Low-field diamagnetic response of granular superconductors at finite temperatures

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We study the low-field diamagnetic response of granular superconductors at finite temperatures by means of a simple two-dimensional Josephson-junction array. The temperature effects are taken into account by inserting white-noise current sources in parallel to the resistively shunted junction circuit models of the Josephson junctions of the network. By this analysis we argue that a simplified one-dimensional description of the equivalent circuit, proposed by the authors for cylindrical granular superconductors, is still valid even in the presence of thermally activated flux jumps. A flux-creep picture for intergranular flux motion follows.

The diamagnetic response of granular superconductors can be described by means of Josephson-junction (JJ) array models.[1,2] In particular, Majhofer, Wolf, and Dieterich,[2] taking into account shielding-current effects, have studied the stationary magnetic states of a two-dimensional (2D) network of JJ's coupled by inductances. They found that the field distributions inside these junction networks is quite similar to those obtained analytically using a critical-state picture.[3] When one considers cylindrically symmetric samples in the presence of an axial external magnetic field, a 1D circuit model can be adopted under the assumption of strong enough grain coupling.[4] This simplified picture consists of concentric superconducting rings interrupted by JJ's. In this model, the structural inhomogeneity of the granular sample is taken into account by averaging the coupling parameters and the size of the integranular regions over an annulus of width of the order of the grain radius.

In granular superconductors we make a distinction between intrinsic and extrinsic temperature dependence of the magnetic variables. We denote as intrinsic the temperature dependence of the superconducting variables as, for example, the maximum Josephson current between two adjacent grains. On the other hand, we denote as extrinsic the additional T dependence coming from thermal fluctuation effects, which are relevant due to the presence of weak links among the grains. In order to analyze the finite-temperature problem, in the present work we generalize the analytical and numerical study done in a previous work[4] at $T=0$ K on a simple 2D system consisting of three concentric granular rings, each one containing N grains (Fig. 1). This generalization is straightforward, since it consists in simply extending the resistively shunted junction (RSJ) model[5] for each weak link by adding a white-noise current source which simulates thermal fluctuation effects. Finally, we adopt the 1D model to derive a flux-creep picture for intergranular flux motion in granular superconductors.

In the system of Fig. 1, the coupling Josephson energy among the grains is taken to vary according to a Gaussian distribution about the mean value $\langle E_J \rangle = \langle I_{J0} \rangle \Phi_0 / 2\pi$, where $\langle I_{J0} \rangle$ is the average maximum Josephson current at zero field. Two concentric levels of intergranular regions of area S_0 enclosing an inner normal region of area S_h are present. The physical grains are placed at each node of the equivalent network of JJ's and inductances,[4] so that each pair of nodes is separated by a JJ, whose phase difference is denoted by $\varphi_{k,i}$, where k is a radial index ranging from 1 to 5 and i denotes the angular position of the JJ and ranges from 1 to N. The dynamic equations, neglecting capacitive

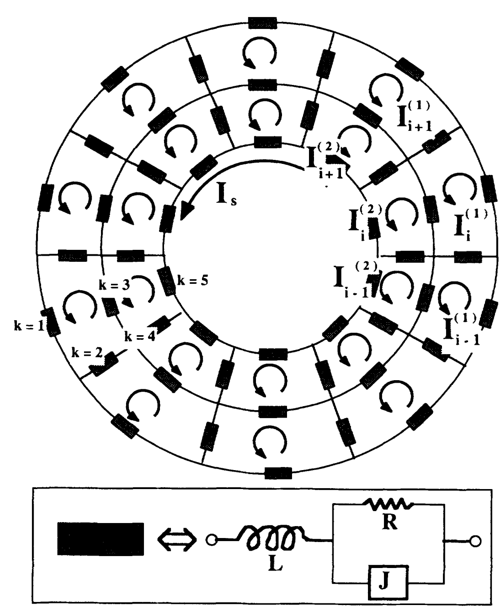

FIG. 1. Simplified 2D network of Josephson junctions and inductances. One rectangle contains an inductance L and a JJ, schematized through the RSJ model, as shown in the inset. The currents $I_i^{(j)}$ circulating in each integranular loop of area S_0 and the inner loop current I_s are also shown.

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effects, can be written, following the same steps in previous work,[4] as follows:

$$\left[\frac{\Phi_0}{2\pi R_n}\right]\frac{d\varphi_{1,i}}{dt}+\alpha_{1,i}(T)\langle I_{J0}\rangle \sin\varphi_{1,i} = \langle I_{J0}\rangle i_i^{(1)}+i_{fi}^{(1)}(t) , \quad (1a)$$

$$\left[\frac{\Phi_0}{2\pi R_n}\right]\frac{d\varphi_{2,i}}{dt}+\alpha_{2,i}(T)\langle I_{J0}\rangle \sin\varphi_{2,i} = \langle I_{J0}\rangle\{(i_{i-1}^{(1)}-i_i^{(1)})+[i_{fi-1}^{(1)}(t)-i_{fi}^{(1)}(t)]\} , \quad (1b)$$

$$\left[\frac{\Phi_0}{2\pi R_n}\right]\frac{d\varphi_{3,i}}{dt}+\alpha_{3,i}(T)\langle I_{J0}\rangle \sin\varphi_{3,i} = \langle I_{J0}\rangle\{(i_i^{(2)}-i_i^{(1)})+[i_{fi}^{(2)}(t)-i_{fi}^{(1)}(t)]\} , \quad (1c)$$

$$\left[\frac{\Phi_0}{2\pi R_n}\right]\frac{d\varphi_{4,i}}{dt}+\alpha_{4,i}(T)\langle I_{J0}\rangle \sin\varphi_{4,i} = \langle I_{J0}\rangle\{(i_{i-1}^{(2)}-i_i^{(2)})+[i_{fi-1}^{(2)}(t)-i_{fi}^{(2)}(t)]\} , \quad (1d)$$

$$\left[\frac{\Phi_0}{2\pi R_n}\right]\frac{d\varphi_{5,i}}{dt}+\alpha_{5,i}(T)\langle I_{J0}\rangle \sin\varphi_{5,i} = \langle I_{J0}\rangle\{(i_s-i_1^{(2)})+[i_{fs}(t)-i_{fi}^{(2)}(t)]\} , \quad (1e)$$

where the index i ranges from 1 to N. The normalized currents $i_{fi}^{(j)}(t)$ $(j=1,2)$ represent the contribution of the white-noise current sources. In Eqs. (1) we use the normalized quantities $i_i^{(j)}=I_i^{(j)}/\langle I_{J0}\rangle$ and $i_s=I_s/\langle I_{J0}\rangle$, which represent, respectively, the currents circulating in the outer intergranular loops and that circulating in the innermost loop of area S_h. These currents are linked to an appropriate linear combination of the phase differences $\varphi_{k,i}$ via fluxoid quantization.[5] The quantity $\alpha_{k,i}(T)=I_{J(k,i)}(T)/\langle I_{J0}\rangle$ is the normalized maximum Josephson current of the junction labeled with the index pair (k,i). The currents $I_{J(k,i)}$ have been taken to be field independent for simplicity. The white-noise term $i_{fi}^{(j)}(t)$, by definition, is realized by the following conditions:

$$\langle i_{fi}^{(j)}(t)\rangle_t=0 \; ; \quad i=1,\ldots,N \; ; \quad j=1,2 , \quad (2)$$

$$\langle i_{fi}^{(j)}(t) i_{fm}^{(n)}(t')\rangle_t=\frac{2k_BT}{R_nI_{J0}^2}\delta_{im}\delta_{jn}\delta(t-t') \; ; \quad i,m=1,\ldots,N \; ; \quad j,n=1,2 , \quad (3)$$

where the symbol $\langle \cdots \rangle_t$ stands for time average and k_B is the Boltzmann constant.

We recall that in Ref. 4 the stationary solutions of this system were obtained by starting from a stationary state under zero-field-cooled conditions. After each small enough increment of the external normalized flux, the system was allowed to evolve to a new stationary state. For the $T\neq0$ K case, on the other hand, there do not exist stationary states for a single history of the system, since the metastable magnetic states are expected to decay with time. Therefore, the particular magnetic state realized must depend on the total elapsed integration time t_f. In this case, then, our numerical analysis proceeds as follows: we give a small enough increment to the normalized external flux $\Psi_{ext}=\mu_0HS_0/\Phi_0$ and let the system evolve for a certain time t_f, after which we record the flux distribution inside the sample, which we take as the starting point for the next step, as before.

We first solved Eqs. (1) for the $T=0$ K case by means of an adaptive multistep Runge-Kutta-Merson algorithm and by a standard Runge-Kutta (RK) method with practically the same results. In order to simulate the same stochastic equations for $T\neq0$ K, a discrete approximation based upon a standard fourth-order RK method was adopted. The stochastic part of the forcing term in Eqs. (1) was taken to be a white-noise Gaussian sequence with zero average value and with variance equal to $\sigma^2\Delta t$, where σ is the amplitude of the variance given in Eq. (3), and Δt is the adopted integration step. The following choice of parameters was made: $\beta_0=L_0\langle I_{J0}\rangle/\Phi_0=3.0$; $z=S_h/S_0=20.0$. The results are shown in terms of the following normalized quantities: $\Psi_i^{(j)}=\Phi_i^{(j)}/\Phi_0$ and $\Psi_s=\Phi_s/\Phi_0$, where $\Phi_i^{(j)}$ is the flux linked to the ith loop in the jth row, and Φ_s is the flux linked to the innermost loop.

One run was done at $T=0$ K, and two were done assuming the system to be immersed in a finite-temperature thermal bath. The first run ($T=0$ K) was performed for a system in which the values of the coefficients $\alpha_{k,i}$ are taken to have a Gaussian distribution about the unitary mean and to be randomly spread over the network. The second two runs ($T\neq0$ K) were performed, respectively, for a completely uniform system $[\alpha_{k,i}(0)=1]$ and for a system in which the values of the coefficients $\alpha_{k,i}$ are taken as in the first case. The value of the temperature was chosen in such a way as to give $\sigma=0.05$. In the second two runs the following normalized integration-time value was used: $\tau_f=2\pi R_nt_f/L_0=100\tau_0$, where $\tau_0=2\pi L_0/R_n$. The resulting curves for the flux numbers Ψ_{N-k} versus the applied flux are shown in Figs. 2, 3(a), and 3(b) for $T=0$ K and for $T>0$ K. At $T=0$ K we reported the presence of cylindrically symmetric stationary (CSS)

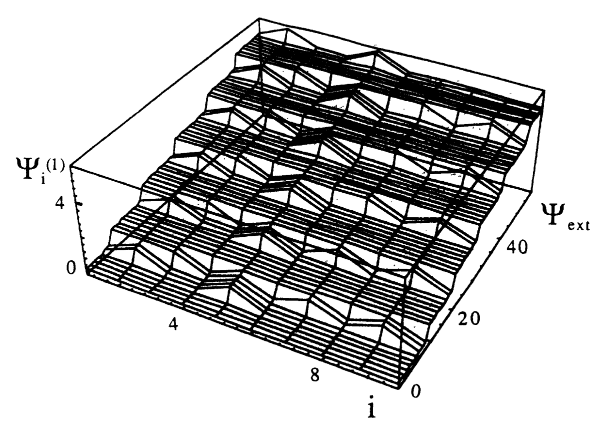

FIG. 2. Normalized flux $\Psi_i^{(1)}$ in the external row of intergranular loops of area S_0 versus the normalized applied flux Ψ_{ext}. The i label represents the angular position of the elementary intergranular loops. The system has been assumed to be completely uniform [$\alpha_{ki}=1$ for any pair (k,i)] and immersed in a finite-temperature bath.

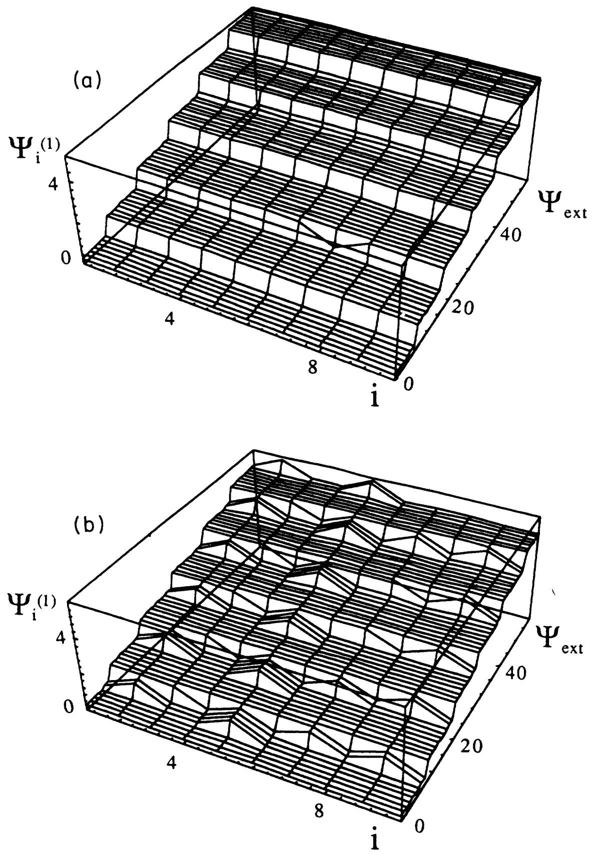

FIG. 3. Normalized flux $\Psi_i^{(1)}$ in the external row of intergranular loops of area S_0 versus the normalized applied flux Ψ_{ext}. The i label represents the angular position of the elementary intergranular loops. The α_{ki} coupling coefficients between adjacent grains of the system are taken to be randomly spread over the network and to have a Gaussian distribution about the unitary mean. (a) and (b) are the results of two runs, one at $T=0$ K and the other at $T\neq 0$ K.

states only for a completely uniform system,[4] since the JJ's are perfectly identical and the flux quanta penetrate simultaneously in the external loops. One cylindrically asymmetric stationary (CAS) state appears in Fig. 3(a) because of thermal noise, even in the homogeneous case. In Fig. 2, where we report the stationary magnetic states at $T=0$ K in the presence of inhomogeneity, we notice a greater number of CAS states, which link two ranges of filed values in which only CSS states are present. The magnetic states realized in the presence of thermal fluctuations in the inhomogeneous case represented in Fig. 3(b) are similar to those of Fig. 2 except for a few states near the transitions, in which some junctions tend to allow flux quanta into the system at lower values of the external flux.

From the above numerical results it can be argued that, for increasing temperature, the value of the external flux at which the system irreversibly admits flux quanta decreases. At a constant temperature value one expects that a similar effect could be detected with increasing integration times t_f. We also expect a smoothing of the unrealistic steps of Fig. 3(a) due to thermal fluctuations. In fact, in order to obtain statistically significant information, we should average over a sufficiently high number of histories of the type obtained for single values of T and t_f. When this average is performed, on the basis of the results shown in Figs. 3(a) and 3(b), we expect that the indeterminacy of the value at which flux transitions occur will lead to a smoothing of the Ψ_{N-k} vs Ψ_{ext} curves.

We now see that the concentric-ring model[4] can be adopted for the study of the diamagnetic response of superconducting granular systems even at finite temperatures. In fact, the sequence of states realized in the presence of thermal fluctuations in the inhomogeneous case presents the same CSS states as the completely symmetric model. The only difference between these two cases is an upward shift with increasing temperature of the flux values at a fixed value of the external field H. This shift is a consequence of the decrease with temperature of the external flux values at which the system irreversibly admits flux quanta. Therefore, in order to take account of the increase of the flux variables with increasing T for a fixed value of H, one can opportunely readjust the characteristic parameters of the model, that is, one can introduce an extrinsic temperature dependence in the system.

Let us now analytically study the phenomenon of thermally activated flux jumps between adjacent intergranular sites by the concentric-ring model. Defining the activation energy for flux motion E_B as the energy difference between a relative minimum and the successive maximum in the Gibbs potential, we may write, in general, $E_B=E_B(h,dh/dr,T)$. If some of the local states of the system are located in the irreversible diamagnetic region of the H vs T diagram, the field distribution inside the sample can be derived by some appropriate critical-state model.[6,7]

We can start our analysis by writing the equation for the relaxation time τ for flux jumps as follows:[8]

$$\frac{\tau(h,T)}{\tau_0}=\exp\left[\frac{E_B(h,T)}{k_B T}\right] . \tag{4}$$

We take this expression as the rate of jump of flux quanta for any loop of infinitesimal thickness dr at a distance r from the center of the sample containing $n(r)$ JJ's. One can suppose that this region is contained between two loops of the circuit model adopted, one located at r, the other at $r+dr$. The time rate of change of the flux in that annulus is proportional to the difference between the incoming and the outcoming rate of flux quanta, so that

$$2\pi r\, dr\frac{dh(r)}{dt}=\frac{\Phi_0}{\mu_0}\left[\frac{n(r+dr)}{\tau(h(r+dr),J_c,T)}-\frac{n(r)}{\tau(h(r),J_c,T)}\right] , \tag{5}$$

where we assume that the penetration of flux quanta proceeds inward in the sample. In order to obtain the total rate of change of the flux in the sample, we can integrate Eq. (5) with respect to the radial variable, obtaining the following:

$$\frac{d\langle h\rangle}{dt}=\frac{\Phi_0}{p\pi R^2\mu_0}\left[\frac{n(r)}{\tau(h(r),J_c,T)}\right]_0^R , \quad (6)$$

where $\langle h\rangle$ is the average field in the sample of radius R, and $(1/p)$ is the ratio of the effective normal-region area to the total area πR^2. From Eq. (6) and from the expression of the characteristic time τ given in Eq. (4), we find the logarithmic time rate of change of the field in the sample,

$$\frac{d\langle h\rangle}{d\ln(t/\tau_0)}=\frac{N\Phi_0}{p\pi R^2\mu_0}\frac{1}{\ln[\tau(H,J_c,T)/\tau_0]} = \frac{N\Phi_0}{p\pi R^2\mu_0}\frac{k_B T}{E_B(H,J_c,T)/\tau_0} . \quad (7)$$

A similar result can be obtained for hollow cylinders. Equation (7) is formally similar to the result of Kim, Hempstead, and Strnad,[9] despite the fact that we are considering a superconducting system of different nature. We therefore argue that the problem of time decay of the diamagnetic properties of granular systems seen as arrays of Josephson junctions[10,11] is formally similar to that of type-II classical superconductors.

In summary, by taking account of finite-temperature effects, we study the low-field diamagnetic response of a simple two-dimensional Josephson-junction array. Thermal effects are introduced by adding white-noise current sources in parallel to the RSJ circuit models of the Josephson junctions of the network. By this analysis we argue that a simplified one-dimensional description of the equivalent circuit, proposed by us for cylindrical granular superconductors, is still valid even in the presence of thermally activated flux jumps. The overall effect of thermal noise on this 1D system consists in modifying the characteristic intrinsically defined parameters of the model in such a way that an additional extrinsic T dependence coming from thermal fluctuations is added to their intrinsic temperature dependence. It is finally shown that, from the concentric-ring model, a flux-creep picture for intergranular flux motion follows. This picture is found to be formally identical to the classical Anderson-Kim flux-creep model.[12]

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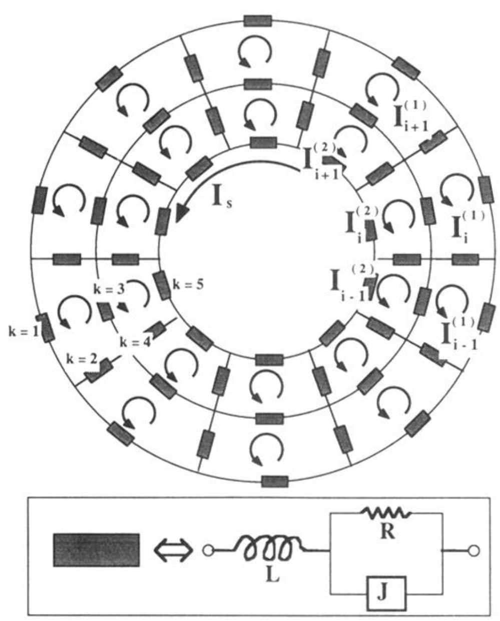

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