Comments

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Theory of impure superconductors: Anderson versus Abrikosov and Gor'kov"

A. A. Abrikosov
Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439

L. P. Gor'kov
National High Magnetic Field Laboratory, Florida State University, Tallahassee, Florida 32306 and L.D. Landau Institute for Theoretical Physics, Moscow, Russia

(Received 17 May 1993)

A response is given to a criticism in a paper of Kim and Overhauser of an earlier work by Abrikosov and Gor'kov on superconductors with impurities. The criticism is based on a primitive misunderstanding of the nature of the attraction of electrons achieved by phonons. As it was explained by the present authors in several publications, there exists a complete equivalence of their results with the later work by Anderson.

In their recent paper[1] Kim and Overhauser claim that they have found a mistake in our old article.[2] It would be more appropriate to discuss other papers[3] where it was first shown that in the BCS weak-coupling limit and for an isotropic model nonmagnetic impurities do not change the order parameter and temperature of the superconducting transition [apart from corrections of the order $(\tau\epsilon_F)^{-1} \sim c$, where τ is the scattering time, ϵ_F is the Fermi energy, and c is the atomic concentration of the impurity]. Independently the same result was obtained by Anderson[4] and it is well known as the "Anderson theorem." After considerable hesitation we have decided finally to comment on the statement made in Ref. 1, which could give a wrong impression that there are two conflicting results in the theory of superconductivity.

As is well known, T_c is determined from an integral equation of the form

$$\Delta(\omega_n, \mathbf{p}) = T \sum_{\omega_m} \int \nu(\epsilon_F)\, d\xi'\, (d\Omega'/4\pi)\, K(\omega_n - \omega_m, \mathbf{p} - \mathbf{p}')G(\omega_m, \mathbf{p}')G(-\omega_m, -\mathbf{p}')\Delta(\omega_m, \mathbf{p}'). \tag{1}$$

In the weak-coupling limit, the right-hand side of Eq. (1) contains a logarithmic divergence

$$T \sum_{\omega_m} \int \frac{d\xi}{\omega_m^2 + \xi^2} \tag{2}$$

and requires a cutoff which then becomes absorbed by the definition of T_c well known from BCS theory. The symmetric form of the divergence in Eq. (2) permits one to introduce such a cutoff either for ω_m or ξ; all the results being expressed in terms of T_c become cutoff independent.

Now, it has been shown in Ref. 3 (see also Ref. 5) that for isotropic s pairing (Δ independent of $\mathbf{p}$) Eq. (2) in the presence of nonmagnetic impurities transforms into

$$1 = \lambda T \sum_{\omega_m} \int \frac{d\xi\, \eta(\omega_m)}{\omega_m^2 \eta^2(\omega_m) + \xi^2}, \qquad \eta = 1 + \frac{1}{2\tau|\omega_m|}, \tag{3}$$

where λ is a dimensionless interaction constant. By integrating over $d\xi$ first (under the assumption that the density of states does not change) one concludes that the sum over frequencies in Eq. (3) is the same as in Eq. (2). This result has been expressed in Ref. 3 in an even more explicit way by demonstrating the exact form of the new Green functions:

$$G(x, x') = G_0(x, x')e^{-|r-r'|/2l}, \qquad F(x, x') = F_0(x, x')e^{-|r-r'|/2l}, \tag{4}$$

$l = v\tau$ being the mean free path. Since the gap is proportional to $F(x, x)$, there is no change due to impurities. What has not been realized in Ref. 1 is that in more complicated cases (such as magnetic impurities) or in the truncated model Hamiltonian used in the original BCS formulation, one often uses the trick of introducing a cutoff for the continuous variable ξ instead of the discrete ω_m; once the divergent terms are removed all convergent frequency sums can be performed more easily.

Of course, it was known even before the BCS theory that phonons provide an attractive interaction between electrons due to their retarded nature; the interaction is always local in space but is nonlocal in the time variable on a scale $1/\omega_D$. This can be described more physically as follows: one electron distorts the lattice vibrational state at one point; after some time another one comes to the same point and is affected by this distortion. The kernel in $K(\omega_n - \omega_m, \mathbf{p} - \mathbf{p}')$ in Eq. (1) is nothing but the phonon D function, and this allowed Eliashberg later to extend the BCS theory beyond the weak-coupling approximation. The present authors never thought that they would have to explain these obvious details in a scientific journal. However, for students, we made some explanatory remarks in Ref. 5 (see bottom of p. 337 and top of p. 338).

To summarize, there is no contradiction between the two theories (by Abrikosov and Gor'kov[2,3] and the Anderson Theorem[4]). Solely due to the isolation of Russian science in those days, the result in Ref. 3 was independently obtained in Ref. 4. It also explains why, instead of discussing Ref. 3, Kim and Overhauser criticize the later paper[2] where the focus was on magnetic impurities, and the question about the cutoff was assumed to be clear to the reader. We would like to mention that, since the correct understanding of the nature of the interaction permits one to avoid wrong conclusions, there is no need to use inconvenient and unjustified "projected" Green functions (see Ref. 1).

This work was supported by the Department of Energy under Contract No. W-31-109-ENG-38 (A.A.) and the National High Magnetic Field Laboratory (L.G.).

[1] Y.-J. Kim and A. W. Overhauser, Phys. Rev. **B 47**, 8025 (1993).
[2] A. A. Abrikosov and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **39**, 1781 (1961) [Sov. Phys. JETP **12**, 1243 (1961)].
[3] A. A. Abrikosov and L. P. Gor'kov, Zh. Eksp. Teor. Fiz. **35**, 1558 (1958); **36**, 319 (1959) [Sov. Phys. JETP **8**, 1090 (1959); **9**, 220 (1959)].
[4] P. W. Anderson, J. Phys. Chem. Solids **11**, 26 (1959).
[5] A. A. Abrikosov, L. P. Gor'kov, and I. Ye. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics* (Dover Publications, New York, 1975).