})]\delta(E_{\mathbf{q}q_z} - eV - E_{\mathbf{q}k_z}) + [n_F(E_{\mathbf{q}q_z}) - n_F(E_{\mathbf{q}k_z})]\delta(E_{\mathbf{q}k_z} - eV - E_{\mathbf{q}q_z}) \quad (19)$$

where we have assumed that there are N_{gR} and N_{gL} gapless branches in the excitation spectra of the right and left superconductor, respectively.

Differential conductance $\sigma = \partial I_{||}/\partial V$ can be evaluated numerically from (19) for finite temperatures. For $T = 0$ we can find the analytical expression for the differential conductance as

$$\frac{\sigma(V, T=0)}{\sigma_0} = N_{gL} N_{gR} \left[\frac{\Delta_R a_{1R}}{\Delta_L a_{1L}} \left(\frac{eV}{\Delta_R a_{1R}} \right)^2 B_1 \left(\frac{\Delta_L a_{1L}}{\Delta_R a_{1R}} \right) + \left(\frac{\Delta_R a_{3R}}{\Delta_L a_{3L}} \right) \left(\frac{eV}{\Delta_R a_{3R}} \right)^{2/3} B_3 \left(\frac{\Delta_L a_{3L}}{\Delta_R a_{3R}} \right) \right] \quad (20)$$

where σ_0 is the tunneling differential conductance between two normal metals in a magnetic field and is given by

$$\sigma_0 = \frac{4\pi e^2}{\hbar} \frac{N_{1L}(0)}{2\pi l^2} \frac{N_{1R}(0)}{2\pi l^2} |T_0|^2. \quad (21)$$

$N_{1L}(0)$ and $N_{1R}(0)$ are one-dimensional densities of states of a normal metal. B_α in (20) are numerical factors associated with the geometry of the vortex lattice and the order of zeros α in the excitation spectrum (18). They are equal to

$$B_\alpha(Z) = 2\pi^2 (2+\alpha) 2^{\frac{2}{\alpha}} \frac{\Gamma^2(\frac{1}{\alpha})}{\Gamma(\frac{2}{\alpha})} \int_0^{\frac{1}{Z+1}} x^{\frac{2}{\alpha}} F\left[\frac{1}{\alpha}, \frac{1}{2}; \frac{1}{\alpha} + \frac{1}{2}; Z^2 \left(\frac{x}{x-1} \right)^2 \right] dx \quad (22)$$

where $F(a,b;c;x)$ is the hypergeometric function and $\Gamma(x)$ is the gamma function. Comparing (20) and (15) we see that the differential conductance has the same algebraic dependence on voltage as the density of states on energy. This dependence is a consequence of the presence of gapless points in the quasiparticle excitation spectrum. $(eV)^2$ dependence in (20) comes from the presence of first order zeros while term $(eV)^{2/3}$ comes from third order zeros in a spectrum of a triangular vortex lattice. At finite temperatures this algebraic dependence will acquire the exponential tail due to the thermal excitations of the quasiparticles over the Fermi surface.

In order to discuss the possible results of scanning tunneling microscope (STM) experiment in the presence of a vortex lattice, we need a model of the tunneling current between the microscope tip and surface of the superconductor in a magnetic field. We will consider the same geometry of the experiment as described for S-N-S tunneling above, i.e., electrons from the tip can tunnel only along vortex lines. The tunneling current will be related to the spectral functions $A_S(\mathbf{r}, \varepsilon)$ and $A_N(\mathbf{r}, \varepsilon)$ in a way described by the expression (17) with the difference that these functions now depend on the position of the tip $\mathbf{r}$. In our calculation we will assume that the presence of the surface does not affect the quasiparticle wave functions of the superconductor. The differential conductance is then given by

$$\frac{\sigma(\mathbf{r}, V, T)}{\sigma_o} = -\frac{2\pi l^2}{N_1(0)} \sum_{q_z > 0} \sum_{\mathbf{q}} \sum_{N} \left[|u^N_{\mathbf{q}q_z}(\mathbf{r})|^2 n'_F(E^N_{\mathbf{q}q_z} - eV) + |v^N_{\mathbf{q}q_z}(\mathbf{r})|^2 n'_F(E^N_{\mathbf{q}q_z} + eV) \right] \quad (23)$$

where $n'_F(E)$ is a derivative of a Fermi distribution and $\mathbf{r} = (x,y)$ is a position of a tip. $u^N_{\mathbf{q}q_z}(\mathbf{r}) = u^N_{\mathbf{q}q_z} \phi_{\mathbf{q}q_z}(\mathbf{r})$ and $v^N_{\mathbf{q}q_z}(\mathbf{r}) = v^N_{\mathbf{q}q_z} \phi_{\mathbf{q}q_z}(\mathbf{r})$ are the solutions of BdG equations (8) for the superconductor in a magnetic field. Sum $\sum_N$ in (23) goes over the gapless branches of the quasiparticle excitation spectrum, since the most important contribution to the tunneling current at small bias voltages comes from the gapless points at the Fermi surface.

Figure 3 shows the differential conductance (23) at zero temperature as a function of the bias voltage when the microscope tip is at the position of the vortex. For small tunneling voltages it has the algebraic voltage dependence, reflecting the behavior of the low-energy density of states (15). It has a peak for the value of the voltage approximately equal to the maximum value of the order parameter. Note that this result differs from the one obtained in Ref. 4. for the isolated vortex case where the conductance is strongly enhanced at zero bias. It is important to understand that our calculation is done for the fundamentally different physical situation than the isolated vortex one considered in Ref. 4. In high magnetic fields vortices are very close to each other so that at low temperatures quasiparticles can propagate coherently over many unit cells. This coherent propagation will influence all the properties of the superconductor as we have shown in this paper: In particular, the formation of the coherent quasiparticle band leads to the *minimum* in differential conductance at zero bias. In lower magnetic fields, where the electronic mean free path is much shorter than the separation of vortices, the quasiparticle

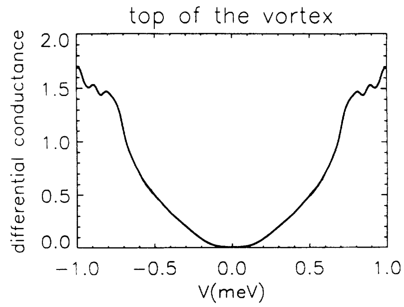

FIG. 3. The differential conductance as a function of a bias voltage when the STM tip is at the position of the vortex (in arbitrary units).

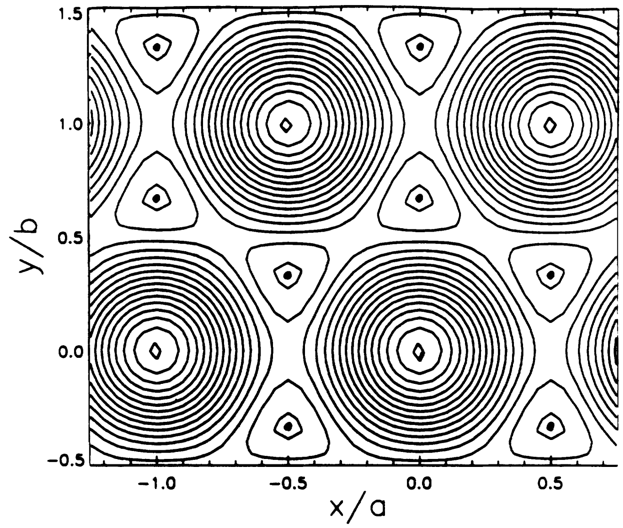

FIG. 4. The differential conductance as a function of the position of a STM tip for $V/\Delta = 0.2$. Full circles represent minima of differential conductance while diamonds show the maxima.

states are effectively bound to a single vortex exhibiting a well-known local *maximum* in differential conductance at zero bias, characterizing localized midgap states. The relationship between these two limits is essentially that of a single-impurity problem versus an ordered lattice of scatterers. Figure 4 shows how the differential conductance (23) depends on a tip position for the fixed value of a biased voltage $V/\Delta = 0.2$. This figure has a sixfold symmetry pattern of a triangular vortex lattice. Differential conductance has maxima at the positions of the vortices. The conductance decreases as the tip moves away from the vortex and reaches its minimum value at the positions where the order parameter has its maxima (these points form the hexagonal lattice).

V. CONCLUSION

In this paper we have shown that the BCS theory of a superconductor in a magnetic field can be solved exactly leading to the gapless behavior of the quasiparticle excitation spectrum in high magnetic fields. We have also discussed the mechanism of gap opening in the lower fields. As a result of the strong dispersion around gapless points, we have found the algebraic behavior of the heat capacity at low temperatures as well as density of states at low energies. The differential conductance of an *S*-*N*-*S* junction and differential conductance between the STM tip and the superconductor are found to have an algebraic dependence on biased voltage. This unusual behavior of a type-II superconductor in a high magnetic field can be in principle detected in suitably designed experiments at low temperatures.

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