# Nonlinear interaction between an external microwave field and weakly coupled superconducting grains

S. E. Shafranjuk
*Institute for Metal Physics, Academy of Sciences, 36 Vernadskii street, 252142, Kiev, Ukraine*

H. van Kempen
*Research Institute for Materials, Toernooiveld, 6525 ED Nijmegen, The Netherlands*
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The interaction between an external microwave field and a system of weakly coupled superconducting grains is considered. It is demonstrated that intensive fields can substantially modify the intergrain Josephson coupling leading to additional residual resistance and absence of a coherence peak in granular superconductors.

## I. INTRODUCTION

The first experiments[1-7] on the interaction between external microwave fields and metal-oxide superconductors (MOS), often having a granular structure, demonstrated unusual behavior in comparison with ordinary homogeneous superconductors. For example, the temperature dependency of the high-frequency surface resistance $R_s(T)$ does not show the typical activation behavior with an energy gap $\Delta(T)$ and contains residual resistance at low temperatures while the real part of ac conductivity $\sigma_1(T)$ does not exhibit a coherence peak.[1-11]

Usually such anomalous residual absorption was described within a phenomenological two-fluid model or within an anisotropic pairing model (see Refs. 9 and 12) which implies nodes in the gap function and predicts the existence of quasiparticles even at $T=0$. In fact these models are applicable to bulk homogeneous superconductors (SC), while the absorption in the granular MOS is affected by the sample quality and by weak magnetic fields.[7-9] Therefore, the above-mentioned properties cannot only be accounted for by the existence of excited quasiparticles at low temperatures, or by the intrinsic nature of metal oxides and one has to take into account also the inhomogeneity of the MOS. Since the MOS samples are frequently granular systems, the natural reason for the residual absorption seems to be the direct absorption inside the intergrain dielectric regions[1-7] which is additional to the absorption by the current carriers.[13,14] However, the absorption inside the dielectric fraction is proportional to the relative volume of this fraction in the sample and could be negligible when the dielectric fraction is small, while the grains could still remain isolated.

The granular structure itself can cause an additional specific contribution to the residual resistance $R_s(T)$ which is connected with the current carriers, which was not considered before and which consists of the following. Since in the granular superconductor the resistance of intergrain borders is much higher in comparison with resistance of the superconducting grains itself, the field penetrated in the system is concentrated inside the intergrain borders. This happens already at a relatively low external field intensity and causes the appearance of nonlinear effects[15] which are not present in ordinary homogeneous superconductors[13] and which can be described in terms of photon-assisted intergrain tunneling. This mechanism was suggested before in Ref. 16 to explain the steps in I-V characteristics of single tunnel junctions arising under the influence of external electromagnetic fields and was used in Ref. 15 to calculate the energy losses in granular SC systems due to the intergrain quasiparticle hopping driven by the harmonically alternating intergrain bias voltage. Physically the nonlinear photon-assisted effects mean that a Cooper pair can be broken by $n^*$ photons with energy $\omega=2\Delta(T)/n^*$ [where $\Delta(T)$ is the energy gap value at temperature $T$] which can be much less than the energy gap value $2\Delta(T)$ when the effective number of photons $n^* \gg 1$. Therefore these nonlinear effects could serve as a cause for residual resistance (because of finite field energy absorption) as well as for disappearance of a coherence peak even at low temperatures $T \ll T_c$ ($T_c$ is the critical temperature of superconducting transition) and at low external field frequency $\omega \ll 2\Delta(T)$ and shall be discussed in this paper.

Another problem which we shall consider here concerns the role of the supercurrent (Josephson) component driven by the alternating external field. The experiments on transport properties of MOS have indicated the presence of Josephson phenomena in granular and even in single-crystal samples.[7,9,17,18] In granular SC the critical current is limited by the critical value of the intergrain Josephson current, while in single crystal metal-oxide SC it is determined mostly by the pinning force of the Abrikosov vortices (if the applied magnetic field $H \neq 0$) or by the critical Josephson current at the twinning boundaries. Phenomenological models of granular SC with Josephson coupling between the grains were proposed in Ref. 10. Since the intergrain Josephson coupling determines the dc electromagnetic properties of the granular SC, it seems to be interesting to clarify its role in the case of an ac field, where the intergrain Josephson coupling itself is responsible for the absorption channel caused by photon-assisted Cooper pair tunneling, converting the field energy into energy of the condensate (in conditions of the Shapiro effect[19]). In addition, photon-assisted tunneling appears as a result of the harmonically alternating external field. However in case of an intensive field and small intergrain capacitance the nonlinear Josephson interaction itself leads to a different situation when the intergrain voltage does not follow the intergrain current and can show chaotical time dependence, instead of harmonical behavior. Of course, the character of the absorption in the latter case can be unusual in comparison with the photon-assisted tunneling picture.[15,20,21] Thus, despite the fact that the quasiparticle channel could be expected to be more effective in the sense of absorption intensity, the Josephson channel at certain conditions is able to modify qualitatively the character of interaction between the external field and a granular superconducting system.

In the present paper we consider the effect of nonlinear interaction between an arbitrary alternating external electromagnetic field and weakly coupled superconducting grains. We shall demonstrate that such nonlinear interaction, caused by photon-assisted quasiparticle and Cooper pair intergrain tunneling, is responsible for the residual absorption at low temperatures as well as for disappearance of a coherence peak. We shall also compare the relative contributions to the absorption due to the Josephson and quasiparticle channels.

## II. DESCRIPTION OF THE MODEL

We shall apply here the model of Ref. 15, describing weakly coupled superconducting grains under influence of an external ac field assuming the importance of the Cooper pair hoppings for the electromagnetic properties of the granular SC, at least in the low-frequency range (i.e., $\omega < 2\Delta$). The granular structure implies an essential inhomogeneity of the field inside the sample, because there is an important difference between the properties of grains and intergrain regions. Hence the electric field value on the intergrain borders can be much larger than that inside the superconducting grains, causing the nonlinear character of the interaction between the field and the granular superconductor. For simplicity we assume here that the granular sample consists of a thin film with thickness $\sim \min\{\lambda_\sigma \lambda_{\rm pen}\}$; where $\lambda_\sigma$ and $\lambda_{\rm pen}$ are the skin depth and the ac field penetration depth, respectively.

The field vector potential $\mathbf{A}(\mathrm{r})$, penetrating in such a system lies in the plane of the film and, as was mentioned before, is essentially inhomogeneous inside the granular superconducting film in contrast to the case of a homogeneous film.[13,14] We shall consider the simplified case when the film thickness is limited only by one grain and the intergrain contact area is comparable to the magnetic penetration depth, thus neglecting the Josephson flux (fluxon) movement.

The external field is described in terms of the "high-frequency intergrain voltage"[15] $V_{ij}$ applied between two neighboring grains marked by $i$ and $j$, respectively. The nonlinear effects are important when $\langle eV_{ij}\rangle_{\rm sample}/\omega > 1$ (where $\langle V_{ij}\rangle_{\rm sample}$ is the value of intergrain voltage amplitude, averaged over the sample; $\omega$ is the external field frequency). If one takes the electric field amplitude value typically used in many experiments, to be $E_0 \simeq 10^6 V/m$ at frequency $\omega = (10^{11} - 10^{13})\ \mathrm{s}^{-1}$, and the typical grain size to be $d_{\rm grain} \approx 1\ \mu$m, a rough estimation for the nonlinearity parameter (see Ref. 15) yields $\langle eV_{ij}\rangle_{\rm sample}/\omega = (10 - 10^2)$. This indicates that the simple linear response picture is not applicable to describe the interaction between the external field and the granular superconductor, because the nonlinear limit $\langle eV_{ij}\rangle_{\rm sample}/\omega \gg 1$ occurs in many experiments.

## III. THEORETICAL APPROACH

As we already mentioned in the Introduction, generally speaking the intergrain bias voltage at certain conditions may not always follow the driving intergrain current which is induced by the harmonically alternating external field and may be alternating in time in a more complex way. Thus the approach required here should be able to describe the properties of granular SC systems in more general terms of intergrain voltage which is of arbitrary amplitude and is arbitrary alternated in time. To consider the nonlinear effects causing the residual resistance we extend here the approach of Ref. 15 (see also Refs. 20 and 21) which assumed the intergrain voltage $V_{ij}(t)$ to be harmonically alternating in time $t$ to the case of arbitrary dependence $V_{ij}(t)$ (e.g., chaotic).

Expressions for the losses in terms of nonequilibrium sources will be sought describing the influence of the quasiparticle and Cooper pair intergrain hopping on the electron distribution function in the kinetic equation. The losses in the superfluid channel are physically connected with excitations in the granular superconducting system through acceleration of the superfluid condensate.

Let us consider the simplest situation that the coupling is important only for neighboring grains. The intergrain bias voltage $V_{ij}$ caused by the external electromagnetic field is described in this case by the equation for intergrain Josephson phase difference $\phi_{ij}(t)$ (see Ref. 22)

$$\omega_p^{-2}\ddot{\phi}_{ij} + J_{qp}(\dot{\phi}_{ij}) + \sin\phi_{ij} = j_{\rm ext}e^{i\omega t} \tag{1}$$

and

$$\phi_{ij}(t) = 2e\int_{\infty}^{t} dt\ V_{ij}(t)\ . \tag{2}$$

In the above formula $\omega_p = (2eI_c/\hbar C)^{1/2}$ is the intergrain Josephson plasma frequency; $e$ is the electron charge; $I_c(\dot{\phi}_{ij})$ is the intergrain Josephson critical current which, generally speaking, depends on the intergrain voltage $V_{ij} = \dot{\phi}_{ij}/2e$; $C$ is the junction capacity (here we assume the intergrain capacity $C$ to be large enough to neglect the single electron tunneling effect, which was considered in detail in respect to the Josephson junctions array elsewhere; see, for instance, Ref. 23 and references there); $J_{qp} = j_{qp}/I_C$; $j_{qp}$ is the quasiparticle current density; $j_{\rm ext} = I_{\rm ext}/I_C$ and $I_{\rm ext}$ is the total current density amplitude, induced by the external field. We note that near $T_c$ or in the limit of $\langle eV_{ij}\rangle_{\rm sample}/\omega \gg 1$, the simple expression $J_{qp} = \omega_c^{-1}\dot{\phi}_{ij}$($\omega_c = R_N/L_S$ is the characteristic junction frequency; $R_N$ is the normal resistivity of the junction; $L_S$ is the supercurrent inductivity) holds, therefore the last equation in fact becomes the equation for a pendulum with a periodic external force[24] and describes a wide set of phenomena in the Josephson junction. For instance, when the amplitude of the driving current is high enough, Eq. (1) yields chaotic behavior of $\phi_{ij}(t)$ [and hence $V_{ij}(t)$], or shows nonlinear resonance (for details, see Ref. 24). In the above formulas we neglected the space term, since the typical size of the intergrain junction usually is less than the London penetration depth. The question what kind of solution is appropriate for the real granular system, is still open and requires experimental data on the parameters of the intergrain coupling. We shall not consider here all the possibilities for different solutions of (1) but restrict ourselves only to the two simple cases that $V_{ij}(t)$ is alternating harmonically, i.e., $V_{ij}(t)=V_{ij}\cos(\omega t)$, or chaotically.

Following the same procedure as in Refs. 15, 20, and 21 we can, in the case of arbitrary alternating intergrain bias voltage, write the nonequilibrium source in the kinetic equation for the quasiparticle channel which depends on energy variable $\varepsilon$ and time variable $\mathcal{T}$ as

$$I_{qp}^{(i)}(\varepsilon,\mathcal{T})=\Gamma_i \operatorname{Im}\sum_j \int d\varepsilon_1 \int d\omega_1 \int d\omega_2 e^{i\mathcal{T}(\omega_1-\omega_2)/2} \times K^{ij}(\omega_1)(K^{ij}(\omega_2))^* N_i(\varepsilon_1) N_j\left[\varepsilon+\frac{\omega_1+\omega_2}{2}\right]\frac{f(\varepsilon_1)-f\{\varepsilon+[(\omega_1+\omega_2)/2]\}}{\varepsilon-\varepsilon_1+(\omega_1-\omega_2)/2-i\delta} \tag{3}$$

where the tunnel density of states is

$$N_i(\varepsilon)=\operatorname{Re}\left\{\frac{\varepsilon}{\sqrt{\varepsilon^2-\Delta_i^2}}\right\}, \tag{4}$$

where $\Delta_{i(j)}$ is the energy gap value in the grain $i$ (or $j$) and the Josephson source is:

$$I_{\rm Jos}^{(i)}=\operatorname{Re}\{I_{\rm Jos}^{(i),\rm tot}\} \tag{5}$$

and where

$$I_{\rm Jos}^{(i),\rm tot}(\varepsilon,\mathcal{T})=-\Gamma_i\sum_j \int d\varepsilon_1 \int d\omega_1 \int d\omega_2 e^{i\mathcal{T}(\omega_1+\omega_2)/2} \times K^{ij}(\omega_1)K^{ij}(\omega_2)M_i(\varepsilon_1)M_j\left[\varepsilon-\frac{\omega_1-\omega_2}{2}\right]\frac{f(\varepsilon_1)-f[\varepsilon-(\omega_1-\omega_2)/2]}{\varepsilon-\varepsilon_1+(\omega_1+\omega_2)/2-i\delta} \tag{6}$$

with $\Gamma_i$ the hopping frequency;

$$M_i(\varepsilon)=\operatorname{Re}\left\{\frac{\Delta_i}{\sqrt{\varepsilon^2-\Delta_i^2}}\right\}. \tag{7}$$

The external field factor is

$$K^{ij}(\omega)=\int dt\, e^{-i\omega t+i\phi_{ij}(t)}. \tag{8}$$

### A. Harmonic solution

If the solution of Eq. (1) yields the harmonically alternating intergrain ac voltage, we get the following expression for $K^{ij}(\omega)$:

$$K^{ij}(\omega)=i\sum_n J_n\left[\frac{eV_{ij}}{\Omega}\right]\delta(\omega-n\Omega). \tag{9}$$

For the Josephson source one finds

$$I_{\rm Jos}^{(i),\rm tot}(\varepsilon,\mathcal{T})=-\Gamma_i\sum_j\sum_{n,n_1,k} J_{n_1}\left[\frac{eV_{ij}}{\omega}\right]J_{n_1+n+2k}\left[\frac{eV_{ij}}{\omega}\right]J_n\left[\frac{2eV_{ij}}{\omega}\right]e^{ik\omega\mathcal{T}} \times\int d\varepsilon_1 M_i(\varepsilon_1)M_j[\varepsilon-(n_1+k)\omega]\frac{f(\varepsilon_1)-f[\varepsilon-(n_1+k)\omega]}{\varepsilon-\varepsilon_1-k\omega-i\delta} \tag{10}$$

$J_n(x)$ is the Bessel function of order $n$. In accordance with (5) we should take the real part in (10). As one can see, the resulting expression consists of two different parts. The first one is proportional to $\sin(k\omega\mathcal{T})$ and is responsible for the Cooper pair tunneling under the electromagnetic field influence in the conditions of the Shapiro effect.[19] The second part is proportional to $\cos(k\omega\mathcal{T})$ and can be attributed to the interference processes of the Cooper pair and quasiparticle hoppings. The obtained source yields the ordinary expression for the Shapiro steps in the $I$-$V$ Josephson characteristics[19] if a constant bias voltage is applied additionally.

The found nonequilibrium sources (3), (5), and (6) allow the expression for the quasiparticle *ac*-current density in the considered weakly coupled system to be derived:

$$j_{\rm ac}^{(i),qp}(\omega,t)=\sum_{j=i\pm1}\frac{1}{eR_g^{ij}}\sum_{nl}e^{i\omega lt}J_n\left[\frac{eV_{ij}}{\omega}\right]J_{n+2l}\left[\frac{eV_{ij}}{\omega}\right]\int d\varepsilon N_i(\varepsilon)N_j(\varepsilon+n\omega)[f(\varepsilon)-f(\varepsilon+n\omega)]\ , \tag{11}$$

where $R_g^{ij}$ is the intergrain resistivity.

### B. Chaotic solution

Near $T\approx T_c$ or in the limit of $\langle eV_{ij}\rangle_{\rm sample}/\omega\gg1$ Eq. (1) is transformed to the pendulum equation[24] and yields an example of a solution which is connected with the chaotic behavior of the Josephson phase. This can be the origin of an additional nonresonance interaction between the external field and the weakly coupled junctions. The chaotic solution itself could arise from a chaotic layer near the separatrix of the solution of Eq. (1) in intensive fields. The width of the chaotic layer in the vicinity of the separatrix can be estimated to be (see Ref. 24):

$$\frac{\Delta E}{E_c}\approx\left[\frac{\omega_c}{\omega_p}\right]e^{-\pi\omega/2\omega_p}\ . \tag{12}$$

That means that the thickness of the chaotic layer can be compared with the Josephson junction energy $E_c$ if the external field frequency $\omega<\omega_p$. Here $E_c=I_c\Phi_0/2\pi$ where $I_c$ is the critical current and $\Phi_0$ is the elementary flux quantum. In this case we use $\langle\phi(t_1)\phi(t_2)\rangle_{\rm stat}=\delta(t_1-t_2)$ (where $\langle\cdots\rangle_{\rm stat}$ is the statistical average over the all possible realizations), thus $K^{ij}(\omega)=$constant and does not depend on the frequency $\omega$. Hence, in the stationary situation we get

$$I_{qp}^{(i)}(\varepsilon)=\Gamma_i\sum_j\int d\omega\, N_i(\varepsilon)N_j(\varepsilon-\omega)\{f(\varepsilon)-f(\varepsilon-\omega)\} \tag{13}$$

and

$$I_{\rm Jos}^{(i),\rm tot}(\varepsilon)=-\Gamma_i\sum_j\int d\varepsilon_1\int d\omega\, M_i(\varepsilon_1)M_j(\varepsilon-\omega)\times\frac{f(\varepsilon_1)-f(\varepsilon-\omega)}{\varepsilon-\varepsilon_1-i\delta}\ . \tag{14}$$

The sources (6), (10), and (14) take into account the influence of the external field on the Josephson current and describe the nonequilibrium pumping of the quasiparticle system through the supercurrent channel.

To write down the expressions (3), (6), (10), (13), and (14) we have used the ordinary BCS approximations to include the superconducting pairing and assumed small transparency of the intergrain borders $D\ll1$ and a field frequency $\omega\gg1/\tau_\varepsilon$ (where $1/\tau_\varepsilon$ is the inelastic relaxation rate).

The above obtained expressions for the nonequilibrium sources allow the calculation of the electromagnetic field energy losses, $Q_{\rm Jos,qp}$, in a system of weakly coupled superconducting grains:[15,21]

$$Q_{{\rm Jos},qp}=\left\langle\sum_p E_pI_{{\rm Jos},qp}(E_p,\mathcal{T})\right\rangle_{\rm time}\ , \tag{15}$$

where $E_p=\sqrt{\xi_p^2+\Delta_p^2}$; $\xi_p$ is the quasiparticle kinetic energy and $\langle\cdots\rangle_{\rm time}$ means averaging over time.

### IV. NUMERICAL CALCULATIONS

To perform the numerical calculations let us consider the BCS superconducting grains with a single isotropic energy gap, which is uniform inside each grain. We take into account the effect of the inelastic electron collisions inside the grain, introducing the complex gap $\Delta=\Delta_1+i\Gamma_{\rm inel}$ ($\Gamma_{\rm inel}=1/\tau_\varepsilon$; $\tau_\varepsilon$ is the inelastic relaxation time). Since we would like to calculate the temperature dependencies of the energy absorption, we used the BCS temperature dependence of the energy gap and take $2\Delta/T_c=3.5$. We also suppose all the grain sizes and the intergrain normal state resistances to be equal in value; the intergrain interaction is nonzero only for the neighboring grains; the grain size is much less in comparison with the field wavelength and we describe the distribution function by the effective temperature approximation (see also Ref. 15).

The results for the numerically calculated dependence of the ac current density on the effective electron temperature $T^*$ are shown in Fig. 1, demonstrating nonlinear behavior in the external field amplitude dependence of the first harmonic $j_{\rm ac}^{(i),\rm tot}(\omega)=j_{\rm ac}^{(i),qp}(\omega)+j_{\rm ac}^{(i),\rm Jos}(\omega)$. Here and below all the energy and temperature quantities are measured in units of $2\Delta(0)$. So we take $2\Delta(T=0)=1$;

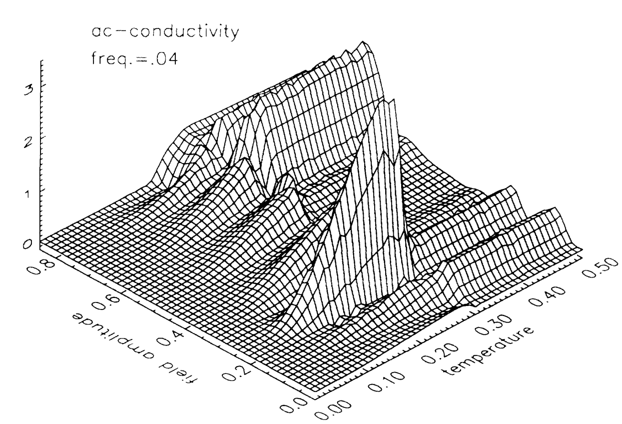


FIG. 1. The dependence of the ac current density on the effective electron temperature $T^*$ and on the external field amplitude parameter $\langle eV_{ij}\rangle_{\rm sample}$.

$T_c=0.3$; $\Gamma_{\text{inel}}=0.1\ \Delta(T^*)$; the external field frequency $\omega=0.04$. Figure 1 shows that the first harmonic of the current density displays a wide coherence peak just below $T_c$ only at low enough field amplitudes $\langle eV_{ij}\rangle_{\text{sample}}\leq 0.03$. When the external field amplitude is increased the temperature dependence of $\langle j_{\text{ac}}^{(i),\text{tot}}(\omega)\rangle_{\text{sample}}$ is substantially modified showing an irregular oscillatory behavior *versus* $\langle eV_{ij}\rangle_{\text{sample}}$.

In Fig. 2 we present the results of numerical calculations of the electromagnetic field energy absorption characteristics $Q_{\text{tot}}=Q_{\text{Jos}}+Q_{qp}$ from formula (15). The parameters we have used for the calculations are the same as before, except that we take here $\omega=0.06$. One can see that an accordance with the homogeneous SC case[13] exists only for low values of the external field amplitude ($\langle eV_{ij}\rangle_{\text{sample}}\leq 0.8$): a coherence peak just below $T_c$ is visible and the absorption is frozen out at $T^*\to 0$. However, when the field amplitude is increased (up to $\langle eV_{ij}\rangle_{\text{sample}}\sim 1.5$), a finite absorption appears even at $T\to 0$ and the coherence peak disappears. Note the anomaly arising at $T\approx T_c$ for $\langle eV_{ij}\rangle_{\text{sample}}\geq 0.9$, which is a consequence of the nonlinear effects.

The evolution of the coherence peak at $\omega=0.06$ is demonstrated in Fig. 3 for low field amplitudes $\langle eV_{ij}\rangle_{\text{sample}}=0.05$; $\langle eV_{ij}\rangle_{\text{sample}}=0.17$ and $\langle eV_{ij}\rangle_{\text{sample}}=0.31$. We emphasize that the above discussed disappearance of the coherence peak at $\langle eV_{ij}\rangle_{\text{sample}}\geq 0.9$ is of completely different origin from the origin that was reported in Ref. 25 as occurring due to a variation in the coupling constant and impurity scattering rate.

The role of the mentioned nonlinear effects is evident from Fig. 4, where the energy absorption is plotted *versus* the external field amplitude for three fixed frequencies $\omega=0.2\times 2\Delta(0)$ (solid curve 1); $\omega=0.4\times 2\Delta(0)$ (solid curve 2) and $\omega=1.2\times 2\Delta(0)$ (solid curve 3) at effective temperature $T^*=0.1$. As can be seen in Fig. 4 a linear behavior versus field intensity takes place only when the external field frequency exceeds the value of $2\Delta(T^*)$.

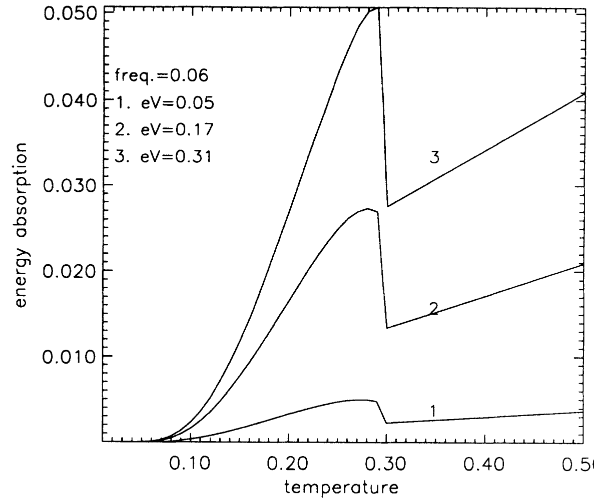


FIG. 3. The evolution of the coherence peak at low external field amplitudes. The external field frequency $\omega=0.06$.

Indeed, the solid curve 3 is well fitted by parabola $\sim 0.023\langle eV_{ij}\rangle_{\text{sample}}^2$ (dashed curve 3) while the solid curves 1 and 2 are not.

Thus from comparison of Figs. 1, 2, and 4 one can see that the nonlinear effect is directly manifest only for each harmonic separately while the time averaged characteristics look more linear.

For the case of chaotic solution of Eq. (1) the energy absorption is calculated using the formulas (12), (13), and (14). Figure 5 shows an example, where part of the absorption spectrum is chaotized due to the condition $\langle eV_{ij}\rangle_{\text{sample}}/\omega \gg 1$, which takes place for the frequency region up to $\omega\sim\langle eV_{ij}\rangle_{\text{sample}}=1$. For frequencies $\omega\geq 1$ the solution of Eq. (1) is transformed to the harmonically alternating solution implying that the intergrain voltage

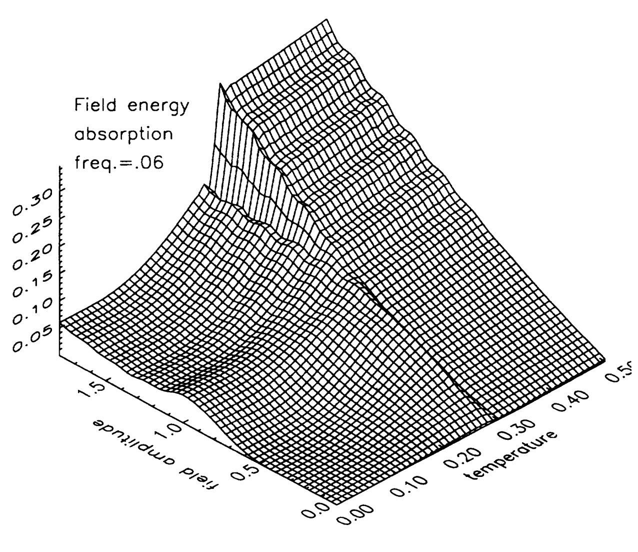


FIG. 2. The temperature-field amplitude dependence of the electromagnetic energy absorption. The parameters of calculations are presented in the text.

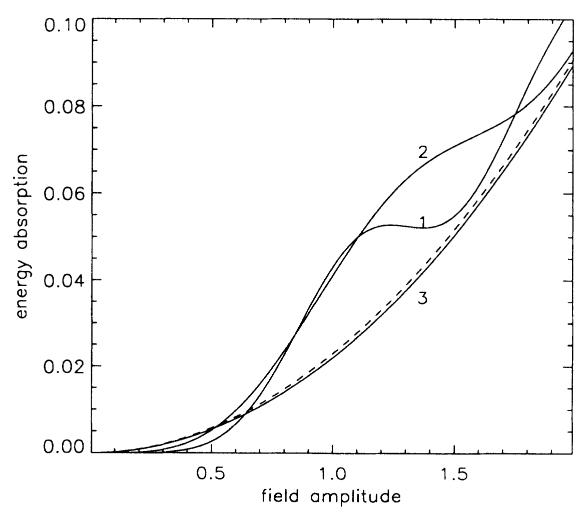


FIG. 4. The energy absorption at $T^*=0.1$, plotted vs the external field amplitude parameter $\langle eV_{ij}\rangle_{\text{sample}}$ at fixed field frequencies $\omega=0.2\times 2\Delta(0)$ (solid curve 1); $\omega=0.4\times 2\Delta(0)$ (solid curve 2) and $\omega=1.2\times 2\Delta(0)$ (solid curve 3), respectively. The dashed curve is corresponding fit by parabola with parameter presented in text.

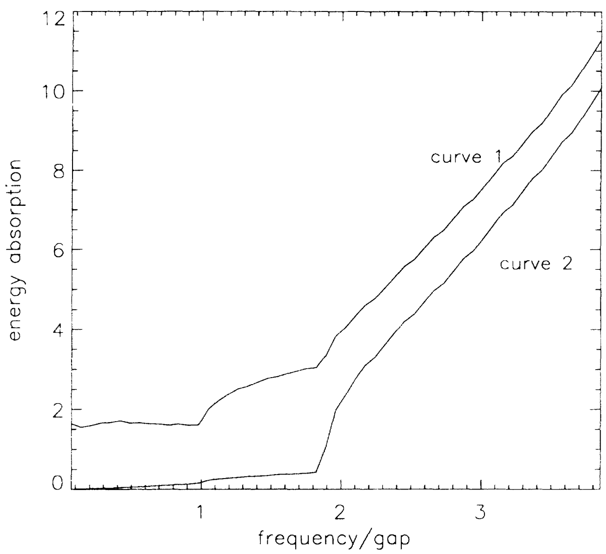


FIG. 5. The partially chaotized absorption spectra, which also display the finite field energy losses inside the gap. $\langle eV_{ij} \rangle_{\mathrm{sample}} = 0.3\Delta(0)$ (curve 1); $\langle eV_{ij} \rangle_{\mathrm{sample}} = 2.4\Delta(0)$ (curve 2).

follows the induced intergrain current, which yields the multiphoton absorption picture considered before. The frequency dependence of absorption, calculated for the chaotic part of the solution is supposed to have a white noise spectrum. The external field frequency in this picture is measured in units of $\Delta(0)$. As is seen, because of the chaotic solution, the absorption in the region $0 < \omega \leq 1$ does not depend on the external field frequency. At $\omega = 1$ the solution is switched to the harmonically alternating one that is immediately reflected in the absorption characteristics becoming frequency dependent. As one can see, the value of residual absorption at $0 < \omega \leq 1$ is much smaller than the residual absorption due to the photon-assisted processes.

Thus, there are two ways to explain the residual resistance and the disappearance of a coherence peak in the current carrier absorption characteristics of weakly coupled superconductors. The first one is connected with the nonlinear photon-assisted processes occurring in the inhomogeneous superconducting system due to the high value of the local electric ac field in the intergrain regions. Physically, the presence of residual absorption at $T^* \rightarrow 0$ due to the quasiparticle multiphoton intergrain jumping means that the Cooper pair breaking in the intensive field is caused by many photons with frequency $\omega \ll 2\Delta$ instead of one photon breaking with frequency $\omega = 2\Delta$ as it happens in the linear case. The second way is attributed to the development of chaotic behavior of the Josephson phase in the intergrain weakly coupled junctions.

## V. DISCUSSION

Of course, the above considered model is simplified in many respects. For example, the high field intensities inside the granular SC causing the above described nonlinear photon-assisted tunneling effects may, for instance, lead to heating or to nonequilibrium phenomena, appearing due to the modification of electron distribution function $f(\varepsilon)$, incorporated in Eqs. (3), (6), (10), (13), and (14). Then, due to electron-phonon relaxation and due to electron diffusion, the absorbed energy is transferred away from the system by the emission of nonequilibrium phonons. The rate of such energy dissipation is determined by many factors, for instance, by the temperature, by electron-phonon and phonon-electron coupling constants, by purity of material, etc. Despite the fact that the mentioned nonequilibrium effects could, in principle, modify the above considered picture, they can be separated from the purely nonlinear effects considered in this paper, by changing the heat transfer conditions used in experiments. The other simplification is connected with neglecting the dispersion of grain sizes and of intergrain resistivities in our model calculations. This dispersion surely would contribute to all the electromagnetic characteristics while summing over the grain indices in the obtained formulas. Additionally, we neglected here gap anisotropy effects (see, for instance, Ref. 9 and 12), the fine electron structure inside the grains itself[26] and we considered only the two-grain interactions (multigrain effects were not included) that also could modify the final results.

Since the contribution of the nonlinear effects considered above is essential at external field frequencies $\omega \simeq \langle eV_{ij} \rangle_{\mathrm{sample}}$, we can estimate the minimal field intensity when it is important in the case of MOS. In the metal-oxide energy gap region, for instance, one could take $\omega \simeq \Delta \simeq 30$ meV. Thus the nonlinear effects are important when the field intensity is of order $W_{\mathrm{field}} \simeq 10$ W/cm$^2$ (for the mean grain size $\sim 1\ \mu$m).

When the temperature is close to $T_c$, the situation is complicated also by the appearance of fluctuations which are able to modify the final absorption spectra crucially. The Josephson phase fluctuations caused by the random supercurrent $I_s(t)$ are described by the Focker-Plank equation for the density of probability with the normalized energy of the ac driven Josephson junction:[22]

$$u(\phi_{ij}) = U(\phi_{ij})/E_c = 1 - \cos\phi_{ij} - \mathcal{J}\phi_{ij} \ , \tag{16}$$

where $\mathcal{J} = \mathcal{J}_0 \cos(\omega t)$; $\mathcal{J}_0$ is the intergrain current density amplitude.

$U(\phi_{ij})$ is the potential energy. Using the numerical procedure described in Ref. 22 we have found the density of probability $\sigma(\phi)$ and then averaged the solutions of (1) over $\sigma(\phi)$. The results show the smoothing of sharp peaks and valleys in the frequency and field amplitude dependencies of $\sin\phi$ that leads again to a smooth addition to the absorption in the Josephson channel.

The simplified model picture with the phase fluctuations, where we neglected the dispersion in the values of grain size and of intergrain resistivity (i.e., the intergrain critical current), can straightforwardly be extended to include, for instance, the dispersion of grain size or intergrain border resistivity. This will result in further smoothing of the absorption characteristics.

Thus, the model applied above to describe the peculiarities of interaction between the granular superconductor and external fields show different SC electrodynamic properties compared to the homogeneous case, described by linear response theory.[13] The granular structure itself causes nonlinear effects resulting in multiphoton absorption of the field energy or in the development of chaotic intergrain voltage. Both consequences can serve as causes for the presence of residual resistance at low temperatures.

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