Mesoscopic Josephson junctions in the presence of nonclassical electromagnetic fields

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Circular superconducting devices with mesoscopic Josephson junctions are considered in the presence of a dc voltage bias V_0 and of nonclassical electromagnetic fields. It is shown that when the voltage V_0 is an integer multiple of a certain value, the current through the junction has a dc component. This is broadly similar to Shapiro steps in macroscopic Josephson junctions, but the details depend on the nature of the nonclassical electromagnetic field. Coherent states, squeezed states, and number eigenstates are considered and the corresponding Shapiro steps are studied in detail. The presence of thermal noise in the nonclassical fields is also considered and its effect on the results is studied.

I. INTRODUCTION

Nonclassical light (squeezed states, number eigenstates, etc.) have been studied extensively in the last few years and produced experimentally in several laboratories (for reviews, see Refs. 1). The emphasis in these studies is on the properties which are due to the quantum nature of light and which cannot be understood classically. In this paper we consider the interaction of nonclassical electromagnetic fields with mesoscopic Josephson junctions. The effect of classical electromagnetic fields on (macroscopic) Josephson junctions is of course well known.[2] In the last few years mesoscopic Josephson junctions (with very small capacitance C and at very low temperatures) have been studied extensively.[3–6] Here we study their behavior in the presence of nonclassical electromagnetic fields.

In Sec. II we study the Hamiltonian that describes mesoscopic Josephson junctions. The phase θ and the charge q are dual quantum variables and therefore they can feel the quantum nature of external nonclassical electromagnetic fields. In Sec. III we consider these junctions with a dc bias voltage V_0 and in the presence of coherent states, squeezed states and number eigenstates. It is shown that when the voltage V_0 is an integer multiple of a certain value, the current through the junction has a dc component which could be observed experimentally. This is of course similar to Shapiro steps[7] in macroscopic Josephson junctions, but we show that the details depend on the nature of the external nonclassical field. For squeezed vacua for example, the voltage steps are double in size than for other squeezed states. For number eigenstates we do not get any dc currents at all. The presence of noise[8] in the external nonclassical fields is also considered and its effect on the results is studied in detail. We conclude in Sec. IV with a discussion of our results. Details about the calculations are given in Appendix A, while Appendix B presents the units that have been used.

II. MESOSCOPIC JOSEPHSON JUNCTIONS

In the last few years mesoscopic Josephson junctions have been studied extensively.[3–6] Most of this work considers mesoscopic Josephson junctions with a dc current bias and studies the Coulomb blockade of Cooper pair tunneling. In this paper the behavior of these junctions in the presence of nonclassical electromagnetic fields is studied. The mesoscopic nature of the junction is needed in order to have a sensitive enough system which will feel the quantum mechanical nature of the external electromagnetic field. More specifically the phase difference θ across the junction should be a quantum variable.

The mesoscopic junction has a very small capacitance C and operates at a very low temperature T. It is described by the Hamiltonian ($\hbar = k_B = c = 1$):

$$H_0 = \frac{(q+Q)^2}{2C} + E_J(1-\cos\theta) = -\frac{1}{2}E_C\left[\partial_\theta + i\frac{Q}{2e}\right]^2 + E_J(1-\cos\theta) = -\frac{1}{2}E_C D_\theta^2 + \Omega^2 E_C^{-1}(1-\cos\theta)\ , \tag{1}$$

$$q = -i(2e)\partial_\theta\ , \tag{2}$$

$$[\theta, q] = 2ei\ , \tag{3}$$

$$D_\theta = \partial_\theta + i\frac{Q}{2e} = \exp\left[-\frac{iQ\theta}{2e}\right]\partial_\theta \exp\left[\frac{iQ\theta}{2e}\right]\ , \tag{4}$$

$$E_J = \frac{I_{cr}}{2e}\ , \tag{5}$$

$$E_C = \frac{(2e)^2}{C}\ , \tag{6}$$

$$\Omega^2 = E_J E_C = \frac{I_{cr}(2e)}{C}\ . \tag{7}$$

The factor 2 in the charge $2e$ comes from the Cooper pairs. E_C, E_J are the Josephson and Coulomb coupling constants, respectively. In ordinary (macroscopic) Josephson junctions $E_C \to 0$ and E_J is much greater than E_C. In this case the first term in the Hamiltonian (1) is negligible and θ behaves as a classical variable. Mesoscopic Josephson junctions have very small capacitance C so that E_C is not negligible. In this case the charge q tunneling through the junction and the phase difference θ across the junction are quantum variables. The quantum nature of these junctions is apparent at low temperatures.

$$T \ll \Omega = (E_C E_J)^{1/2} . \tag{8}$$

In mesoscopic Josephson junctions used in connection with the Coulomb blockade phenomenon, the constraint $E_J < E_C$ is also sometimes required, but this is not necessary for our purposes.

Q is the external charge and it depends on how the junction is coupled to the external world. When $Q=0$ the system is approximately a harmonic oscillator with frequency Ω. Caution is required when this approximation is used, in the sense that the $(1-\cos\theta)$ has an infinite number of minima and instanton tunneling solutions between these minima might distort results obtained through the harmonic oscillator approximation. The eigenfunctions of H_0 are periodic functions of θ.

$$H_0 \chi_{n,Q}(\theta) = E_n(Q) \chi_{n,Q}(\theta) , \tag{9}$$

$$\chi_{n,Q}(\theta+2\pi) = \chi_{n,Q}(\theta) . \tag{10}$$

Using Eq. (4) we see that Q can be absorbed in quasiperiodic boundary conditions, and then it does not appear explicitly in the Hamiltonian:

$$\begin{aligned} H_1 &= -\tfrac{1}{2} E_C \partial_\theta^2 + E_J(1-\cos\theta) \\ &= \exp\left[\frac{iQ\theta}{2e}\right] H_0 \exp\left[-\frac{iQ\theta}{2e}\right] , \end{aligned}$$

$$H_1 \Psi_{n,Q}(\theta) = E_n \Psi_{n,Q}(\theta) , \tag{11}$$

$$\Psi_{n,Q}(\theta) = \exp\left[\frac{iQ\theta}{2e}\right] \chi_{n,Q}(\theta) , \tag{12}$$

$$\Psi_{n,Q}(\theta+2\pi) = \exp\left[i\frac{\pi Q}{e}\right] \Psi_{n,Q}(\theta) . \tag{13}$$

In the Heisenberg picture the operators θ, q evolve as follows

$$I = -\partial_t q = -i[H_0, q] = 2eE_J \sin\theta = I_{cr} \sin\theta , \tag{14}$$

$$\partial_t \theta = i[H_0, \theta] = -iE_C \partial_\theta + 2eV_{ex} = -iE_C D_\theta , \tag{15}$$

where V_{ex} is the external voltage:

$$V_{ex} = \frac{Q}{C} . \tag{16}$$

In the next section nonclassical external electromagnetic fields together with a dc voltage V_0 will be considered and the behavior of the above equations will be studied.

III. SHAPIRO STEPS IN THE PRESENCE OF NONCLASSICAL ELECTROMAGNETIC FIELDS

We consider a circular superconducting device with a mesoscopic Josephson junction (Fig. 1) and we apply an ac nonclassical magnetic field of angular frequency ω_1. The Cooper pairs feel in this case both an ac vector potential A_i and an ac electric field E_i induced by the magnetic field according to Faraday's law. Integration of A_i, E_i in a closed loop C gives the magnetic flux ϕ, and the electromotive force V_{EMF}, correspondingly. For nonclassical fields[1] considered here ϕ, V_{EMF} (and also A_i, E_i) are dual quantum variables.

$$V_{EMF} = -i\omega_1 \partial_\phi , \tag{17}$$

$$[\phi, V_{EMF}] = i\omega_1 . \tag{18}$$

The Hamiltonian of the electromagnetic field in the monochromatic case is the harmonic oscillator Hamiltonian:

$$H = \omega_1 (a^+ a + \tfrac{1}{2}) , \tag{19}$$

where

$$\begin{aligned} a &= 2^{-1/2}[\phi + i\omega_1^{-1} V_{EMF}] , \\ a^+ &= 2^{-1/2}[\phi - i\omega_1^{-1} V_{EMF}] , \\ [a, a^+] &= 1 . \end{aligned} \tag{20}$$

Using this Hamiltonian we find that in the Heisenberg picture, V_{EMF}, ϕ evolve in time as follows:

$$\begin{aligned} V_{EMF}(t) &= 2^{-1/2}\omega_1 i[\exp(i\omega_1 t)a^+ - \exp(-i\omega_1 t)a] , \\ \phi(t) &= 2^{-1/2}[\exp(i\omega_1 t)a^+ + \exp(-i\omega_1 t)a] . \end{aligned} \tag{21}$$

These equations are derived for a free electromagnetic field. In the presence of electrons there are "back-reaction" corrections which we neglect for mesoscopic junctions with very small currents.

Apart from the nonclassical ac electromagnetic field, we also impose a classical static electromotive force (voltage) V_0. This can be done in practice in various ways. For example we can apply a classical magnetic flux

$$\phi = V_0 t . \tag{22}$$

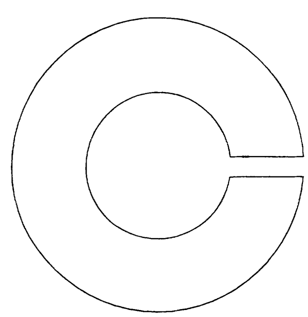

FIG. 1. Circular superconductor with mesoscopic Josephson junction. The classical magnetic flux $\phi = V_0 t$ of Eq. (22) and the nonclassical magnetic field $\phi(t)$ of Eq. (21), are perpendicular to the plane of the diagram.

We study the case

$$\omega_1 \gg \Omega \ , \tag{23}$$

where Ω is the angular frequency of the mesoscopic junction when it is approximated as a harmonic oscillator [Eq. (7)]. In this case the Hamiltonian of the junction itself produces an evolution which is much slower than the Hamiltonian describing the effect from external fields. For time intervals:

$$\omega_1^{-1} < \Delta t < \Omega^{-1} \tag{24}$$

we can ignore the first term on the right-hand side of Eq. (15):

$$\partial_t \theta = 2eV_{\rm ex} = 2e(V_0 + V_{\rm EMF}) \ . \tag{25}$$

Taking into account Eq. (21) which describes the time evolution of the free electromagnetic fields, we get

$$\begin{aligned}\theta(t) &= 2eV_0 t + 2e\int V_{\rm EMF}(t)dt \\ &= \omega_0 t + 2^{1/2}e\exp(i\omega_1 t)a^+ \\ &\quad + 2^{1/2}e\exp(-i\omega_1 t)a + \text{const} \ ,\end{aligned} \tag{26}$$

where

$$\omega_0 = 2eV_0 \ . \tag{27}$$

Therefore

$$\exp[i\theta(t)] = \exp(i\omega_0 t)D[2^{1/2}e\exp(i\omega_1 t)] \ , \tag{28}$$

where D is the displacement operator

$$D(A) = \exp[Aa^+ - A^*a] \ . \tag{29}$$

For simplicity we have moved the time origin by $\pi/2\omega_1$, so that a factor i which should be in (28) is eliminated. We now use Eq. (14) to calculate the expectation value of the current in the junction, by taking the trace of the operator I multiplied with the density matrix ρ describing the nonclassical electromagnetic field:

$$\begin{aligned}\langle I\rangle &= \text{Tr}(\rho I) = I_{cr}\text{Im}\{\text{Tr}[\rho\exp(i\theta(t))]\} \ , \\ &= I_{cr}\text{Im}\{\exp(i\omega_0 t)\text{Tr}\{\rho D[2^{1/2}e\exp(i\omega_1 t)]\}\} \ .\end{aligned} \tag{30}$$

It is seen that the density matrix ρ of the external nonclassical electromagnetic field controls the operation of the junction in the sense that the $\langle I\rangle$ and also all the other quantities somebody might wish to calculate, depend on ρ. By changing ρ we change the state in which the junction operates. Note that in the case of a macroscopic Josephson junction with dc voltage bias, the phase θ has a definite value while the charge q has an indefinite value and tunneling occurs randomly. In the case of mesoscopic Josephson junctions with dc current bias, the phase θ has an indefinite value while the charge q has a definite value and tunneling occurs regularly. These are two extreme cases to be compared and contrasted with the experiment proposed here where suitable choice of ρ can make the junction to operate in any desirable state inbetween. Of course in practice somebody will have to use one of the density matrices describing nonclassical fields which are experimentally available. In the following we consider nonclassical fields that have been produced experimentally by the Quantum Optics Community.

Using the results from Appendix A [Eqs. (A23), (A24), (A25)], we conclude that for squeezed states:

$$\begin{aligned}\langle I\rangle = I_{cr}\exp[-e^2\cosh(r)] \sum_{K=-\infty}^{\infty}\sum_{\Lambda=-\infty}^{\infty}\sum_{M=-\infty}^{\infty} & I_K[-e^2\sinh(r)]J_\Lambda[2^{3/2}e|A|\cosh(\tfrac{1}{2}r)] \\ & \times J_M[-2^{3/2}e|A|\sinh(\tfrac{1}{2}r)] \\ & \times\sin\{[\omega_0+(2k+\Lambda+M)\omega_1]t+K(2\lambda+\gamma) \\ & \quad +\Lambda(\lambda-\theta_A)+M(\lambda+\theta_A+\gamma)\}\end{aligned} \tag{31}$$

for coherent states:

$$\langle I\rangle = I_{cr}\exp(-e^2)\sum_{k=-\infty}^{\infty} J_K(2^{3/2}e|A|)\sin[(\omega_0+K\omega_1)t-K\theta_A] \tag{32}$$

and for squeezed vacua:

$$\langle I\rangle = I_{cr}\exp[-e^2\cosh(r)]\sum_{k=-\infty}^{\infty} I_K[-e^2\sinh(r)]\sin[(\omega_0+2K\omega_1)t+K(2\lambda+\gamma)] \ . \tag{33}$$

It is seen from Eqs. (31), (32) that for squeezed states (with $A\neq 0$) and also for coherent states, when the dc bias voltage V_0 is such that:

$$\omega_0 = N\omega_1 \rightarrow V_0 = N\frac{\omega_1}{2e} \ , \tag{34}$$

where N is an integer, we get Shapiro dc currents similar to those in macroscopic junctions.[7] Their values are

$$I_{dc}=I_{cr}\exp[-e^2\cosh(r)]\sum_{K=-\infty}^{\infty}\sum_{\Lambda=-\infty}^{\infty}I_K[-e^2\sinh(r)]J_\Lambda[2^{3/2}e|A|\cosh(\tfrac{1}{2}r)]$$
$$\times(-1)^{N+2K+\Lambda}J_{(N+2K+\Lambda)}[-2^{3/2}e|A|\sinh(\tfrac{1}{2}r)]$$
$$\times\sin[-N(\lambda+\theta_A+\gamma)-(K+\Lambda)(\gamma+2\theta_A)] , \quad (35)$$

$$I_{dc}=I_{cr}\exp(-e^2)(-1)^N J_N(2^{3/2}e|A|)\sin(N\theta_A) \quad (36)$$

for squeezed and coherent states correspondingly. The interpretation of this result is that the electrostatic energy $\omega_0=2eV_0$ that the Cooper pair loses during tunneling is compensated by N photons. It is also seen from Eq. (33) that for squeezed vacua, when the dc voltage V_0 is such that

$$\omega_0=2N\omega_1\rightarrow V_0=N\frac{\omega_1}{e} , \quad (37)$$

where N is an integer we also get Shapiro dc currents. They are given by the equation

$$I_{dc}=I_{cr}\exp[-e^2\cosh(r)]I_N[-e^2\sinh(r)]$$
$$\times\sin[-N(2\lambda+\gamma)] . \quad (38)$$

Note that the voltage jumps here are double in size in comparison with the voltage jumps in Eq. (34). This reflects the fact that

$$\langle 2N+1|0;r\gamma\lambda\rangle=0 , \quad (39)$$

where $|2N+1\rangle$ is an odd number eigenstate, and consequently the electrostatic energy $\omega_0=2eV_0$ that the Cooper pair looses is compensated by an even number of photons. It is clear from Eqs. (34), (37) that we can distinguish Shapiro steps due to coherent states from Shapiro steps due to squeezed vacua by looking at the size of the voltage steps in the two cases.

We see from Eqs. (35), (36) that the dc currents depend on $e|A|$ through Bessel functions. In the case of coherent states [Eq. (36)], when $e|A|$ takes values such that:

$$2^{3/2}e|A|=j_{NM} , \quad (40)$$

where j_{NM} are the roots of the Bessel functions ($J_N(j_{NM})=0$), the dc current becomes equal to zero.

For the density matrix of Eq. (A6) we get [using Eq. (A27)]

$$\langle I\rangle=I_{cr}\exp(-e^2)\sin(\omega_0 t)\sum_{N=0}^{\infty}p_N L_N(2e^2) . \quad (41)$$

It is clear that in this case there are no Shapiro dc currents. Number eigenstates [Eq. (A7)], thermal states [Eq. (A8)], and coherent states with randomized phase [Eq. (A9)], are all special cases of the density matrices (A6) and therefore they also do not produce Shapiro dc currents.

We next consider the density matrix (A10) describing coherent states with thermal light and using Eq. (A30) we prove:

$$\langle I\rangle=I_{cr}\exp[-e^2\coth(\tfrac{1}{2}\beta\omega_1)]$$
$$\times\sum_{K=-\infty}^{\infty}J_K(2^{3/2}e|A|)\sin[(\omega_0+K\omega_1)t-K\theta_A] \quad (42)$$

Apart from a factor (which is less than one), this is the same result as in Eq. (32) for coherent states. Therefore the existence of noise in the electromagnetic field is not going to affect the Shapiro staircase. The voltage steps will be exactly the same as in the noiseless case but the dc currents will be smaller by a factor

$$f=\exp\{e^2[1-\coth(\tfrac{1}{2}\beta\omega_1)]\} \quad (43)$$

and therefore it will be more difficult to observe them.

The results of Eqs. (34), (37) which indicate that the ratio V_0/ω_1 depends only on fundamental constants, could be useful in metrology. In this direction it is very important that these results are not going to be affected by the presence of thermal noise in the nonclassical electromagnetic fields.

We also consider the density matrix (A11) describing coherent states with partially randomized phase and using Eq. (A31) we prove:

$$\langle I\rangle=\mathrm{Tr}(\rho I)=I_{cr}\exp(-e^2)\sum_{N=-\infty}^{\infty}J_N[2^{3/2}e|A|]$$
$$\times\int\frac{d\theta_A}{2\pi}p(\theta_A)\sin[(\omega_0+N\omega_1)t-N\theta_A] . \quad (44)$$

It is seen that Eq. (34) of the voltage steps is not affected by the fact that the phase of the coherent state might be partially randomized. Only in the case of coherent states with fully randomized phase, the Shapiro dc currents disappear altogether.

IV. DISCUSSION

Mesoscopic Josephson junctions are very sensitive devices that can respond to the quantum nature of external nonclassical electromagnetic fields. In this paper we have studied their behavior in the presence of a dc voltage bias V_0 and various types of nonclassical fields. The basic idea is to treat the I,θ of Eq. (14) as quantum mechanical operators. The phase operator θ is then expressed in terms of the electromagnetic field operators in Eq. (26), and the expectation value of the current $\langle I\rangle$ is given in Eq. (30) as the trace of the density matrix ρ multiplied by the operator $I_{cr}\sin\theta$. We have shown that in the case of coherent states and squeezed states (with $A\neq 0$) and for values of the voltage V_0 given by Eq. (34), the current has a dc component that could be observed experimentally. For squeezed vacua the same result holds, but the voltage steps are double in size [Eq. (37)]. For number eigenstates or pure thermal noise there are no dc currents for any value of the voltage V_0. The case of coherent states with thermal noise has also been considered and it has been shown that the voltage steps of Eq. (34) are not affected by the noise, but the value of the dc current is reduced by the factor of Eq. (43). The results of Eqs. (34), (37) could be useful in metrology. The ratio V_0/ω_1 depends only on fundamental constants and is not affected by the presence of thermal noise in the nonclassical electromagnetic fields.

In this paper we have concentrated on the quantity $\langle I \rangle$ and more specifically on its dc component because it can be observed experimentally providing a confirmation of the theory. Another interesting question would be to study the statistics of the electron tunneling and see how it is related to the statistics of the photons in the nonclassical electromagnetic field. It is well known that coherent states of light exhibit Poissonian statistics, squeezed states exhibit sub-Poissonian statistics (at least for some values of the parameters) etc. How the statistics of photons in the nonclassical light affects the statistics of electrons tunneling through the junction is an interesting question that should be explored.

The above results have been presented in the context of Josephson mesoscopic junctions described by the Hamiltonian (1), but they could be generalized for other mesoscopic junctions. There has been a lot of research in this area in the last few years[9] and the purpose of this paper is to present some interdisciplinary research which exploits the quantum nature of the nonclassical electromagnetic fields studied in quantum optics, in order to control the behavior of mesoscopic quantum devices.

APPENDIX A

In this appendix we consider several types of nonclassical electromagnetic fields described by density matrices ρ and calculate the trace which appears in Eq. (30):

$$\mathrm{Tr}\{\rho D[\xi \exp(i\omega_1 t)]\} , \tag{A1}$$

where ξ is a real number. We use the notation and some of the formulas of Ref. 8. We first consider squeezed states:

$$|A;r\gamma\lambda\rangle = S(r\gamma\lambda)|A\rangle = S(r\gamma\lambda)D(A)|0\rangle \tag{A2}$$

$$S(r\gamma\lambda) = \exp[-\tfrac{1}{4}re^{-i\gamma}(a^+)^2 + \tfrac{1}{4}re^{i\gamma}a^2]\exp(i\lambda a^+ a) . \tag{A3}$$

Ordinary coherent states are a special case ($r=\gamma=\lambda=0$) of these states:

$$|A\rangle = D(A)|0\rangle . \tag{A4}$$

The squeezed vacuum ($A=0$) is also another special case

$$|0;r\gamma\lambda\rangle = S(r\gamma\lambda)|0\rangle . \tag{A5}$$

We also consider states described by the density matrices

$$\rho = \sum_{N=0}^{\infty} p_N |N\rangle\langle N| , \tag{A6}$$

$$\Sigma p_N = 1; 0 \le p_N \le 1 ,$$

where $|N\rangle$ are number eigenstates and p_N some probability distribution. Special cases of these states are the following:

(i) The number eigenstates

$$\rho = |N\rangle\langle N| . \tag{A7}$$

(ii) The thermal states

$$\rho_{\mathrm{th}} = [1-\exp(-\beta\omega_1)]\exp[-\beta\omega_1 a^+ a] = [1-\exp(-\beta\omega_1)] \sum_{N=0}^{\infty} \exp(-\beta\omega_1 N)|N\rangle\langle N| . \tag{A8}$$

(iii) The "coherent states with randomized phase"

$$\rho = \int \frac{d\theta_A}{2\pi} ||A|\exp(i\theta_A)\rangle\langle |A|\exp(i\theta_A)| = \exp(-|A|^2) \sum_{N=0}^{\infty} \frac{|A|^{2N}}{N!} |N\rangle\langle N| . \tag{A9}$$

We consider coherent states with thermal noise, described by the density matrix

$$\rho = D(A)\rho_{\mathrm{th}} D^+(A) = [1-\exp(-\beta\omega_1)]\exp[-\beta\omega_1(a^+ - A^*)(a-A)] \tag{A10}$$

and "coherent states with partially randomized phase," described by the density matrix

$$\rho = \int \frac{d\theta_A}{2\pi} p(\theta_A) ||A|\exp(i\theta_A)\rangle\langle |A|\exp(i\theta_A)| , \tag{A11}$$

$$\int \frac{d\theta_A}{2\pi} p(\theta_A) = 1 . \tag{A12}$$

We calculate Eq. (A1) for all these density matrices. We start with squeezed states and use the relations:

$$S^+(r\gamma\lambda) a S(r\gamma\lambda) = \mu a + \nu a^+ , \tag{A13}$$

$$S^+(r\gamma\lambda) a^+ S(r\gamma\lambda) = \nu^* a + \mu^* a^+ , \tag{A14}$$

$$\mu = \exp(-i\lambda)\cosh(\tfrac{1}{2}r) , \tag{A15}$$

$$\nu = -\exp[-i(\lambda+\gamma)]\sinh(\tfrac{1}{2}r) \tag{A16}$$

to prove

$$\langle A;r\gamma\lambda|D[\xi\exp(i\omega_1 t)]|A;r\gamma\lambda\rangle = \exp[-Y+iX] , \tag{A17}$$

$$X=2\xi|A|[\cosh(\tfrac{1}{2}r)\sin(\omega_1 t+\lambda-\theta_A) - \sinh(\tfrac{1}{2}r)\sin(\omega_1 t+\lambda+\theta_A+\gamma)] , \quad (A18)$$

$$Y=\tfrac{1}{2}\xi^2[\cosh(r)+\sinh(r)\cos(2\omega_1 t+2\lambda+\gamma)] , \quad (A19)$$

$$\theta_A=\arg(A) . \quad (A20)$$

Using the formulas

$$\exp[iA\sin\theta]=\sum_{N=-\infty}^{\infty} J_N(A)\exp(iN\theta) , \quad (A21)$$

$$\exp[A\cos\theta]=\sum_{N=-\infty}^{\infty} I_N(A)\exp(iN\theta) , \quad (A22)$$

where $J_N(A), I_N(A)$ are Bessel and modified Bessel functions, respectively, we expand this result into

$$\exp(-Y+iX)=\exp[-\tfrac{1}{2}\xi^2\cosh(r)]\sum_{K=-\infty}^{\infty}\sum_{\Lambda=-\infty}^{\infty}\sum_{M=-\infty}^{\infty} I_K[-\tfrac{1}{2}\xi^2\sinh(r)]J_\Lambda[2\xi|A|\cosh(\tfrac{1}{2}r)] \times J_M[-2\xi|A|\sinh(\tfrac{1}{2}r)] \times \exp[iK(2\omega_1 t+2\lambda+\gamma)+i\Lambda(\omega_1 t+\lambda-\theta_A) +iM(\omega_1 t+\lambda+\theta_A+\gamma)] . \quad (A23)$$

In the special case of coherent states Eq. (A17) reduces to

$$\langle A|D[\xi\exp(i\omega_1 t)]|A\rangle=\exp[-\tfrac{1}{2}\xi^2+i2\xi|A|\sin(\omega_1 t-\theta_A)] =\exp(-\tfrac{1}{2}\xi^2)\sum_{K=-\infty}^{\infty} J_K(2\xi|A|)\exp[iK(\omega_1 t-\theta_A)] \quad (A24)$$

In the special case of a squeezed vacuum Eq. (A17) reduces to

$$\langle 0;r\gamma\lambda|D[\xi\exp(i\omega_1 t)]|0;r\gamma\lambda\rangle=\exp\{-\tfrac{1}{2}\xi^2[\cosh(r)-\sinh(r)\cos(2\omega_1 t+2\lambda+\gamma+\pi)]\} =\exp[-\tfrac{1}{2}\xi^2\cosh(r)]\sum_{K=-\infty}^{\infty} I_K[\tfrac{1}{2}\xi^2\sinh(r)]\exp[iK(2\omega_1 t+2\lambda+\gamma+\pi)] . \quad (A25)$$

For number eigenstates we get

$$\langle N|D[\xi\exp(i\omega_1 t)]|N\rangle=\exp(-\tfrac{1}{2}\xi^2)L_N(\xi^2) , \quad (A26)$$

where L_N are Laguerre polynomials. The matrix elements appearing in (A26) have been calculated in Refs. 10. Using this result we calculate the quantity of Eq. (A1) for the density matrix (A6):

$$\mathrm{Tr}\{\rho D[\xi\exp(i\omega_1 t)]\}=\exp(-\tfrac{1}{2}\xi^2)\sum_N p_N L_N(\xi^2) . \quad (A27)$$

We can now calculate this quantity for the thermal states of Eq. (A8)

$$\mathrm{Tr}\{\rho_{\mathrm{th}} D[\xi\exp(i\omega_1 t)]\}=\exp(-\tfrac{1}{2}\xi^2)[1-\exp(-\beta\omega_1)]\sum_N \exp(-\beta\omega_1 N)L_N(\xi^2)=\exp[-\tfrac{1}{2}\xi^2\coth(\tfrac{1}{2}\beta\omega_1)] \quad (A28)$$

and also for the coherent states with randomized phase of Eq. (A9).

$$\mathrm{Tr}\{\rho D[\xi\exp(i\omega_1 t)]\}=\exp(-\tfrac{1}{2}\xi^2)\exp(-|A|^2)\sum_N \frac{|A|^{2N}}{N!}L_N(\xi^2)=\exp(-\tfrac{1}{2}\xi^2)J_0(2\xi|A|) , \quad (A29)$$

where J_0 is a Bessel function.

In the case of coherent states with thermal noise [Eq. (A10)] we get

$$\mathrm{Tr}\{\rho D[\xi\exp(i\omega_1 t)]\}=\mathrm{Tr}\{D(A)\rho_{\mathrm{th}}D^+(A)D[\xi\exp(i\omega_1 t)]\} =\exp[i2\xi|A|\sin(\omega_1 t-\theta_A)]\mathrm{Tr}\{\rho_{\mathrm{th}}D[\xi\exp(i\omega_1 t)]\} =\exp[i2\xi|A|\sin(\omega_1 t-\theta_A)]\exp[-\tfrac{1}{2}\xi^2\coth(\tfrac{1}{2}\beta\omega_1)] =\exp[-\tfrac{1}{2}\xi^2\coth(\tfrac{1}{2}\beta\omega_1)]\sum_{K=-\infty}^{\infty} J_K(2\xi|A|)\exp[iK(\omega_1 t-\theta_A)] . \quad (A30)$$

In the case of coherent states with partially randomized phase [Eq. (A11)] we use Eq. (A24) to prove:

$$\mathrm{Tr}\{\rho D[\xi\exp(i\omega_1 t)]\}=\int\frac{d\theta_A}{2\pi}p(\theta_A)\exp[-\tfrac{1}{2}\xi^2+2\xi|A|\sin(\omega_1 t-\theta_A)]$$
$$=\exp(-\tfrac{1}{2}\xi^2)\sum_{K=-\infty}^{\infty}J_K(2\xi|A|)\exp(iK\omega_1 t)\int\frac{d\theta_A}{2\pi}p(\theta_A)\exp(-iK\theta_A)\ . \quad \text{(A31)}$$

APPENDIX B

In the system of units that we use $K_B=\hbar=c=1$ and all quantities are expressed in appropriate powers of eV. The charge of the electron is dimensionless and is equal to

$$e=\left[\frac{4\pi}{137}\right]^{1/2}.$$

The following relations help the conversion from the ordinary units:

$$1\ \mathrm{A}=1230\ \mathrm{eV}\ ,$$
$$1\ \mathrm{V}=3.3\ \mathrm{eV}\ ,$$
$$1\ \Omega=2.6\times10^{-3}\ ,$$
$$1\ \mathrm{Wb}=5\times10^{15}\ ,$$
$$1^\circ\ \mathrm{K}=0.86\times10^{-4}\ \mathrm{eV}\ ,$$
$$1\ \mathrm{cm}=0.5\times10^{5}\ \mathrm{eV}^{-1}\ ,$$
$$1\ \mathrm{sec}=1.53\times10^{15}\ \mathrm{eV}^{-1}\ .$$

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