Quasiparticle lifetimes and tunneling times in a superconductor-insulator-superconductor tunnel junction with spatially inhomogeneous electrodes

A. A. Golubov,* E. P. Houwman, J. G. Gijsbertsen, J. Flokstra, and H. Rogalla
University of Twente, Department of Applied Physics, P.O. Box 217, 7500 AE Enschede, The Netherlands

J. B. le Grand and P. A. J. de Korte
Laboratory for Space Research, Sorbonnelaan 2, 3584 CA Utrecht, The Netherlands

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The low-energy quasiparticle scattering and recombination lifetimes for a proximity sandwich of two superconductors S and S' with different bulk energy gaps, are calculated as a function of the spatial coordinate and temperature. The spatial dependence of the order parameter and density of states are calculated on the basis of a microscopic model of the proximity effect, based on the Usadel equations, for dirty superconductors in thermal equilibrium. A zero boundary resistance between S and S' and a Boltzmann-like energy distribution of the excess quasiparticles are assumed. In the case of a small diffusion time constant an effective quasiparticle relaxation rate into and excitation rate out of the reduced gap region in the SS' sandwich are obtained as a function of (finite, but low) temperature and strength of the proximity effect, determined by the parameter γ_m, by averaging over the energies and positions of the quasiparticles. In the same way effective tunneling times for electrons and holes tunneling out of the trap in the SS' sandwich to the other electrode of an $SS'IS''S$ junction are determined as a function of temperature, voltage, and γ_m.

I. INTRODUCTION

Nonequilibrium processes in superconductivity have been studied extensively in the past in thin superconducting films. Excess quasiparticle or phonon densities were induced by α particles, laser light or the injection of a (large) current of quasiparticles or phonons by a tunnel junction.[1-6] The excess quasiparticle densities are measured with a (second) junction. In fact, the tunneling current in any Josephson tunnel junction is an example of such injection current. For low current-density devices the effect on the equilibrium density of states in the electrodes is negligible, but for the very high-density devices that have been fabricated recently,[7] nonequilibrium effects are likely to give rise to important deviations from standard tunneling models that assume thermal equilibrium densities of states and distribution functions in the electrodes.

The development of superconducting detectors for the quantitative measurement of the energy of high-energetic (nuclear) particles or photons has renewed the interest in nonequilibrium superconductivity.[8,9] The working principle of such detectors is that the energy of the particle or photon, after absorption in a superconducting layer (the absorber), is used to break up Cooper pairs into excess quasiparticles and in the creation of nonequilibrium phonons. As time evolves the energy is distributed over an increasing number of excess quasiparticles and phonons in a cascade, until the energy of an excess quasiparticle is equal or slightly larger than the gap energy Δ_g of the superconductor, and the excess phonon has an energy of about $2\Delta_g$. This energy cascade takes place on a psec time scale and ends within less than a nsec. The excess energy is also spreading out in space, due to diffusion of the excess quasiparticles and phonons on a slower time scale.

The measurement of excess quasiparticles in the superconductor can be achieved by the detection of the breaking of superconductivity due to local heating (e.g., in superconducting granule detectors or superconducting strip detectors) or by measuring the excess current due to the excess quasiparticles through the barrier of a tunnel junction (see, e.g., Refs. 8–10). This paper is concerned with the latter detection method.

The intrinsic energy resolution of a junction detector for a particle which deposits energy E, is given by $\sigma(E)/E=(F/N)^{1/2}$, where $N=E/\omega$ is the number of created particles, ω the average energy to create one excess particle, and F is the Fano factor ($F\le 1$). In a practical device the energy resolution will therefore strongly depend on the number of excess quasiparticles that is detected by the junction. A large fraction of the excess quasiparticles may be lost before the quasiparticles have reached the barrier and can tunnel. Apart from loss due to recombination, an important loss mechanism is the trapping in regions with a lower energy gap, due to energy relaxation. Such regions may be, e.g., (1) induced by the proximity effect; (2) the core of a trapped Abrikosov vortex; (3) due to pair breaking caused by screening currents induced by an external magnetic field.

In order to have low losses one should extract the excess quasiparticles as fast as possible from the absorbing layer by the tunneling process. In principle, this can be achieved by using a junction with a highly transmissive barrier. However, the fabrication of such devices, which should also have very low leakage currents down to the low operating temperatures, poses severe or even insurmountable difficulties. An artificial trapping layer adja-

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cent to the tunnel barrier can be used advantageously to collect the excess quasiparticles from the absorber very fast and effectively.[11] Larger absorber volumes can therefore be used. This increases the detection efficiency, i.e., the percentage of incoming particles or photons that is absorbed. This can be shown simply as follows. Assume a bilayer with total volume V, consisting of a thick layer S with volume V-v with band gap Δ_{g1}, and a thin layer S' with small volume v ($v \ll V$) and band gap Δ_{g1} ($\Delta_{g1} > \Delta_{g2}$). Particles in layer S' with energy $\varepsilon \approx \Delta_{g1}$ can become trapped in the S' region by energy relaxation due to phonon emission, with a relaxation rate $1/\tau_e$, obtaining an energy $\Delta_{g2} < \varepsilon < \Delta_{g1}$. The relaxation rate is largest for the largest energy difference, thus the final energy of the trapped particle is predominantly equal to Δ_{g2}. Assuming that the fraction of time that the particles with energy Δ_{g1} spend in S' is just the volume fraction v/V, the effective trapping rate is $1/\tau_{\rm tr} \approx (v/V\tau_e)$. Thus the number of quasiparticles with energy Δ_{g1}, decreases initially as $N(\Delta_{g1}) \sim \exp(-t/\tau_{\rm tr})$ due to trapping. This process can be very fast compared to tunneling of particles with energy Δ_{g1} or other loss processes. The trapped particles can be excited out of the trap with rate $1/\tau_{\rm exc}$ and a dynamical equilibrium between particles with energy Δ_{g1} and those with energy Δ_{g2} will be established, such that

$$dN(\Delta_{g1})/dt\,|_{\rm trapping} = N(\Delta_{g1})/\tau_{\rm tr} = dN(\Delta_{g2})/dt\,|_{\rm excitation} = N(\Delta_{g2})/\tau_{\rm exc}$$

(neglecting other loss processes). From the results of this paper it follows that at low temperatures $\tau_{\rm exc} \gg (V/v)\tau_e$, even for large ratios V/v, thus the number of trapped particles, $N(\Delta_{g2}) = (\tau_{\rm exc}/\tau_{\rm tr})N(\Delta_{g1})$, is much larger than the number of particles in the bulk. The second large advantage is that the tunneling rate out of such a trapping layer is much larger than out of the bulk of the absorbing layer. Since the (signal) current (arising from the tunneling process) $i_s = dN/dt\,|_{\rm tun}$ is proportional to the particle density it follows that the tunneling rate for particles trapped in the small volume v of layer S',

$$(1/\tau_{\rm tun})_{\rm tr} = (1/N(\Delta_{g2}))(dN(\Delta_{g2})/dt)|_{\rm tun} \sim 1/v \ ,$$

is much larger than for particles with energy Δ_{g1}, which have a tunneling rate proportional to $1/V$.

There is a vast literature on the theory of nonequilibrium processes in superconducting films and devices.[12,13] Nonequilibrium processes in superconducting thin films and devices are generally described with the Rothwarf-Taylor (RT) equations.[14] These are rate equations for the number densities of nonequilibrium quasiparticles and phonons, which, in principle, describe the energy exchange between these systems and the environment. Some of the time constants that describe these processes can be obtained from the important paper of Kaplan *et al.*[15] They calculated the lifetimes of low-energy quasiparticles and phonons due to electron-phonon interaction in a spatially homogeneous superconductor in nearly thermal equilibrium: The scattering or thermalization lifetime of a quasiparticle with energy ε and at temperature T, $\tau_s(\varepsilon,T)$, is due to the absorption or emission of a phonon. The recombination lifetime of a quasiparticle, $\tau_r(\varepsilon,T)$, is determined by the recombination with another quasiparticle, forming a Cooper pair. The branch-mixing time, $\tau_Q(\varepsilon,T)$, describes the relaxation of the population imbalance of the two branches of the quasiparticle excitation curve, corresponding to quasiparticle wave vectors less and greater than the Fermi wave vector. The phonon system can be described with two lifetimes: the scattering lifetime of a phonon with energy Ω due to scattering with a quasiparticle, $\tau_{\rm phs}(\Omega,T)$, and the lifetime against Cooper pair breaking, $\tau_B(\Omega,T)$.

The RT equations and the Kaplan time constants have been applied by many authors to describe the nonequilibrium processes in junction particle detectors.[16–18] However, both the RT equations and the Kaplan theory assume a superconductor with spatially homogeneous properties, so that there is no dependence of the number densities on space coordinates. Such an oversimplified model is inadequate to describe quantitatively the nonequilibrium processes in many particle devices, as, e.g., particle or photon detectors in which reduced gap regions or trapping layers are present.

In this paper some of the results of Kaplan *et al.* are extended to the case of a spatially inhomogeneous superconductor. We consider the practically important case of a thin-film proximity sandwich of two superconducting metals with different bulk energy gaps. Such a sandwich may then be described by two sets of RT equations: one for the bulk layer and one for the trap, which are coupled by time constants describing the relaxation and excitation processes. In Ref. 19 the trapping rates due to a proximity layer and due to an Abrikosov vortex were studied for zero temperature. Here we will extend the calculations for the proximity sandwich to finite temperatures and will also calculate the excitation rate out of the trap, which was equal to zero in the zero-temperature case.

In a junction detector the quasiparticles are extracted from the absorber electrode by tunneling. The tunneling time constant $\tau_{\rm tun}$ was given by Ginsberg[20] for a homogeneous superconductor at high voltages. The validity range was extended to low voltages later.[21,22] $\tau_{\rm tun}$ is proportional to the thickness of the superconductor from which the tunneling electrons are extracted. However, in the case that the trapping layer is adjacent to the barrier, this tunneling length for quasiparticles in the trap is much smaller than that of the quasiparticles in the bulk electrode. This means that tunneling of the trapped quasiparticles is much faster than that of the bulk quasiparticles.

Due to the proximity effect the order parameter and consequently the densities of states vary in space. This has a pronounced effect on the quasiparticle lifetimes as well as the tunneling times, which also become a function of the space coordinates. Previously this effect was taken into account using oversimplified models: either BCS-like densities of states in the bulk and in the trap, or in the McMillan-tunneling model of the proximity effect.[23] Here we calculate the time constants for a proximity junction at a finite temperature using the solutions of a microscopic proximity-effect model[24] based on the Usadel equations.[25] Some aspects of this model are described in Sec. II.

In Sec. III the equations for the quasiparticle lifetimes in a proximity sandwich are developed as an extension of the equations of Ref. 15. The results are compared with those given there.

Under the assumption that the diffusion time constant over the thickness of the sandwich is much smaller than the quasiparticle lifetime constants, these lifetimes can be averaged over the position and the energies of the quaisparticles. In this way effective quasiparticle lifetimes are defined by the introduction of the size of an effective trapping region, which gives the same total scattering rate as the reduced gap region. This averaging procedure was done earlier in the limit of zero temperature.[19] In Sec. IV we will extend the formalism developed there to the finite (but low) temperatures at which most of the practical devices are used. The quasiparticle scattering rate will be divided into two rates τ_e^{-1} and τ_a^{-1}, due to, respectively, phonon emission and absorption. Effective rates are calculated for quasiparticles in and above the trap separately, as functions of the strength of the proximity effect that determines the size of the trap.

In Sec. V we calculate the effective tunneling time for quasiparticles in the trap as a function of the strength of the proximity effect, temperature, and voltage bias of the junction.

It is noted that the proximity effect model that is used applies only to sandwiches of dirty superconductors, so that, in principle, all calculated time constants are valid for that case only. However, it is expected that most of the results will also apply, at least qualitatively, for superconductors near or in the clean limit.

II. THE PROXIMITY-EFFECT MODEL

The inhomogeneous state of a dirty superconductor in the weak-coupling limit can be described by the Usadel equations:[25]

$$\phi(\omega_n,\mathbf{r})=\Delta(\omega_n,\mathbf{r})+(\xi_s)^2\frac{\pi T_c}{\omega_n G(\omega_n,\mathbf{r})} \times\nabla[G^2(\omega_n,\mathbf{r})\nabla\phi(\omega_n,\mathbf{r})] \ , \quad (1a)$$

$$\Delta(\omega_n,\mathbf{r})\ln\frac{T}{T_c}+2\pi T\sum_{\omega_n}[\Delta(\omega_n,\mathbf{r})/\omega_n-F(\omega_n,\mathbf{r})]=0 \ , \quad (1b)$$

$$\phi=\omega_n F/G,\quad G=\omega_n[\omega_n^2+\phi^2]^{-1/2},\quad F=\phi[\omega_n^2+\phi^2]^{-1/2} \ , \quad (1c)$$
$$\omega_n=\pi T(2n+1),\quad n=0,1,2,\ldots ,$$

where G and F are the Green's functions of a superconductor, Δ is the order parameter, and ω_n the Matsubara frequency. Equation (1b) is the self-consistency relation for the determination of the order parameter $\Delta(\omega_n,\mathbf{r})$.

As a model for the inhomogeneous state we consider the proximity effect between a thick superconducting layer S (extending along the x axis perpendicular to the SS' interface from 0 to d_s) and a thin layer of another superconducting material S' ($-d<x<0$), which are in a good electrical contact. We assume that

$$T_c>T_c^*;\quad d_s\gg\xi_s\gg l_s;\quad l\le d\ll\xi^* \ , \quad (2)$$

i.e., both materials are in the dirty limit, where T_c (T_c^*), d_s (d), l_s (l), and $\xi_s=(D_s/2\pi T_c)^{1/2}$ [$\xi^*=(D/2\pi T_c)^{1/2}$] are the critical temperature, the thickness, the electron mean free path, and the coherence length of the $S(S')$ layer, respectively. D_s and D are the normal-state diffusion coefficients of electrons in these materials. The coherence length ξ^* in the S' layer is related to the bulk coherence length $\xi_{s'}$ of the S' material by $\xi_{s'}=\xi^*(T_c/T_c^*)^{1/2}$. In this way one can treat T_c^* as an independent parameter. The last condition in Eq. (2) also implies that the functions G, F, and Δ can be assumed to be constant over the S' layer. Details on this model of the proximity effect are given in Ref. 26 for an SN sandwich ($T_c^*=0$) and in Refs. 24 and 27 for the more general case of an SS' sandwich.

It was shown that in both cases the extent of the influence of the proximity effect is determined by two parameters:

$$\gamma_m=(\rho_s\xi_s/\rho\xi^*)(d/\xi^*) \ , \quad (3a)$$

$$\gamma_B=(R_B/\rho\xi^*)(d/\xi^*) \ . \quad (3b)$$

Here, ρ_s (ρ) is the normal-state specific resistivity of the S (S') metal and R_B is the product of the SS' boundary resistance with its area.

The parameters γ_m and γ_B have a simple physical interpretation. The value of γ_m is largely determined by the electron densities in the S and S' metals in the SS' sandwich. A large value of γ_m corresponds to a high density of quasiparticles in S' compared to that in S near the SS' boundary. In this case the diffusion of these quasiparticles into the superconductor leads to a strong suppression of the order parameter in the S region at distances of the order of ξ_s from the boundary. In the opposite case ($\gamma_m\ll 1$) the influence of the S' layer on the superconducting properties of S metal is weak and the order parameter in the S region is nearly spatially homogeneous. Thus γ_m plays the role of an effective pair-breaking parameter near the SS' boundary, as discussed in Ref. 24.

The parameter γ_B determines the effect of a finite transparency of the SS' boundary. The case $\gamma_B\ll 1$ corresponds to a vanishing potential barrier at the SS' boundary (vanishing barrier resistance, $R_B=0$), i.e., the S and S' metals are in good electric contact. The opposite situation, $\gamma_B\gg 1$, corresponds to a low transparency of the potential barrier, i.e., the S and S' metals are weakly coupled. In the latter case the McMillan model of the proximity effect is applicable. The relation between both proximity-effect models was discussed recently in Ref. 24.

The calculations presented in this paper apply to the case of a Nb($T_c=9.2$ K)/Al($T_c^*=1.3$ K) sandwich. It is known that the Nb/Al interface has a low resistance.[28] Therefore it is assumed below that no barrier exists at the SS' interface, hence $\gamma_B=0$. In this case the superconducting properties of the SS' sandwich are determined by the parameter γ_m and the critical temperature ratio T_c^*/T_c. As was shown in Refs. 24 and 27 a variation of the T_c^*/T_c ratio can easily be taken into account by a renormalization of γ_m by introducing an effective proximity parameter $\gamma_m^{\rm eff}=\gamma_m \alpha$, where α is a function of the ratio T_c^*/T_c. In Fig. 1 of Ref. 24, $\alpha=\alpha(T_c^*/T_c)$ is presented. The results, obtained in this paper for the Nb/Al system, can therefore straightforwardly be applied to other material combinations under the condition $\gamma_B \ll 1$.

The set of equations (1) for the S and S' layers under the proper boundary conditions, were solved numerically. As an illustration the density of states in S, $\bar{N}(\varepsilon=-i\omega_n)={\rm Re}G_S(\varepsilon)$, at the SS' boundary, is plotted in Fig. 1 for different γ_m values. $\bar{N}$ is normalized to the total density of states at the Fermi level in the normal state $N(0)$ (spin up and down). In the S' region the density of states, $\bar{N}'(\varepsilon)$ [normalized to $N'(0)$], is spatially homogeneous due to the condition $d \ll \xi^*$, and takes the value at the boundary $\bar{N}(\varepsilon,0)$ because $\gamma_B=0$. It is seen that the energy gap in the density of states Δ_g is suppressed relative to the bulk value Δ_0. [Δ_0 (Δ_0') is the bulk BCS equilibrium value of the order parameter at temperature T of metal S (S')]. In the S region the behavior of $\bar{N}$ depends on the spatial coordinate x. $\bar{N}(\varepsilon,x)$ is plotted in Fig. 2 of Ref. 19 for $\gamma_m=10$ at different distances x/ξ_s from the SS' boundary (at $x=0$) in the S region. The value of Δ_g is the same for all points in S, the difference being in the values of $\bar{N}(\varepsilon,x)$ at $\Delta_g<\varepsilon<\Delta_0$, which become small as $x/\xi_s \geq 3.5$. This means that a large region of the S material near the SS' boundary has a reduced gap value $\Delta_g<\Delta_0$. On the other hand the energy gap in the S' layer is increased compared to the bulk value $\Delta_g>\Delta_0'$.

The gap Δ_g at zero temperature is shown in Fig. 2 as a function of γ_m. It is seen that Δ_g decreases with increasing γ_m in accordance with the pair-breaking nature of this parameter, as discussed above. The physical consequence of the gap suppression by the proximity effect is that quasiparticles with energy equal to the bulk gap Δ_0 have a finite lifetime, even at zero temperature, because of the presence of a finite density of states below Δ_0 in the energy interval $\Delta_g \leq \varepsilon \leq \Delta_0$. At finite temperature $T>0$ this will lead to quasiparticle lifetimes that are different from those calculated by Kaplan *et al.*[15] for a spatially homogeneous superconductor. This effect is the main subject of investigation in this paper. First we extend the formalism and calculations of Ref. 15 to the spatially inhomogeneous case of an SS' sandwich. Then we apply the formalism to calculate effective quasiparticle trapping and tunneling times in a tunnel junction with electrodes consisting of SS' sandwiches.

III. QUASIPARTICLE LIFETIMES

Finite quasiparticle lifetimes in a metal are due to inelastic scattering of electrons. Therefore one should first specify a mechanism of inelastic scattering to introduce a general expression for these lifetimes. Two processes are usually considered: emission or absorption of real phonons (electron-phonon interaction) and direct electron-electron interaction.[6,15,29] Here we make the usual assumption that the main scattering mechanism is due to electron-phonon interaction. This is true for most metals for which the condition $\Omega_D^2/\mu_F T_c \ll 1$ holds, where Ω_D is the Debye energy and μ_F the Fermi energy.[15] In this case the self-consistency equation (1b) should be substituted by the set of Eliashberg equations.[30] In the case of a spatially inhomogeneous superconductor these equations have the following general form:[19,31]

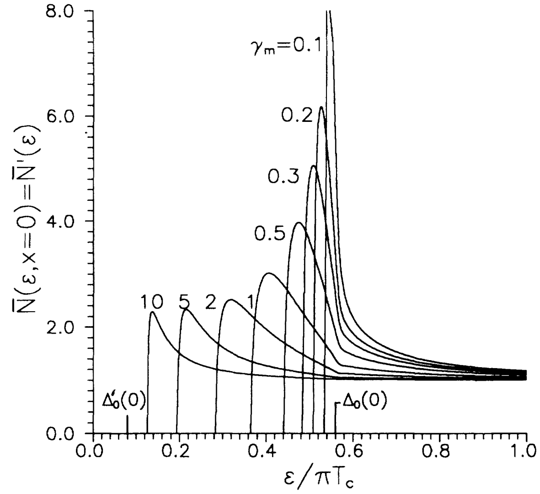

FIG. 1. Normalized quasiparticle densities of states at the SS' interface of an SS' sandwich, with $T_c^*/T_c=0.14$ (corresponding to a Nb/Al sandwich) at temperature $T \ll T_c$, for γ_m ranging from 0.1 to 10 and $\gamma_B=0$. The bulk BCS energy gaps at $T=0$ of Nb and Al are indicated with $\Delta_0(0)$ and $\Delta_0'(0)$, respectively.

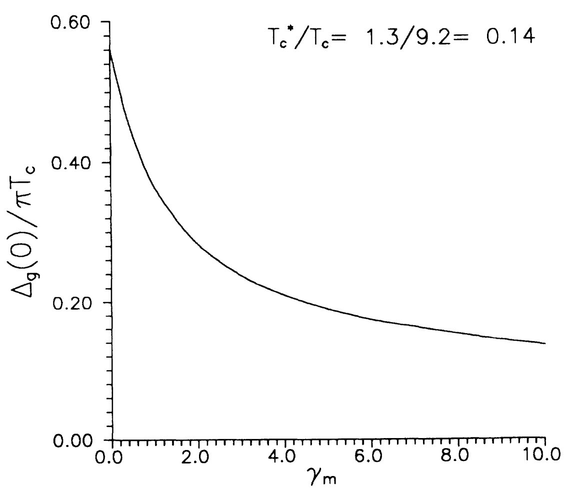

FIG. 2. The energy gap Δ_g, of an SS' proximity sandwich with $T_c^*/T_c=0.14$ and $\gamma_B=0$, as function of the proximity parameter γ_m.

$$\tilde{\varepsilon}(\varepsilon,\mathbf{r})=\varepsilon Z(\varepsilon,\mathbf{r})=\varepsilon-\int_0^\infty d\varepsilon' \,\mathrm{Re}G(\varepsilon',\mathbf{r})\int_0^\infty d\Omega\,\alpha^2(\Omega)F(\Omega)\left[\frac{f(-\varepsilon')+n(\Omega)}{\varepsilon'+\varepsilon+\Omega+i\delta}-\frac{f(-\varepsilon')+n(\Omega)}{\varepsilon'-\varepsilon+\Omega-i\delta}+\frac{f(\varepsilon')+n(\Omega)}{-\varepsilon'+\varepsilon+\Omega+i\delta}-\frac{f(\varepsilon')+n(\Omega)}{-\varepsilon'-\varepsilon+\Omega-i\delta}\right], \tag{4a}$$

$$\tilde{\Delta}(\varepsilon,\mathbf{r})=\Delta(\varepsilon,\mathbf{r})Z(\varepsilon,\mathbf{r})=\int_0^\infty d\varepsilon' \,\mathrm{Re}F(\varepsilon',\mathbf{r})\int_0^\infty d\Omega\,\alpha^2(\Omega)F(\Omega)\left[\frac{f(-\varepsilon')+n(\Omega)}{\varepsilon'+\varepsilon+\Omega+i\delta}+\frac{f(-\varepsilon')+n(\Omega)}{\varepsilon'-\varepsilon+\Omega-i\delta}-\frac{f(\varepsilon')+n(\Omega)}{-\varepsilon'+\varepsilon+\Omega+i\delta}-\frac{f(\varepsilon')+n(\Omega)}{-\varepsilon'-\varepsilon+\Omega-i\delta}\right]$$
$$-\mu_C\int_0^{\varepsilon_c} d\varepsilon' \,\mathrm{Re}F(\varepsilon',\mathbf{r})\tanh(\varepsilon'/2T)\,. \tag{4b}$$

Here, $\Delta(\varepsilon,\mathbf{r})$ and $Z(\varepsilon,\mathbf{r})$ are the energy and space-dependent order parameter and renormalization function, respectively. The function $\alpha^2(\Omega)F(\Omega)$ is the spectral function of the electron-phonon interaction, Ω being the phonon energy. μ_C is the Coulomb pseudopotential, ε_c is a cutoff energy of the order of the Debye energy Ω_D, and $f(\varepsilon)$, $n(\varepsilon)$ are the Fermi and Bose distribution functions, respectively.

In the Eliashberg formalism the normal and anomalous parts of the electron self-energy, Z and Δ, respectively, have imaginary parts (proportional to the electron-phonon coupling constant λ for small λ). Generally the imaginary part of the self-energy determines the quasiparticle lifetime. As was shown in Ref. 19, one can introduce an electron scattering rate $\Gamma(\varepsilon,\mathbf{r})$ in a spatially inhomogeneous superconductor following the approach of Ref. 15 and obtain

$$\Gamma(\varepsilon,\mathbf{r})=\varepsilon Z_2(\varepsilon,\mathbf{r})/Z_1(\varepsilon,\mathbf{r})-\tilde{\Delta}_1(\varepsilon,\mathbf{r})\tilde{\Delta}_2(\varepsilon,\mathbf{r})/(Z_1^2(\varepsilon,\mathbf{r})\varepsilon)\,. \tag{5}$$

Here, $\tilde{\Delta}_{1,2}$ and $Z_{1,2}$ are the real and imaginary parts of $\tilde{\Delta}$ and Z, respectively, and are determined directly by Eqs. (4a) and (4b) $\Gamma(\varepsilon,\mathbf{r})$ determines a quasiparticle decay rate

$$\frac{1}{\tau(\varepsilon,\mathbf{r})}=2\Gamma(\varepsilon,\mathbf{r})\,, \tag{6}$$

where $\tau(\varepsilon,\mathbf{r})$ is the coordinate and energy-dependent quasiparticle lifetime. Here, we consider the case of rather weak coupling, when the typical electron energy of interest, $\varepsilon\approx kT_c$, is small compared to typical phonon frequencies, $\Omega\approx\Omega_D$, so that the energy dependence of Z_1 and $\tilde{\Delta}_1$ can be neglected. Then the real part of the order parameter, $\Delta_1(\mathbf{r})=\tilde{\Delta}_1(\mathbf{r})/Z_1(0)$, can be determined from the Usadel equations, Eqs. (1). The functions Z_2 and $\tilde{\Delta}_2$ are obtained from Eq. (4), so that a generalization of Eq. (8) of Ref. 15 for τ^{-1} in the spatially inhomogeneous case is obtained:

$$\tau^{-1}(\varepsilon,\mathbf{r})=\tau_e^{-1}(\varepsilon,\mathbf{r})+\tau_a^{-1}(\varepsilon,\mathbf{r})+\tau_r^{-1}(\varepsilon,\mathbf{r})\,, \tag{7a}$$

$$\tau_e^{-1}(\varepsilon,\mathbf{r})=\frac{2\pi}{\hbar Z_1(0)[1-f(\varepsilon)]}\int_0^{\varepsilon-\Delta_g(\mathbf{r})} d\Omega\alpha^2(\Omega)F(\Omega)\left[\mathrm{Re}G(\mathbf{r},\varepsilon-\Omega)-\frac{\Delta(\mathbf{r})}{\varepsilon}\mathrm{Re}F(\mathbf{r},\varepsilon-\Omega)\right]\times[n(\Omega)+1][1-f(\varepsilon-\Omega)]\,, \tag{7b}$$

$$\tau_a^{-1}(\varepsilon,\mathbf{r})=\frac{2\pi}{\hbar Z_1(0)[1-f(\varepsilon)]}\int_0^\infty d\Omega\alpha^2(\Omega)F(\Omega)\left[\mathrm{Re}G(\mathbf{r},\varepsilon+\Omega)-\frac{\Delta(\mathbf{r})}{\varepsilon}\mathrm{Re}F(\mathbf{r},\varepsilon-\Omega)\right]n(\Omega)[1-f(\varepsilon+\Omega)]\,, \tag{7c}$$

$$\tau_r^{-1}(\varepsilon,\mathbf{r})=\frac{2\pi}{\hbar Z_1(0)[1-f(\varepsilon)]}\int_{\varepsilon+\Delta_g(\mathbf{r})}^\infty d\Omega\alpha^2(\Omega)F(\Omega)\left[\mathrm{Re}G(\mathbf{r},\Omega-\varepsilon)-\frac{\Delta(\mathbf{r})}{\varepsilon}\mathrm{Re}F(\mathbf{r},\Omega-\varepsilon)\right][n(\Omega)+1]F(\Omega-\varepsilon)\,. \tag{7d}$$

Here τ_e^{-1} is the scattering rate with emission of phonons, τ_a^{-1} is the rate with absorption of phonons, and τ_r^{-1} is the recombination rate, in which two quasiparticles form a Cooper pair with the excess energy emitted as a phonon. Summing up τ_e^{-1} and τ_a^{-1} one can define the quasiparticle scattering rate τ_s^{-1}:

$$\tau_s^{-1}(\varepsilon,\mathbf{r})=\tau_e^{-1}(\varepsilon,\mathbf{r})+\tau_a^{-1}(\varepsilon,\mathbf{r})\,. \tag{8}$$

Further simplification is possible because the electron energy is small compared to the typical phonon energy. In this case the function $\alpha^2(\Omega)F(\Omega)$ can be approximated by its low-frequency form $b\Omega^2$ and a characteristic time constant for a given material can be introduced:

$$\tau_0=Z_1(0)\hbar/2\pi b(kT_c)^3\,. \tag{9}$$

Values for τ_0 for several materials are tabulated in Ref. 15.

For the determination of $\tau_0/\tau_s(\varepsilon,\mathbf{r})$ and $\tau_0/\tau_r(\varepsilon,\mathbf{r})$ one needs to determine the generalized densities of states $\mathrm{Re}G(\varepsilon,\mathbf{r})$ and $\mathrm{Re}F(\varepsilon,\mathbf{r})$. In a spatially homogeneous superconductor, which is in thermal equilibrium, these are given by the well-known expressions

$$\mathrm{Re}G(\varepsilon)=\mathrm{Re}\frac{\varepsilon}{[\varepsilon^2-\Delta^2(\varepsilon)]^{1/2}}\,,$$
$$\mathrm{Re}F(\varepsilon)=\mathrm{Re}\frac{\Delta(\varepsilon)}{[\omega^2-\Delta^2(\varepsilon)]^{1/2}}\,. \quad (10)$$

Employing these equations, the calculations of lifetimes were performed by Kaplan *et al.*[15] In order to determine $\mathrm{Re}G(\varepsilon)$ and $\mathrm{Re}F(\varepsilon)$ and to calculate the lifetimes according to Eqs. (7) and (8) in the general, spatially inhomogeneous case one should solve the equations of the microscopic theory, applying them to any specific model under consideration. In our case of the proximity effect in the SS' sandwich this was done in the framework of the Usadel equations.

In Figs. 3 and 4 the scattering $[\tau_s(\varepsilon,x,T)/\tau_0]$ and recombination time $[\tau_r(\varepsilon,x,T)/\tau_0]$ at the SS' boundary are plotted as function of the reduced temperature T/T_c for different γ_m, for the energies $\varepsilon/\Delta_0(0)=1.0$ and 1.5, respectively. For $\gamma_m=0$, i.e., the homogeneous case, the results of Kaplan *et al.* are regained. At low temperatures a quasiparticle at the gap edge with energy $\varepsilon=\Delta_0(T)$ $[\approx\Delta_0(0)$ for $T/T_c\leq 0.5]$ cannot scatter to a lower-lying state [the upper limit of the integral in Eq. (7b) is zero], so that $\tau_e=\infty$ and $\tau_s=\tau_a$. Since the number of phonons decreases exponentially with decreasing temperature, τ_a and thus τ_s increase strongly. For $\gamma_m>0$ there is a finite number of states with energy less than Δ_0 to which the quasiparticles can relax by phonon emission. Consequently, τ_e is finite, even at zero temperature. With increasing γ_m the number of these states grows and τ_e and thus τ_s decreases. Analogously one finds a finite τ_e for quasiparticle energies larger than Δ_0 in the case of $\gamma_m=0$, as is seen in Fig. 4.

The recombination time increases strongly with decreasing temperature, because the number of quasiparticles with which a given quasiparticle can form a Cooper pair decreases exponentially. With increasing γ_m the number of occupied states (mainly with energies less than the quasiparticle energies considered in the figures) increases, giving rise to a decreasing τ_r.

From both figures it is seen that for particles with $\varepsilon>\Delta_0$ and at low T, τ_r is much larger than τ_s. The crossover temperature (where $\tau_s=\tau_r$) increases with increasing quasiparticle energy. This is also the case for particles above the trap.

Figure 5 gives τ_s and τ_r in S for quasiparticles with energy $\varepsilon=1.5\Delta_0(0)$ as function of temperature for different distances from the SS' boundary in the case $\gamma_m=1$. Deep in S the results coincide with those of Kaplan. On approaching, the SS' boundary the number of empty states with energy less than $1.5\Delta_0(0)$ at a given tempera-

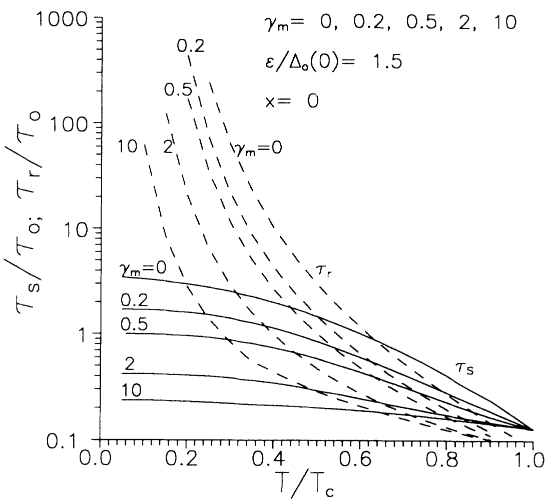

FIG. 4. As Fig. 3, but for $\varepsilon/\Delta_0(0)=1.5$.

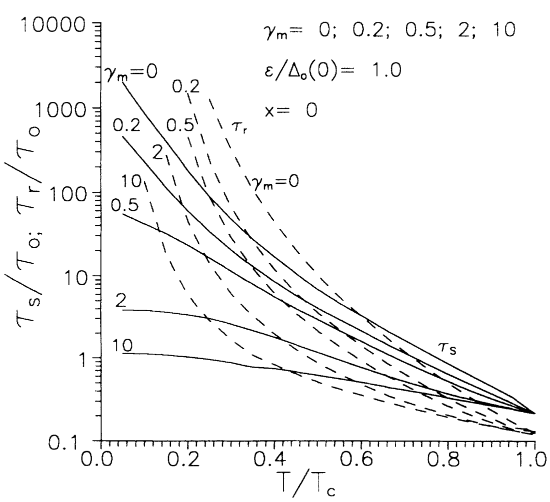

FIG. 3. Quasiparticle scattering (τ_s) and recombination rate (τ_r) at the SS' interface of an SS' sandwich $(T_c^*/T_c=0.14,\gamma_B=0)$ for quasiparticle energy $\varepsilon/\Delta_0(0)=1$ and $\gamma_m=0,0.2,0.5,2,10$.

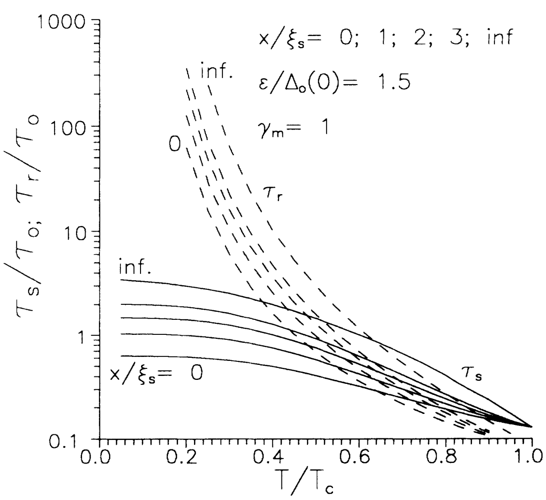

FIG. 5. Quasiparticle scattering (τ_s) and recombination rate (τ_r) at different distances, $x/\xi_s=0,1,2,3,\infty$ from the SS' interface into the S layer for quasiparticle energy $\varepsilon/\Delta_0(0)=1.5$ and $\gamma_m=1$ $(T_c^*/T_c=0.14,\gamma_B=0)$.

ture, increases, so that the scattering rate becomes larger. The number of occupied states also grows when coming closer to the SS' boundary and the recombination rate therefore increases as well.

IV. EFFECTIVE QUASIPARTICLE RELAXATION AND EXCITATION TIMES IN SS' SANDWICHES

In this section we calculate the average lifetime of a quasiparticle against the trapping in the reduced gap region of the SS' sandwich, as well as the lifetime against excitation out of the trap.

A. Effective trapping time

The time constant τ_{tr} for a single electronlike quasiparticle with energy $\varepsilon > \Delta_0$ to relax into the reduced gap region so that it can be considered to be trapped, is given by the rate τ_{tr}^{-1} by which the total number of quasiparticles with energy $\varepsilon > \Delta_0$, $N^{tot}_{\varepsilon>\Delta_0}$, changes due to phonon emission:

$$\frac{1}{\tau_{tr}} = \frac{1}{N^{tot}_{\varepsilon>\Delta_0}} \left| \frac{dN^{tot}_{\varepsilon>\Delta_0}}{dt} \right|$$
$$= \left[\frac{1}{N_{\varepsilon>\Delta_0}} \left| \frac{dN_{\varepsilon>\Delta_0}}{dt} \right| \right] \frac{N_{\varepsilon>\Delta_0}}{N_{\varepsilon>\Delta_0}+N'_{\varepsilon>\Delta_0}} + \left[\frac{1}{N'_{\varepsilon>\Delta_0}} \left| \frac{dN'_{\varepsilon>\Delta_0}}{dt} \right| \right] \frac{N'_{\varepsilon>\Delta_0}}{N_{\varepsilon>\Delta_0}+N'_{\varepsilon>\Delta_0}} . \tag{11}$$

Here, $N_{\varepsilon>\Delta_0}$ and $N'_{\varepsilon>\Delta_0}$ are, respectively, the number of particles in S and S' with $\varepsilon > \Delta_0$.

First we consider the trapping process in the S layer, with $d_s \gg \xi_s$, where $(\tau_{tr}^{-1})_S$ is defined as

$$\left[\frac{1}{\tau_{tr}} \right]_S = \frac{1}{N_{\varepsilon>\Delta_0}} \left| \frac{dN_{\varepsilon>\Delta_0}}{dt} \right| \tag{12a}$$
$$= \frac{\int_0^{d_s} dx \int_{\Delta_0}^{\infty} d\varepsilon\, \tau_e^{-1}(\varepsilon,x) \bar{N}(\varepsilon,x) f(\varepsilon,x)}{\int_0^{d_s} dx \int_{\Delta_0}^{\infty} d\varepsilon\, \bar{N}(\varepsilon,x) f(\varepsilon,x)} . \tag{12b}$$

The relaxation time constant $\tau_e^{-1}(\varepsilon,x)$ is defined by Eq. (7b), but with the condition that the energy of the emitted phonon is large enough, that the quasiparticle becomes trapped: $\Omega \geq \varepsilon - \Delta_0 \equiv \Omega_{min}$. The lower integration limit in Eq. (7b) is thus equal to Ω_{min}. It is easy to verify that for the holelike quaisparticles the same time constant $(\tau_{tr})_S$ is found.

We consider the case that excess quasiparticles may be present. The nonequilibrium distribution function is assumed to be given by[32]

$$f(\varepsilon,x,t) = f_0(\varepsilon) + c(x,t) e^{-(\varepsilon-\Delta_g)/T} , \tag{13}$$

where $f_0(\varepsilon)$is the equilibrium Fermi-Dirac distribution function. The space and time dependence of the nonequilibrium part of $f(\varepsilon,x,t)$ [the second term in the right-hand side of (13)] is given by the factor $c(x,t)$, whereas the energy dependence is determined by the Boltzmann-like factor. We will assume that the quasiparticle diffusion is so fast that $c(x,t)$ is not spatially dependent on the time scale of the trapping process. This is mostly the case since the diffusion time constant $\tau_D \approx d_s^2/D_s$ is much smaller than τ_{tr}. Further the low-temperature limit is considered, $\varepsilon \geq \Delta_g \gg T$. Then $f(\varepsilon,x,t)$ reduces to

$$f(\varepsilon,x,t) \approx f(\varepsilon) = [e^{-\Delta_0/T} + c(t) e^{(\Delta_g-\Delta_0)/T}] e^{-(\varepsilon-\Delta_0)/T} \equiv C(t) e^{-(\varepsilon-\Delta_0)/T} . \tag{14}$$

In Eq. (12b) the factors C cancel, thus $(\tau_{tr}^{-1})_S$ becomes independent of the excess quasiparticle density, and the trapping rate for excess and equilibrium quasiparticles is equal.

$N_{\varepsilon>\Delta_0}$, given by the numerator in (12b), is approximately equal to

$$N_{\varepsilon>\Delta_0} \simeq N(0) d_s C \int_{\Delta_0}^{\infty} d\varepsilon\, \bar{N}^{BCS}(\varepsilon) e^{-(\varepsilon-\Delta_0)/T} = N(0) d_s C B(T) ; \tag{15}$$

where $\bar{N}^{BCS}(\varepsilon)$ is the normalized BCS density of states and $B(T)$ is the function

$$B(T) = \int_{\Delta_0}^{\infty} d\varepsilon \frac{\varepsilon}{\sqrt{\varepsilon^2-\Delta_0^2}} e^{-(\varepsilon-\Delta_0)/T} = T \int_0^{\infty} \sqrt{x(x+2\Delta_0/T)} e^{-x} dx = \begin{cases} \sqrt{(\pi/2)\Delta_0 T} , & T \ll 2\Delta_0(T) \\ T, & T \geq \Delta_0(T) . \end{cases} \tag{16}$$

In the limit $T \to 0$, $\tau_e^{-1}(\varepsilon,x)$ is approximately equal to $\tau_e^{-1}(\Delta_0,x)$, and can be taken out of the integral over the energy in Eq. (12b), which then reduces to

$$\left[\frac{1}{\tau_{tr}} \right]_S = \frac{1}{d_s} \int_0^{d_s} \frac{dx}{\tau_e(\Delta_0,x)} . \tag{17}$$

This result was obtained earlier.[19]

In Ref. 19 an effective length L_{eff} was defined, as the thickness of a layer in the S material that is in the normal state ($\Delta_g = 0$), which gives the same total scattering rate as the reduced gap region. L_{eff} is given by

$$\int_0^{d_s} \frac{dx}{\tau_e(\Delta_0,x)} = L_{eff}(T=0) \frac{1}{\tau_e(\Delta_g=0)} , \tag{18a}$$

where

$$\tau_e^{-1}(\Delta_g=0) = \frac{2\pi b(\Delta_0)^3}{3Z_1(0)\hbar} \simeq \frac{1.82}{\tau_0} \tag{18b}$$

is the relaxation rate for particles in S with energy Δ_0 down to the Fermi energy (use was made of the BCS relation $2\Delta_0/T_c = 3.52$). Here τ_0 is the material-dependent time constant for S, given by Eq. (9).

The definition of L_{eff} can be extended to finite temperatures in a natural way. For $\varepsilon \geq \Delta_g \gg T$, Eq. (12b) is written as

$$\left[\frac{1}{\tau_{tr}}\right]_S = \frac{1.82}{d_s \tau_0} L_{\rm eff}(T) \ , \tag{19a}$$

with

$$L_{\rm eff}(T) = \frac{\int_0^{d_s} dx \int_{\Delta_0}^{\infty} d\varepsilon\, \tau_0 \tau_e^{-1}(\varepsilon, x) \bar{N}(\varepsilon, x) e^{-(\varepsilon - \Delta_0)/T}}{1.82 B(T)} \ . \tag{19b}$$

This material-independent parameter is only a function of γ_m (and the T_c^*/T_c ratio). In order to find the temperature dependence of $L_{\rm eff}(T)$ in different regions, the expression for $\tau_s^{-1}(T=0)$ (which at low temperatures is equal to τ_e^{-1}), as given in Eq. (17) of Ref. 15, is expanded for small energies ε around Δ_0, $1/\tau_e \sim (\varepsilon - \Delta_g)^{3/2}$. For very low temperatures, $T \ll (\Delta_0 - \Delta_g)$, the quasiparticles have energies close to the lower energy limit of (19b), $\varepsilon \approx \Delta_0$, giving $\tau_e^{-1} \sim (\Delta_0 - \Delta_g)^{3/2}$. Using the approximation $\bar{N}(\varepsilon, x) \approx \bar{N}^{\rm BCS}(\varepsilon)$, we have

$$L_{\rm eff}(T) \sim (\Delta_0 - \Delta_g)^{3/2} = \text{const}, \quad T \ll (\Delta_0 - \Delta_g) \ . \tag{20}$$

In the intermediate-temperature regime, $(\Delta_0 - \Delta_g) \ll T \leq \Delta_0$, and for small γ_m one can write $\tau_e^{-1} \sim (\varepsilon - \Delta_0)^{3/2}$ and $\bar{N} \approx \bar{N}^{\rm BCS}$, i.e., $\Delta_g \approx \Delta_0$, so that Eq. (19b) reduces to

$$L_{\rm eff}(T) \sim \frac{1}{B(T)} \int_{\Delta_0}^{\infty} d\varepsilon\, \varepsilon(\varepsilon - \Delta_0) e^{-(\varepsilon - \Delta_0)/T}$$
$$\sim \frac{T^3}{B(T)} \sim T^2, \quad (\Delta_0 - \Delta_g) \ll T \leq \Delta_0 \ . \tag{21}$$

For high temperatures and energies, $\varepsilon \sim T \gg \Delta_0(T)$, $\tau_e^{-1}(\varepsilon)$ has the energy dependence

$$\frac{1}{\tau_e(\varepsilon)} \sim \varepsilon^3 + T\varepsilon^2 \ , \tag{22}$$

so that the temperature dependence of $L_{\rm eff}$ in this limit reduces to

$$L_{\rm eff} \sim \frac{T^4}{B(T)} \sim T^3, \quad T \gg \Delta_0 \ . \tag{23}$$

From Eqs. (20), (21), and (23) it follows that the effective size of the trap increases stronger with temperature, the higher the temperature.

An analog procedure can be applied to the trapping layer S' by introducing an effective length $L'_{\rm eff}(T)$ for the S' material in the normal state ($\Delta_g = 0$). The corresponding equation for the trapping is

$$\left[\frac{1}{\tau_{tr}}\right]_{S'} = \frac{1.82}{d\tau_0'} L'_{\rm eff}(T) \ , \tag{24a}$$

with

$$\frac{L'_{\rm eff}(T)}{d} = \frac{\int_{\Delta_0}^{\infty} d\varepsilon\, \tau_0 \tau_e^{-1}(\varepsilon, 0) \bar{N}'(\varepsilon, 0) e^{-(\varepsilon - \Delta_0)/T}}{1.82 \int_{\Delta_0}^{\infty} d\varepsilon\, \bar{N}'(\varepsilon, 0) e^{-(\varepsilon - \Delta_0)/T}} \ . \tag{24b}$$

Here $\bar{N}'(\varepsilon, 0)$ is the normalized density of states in S', which is equal to $\bar{N}(\varepsilon, 0)$ due to the vanishing resistance of the SS' boundary ($\gamma_B = 0$). Because $\bar{N}'(\varepsilon, 0)$ generally differs fairly much from $\bar{N}^{\rm BCS}(\varepsilon)$, the integral in the numerator of Eq. (24) cannot be approximated by $B(T)$ when $L'_{\rm eff}(T)$ is calculated. Also here one finds in the low-temperature regime, $T \ll (\Delta_0 - \Delta_g)$,

$$L'_{\rm eff}(T) \sim (\Delta_0 - \Delta_g)^{3/2}, \quad T \ll (\Delta_0 - \Delta_g) \ . \tag{25a}$$

In the intermediate range, $(\Delta_0 - \Delta_g) \ll T \leq \Delta_0$, and for small γ_m, approximating $\bar{N}'(\varepsilon, 0)$ by $\bar{N}^{\rm BCS'} = {\rm Re}[\varepsilon/(\varepsilon^2 - \Delta_g^2)^{1/2}]$, one obtains

$$L'_{\rm eff}(T) \sim T^2, \quad (\Delta_0 - \Delta_g) \ll T \leq \Delta_0 \ . \tag{25b}$$

In Fig. 6, $L_{\rm eff}(T)/\xi_s$ and $L'_{\rm eff}(T)/d$ are given as a function of temperature for different values of γ_m, calculated from Eqs. (19b) and (24b), respectively. It is seen that in accordance with Eqs. (20) and (25b), $L_{\rm eff}$ and $L'_{\rm eff}$

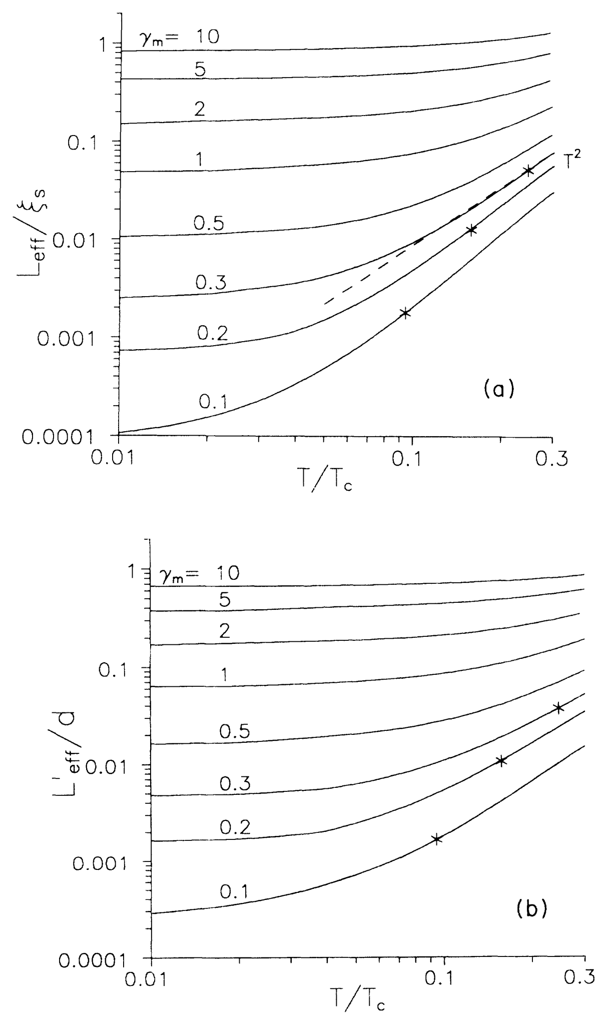

FIG. 6. Temperature dependence of the effective trapping lengths of an SS' sandwich ($T_c^*/T_c = 0.14, \gamma_B = 0$) with γ_m ranging from 0.1 to 10: (a) effective length $L_{\rm eff}/\xi_s$ in S; (b) effective length $L'_{\rm eff}/d$ in S'. The crossover temperature $T^* = \Delta_0 - \Delta_g(\gamma_m)$ is indicated with a star (*).

are constant at low temperatures and that their values only depend on Δ_g determined by γ_m. L_{eff} and L'_{eff} increase with increasing temperature, reaching the T^2 dependence in the intermediate temperature range. The crossover temperature, $T^* \equiv \Delta_0 - \Delta_g(\gamma_m)$, is indicated by a star in the corresponding curve. In Fig. 4 of Ref. 17 L_{eff} and L'_{eff} are given as a function of γ_m at $T=0$.

To calculate the effective trapping time τ_{tr}, for an electronlike quasiparticle in the SS' bilayer with energy $\varepsilon > \Delta_0$, one needs to find the quasiparticle number ratios in Eq. (11). These ratios read

$$\frac{N_{\varepsilon>\Delta_0}}{N_{\varepsilon>\Delta_0}+N'_{\varepsilon>\Delta_0}}=(1+\alpha)^{-1}\approx 1\ ;$$

$$\frac{N'_{\varepsilon>\Delta_0}}{N_{\varepsilon>\Delta_0}+N'_{\varepsilon>\Delta_0}}=(1+\alpha^{-1})^{-1}\approx\alpha\ , \tag{26a}$$

with

$$\alpha=\frac{N'_{\varepsilon>\Delta_0}}{N_{\varepsilon>\Delta_0}}=\frac{C'N'(0)}{CN(0)}\frac{B'(T)}{B(T)}\frac{d}{d_s}\ll 1\ . \tag{26b}$$

The latter inequality is obtained from the fact that $d \ll d_s$ and the other factors in (26b) are of the order 1, as will be shown further on. C' is the constant in Eq. (14) for the distribution function in the S' layer. $B'(T)$ is the function

$$B'(T,\gamma_m)=\int_{\Delta_0}^{\infty} d\varepsilon\,\bar{N}'(\varepsilon,0)e^{-(\varepsilon-\Delta_0)/T}\ , \tag{27a}$$

in analogy with Eq. (16). Because $\gamma_B=0$, we have $\bar{N}'(\varepsilon,0)=\bar{N}(\varepsilon,0)$. For not too small γ_m values the density of states $\bar{N}(\varepsilon,0)\approx 1$ for $\varepsilon \ge \Delta_0$, so that

$$B'(T,\gamma_m)\approx T,\quad \gamma_m \ge 1\ . \tag{27b}$$

For $\gamma_m \le 1$ we have $\bar{N}'\approx \bar{N}^{\text{BCS}'}$. At temperatures $T \ll (\Delta_0-\Delta_g)$ one can replace $N'(\varepsilon,0)$ in (27a) by its lower limit value, so that $B'(T,\gamma_m)$ is

$$B'(T,\gamma_m)\approx T\Delta_0(\Delta_0^2-\Delta_g^2)^{-1/2},\quad T\ll(\Delta_0-\Delta_g),\quad \gamma_m\le 1\ . \tag{27c}$$

For small γ_m we have approximately $\bar{N}(\varepsilon,0)\approx\bar{N}^{\text{BCS}}(\varepsilon)$, from which it follows that

$$B'(T,\gamma_m)\approx B(T),\quad \gamma_m \ll 1\ . \tag{27d}$$

A relation between C' and C is obtained from the condition that the quasiparticle flow across a plane parallel to the SS' interface should be continuous (conservation of charge). Applying this condition in particular to the current $I(S\to S')$ from S to S' across the SS' interface, we have the equality for I_S in S and $I_{S'}$ in S' at the interface

$$\begin{aligned} I_S(S\to S')&=N(0)C\int_{\Delta_0}^{\infty} d\varepsilon\,\bar{N}(\varepsilon,0)e^{-(\varepsilon-\Delta_0)/T}v_F|\partial\bar{N}/\partial\varepsilon|^{-1}\\ &=I_{S'}(S'\to S')=N(0)C'\int_{\Delta_0}^{\infty} d\varepsilon\,\bar{N}'(\varepsilon,0)e^{-(\varepsilon-\Delta_0)/T}v'_F|\partial\bar{N}'/\partial\varepsilon|^{-1}\ , \end{aligned} \tag{28}$$

where the group velocity is given by $v=v_F|\partial\bar{N}/\partial\varepsilon|^{-1}$, and v_F (v'_F) is the Fermi velocity in S (S'). In equilibrium $I(S\to S')$ is counterbalanced by the quasiparticle flow in the reverse direction. Since $\bar{N}=\bar{N}'$ at $x=0$ it follows immediately that $N'(0)C'/N(0)C)=v_F/v'_F$. The ratio α is thus given by

$$\alpha=\beta\frac{d}{d_s},\quad \beta=\frac{v_F}{v'_F}\frac{B'(T,\gamma_m)}{B(T)}\ , \tag{29a}$$

where

$$\frac{B'(T,\gamma_m)}{B(T)}\approx\begin{cases}(2T/\pi\Delta_0)^{1/2}=0.60(T/T_c)^{1/2},\quad \gamma_m\ge 1,\quad T\ll 2\Delta_0(0) & \text{(29b)}\\ (2\Delta_0 T/\pi)^{1/2}(\Delta_0^2-\Delta_g^2)^{1/2},\quad \gamma_m \le 1,\quad T\ll(\Delta_0-\Delta_g) & \text{(29c)}\\ 1,\quad \gamma_m\ge 1,\quad T\ge\Delta_0(T)\ \text{ or }\ \gamma_m\ll 1\ . & \text{(29d)}\end{cases}$$

For large γ_m one has $\Delta_g \ll \Delta_0$ so that (29c) goes over in the limit of (29b). The total trapping rate [Eq. (11)] is now obtained as

$$\frac{1}{\tau_{\text{tr}}}=\frac{1.82}{d_s}\left[\frac{L_{\text{eff}}(T,\gamma_m)}{\tau_0}+\beta\frac{L'_{\text{eff}}(T,\gamma_m)}{\tau'_0}\right]\ . \tag{30}$$

This equation was obtained before in Ref. 19, but without the factor $\beta(T,\gamma_m)$. The effective trapping time is reduced if the area A of the superconductor S is larger than the area A' of S'. This effect is easily taken into account by multiplying the right-hand side of Eq. (30) with A'/A, assuming (a) that the fraction of time the quasiparticles spend in the area with the reduced gap region is proportional to the fraction of the area to the total area of the S electrode, and (b) that all densities of states and distribution functions in the area with the reduced gap region are not dependent on the coordinates in the plane of the sandwich.

Since the time constants for the holelike quasiparticles are also given by Eqs. (12a) and (24a), Eq. (30) is also applicable to the trapping of holes.

B. Quasiparticle excitation out of the trap

At finite temperature an electronlike quasiparticle in the trap may absorb a phonon which has enough energy that it can excite the quasiparticle out of the trap. The quasiparticle and phonon cascade, following the absorption of a particle or phonon, ends on a timescale much shorter than the trapping and excitation time constants. At low temperatures the recombination time constants are much larger than the time constants considered here so that recombination processes can be neglected. In that case there is a steady-state quasiparticle distribution (which is not necessarily the thermal equilibrium state since an excess quasiparticle density may be present) during which there is an exchange of particles between trap and bulk. The rate of particles being excited out of the trap is equal to the rate of particles being trapped,

$$\left|\frac{dN^{\rm tot}_{\varepsilon<\Delta_0}}{dt}\right| = \left|\frac{dN^{\rm tot}_{\varepsilon>\Delta_0}}{dt}\right| , \tag{31}$$

where $N^{\rm tot}_{\varepsilon<\Delta_0}$ is the number of particles in the trap, thus with energy $\Delta_g<\varepsilon<\Delta_0$. The time constant for a single trapped particle to be excited out of the trap is defined by

$$\frac{1}{\tau_{\rm exc}} = \frac{1}{N^{\rm tot}_{\varepsilon<\Delta_0}}\left|\frac{dN^{\rm tot}_{\varepsilon<\Delta_0}}{dt}\right| . \tag{32}$$

Combination of Eqs. (31) and (32) gives

$$\frac{1}{\tau_{\rm exc}} = \frac{1}{N^{\rm tot}_{\varepsilon<\Delta_0}}\left|\frac{dN^{\rm tot}_{\varepsilon>\Delta_0}}{dt}\right| \tag{33a}$$

$$= \left[\frac{1}{N_{\varepsilon<\Delta_0}}\left|\frac{dN_{\varepsilon>\Delta_0}}{dt}\right|\right](1+\delta)^{-1} + \left[\frac{1}{N'_{\varepsilon<\Delta_0}}\left|\frac{dN'_{\varepsilon>\Delta_0}}{dt}\right|\right](1+\delta^{-1})^{-1} . \tag{33b}$$

The quasiparticle number ratios in (33b) are defined by δ

$$\delta = \frac{N'_{\varepsilon<\Delta_0}}{N_{\varepsilon<\Delta_0}} = \frac{C'N'(0)}{CN(0)}\frac{E'(T,\gamma_m)}{E(T,\gamma_m)} = \frac{v_F E'(T,\gamma_m)}{v'_F E(T,\gamma_m)} . \tag{34}$$

For the latter equality use has been made of (28). The ratio of the (normalized) number of states of quasiparticles in the trap in S' and in S is

$$\frac{E'(T,\gamma_m)}{E(T,\gamma_m)} = \frac{d\int_{\Delta_g}^{\Delta_0} d\varepsilon\,\overline{N}'(\varepsilon,0)e^{-(\varepsilon-\Delta_g)/T}}{\int_0^{d_s}dx\int_{\Delta_g}^{\Delta_0}d\varepsilon\,\overline{N}(\varepsilon,x)e^{-(\varepsilon-\Delta_g)/T}} . \tag{35a}$$

One can again make use of the relation $\overline{N}'(\varepsilon,0)=\overline{N}(\varepsilon,0)$. In the high-temperature limit $T\gg(\Delta_0-\Delta_g)$ the ratio E'/E reduces to

$$E'(T,\gamma_m)/E(T,\gamma_m)\sim\text{const},\quad T\gg(\Delta_0-\Delta_g) . \tag{35b}$$

This limit corresponds to the situation that the traps in S' and S are nearly filled up with quasiparticles.

The ratio E'/E is also found in the effective tunneling length $L^*_{\text{eff}-t}(T)$ [Eq. (43b)], which is discussed in the next section, and which is related to this quantity as

$$E'(T,\gamma_m)/E(T,\gamma_m) = d/L^*_{\text{eff-}t}(T,\gamma_m) . \tag{35c}$$

$L^*_{\text{eff}-t}(T)$ is shown in Fig. 8 as function of temperature, for different values of γ_m.

The first term in square brackets in (33b) gives the excitation rate out of the trap in S, which will be considered now. From Eqs. (11a), (14), and (18) it follows that

$$|dN_{\varepsilon>\Delta_0}/dt| = 1.82N(0)L_{\rm eff}(T)CB(T)/\tau_0 .$$

Writing out $N_{\varepsilon>\Delta_0}$ and substituting $|dN_{\varepsilon>\Delta_0}/dt|$ one obtains

$$\left[\frac{\tau_0}{\tau_{\rm exc}}\right]_S = e^{-(\Delta_0-\Delta_g)/T} \times\frac{1.82L_{\rm eff}(T)B(T)}{\int_0^{d_s}dx\int_{\Delta_g}^{\Delta_0}d\varepsilon\,\overline{N}(\varepsilon,x)e^{-(\varepsilon-\Delta_g)/T}} . \tag{36}$$

In the limit of low temperatures the temperature dependence of $(\tau_0/\tau_{\rm exc})_S$ is dominated by the exponential factor

$$(\tau_0/\tau_{\rm exc})_S \sim e^{-(\Delta_0-\Delta_g)/T}T^{-1/2} ,$$
$$T\ll(\Delta_0-\Delta_g)\ll 2\Delta_0(0) . \tag{37a}$$

Here we made use of Eqs. (15) and (19) and the fact that in the trap $\overline{N}(\varepsilon,x)$ is bounded and of the order of $\overline{N}(\Delta_0,0)$, so that the integration over energy leaves only a factor $1/T$. From (37a) it is seen that the excitation rate decreases exponentially with decreasing temperature, reflecting the exponential decrease of the number of phonons with decreasing T. With increasing γ_m, hence increasing $\Delta_0-\Delta_g$, $(\tau_0/\tau_{\rm exc})_S$ also decreases strongly in this limit. In the intermediate-temperature region, $T\gg(\Delta_0-\Delta_g)$, the energy integral in the numerator gives approximately $(\Delta_0-\Delta_g)\overline{N}(\Delta_0,0)$, and is thus nearly temperature independent. In this limit only the factors $L_{\rm eff}(T)B(T)$ give rise to a dependence on temperature

$$(\tau/\tau_{\rm exc})_S\sim T^3,\quad (\Delta_0-\Delta_g)\ll T\le\Delta_0(0) . \tag{37b}$$

Since $L_{\rm eff}(T)$ increases with increasing γ_m, $(\tau_0/\tau_{\rm exc})_S$ also becomes larger with γ_m.

In the S' layer an equation for $\tau'_{\rm exc}$, given by the term in the second pair of square brackets in (33b), analogous to that of $\tau_{\rm exc}$ in S, is straightforwardly derived as

$$\left[\frac{\tau'_0}{\tau'_{\rm exp}}\right]_{S'} = e^{-(\Delta_0-\Delta_g)/T}\frac{1.82L'_{\rm eff}(T)B'(T)}{d\int_{\Delta_g}^{\Delta_0}d\varepsilon\,\overline{N}(\varepsilon,0)e^{-(\varepsilon-\Delta_g)/T}} . \tag{38}$$

For $(\tau'_0/\tau'_{\rm exc})_{S'}$ the same dependencies on temperature in the low and intermediate regions hold as for $(\tau_0/\tau_{\rm exc})_S$.

In Figs. 7(a) and 7(b), $(\tau_0/\tau_{\rm exc})_S$ and $(\tau'_0/\tau'_{\rm exc})_{S'}$ are shown as functions of the reduced temperature, calculated from Eqs. (36) and (38). It is seen that the temperature dependence given by (37b) holds nearly for the whole temperature regime considered in the figures, especially for small γ_m. For larger γ_m the stronger temperature and γ_m dependences (37a) are observed.

The total excitation rate of Eq. (33) can now be written

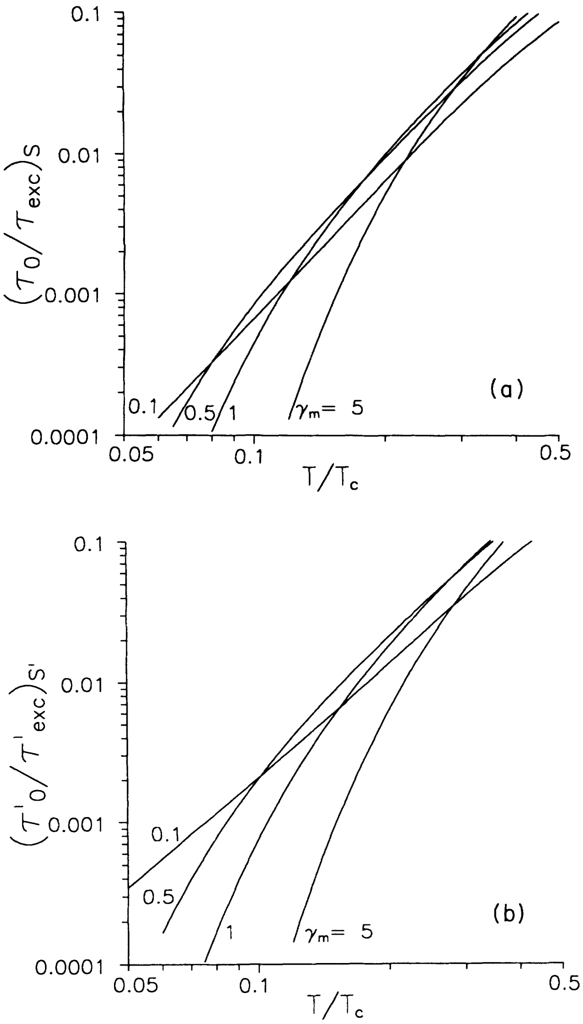

FIG. 7. Temperature dependence of the excitation rate out of the trap in an SS' temperature ($T_c^*/T_c=0.14, \gamma_B=0$) with γ_m ranging from 0.1 to 5: (a) excitation rate $(\tau_0/\tau_s)_S$ out of the trap in S; (b) excitation rate $(\tau'_0/\tau'_s)_{S'}$ out of the trap in S'.

in terms of Eqs. (36) and (38) and δ as

$$\frac{1}{\tau_{exc}}=\left[\frac{\tau_0}{\tau_{exc}}\right]_S\frac{1}{1+\delta}\frac{1}{\tau_0}+\left[\frac{\tau'_0}{\tau'_{exc}}\right]_{S'}\frac{\delta}{1+\delta}\frac{1}{\tau'_0}\ . \qquad (39)$$

For the holes the same excitation rates, Eqs. (36), (38), and (39), are found.

V. TUNNELING TIMES IN AN $SS'IS''S$ JUNCTION

The tunneling time of a junction gives the rate with which a quasiparticle tunnels from one electrode to the other. In a junction detector this tunneling process is used to measure the excess quasiparticle density in one of the electrodes as an excess current. In junctions with spatially homogeneous electrodes the tunneling time τ_{tun} is fairly long, especially if thick electrodes are used, since τ_{tun} is proportional to the thickness. In practical energy-resolving detectors this means that a large fraction of the excess quasiparticle density may be lost, due to loss processes that are faster than the tunneling time. A trapping layer is therefore not only advantageous to collect the quasiparticles efficiently from a bulk superconductor, but also because the tunneling time out of the trap is much shorter, due to the smaller thickness of the trap compared to that of the bulk.

The total tunneling current is the sum of electron and hole currents. As far as the branch imbalance relaxation is a fast process compared to tunneling, the number of electrons and holes in the trap are considered to be equal. However, the dependencies of the corresponding tunneling times on the bias voltage are qualitatively different. Therefore one should consider separately, tunneling times for electrons and for holes both in the trap and in the bulk, and from electrode 1 to electrode 2 and in the reverse direction. Here and further on, the indices 1 and 2 refer to the SS' and $S''S$ electrode, respectively.

The tunneling rate of an electronlike quasiparticle in the trap from electrode 1 to 2, $\tau_{tun}^{(e)}(1\to2)$ is defined in the following way:

$$\frac{1}{\tau_{tun}^{(e)}(1\to2)}=\frac{1}{N_{\varepsilon<\Delta_0,1}^{(e)}}\left|\frac{dN_{\varepsilon<\Delta_{0,1}}^{(e)}}{dt}\right|\ , \qquad (40)$$

where $N_{\varepsilon<\Delta_0,1}^{(e)}$ is the total number of electrons in the trap in electrode 1. The rate can be related to the electron part of the tunneling current from the trap in electrode 1 to electrode 2,

$$I_{\varepsilon<\Delta_0}^{(e)}(1\to2)=e\left|\frac{dN_{\varepsilon<\Delta_{0,1}}^{(e)}}{dt}\right|$$
$$=\frac{1}{eR_N}\int_{\Delta_{1g}}^{\Delta_0}d\varepsilon\,\overline{N}'_1(\varepsilon,0)\overline{N}'_2(\varepsilon+eV,0)$$
$$\times f_1(\varepsilon)[1-f_2(\varepsilon+eV)]\ . \qquad (41)$$

It is noted that the factors $N'_1(0)$ and $N'_2(0)$ are contained in the resistance of the junction R_N. Keeping in mind that the quasiparticles in the trap are both in the reduced gap regions of the S and of the S' layer, we can write

$$\tau_{tun}^{(e)}(1\to2)=e^2R_N\frac{\left[N_1(0)A'\int_0^{d_{21}}dx\int_{\Delta_{1g}}^{\Delta_0}d\varepsilon\,\overline{N}_1(\varepsilon,x)f_1(\varepsilon)+N'_1(0)A'd\int_{\Delta_{1g}}^{\Delta_0}d\varepsilon\,\overline{N}'_1(\varepsilon,0)f_1(\varepsilon)\right]}{\int_{\Delta_{1g}}^{\Delta_0}d\varepsilon\,\overline{N}'_1(\varepsilon,0)\overline{N}'_2(\varepsilon+eV,0)f_1(\varepsilon)[1-f_2(\varepsilon+eV)]} \qquad (42a)$$

$$\equiv\tau_{tun,S}^{(e)}(1\to2)+\tau_{tun,S'}^{(e)}(1\to2)\ . \qquad (42b)$$

The denominator in (42a) gives the number $N^{(e)}_{\varepsilon<\Delta_0,1}$ and A' is the area of the reduced gap region, which is not necessarily equal to the junction area A_J. The latter is contained in the junction resistance, which can be written as $R_N^{-1}=A_J\cdot\sigma_N$, with σ_N the normal-state conductivity per unit area. $\tau^{(e)}_{\mathrm{tun},S}(1\rightarrow2)$ is due to the term containing the integral $N_1(0)A'\int dx\int d\varepsilon(\cdots)$ and $\tau^{(e)}_{\mathrm{tun},S'}(1\rightarrow2)$ due to the term with $N_1'(0)A'd\int d\varepsilon(\cdots)$.

One can now define an effective tunneling length $L_{\text{eff-}t}(V,T)$ by the definition

$$\tau^{(e)}_{\mathrm{tun},S}(1\rightarrow2)=e^2R_NA'N_1(0)L_{\text{eff-}t}(V,T)\ , \tag{43a}$$

with

$$L_{\text{eff-}t}(V,T)=\frac{\int_0^{d_{s1}}dx\int_{\Delta_{1g}}^{\Delta_0}d\varepsilon\,\overline{N}_1(\varepsilon,x)f_1(\varepsilon)}{\int_{\Delta_{1g}}^{\Delta_0}d\varepsilon\,\overline{N}_1(\varepsilon,0)\overline{N}_2(\varepsilon+eV,0)f_1(\varepsilon)[1-f_2(\varepsilon+eV)]}\ , \tag{43b}$$

where we have used the relations $\overline{N}_1=\overline{N}_1'$ and $\overline{N}_2=\overline{N}_2'$. In the limit of low temperatures, $T\ll(\Delta_{2g}-\Delta_{1g})$, Eq. (43b) can be simplified, because $\overline{N}_2(\varepsilon+eV,0)\approx\overline{N}_2(\Delta_{1g}+eV)$ and $[1-f_2(\varepsilon+eV)]\approx1$, so that

$$L_{\text{eff-}t}(v,T)=\frac{L^*_{\text{eff-}t}(T)}{\overline{N}_2(\Delta_{1g}+eV)}\ , \tag{44a}$$

$$L^*_{\text{eff-}t}(T)\approx\frac{\int_0^{d_{s1}}dx\int_{\Delta_{1g}}^{\Delta_0}d\varepsilon\,\overline{N}_1(\varepsilon,x)f_1(\varepsilon)}{\int_{\Delta_{1g}}^{\Delta_0}d\varepsilon\,\overline{N}_1(\varepsilon,0)f_1(\varepsilon)}\ . \tag{44b}$$

Due to the fact that the density of states in S' is spatially constant the ratio $L^*_{\text{eff-}t}(T)/d$ is equal to the ratio of the number of quasiparticles in the trap in S [normalized to $N(0)$] to the corresponding number in S', which is given by E/E' of Eq. (35a).

Analogously one finds for $\tau^{(e)}_{\mathrm{tun},S'}(1\rightarrow2)$,

$$\tau^{(e)}_{\tau,S'}(1\rightarrow2)=e^2R_NN_1'(0)A'\frac{d_1}{\overline{N}_2(\Delta_{1g}+eV)}\ , \tag{45}$$

so that (42b) can now be written as

$$\tau^{(e)}_{\mathrm{tun}}(1\rightarrow2)=e^2R_NA'\frac{N_1(0)L^*_{\text{eff-}t}(\gamma_m,T)+N_1'(0)d_1}{\overline{N}_2(\Delta_{1g}+eV)}\ ,$$

$$T\ll(\Delta_{2g}-\Delta_{1g})\ . \tag{46}$$

In the spatially homogeneous case $d=0$, $L^*_{\text{eff-}t}=d_s$, $A'=A$, and $\overline{N}_2(\Delta_{1g}+eV)=\overline{N}_2^{\mathrm{BCS}}(\Delta_1+eV)$, and (46) reduces to the result obtained by Ivlev et al.[21] for the tunneling of a junction with homogeneous electrodes, but with different bulk gaps Δ_{01} and Δ_{02},

$$\tau^{(e)}_{\mathrm{tun}}(1\rightarrow2)=e^2R_NN_1(0)V_{s1}\left[1-\left[\frac{\Delta_{02}}{\Delta_{01}+eV}\right]^2\right]^{1/2}\ , \tag{47}$$

with $V_{s1}=A'd_{s1}$ the volume of electrode 1. For voltages $eV>(\Delta_{01}-\Delta_{01})$, $\overline{N}_2\approx1$, so that (47) reduces to the well-known Ginsberg equation[20]

$$\tau^{(e)}_{\mathrm{tun}}(1\rightarrow2)=e^2R_NN_1(0)V_{s1}\ . \tag{48}$$

From Eq. (46) it is seen that for low temperatures the temperature dependence of the tunneling time $\tau^{(e)}_{\mathrm{tun}}(1\rightarrow2)$ is described by the effective tunneling length in S, $L^*_{\text{eff-}t}(T)$. For the distribution function $f_1(\varepsilon)$ one can take the nonequilibrium form of Eq. (14). Since the factor C drops out, it follows that the tunneling length is equal for equilibrium and excess quasiparticles in this approximation. In Fig. 8, $L^*_{\text{eff-}t}(T)$ is shown as function of temperature for different strengths of the proximity effect. For small γ_m the shallow trap is quickly filled with quasiparticles when temperature increases. The average length from which the quasiparticle tunnel consequently also increases, until the trap is filled and $L^*_{\text{eff-}t}$ saturates. The larger γ_m, the deeper the trap and saturation occurs at higher temperatures. For the largest γ_m values the latter situation is not reached in the temperature interval considered. It is noted that the temperature range for which these results are valid, is given by the condition $T\ll[\Delta_{2g}(\gamma_{m2})-\Delta_{1g}(\gamma_{m1})]$, and thus depends on the γ_m values of both electrodes. The larger the gap difference, the larger the range of applicability.

From Eq. (46) it follows that the voltage dependence of the tunneling rate at low temperatures is given by the factor $\overline{N}_2(\Delta_{1g}+eV)$. This means that $1/\tau^{(e)}_{\mathrm{tun}}(1\rightarrow2)$ reflects the density of states at the IS'' interface of the opposite electrode. The tunneling rate vanishes for $-eV_g<eV$

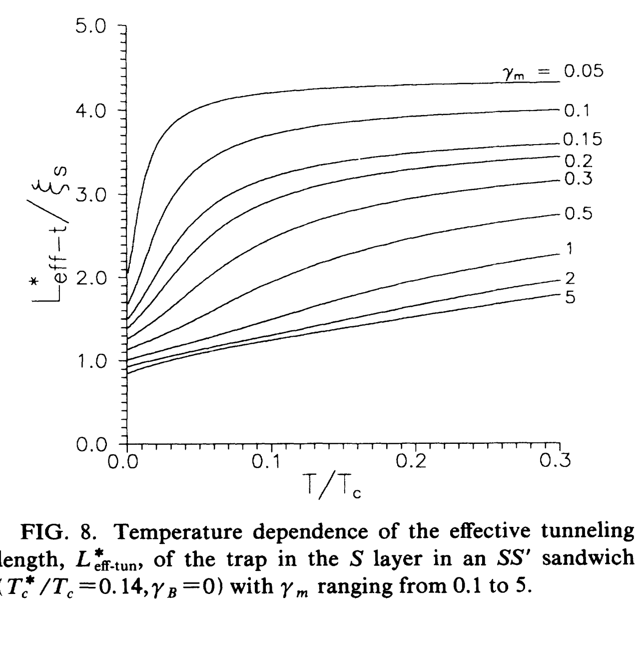

FIG. 8. Temperature dependence of the effective tunneling length, $L^*_{\text{eff-tun}}$, of the trap in the S layer in an SS' sandwich ($T_c^*/T_c=0.14,\gamma_B=0$) with γ_m ranging from 0.1 to 5.

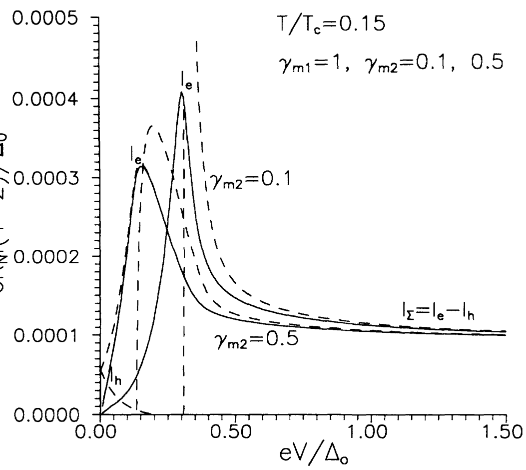

FIG. 9. Electron and hole tunneling currents, $I^{(e)}_{\varepsilon<\Delta_0}(1\rightarrow2)$, resp. $I^{(h)}_{\varepsilon<\Delta_0}(1\rightarrow2)$, of quasiparticles in the trap of the SS' electrode (1) of an $SS'IS''S$ junction ($T^*_{c1}/T_c=T^*_{c2}/T_c=0.14$, $\gamma_{B1}=\gamma_{B2}=0$, corresponding to a Nb/Al junction) to the $S''S$ electrode (2) at a temperature $T/T_c=0.15$, and with proximity parameters $\gamma_{m1}=1$ and $\gamma_{m2}=0.1$. Solid curves: exact voltage dependence [Eq. (41)]; dashed curves: approximate voltage dependence [Eq. (45)].

$<(\Delta_{2g}-\Delta_{1g})$, where $V_g=(\Delta_{2g}+\Delta_{1g})/e$ is the sum gap voltage of the junction. This is due to the fact that in this voltage range there are no states in electrode 2 to which the quasiparticles in the trap in electrode 1 can tunnel.

At higher temperatures one should consider the full expression of the tunneling current, Eq. (41), for describing the voltage dependence of $1/\tau^{(e)}_{\text{tun}}(1\rightarrow2)$. In Fig. 9 the normalized electron current $I^{(e)}_{\varepsilon<\Delta_0}(1\rightarrow2)(eR_N/\Delta_0)$ from Eq. (41) is shown as function of voltage at $T/T_c=0.15$ for $\gamma_{m1}=1$ and $\gamma_{m2}=0.1$ and 0.5. Also is shown the approximate voltage dependence given by $\overline{N}_2[\Delta_{1g}(\gamma_{m1}=1)+eV,\gamma_{m2}=0.1]$ and $\overline{N}_2[\Delta_{1g}(\gamma_{m1}=1)+eV,\gamma_{m2}=0.5]$ at $T\ll T_c$, where $\overline{N}_2$ is normalized such that it has the same amplitude as $I^{(e)}_{\varepsilon<\Delta_0}(1\rightarrow2)$ at $eV=2\Delta_0$. The sharp features in $\overline{N}_2$ are smeared out at this relative high temperature $T/T_c=0.15$, which should be compared with

$$|\Delta_{1g}(\gamma_{m1}=1)-\Delta_{2g}(\gamma_{m2}=0.1)|/T_c=0.52$$

and

$$|\Delta_{1g}(\gamma_{m1}=1)-\Delta_{2g}(\gamma_{m2}=0.5)|/T_c=0.23\ ,$$

respectively, since the voltage-dependent factor in $\tau^{(e)}_{\text{tun}}$ was separated from the temperature-dependent factor under the condition $T\ll(\Delta_{2g}-\Delta_{1g})$.

It can be shown straightforwardly that the tunneling time for holes from the trap in electrode 1 to electrode 2 is related to the electron tunneling time by

$$\tau^{(h)}_{\text{tun}}(1\rightarrow2)(V,T)=\tau^{(3)}_{\text{tun}}(1\rightarrow2)(-V,T)\ . \tag{49}$$

The total current of positive charge tunneling from electrode 1 to 2 is thus

$$I_{\text{tot}}(1\rightarrow2)=I^{(h)}-I^{(e)}\ . \tag{50}$$

Because at low temperatures $1/\tau^{(h)}_{\text{tun}}(1\rightarrow2)$ and consequently $I^{(h)}(1\rightarrow2)$ vanishes for voltages in the range $0<V<V_{g'}$, the tunneling current is then mainly due to electrons

$$I_{\text{tot}}(1\rightarrow2)\simeq-I^{(e)}(1\rightarrow2),\quad 0<V<V_g\ . \tag{51a}$$

In the opposite case of negative voltage, tunneling is mainly caused by holes

$$I_{\text{tot}}(1\rightarrow2)\simeq I^{(h)}(1\rightarrow2),\quad -V_g<V<0\ . \tag{51b}$$

At finite temperatures and positive voltage there is a finite hole current, which decreases strongly with increasing voltage, as is seen in Fig. 9. At zero-bias voltage the hole and electron currents from electrode 1 to 2 cancel, as is reflected in the equal tunneling times at $V=0$.

For the tunneling time of electrons with energy $\varepsilon>\Delta_0$, thus out of the bulk, one has an expression similar to that for electrons in the trap, Eq. (42), but with the following substitution for the limits of the integrals over energy in (42b),

$$\int_{\Delta_{1g}}^{\Delta_0}d\varepsilon\cdots\rightarrow\int_{\Delta_0}^{\infty}d\varepsilon\cdots\ .$$

In order to get a first-order approximation of $L^*_{\text{eff-}t,\text{bulk}}(T)$ in the low-temperature limit, we use for the high-energy part of the densities of states, i.e., $\varepsilon>\Delta_0$, the approximations $\overline{N}_1(\varepsilon,0)\simeq1(\gamma_m\gg1)$ and $\overline{N}_1(\varepsilon,0)\simeq\overline{N}^{\text{BCS}}(\varepsilon)$ $(\gamma_m\ll1)$ in the numerator (corresponding to the density of states at the $S'I$ interface), and $\overline{N}_1(\varepsilon,x)\simeq\overline{N}^{\text{BCS}}(\Delta_0)$ in the denumerator of Eq. (44b) (approximate average density of states in the SS' electrode), resulting in

$$L^*_{\text{eff-}t,\text{bulk}}(T)\simeq\begin{cases}d_{s1}, & \gamma_m\ll1\\ d_{s1}[(\pi/2)(\Delta_0/T)]^{1/2}, & \gamma_m\gg1, T\ll2\Delta_0\,.\end{cases} \tag{52}$$

As one may expect the effective tunneling length becomes equal to the electrode thickness if trapping is negligible. For deep traps, large γ_m, $L^*_{\text{eff-}t,\text{bulk}}$ becomes large than d_{s1} due to the decreasing number of states at the interface with the barrier from which the quasiparticles can tunnel. Thus one finds that $L^*_{\text{eff-}t,\text{bulk}}$ is at least equal to the thickness of the S layer and increases with decreasing temperature. Therefore we have the condition

$$L^*_{\text{eff-}t,\text{bulk}}\gg L^*_{\text{eff-}t,\text{trap}}\approx(1\cdots4)\xi_s\ ,$$

since $d_s\gg\xi_s$, from which it follows that the tunneling rate out of the bulk is negligible compared to that out of the trap. The same argument holds for the tunneling rate of holes out of the bulk.

Now we will consider the tunneling rates from electrode 2 to 1 for a junction with $\Delta_{1g}<\Delta_{2g}$. The corresponding equations for the tunneling times can be derived straightforwardly. This results in the following time constants for tunneling out of the trap

$$\tau^{(e)}_{\text{tun}}(2\rightarrow1)\simeq e^2R_N\frac{N_2(0)L^*_{\text{eff-}t}(\gamma_{m2},T)+N'_2(0)d_2}{\overline{N}_1(\Delta_{2g}-eV)}\ , \tag{53a}$$

and

$$\tau_{\text{tun}}^{(h)}(1\rightarrow 2)(V,T)=\tau_{\text{tun}}^{(e)}(2\rightarrow 1)(-V,T)\ . \tag{53b}$$

In Fig. 10 the tunneling currents $I^{(e)}(2\rightarrow 1) \sim 1/\tau_{\text{tun}}^{(e)}(2\rightarrow 1)$ and $I^{(h)}(2\rightarrow 1)\sim 1/\tau_{\text{tun}}^{(h)}(2\rightarrow 1)$, as well as the sum current $I_{\text{tot}}(2\rightarrow 1)=I^{(h)}-I^{(e)}$ of positive charge from electrode 2 to 1 are shown, for the cases $\gamma_{m1}=1$ and $\gamma_{m2}=0.1$ and 0.5. Also depicted are the approximate voltage dependencies due to the term $\bar{N}_1$. For voltages $0<V<(\Delta_{2g}-\Delta_{1g})$, $I^{(e)}$ reflects the bump in the quasiparticle density of states in electrode 1, given by $\bar{N}_1(\Delta_{2g}-eV)$, which vanishes for $(\Delta_{2g}-\Delta_{1g})<V<V_g$. $I^{(h)}$ is due to the high-energy part of $\bar{N}_1(\Delta_{2g}+eV)$ which goes to 1 for $(\Delta_{2g}+eV)\gg\Delta_{1g}$. For $0<V<(\Delta_{2g}-\Delta_{1g})$, I_{tot} becomes negative, which implies that the positive charge current is flowing from 1 to 2. This can be understood from the fact that in this voltage range there are more empty electron states in electrode 1 to which the electrons from electrode 2 can tunnel, than there are empty hole states in electrode 1 for electrons tunneling from electrode 2 to 1.

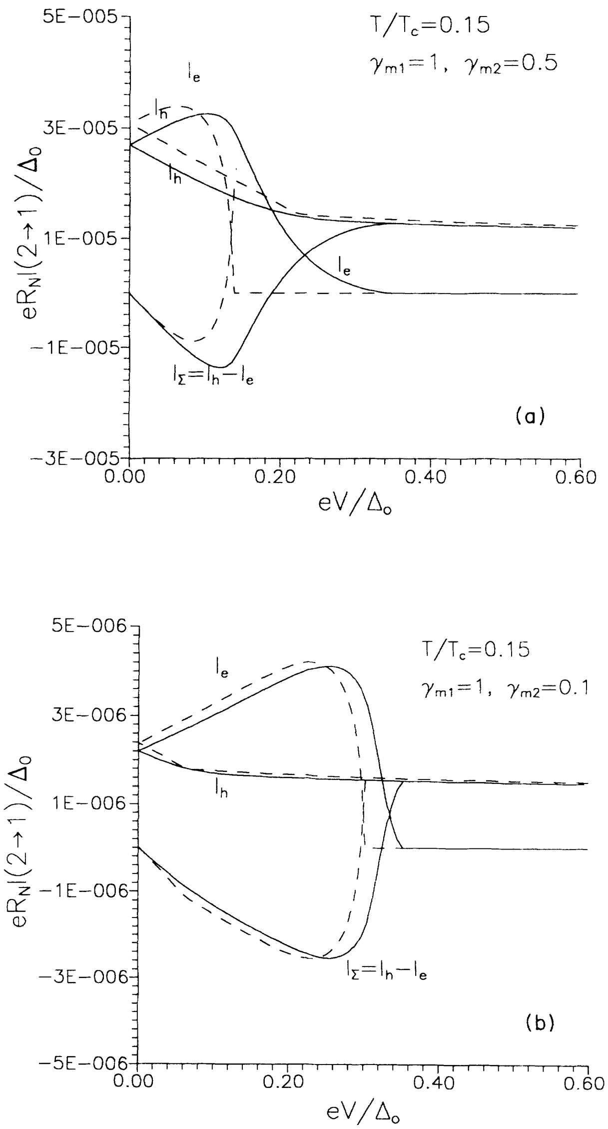

FIG. 10. As Fig. 9, but for tunneling from the trap in the $S''S$ electrode (2) to the SS' electrode (1), $I^{(e)}_{\varepsilon<\Delta_0}(2\rightarrow 1)$, resp. $I^{(h)}_{\varepsilon<\Delta_0}(2\rightarrow 1)$. (a) $\gamma_{m1}=1$, $\gamma_{m2}=0.5$, (b) $\gamma_{m1}=1$, $\gamma_{m2}=0.1$.

VI. DISCUSSION AND CONCLUSIONS

In Secs. IV and V we have derived expressions for the relaxation rate, τ_{tr}^{-1} [Eq. (30)], into, and excitation rate, τ_{exc}^{-1} [Eq. (39)], out of the reduced gap region in a SS' proximity sandwich at finite temperatures and as function of the strength of the proximity effect, given by the parameter γ_m. Second, in Sec. V the tunneling times for electrons and holes, $\tau_{\text{tun}}^{(e)}(1\rightarrow 2)$ [Eq. (46)] and $\tau_{\text{tun}}^{(h)}(1\rightarrow 2)$ (49), $\tau_{\text{tun}}^{(e)}(2\rightarrow 1)$ (53a) and $\tau_{\text{tun}}^{(h)}(2\rightarrow 1)$ (53b) out of the reduced gap region adjacent to the tunnel barrier to the counter electrode of an $SS'IS''S$ junction were calculated as function of temperature and bias voltage. These time constants are averages over all quasiparticles above or in the trap. The derivations have been made under some assumptions, which will be discussed now in somewhat more detail.

First, the proximity model from which the spatially dependent order parameter and Green's functions are calculated, applies to dirty, weak-coupling superconductors [Eq. (2)]. It is evident that in the clean limit the spatial dependencies and the time constants become at least quantitatively different. However, large differences in temperature and voltage dependencies are not expected. Further, it was assumed that there is no interface resistance (i.e., $\gamma_B=0$) at the SS' interface of the proximity sandwich. This assumption is mostly true for clean metal interfaces, as they can be produced by standard techniques for the fabrication of multilayers and junctions. Finite boundary resistances may be produced artificially. Then the spatial dependence of the order parameter changes from the smooth variation for $\gamma_B=0$, to the steplike dependence for large γ_B, corresponding to the McMillan model of the proximity effect. In this limit one can derive the effective trapping and tunneling time constants straightforwardly from the Kaplan theory.[33] The dependence of $\Delta(x)$ on γ_B is discussed extensively in Ref. 19, as are the superconducting properties of an SS' sandwich.

Second, it is assumed that there is no external magnetic field present. In the use of $SS'IS''S$ junctions as detectors, often a magnetic field H parallel to the junction barrier is applied in order to suppress the Josephson current and Fiske resonances (see, e.g., Refs. 33 and 34). The field strengths that are used can be a substantial fraction of H_{c1} (H_c) of type-II (I) superconductors. The field will penetrate over a distance of the order of the penetration depth into the S' (S'') layer of the SS' (SS'') electrode. This will increase the pair-breaking strength of the proximity layer and decrease the order parameter in the S' (S'') layer. Consequently the effective proximity parameter will be increased and becomes a function of H. The magnetic field also introduces a quasiparticle trap at the vacuum-superconductor interfaces of the junction. These aspects are the subject of a further study.[35]

A third aspect is the nonequilibrium energy distribution function for the quasiparticles [Eq. (14)] we assumed in the calculation of the effective time constants. This Boltzmann-like distribution was obtained by Vardanyan and Ivlev[32] for a superconductor under continuous microwave and laser irradiation. They considered the case that the phonons are in thermal equilibrium, i.e., perfect thermal coupling to the bath. This means that the (excess) phonons produced by quasiparticle recombination escape from the superconductor before they can be absorbed in a pair-breaking process. Chang and Scalapino[36] showed that, if phonon trapping is of importance, the phonon system cannot be considered to be in thermal equilibrium. For that case they solved a set of coupled kinetic equations for the quasiparticle and phonon-distribution function, which contain continuous energy injection terms that depend on the excitation mechanism of the quasiparticles and phonons. The resulting quasiparticles distributions are fairly much different from the Boltzmann function, especially in the case of microwave excitation. Our results for the time constants, in principle, only apply if the nonequilibrium distribution function is Boltzmann like. This excludes the case of continuous quasiparticle or phonon injection at high energies, since that will give rise to a hot band in the distribution function. However, if the number of (continuously) injected quasiparticles is small compared to the equilibrium number and the injection energy is low, then the nonequilibrium distribution function is expected to be approximately equal to the equilibrium function, which has a Boltzmann-like behavior at low temperatures, as given by Eq. (14) and the developed formalism may be applied. Other types of nonequilibrium distribution functions have been described in literature using an effective quasiparticle chemical potential μ^* and an effective quasiparticle temperature T^* $(T^*>T)$, giving $f(\varepsilon)=[1+\exp((\varepsilon-\mu^*)/T^*)]^{-1}$.[37,38] Such a distribution function was shown to be very useful to describe nonequilibrium superconductivity in strongly disturbed systems, corresponding to large values of T^*. It is easy to show that for the case of small disturbance [i.e., $(T^*-T)/T \ll 1$] the μ^*-T^* model equation can be written in the form of Eq. (14), which has the advantage that only one parameter is needed, namely $c(t)$, which, in principle, gives the "amplitude" of the nonequilibrium distribution function.

For superconducting energy-resolving detectors one would like the phonon system to be decoupled from the bath as much as possible (perfect phonon trapping), since any loss of deposited energy through phonon loss reduces the energy resolution. After the absorption of a high-energetic particle or photon in the superconductor a cascade of relaxing quasiparticles and phonons is created. This cascade ends finally in some excess quasiparticle and phonon distributions, which are relatively small deviations from the equilibrium distribution functions if the number of excess quasiparticles, respectively, phonons is small compared to the equilibrium number. This is mostly the case, especially if the detector has a large volume, needed for having a large detector area, and/or efficiency. Second, at low temperatures the quasiparticle recombination rate τ_r^{-1} is very much smaller than the phonon pair-breaking rate τ_B^{-1}. This implies that most of the excess energy is stored in the quasiparticle system and only few excess phonons with energy larger than $2\Delta_0(T)$ exist. About the processes that take place during the energy cascade little is known, nor about the final quasiparticle distribution. The Boltzmann distribution function for the excess quasiparticles therefore seems the most probable choice as a first-order approximation of this final distribution. From these arguments we assume that Eq. (14) is a good approximation for the nonequilibrium quasiparticle distribution function, resulting from the energy relaxation process after the absorption of an high-energetic particle or photon.

Nonequilibrium systems are often described with the Rothwarf-Taylor (RT) equations. The derived time constants can be used to describe the quasiparticle exchange between the reduced gap region and the bulk in a proximity sandwich, with two sets of RT equations for the trap and the bulk. In the case of an $SS'IS''S$ junction, in principle, two more sets of RT equations are involved for the description of the $S''S$ electrode, which are coupled with those of the SS' electrode by the tunneling times. The tunneling case is even more complicated, since the tunneling times are different for electrons and holes, contrary to the trapping, excitation, and recombination time constants. In the foregoing sections we have implicitly assumed that electron and hole branches of the energy spectrum are equally populated, so that the same distribution functions can be assumed for the holes and electrons. This assumption is justified if the branch-mixing time τ_Q is much shorter than the average lifetimes against trapping and excitation. In the case that the initial excess quasiparticle density consists of high-energetic particles $(\varepsilon \gg \Delta_0)$ branch mixing and energy relaxation are very fast with time constants of the order of τ_0 or much less: $\tau_Q=\tau_0(2T_c^3/\varepsilon^2\Delta_0)$ and $\tau_e=\tau_0(3T_c^3/\varepsilon^3)$ $(\varepsilon \gg \Delta_0, T \ll T_c)$.[15] One may thus assume that the holes and electron branches are equally populated after the quasiparticles have relaxed to the band gap and before the trapping process sets in with a time constant that is generally much larger than τ_0. This situation is considered to be realistic in the cases where the excess quasiparticle density is due to the impact of a high-energetic particle or photon, which creates first an excess density on the high-energy part of the electron branch of the energy spectrum. Similar arguments hold for particle injection by a junction at high energies. On the other hand if the injection is at energies close to the band gap energy, where branch mixing becomes slow, the branches may be imbalanced from the beginning. However, also if both branches are equally populated after the relaxation process, they may become imbalanced again by the tunneling process. In a tunnel junction, mainly particles from one branch have a large tunneling rate as can be seen from Figs. 9 and 10. At low temperatures the branch-mixing time for low particle energies becomes very large so that an imbalance between the holes and electrons in the trap can occur if the tunneling time out of the trap is shorter than the branch-mixing time. This may be taken into account by splitting up the quasiparticle equation in the set of RT equations in one for electrons and one for holes, which are coupled by the branch-mixing process.

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*On leave from the Institute of Solid State Physics, 142432 Chernogolovka, Moscow Distr., Russia.

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