# Low-temperature electron mobility in a  $\delta$ -doped semiconductor

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The low-temperature electron mobility in 5-doped GaAs is calculated by using the Boltzmann equation and the relaxation-time approximation. It is assumed that the electrons are scattered from ionized impurities. Screening of charged impurities by electrons occupying several subbands is described with the help of (i) the random-phase approximation, (ii) the Thomas-Fermi method, and (iii) the bulk dielectric constant only. Among those methods mentioned above, the random-phase approximation has proved quite successful in studying the screening while the other two methods are inadequate. The mobility exhibits a drop when the excited subbands become occupied. It is shown, however, that as a consequence of the parity of the subband wave functions, the drop in the mobility when the Fermi level coincides with the bottom of the first excited subband is negligible.

#### I. INTRODUCTION

The study of properties of a two-dimensional electron gas (2DEG) in semiconductor structures is motivated by 'both scientific interest and device applications.<sup>1</sup>

One of the effective methods for obtaining 2DEG in semiconductors is the planar doping method (so-called  $\delta$ semiconductors is the planar doping method (so-called  $\delta$  doping).<sup>3-11</sup> Similarly to other two-dimensional carrier systems in semiconductors (e.g., 2DEG in  $\text{Al}_x\text{Ga}_{1-x}\text{As/GaAs heterostructures}$  the  $\delta$ -doped epitaxial layer is characterized by electron mobility enhancement over the corresponding three-dimensional doped semiconductor with identical mean impurity separation. This enhancement, which is more than a factor of 4 in the case of  $\delta$ -doped GaAs<sup>6</sup> has great importance for the applications in electronics.

A typical 2DEG concentration in  $\delta$ -doped GaAs is of the order  $10^{12}$  cm<sup>-2</sup> while in Al<sub>x</sub>Ga<sub>1-x</sub>As/GaAs heterostructures it is only  $10^{11}$  cm<sup>-2</sup>. Thus, in contrast to  $\text{Al}_x\text{Ga}_{1-x}\text{As/GaAs}$ , where usually only one subband is occupied, the electrons in 5-doped GaAs populate several subbands.

In the present work we shall investigate the effect of intersubband scattering on the low-temperature mobilitv<sup>12-18</sup> of the 2DEG in  $\delta$ -doped GaAs.

Low-temperature 2DEG mobility exhibits discontinuities as a function of electron concentration each time the Fermi level coincides with the bottom of a subband. These anomalies reflect the steplike character of the 2DEG density-of-states function. So far they have been studied theoretically as well as experimentally in the case of 2D electron systems in Si/SiO<sub>2</sub> and  $Al_xGa_{1-x}As/GaAs$  heterojunctions<sup>13,15–1</sup> multiple-quantum-well structures.<sup>14,18</sup> and in

Our aim is also to discuss the effect of the occupation of higher subbands on the screening of the scattering potential in the case of  $\delta$  doping. This problem was already considered in connection with semiconductor heterojunctions in Refs. 12, 13, and 19.

Detailed studies have shown that the calculated mobilities of a 2DEG in semiconductor heterojunctions, where usually only one subband is occupied, are sensitive to the model of screening one employs.<sup> $20-23$ </sup> In contrast to the Thomas-Fermi (TF) approach, the random-phase approximation (RPA) has proved quite successful in determining the screening.

In our study we are going to compare results obtained within (i) the RPA and (ii) the TF approximation and (iii) when the screening by 2DEG is neglected and only that of the background is taken into account.

We restrict ourselves to the low-temperature case, and we assume that the mobility is limited by ionized impurity scattering only. To calculate the mobility we solve numerically the coupled Boltzmann equations in the relaxation-time approximation<sup>12,13</sup> using one-, two-, or three-subband models depending on the population of the subbands. Numerical calculations of the mobility as a function of 2DEG concentration,  $n_{2DEG}$ , are performed for  $\delta$ -doped GaAs where the background net accepto density  $n_A$  is  $5 \times 10^{15}$  cm<sup>-3</sup>. It turns out that for a given 2DEG concentration the number of occupied subband tends to decrease with increasing background acceptor density. Hence one-, two-, or three-subband models ought to be applicable in a wider range of 2DEG concentrations if  $n_A$  is higher. Note that for  $n_A = 0$  there are already 5 to 6 subbands occupied when  $n_{2DEG}$  is about  $5 \times 10^{12}$  cm<sup>-2</sup> (see, for example, Refs. 8 and 10).

 $2DEG$  mobility in a  $\delta$ -doped semiconductor was calcu-

lated in Ref. 24 (see also Ref. 11); however, the onesubband version of the TF model used there is inadequate to describe the effects of intersubband contributions on the screening.

## II. SUBBAND STRUCTURE FOR A 5-DOPED SEMICONDUCTOR

The system we deal with is a  $\delta$ -doped semiconductor, say gallium arsenide, containing ionized Si donor impurities localized in one atomic monolayer [the (100) Ga plane of GaAs]. The fractional coverage of the available Ga sites can reach several donors per unit effective Bohr area. Thus we model our system assuming a uniform distribution of a positive charge in a perfect plane. Attractive electrostatic forces tend to keep electrons close to their parent ionized donors. As a result, a quasi-twodimensional electron gas (2DEG) is formed.

In the envelope-function description (see, for example, Ref. 2) which we are going to apply, the electrons are free to move in the doping (e.g.,  $z=0$ ) plane whereas their motion is bound in the direction perpendicular to the doping plane (here in the z direction). In the singleparticle picture the potential  $V(z)$  experienced by the electrons in the z motion should be determined selfconsistently by solving simultaneously both the onedimensional Schrödinger equation and the Poisson equation. Here, however, for the sake of simplicity we shall use the TF approximation which, as was shown in Ref. 10, has proved an effective method of determining  $V(z)$  in the case of  $\delta$  doping in an otherwise intrinsic semiconductor. For the reasons mentioned in Sec. I we focus our attention on a  $\delta$  doped semiconductor with a nonvanishing net acceptor density. The potential  $V(z)$  is shown schematically in Fig. 1. By using the TF approximation to express the charge density of the electrons, the Poisson equation becomes ( $\vartheta$  is the Heaviside step function)

$$
\frac{d^2V(z)}{dz^2} = \frac{-8}{3\pi R y^{*1/2} a^{*2}} [E_F - V(z)]^{3/2} \vartheta [E_F - V(z)] \qquad h_1 = -8\pi R y^* a^* n_A(z) + 8\pi R y^* a^* n_D(z) , \qquad (1) \qquad h_2 =
$$



FIG. 1. Schematic sketch of the potential  $V(z)$  experienced by electrons in a 5-doped semiconductor with nonvanishing background acceptor density.

where  $a^* = \epsilon_0 \hbar^2 /me^2$  is the effective Bohr radius, and  $Ry^* = e^2/2a^* \varepsilon_0$  is the effective Rydberg. If the net three-dimensional acceptor density is  $n_A$ , then

$$
n_A(z) = n_A \vartheta(d - |z|) , \qquad (2)
$$

where d is determined by the condition that for  $z=d$ ,  $V(z)$  approaches its bulk value  $E_c$ , i.e., the bottom of the conduction band. The last term in (1) is due to the  $\delta$  doping. Denoting the two-dimensional donor density by  $N_D$ , one may write

$$
n_D(z) = N_D \delta(z) \tag{3}
$$

In contrast to the  $n_A = 0$  case, where the analytical solution of (1) is available,<sup>10</sup> we must perform simple numer cal calculations to find a value of  $V(z)$  which satisfies (1).

The electronic states of a 2DEG are described by the following envelope functions and the corresponding eigenvalues  $[\boldsymbol{\rho}=(x,y),\mathbf{k}=(k_x,k_y)]$ 

$$
\phi_{n\mathbf{k}}(\mathbf{r}) = \frac{1}{2\pi} \exp(i\mathbf{k}\boldsymbol{\rho}) h_n(z) , \qquad (4)
$$

$$
E_{n\mathbf{k}} = E_n + \frac{\hbar^2 \mathbf{k}^2}{2m} \tag{5}
$$

where  $h_n$  and  $E_n$  are solutions of the one-dimensional Schrödinger equation written in the effective-mass approximation

$$
\left[ -\frac{\hbar^2}{2m} \frac{d^2}{dz^2} + V(z) - E_n \right] h_n(z) = 0 \ . \tag{6}
$$

Note that  $E_{nk}$  represents the nth two-dimensional parabolic subband whose bottom is  $E_n$ .

To determine  $h_n(z)$  and  $E_n$ , we use a variational procedure. We take the following trial wave functions:

$$
h_0 = \phi_0 \t{,} \t(7)
$$

$$
h_1 = \phi_1 \tag{8}
$$

$$
h_2 = \frac{\phi_2 - \langle \phi_0 | \phi_2 \rangle \phi_0}{|\phi_2 - \langle \phi_0 | \phi_2 \rangle \phi_0|}, \qquad (9)
$$

where, for  $n = 0$  and 2  $(\kappa_n = k_n t g k_n a_n)$ ,

$$
\mathbf{E}_{\mathbf{C}}\left\{\n\begin{aligned}\n\phi_n(z) &= \n\begin{cases}\nA_n \exp(-\kappa_n |z|) & \text{if } |z| > a_n \\
A_n \exp(-\kappa_n a_n) & \frac{\cos k_n z}{\cos k_n a_n} & \text{otherwise}\n\end{cases}\n\end{aligned}\n\right.\n\tag{10}
$$

and for  $n = 1$  ( $\kappa_1 = -k_1 ctg k_1 a_1$ ),

$$
\phi_1(z) = \begin{cases} \operatorname{sgn}(z) A_1 \operatorname{exp}(-\kappa_1|z|) & \text{if } |z| > a_1 \\ A_1 \operatorname{exp}(-\kappa_1 a_1) \frac{\operatorname{sin}k_1 z}{\operatorname{sin}k_1 a_1} & \text{otherwise} \end{cases}
$$
(11)

Here  $a_n$  and  $k_n$  are the variational parameters which satisfy the following inequalities:  $n \pi/2 < k_n a_n$ satisfy the following inequalities:  $\langle (n+1)\pi/2, A_n$  are the normalization factors. Note that  $\phi_n$  are the solutions for the square-well problem.

In Fig. 2 we present the resulting eigenvalues  $E_n$  as In Fig. 2 we present the resulting eigenvalues  $L_n$  and functions of 2DEG concentration when  $n_A = 5 \times 10^1$ 



FIG. 2. Subband separation  $E_i - E_F$  vs electron concentration of the  $\delta$ -doped GaAs with background acceptor concentration  $n_A = 5 \times 10^{15}$  cm<sup>-3</sup>.

 $cm<sup>-3</sup>$ . It turns that the first, second, and third excited subbands appear for  $n_{2DEG}$  equal to 1.09, 3.23, and  $6.54 \times 10^{12}$  cm<sup>-2</sup>, respectively. The following GaAs parameters were used:  $m=0.067m_0$ ,  $\varepsilon_0=12.5$ ,  $a^*=98.7$  $A$ , and  $Ry^* = 5.83$  meV.

### III. SCREENING OF IONIZED IMPURITIES

To calculate the mobility, it is necessary to take into account the screening of the scattering potentials. According to the linear-response theory the relation between screened and external potentials can be expressed by a dielectric function which in a many-subband system is given by a matrix  $\varepsilon_{nn',ll'}$ . Let  $V_{nn'}(q)$  denote the twodimensional Fourier transform  $[q=(q_x,q_y);p=(x,y)]$ 

$$
V_{nn'}(\mathbf{q}) = \int d^2 \rho \ V_{nn'}(\rho) \exp(-i\mathbf{q} \cdot \boldsymbol{\rho}) \tag{12}
$$

of the matrix element with respect to z of the potential  $V(\rho, z)$  in 3D:

$$
V_{nn'}(\rho) = \int dz \; h_n^*(z) V(\rho, z) h_{n'}(z) \; . \tag{13}
$$

The effective potential  $V_{\text{eff}}$  experienced by an electron, i.e., the screened potential, and the impurity potential  $\boldsymbol{V}_{\text{imp}}$  are related by an inverse dielectric matrix

$$
V_{\text{eff}_{nn'}}(\mathbf{q}) = \sum_{ll'} \varepsilon_{nn',ll'}^{-1}(\mathbf{q}) V_{\text{imp}_{ll'}}(\mathbf{q}) . \qquad (14)
$$

The RPA dielectric matrix is of the form<sup>13</sup>

$$
\varepsilon_{ll',nn'}(\mathbf{q}) = \varepsilon_0 \delta_{ll',nn'} + \frac{2\pi e^2}{q} F_{ll',nn'}(\mathbf{q}) \Pi_{nn'}(\mathbf{q}) , \qquad (15)
$$

where  $\varepsilon_0$  is the background dielectric constant. The formfactor  $F$  is defined by

$$
F_{ll',nn'}(\mathbf{q})
$$
  
= 
$$
\int dz \int dz' g_{ll'}(z) \exp(-q |z-z'|) g_{nn'}^*(z'), \quad (16)
$$

where  $g_{ll'}(z) = h_l^*(z)h_{l'}(z)$ . The polarization part  $\Pi$  at where  $\mu_i$  is the mobility in the *i*th subband, and  $n_i$  is the

 $T=0$  is given by (see also Ref. 13)

$$
\Pi_{nn'}(\mathbf{q}) = \frac{m}{\pi \hbar^2} \{ 1 + A_{nn'}(\mathbf{q}) + A_{n'n}(\mathbf{q}) + i [B_{nn'}(\mathbf{q}) - B_{n'n}(\mathbf{q})] \},
$$
(17)

where

$$
A_{nn'}(\mathbf{q}) = \frac{1}{2}\vartheta \left[ |a_{nn'}(\mathbf{q})| - \frac{2k_{Fn}}{q} \right] \text{sgn}[a_{nn'}(\mathbf{q})]
$$
  
\n
$$
\times \left[ a_{nn'}^2(\mathbf{q}) - \left( \frac{2k_{Fn}}{q} \right)^2 \right]^{1/2},
$$
\n
$$
B_{nn'}(\mathbf{q}) = -\frac{1}{2}\vartheta \left[ \frac{2k_{Fn}}{q} - |a_{nn'}(\mathbf{q})| \right]
$$
  
\n
$$
\times \left[ \left( \frac{2k_{Fn}}{q} \right)^2 - a_{nn'}^2(\mathbf{q}) \right]^{1/2},
$$
\n(18b)

 $\times \left[ \left( \frac{2n_{Fn}}{q} \right) - a_{nn'}^2(q) \right]$ , (18b)<br>
and  $a_{nn'}(q) = 2m(E_n - E_{n'})/R^2q^2 - 1$  and<br>  $k_{Fn} = [2m(E_F - E_n)/R^2]^{1/2}$ .

In the TF approximation (the small-q limit formula of the RPA expression) the dielectric matrix is reduced to<sup>12</sup>

$$
\varepsilon_{nn',ll'}^{TF}(q) = \begin{cases} \varepsilon_0 \delta_{nn',ll'} + \frac{2me^2}{\hbar^2 q} \delta_{nn'} \delta_{ll'} \text{ when } E_F > E_l = E_{l'}\\ \varepsilon_0 \delta_{nn',ll'} \text{ when } E_F \le E_l = E_{l'} \end{cases} \tag{19}
$$

Finally, when the screening by a 2DEG is neglected,

$$
\varepsilon_{nn',ll'}(\mathbf{q}) = \varepsilon_0 \delta_{nn',ll'} . \tag{20}
$$

Note that in the TF method the screening is overestimated, whereas in the last approximation it is underestimated.

In our case the unscreened scattering potential  $V_{\text{imp}}$  is the Coulomb potential.

### IV. MOBILITY LIMITED BY IONIZED IMPURITY SCATTERING. INTERSUBBAND EFFECTS

We restrict ourselves to the  $T=0$  case, and we calculate the mobility limited by Coulomb scattering. We consider a highly doped  $n^+$  layer in a weakly p-type GaAs. The ionized Si donors are located in a single atomic layer, while charged acceptors are distributed over a distance d (see Fig. 1). Since the ratio of the charged acceptors to the ionized donors is small, and since the electrons are close to their parent donors, we ignore the scattering from the acceptors and take into account the scattering from the donors only.

The low-temperature mobility  $\mu$  is given by

$$
\mu = \frac{\sum_{i} n_i \mu_i(E_F)}{\sum_{j} n_j} , \qquad (21)
$$

concentration of electrons in that subband. The mobility  $\mu_i$  is related to the relaxation time  $\tau_i$  in the *i*th subband by

$$
\mu_i(E) = \frac{|e|}{m} \tau_i(E) \tag{22}
$$

The relaxation times  $\tau_i$ 's satisfy coupled linear equations $^{13,14}$ 

$$
P_n(E)\tau_n(E) - \sum_{n' \neq n} P_{nn'}(E)\tau_{n'}(E) = 1 \tag{23}
$$

where  $P_n(E)$  and  $P_{nn'}(E)$  are  $(E \geq E_n)$ 

$$
P_n(E) = \frac{mN_D}{2\pi\hbar^3} \int_0^{2\pi} d\phi |V_{\text{eff}_{nn}}(q)|^2 (1 - \cos\phi) + \frac{mN_D}{2\pi\hbar^3} \sum_{n' \neq n} \vartheta(E - E_{n'}) \int_0^{2\pi} d\phi |V_{\text{eff}_{nn'}}(q')|^2 ,
$$
\n(24)

$$
P_{nn'}(E) = \frac{mN_D}{2\pi\hbar^3} \vartheta(E - E_{n'}) \left(\frac{E - E_{n'}}{E - E_n}\right)^{1/2} \times \int_0^{2\pi} d\phi \, |V_{\text{eff}_{nn'}}(q')|^2 \cos\phi , \qquad (25)
$$

where  $q = \sqrt{2k}(1 - \cos\phi)^{1/2}$ ,  $q' = (k^2 - 2kk'\cos\phi + k'^2)^{1/2}$ where  $q = v^2 k(1 - \cos\phi)$ ,  $q = k^2 k$ ,  $\cos\phi + k$ <br>and  $k = [2m(E - E_n)/\hbar^2]^{1/2}$ ,  $k' = [2m(E - E_{n'})/\hbar^2]^{1/2}$ 

The mobility given by Eqs.  $(21) - (25)$  (the so-called transport mobility) can be determined from the classical magnetoresistence effect in low magnetic fields.

Low-temperature mobility limited by the Coulomb scattering is calculated numerically from formulas  $(21)$ – $(25)$  and plotted in Fig. 3 as a function of the 2DEG concentration. The effective scattering potential is approximated by (a) the TF method, (b) RPA, and (c} by



FIG. 3. Calculated electron mobility in 5-doped GaAs with background acceptor density  $n_A = 5 \times 10^{15}$  cm<sup>-3</sup> as a function of 2DEG concentration. (a) Thomas-Fermi model. (b) RPA. (c) Without 2DEG contribution to the screening.



FIG. 4. Calculated subband mobilities  $\mu_i$  ( $i = 0, 1$ , and 2) in  $\delta$ -doped GaAs with background acceptor density  $n_A = 5 \times 10^{15}$  $\text{cm}^{-3}$  as a function of 2DEG concentration in the RPA model

neglecting the screening by a 2DEG. When the effect of a 2DEG on the screening is neglected  $[\epsilon(q)=\epsilon_0]$ , the calculated mobility is the lowest. In contrast, in the TF approximation the role of the screening is overestimated, and thus the calculated mobility is the highest.

Low-temperature subband mobilities  $\mu_i$  limited by the Coulomb scattering are calculated from formulas  $(22)$ – $(25)$ . The results are shown in Fig. 4. The rapid growth of the calculated  $\mu_i$ , with 2DEG concentration when the ith subband becomes populated cannot be attributed to the improvement in screening only, since it is also observed when the screening is ignored. This indicates that the effective potential scatters mainly with small wave vectors, which reflects its relatively slow decay with the distance (well-localized Fourier transform). Note that the electrons in a given subband are characterized by a small  $k_F$  when  $n_{2DEG}$  is such that the Fermi level is close to the bottom of this subband.

#### V. DISCUSSION

We calculated the low-temperature 2DEG mobility in the 5-doped semiconductor taking into account the screened Coulomb scattering from ionized donors.

As was expected the mobility examined exhibits drops as a function of the electron concentration when the excited subbands become occupied. It turns that the drop in the mobility appears when the Fermi level coincides with the bottom of the second excited subband, and it is negligible when the first excited subband becomes occupied. This is a consequence of the parity of the subband wave functions, and can be explained as follows.

The form factors  $F_{ij,kl}$  given by formula (16) vanish when  $i+j+k+l$  is equal to an odd number. Thus  $\varepsilon_{i,j,kl} = 0$  when  $i + j + k + l$  equals an odd number, and the only nonvanishing matrix elements of  $V_{\text{eff}}$  are  $V_{\text{eff}_{00}}$ ,  $V_{\text{eff}_{11}}$ ,  $V_{\text{eff}_{02}}$ , and  $V_{\text{eff}_{22}}$ . Now  $(P_n$  and  $P_{nn'}$  are taken at  $E=E_F=0$ 

$$
\mu = \frac{|e|}{m} \frac{1}{P_0} \quad \text{(one-subband model)}\tag{26}
$$

$$
\mu = \frac{|e|}{m} \frac{1}{E_0 + E_1} \left[ \frac{E_0}{P_0} + \frac{E_1}{P_1} \right] \text{ (two-subband model)},
$$
\n
$$
\mu = \frac{|e|}{m} \frac{1}{E_0 + E_1 + E_2} \left[ \frac{E_0 (P_2 + P_{02}) + E_2 (P_0 + P_{20})}{P_0 P_2 - P_{02} P_{20}} + \frac{E_1}{P_1} \right] \text{ (three-subband model)}.
$$
\n(28)

when  $E_1 \rightarrow E_F + 0 = +0$ , then the one-subband model is valid and

$$
\lim_{E_1 \to +0} \mu = \frac{|e|}{m} \frac{1}{P_0} \ . \tag{29}
$$

If  $E_1 \rightarrow E_F - 0 = -0$ , the two-subband model should be used, which gives

$$
\lim_{E_1 \to -0} \mu = \frac{|e|}{m} \frac{1}{P_0} \ . \tag{30}
$$

This means [compare Eq. (29)] that there is no discontinuity in the mobility when  $E_1 = E_F$ , and the screening by a 2DEG is neglected. Note that in the TF approach or the RPA  $P_0(E_F = E_1 - 0)$  differs from  $P_0(E_F = E_1 + 0)$ , which is due to the fact that according, to Eqs. (14) and (24),  $V_{\text{imp}_{11}}$  contributes to  $V_{\text{eff}_{00}}$ , and what follows to  $P_0(E_F = E_1 + 0)$ , while it does not contribute to  $P_0(E_F=E_1=0)$ . However, it turns out that in the RPA the discontinuity in  $\mu$  is negligible when  $E_F = E_1$ , and is about 0.25% only when the TF method is used. The discontinuity in  $\mu$  when  $E_2=E_F=0$  is substantial, and can be calculated from

$$
\lim_{E_2 \to +0} \mu = \frac{|e|}{m} \frac{1}{E_0 + E_1} \left[ \frac{E_0}{P_0} + \frac{E_1}{P_1} \right]
$$
(31)

and

$$
\lim_{E_2 \to -0} \mu = \frac{|e|}{m} \frac{1}{E_0 + E_1} \left[ \frac{E_0 (P_2 + P_{02})}{P_0 P_2 - P_{02} P_{20}} + \frac{E_1}{P_1} \right].
$$
 (32)

Mobility data for some values of 2DEG concentrations in different samples have been given in Refs. 25–33; however, to the best of our knowledge, so far there has been no experimental evidence for the discontinuities in  $\mu$  in 5-doped semiconductors. For that reason a more detailed experimental investigation of the dependence of  $\mu$  on  $n_{2DEG}$  would be very helpful.

We now turn to a discussion of the screening effects. As seen in Fig. 3, theoretical predictions for the electron mobilities are very sensitive to the screening model. Among the methods which we applied, the RPA has proved quite successful in studying the screening. The other two methods (the TF approach and that in which the screening by a 2DEG is neglected} lead to substantial discrepancies between the theory and experimental results.

The symmetry and the extent of the wave functions  $h_i(z)$  strongly influence the theoretical values of subband

mobilities  $\mu_i$ . Note that (1)  $h_1$  has a node at the doping plane, while  $h_2$  and  $h_0$  approach their maximum values at  $z=0$ ; and (2) that  $h_2$  is more extended than  $h_0$ . For that reason one obtains that  $\mu_1 > \mu_2 > \mu_0$  (the potential associated with ionized donors experienced by the electrons in the first excited subband is weaker than that experienced by the electrons in the second subband, etc.). The inequalities  $\mu_1 > \mu_0$  and  $\mu_2 > \mu_0$  are confirmed experimentally, while  $\mu_1 > \mu_2$  is not.

Transport mobilities  $\mu$  deduced from the experimental data given in Refs. 27, 29, 30, and 32 are about two times smaller than the theoretical results.

Hall mobility  $\mu_H = \sum n_i \mu_i^2 / \sum n_j \mu_j$  calculated by using theoretically determined  $n_i$  and  $\mu_i$  is about 2000, 5000, or  $10000 \text{ cm}^2$ /V s when one, two, or three subbands are occupied, respectively. Experimental values of  $\mu_H$  are in the range between 1000 and 7000  $\text{cm}^2/\text{V}$  s (Refs. 25, 26, and 29-33) and are less than predicted by theory by a factor of almost 2. This discrepancy is caused mainly by a very high theoretical value of  $\mu_1$  (see Fig. 4). Only the highest experimental values of  $\mu_1$  reported in literature  $(7000 - 8000, 9100, 9250, \text{ and } 10900 \text{ cm}^2/\text{V s}; \text{ see Refs.}$ 26, 29, and 30, respectively) are comparable with our theoretical predictions.

We believe that the theoretical values of  $\mu$  and  $\mu$ <sub>H</sub> will be much closer to the experimental ones if the scattering from charged acceptors in the depletion region is taken into account. Due to the character of the wave function  $h_1(z)$ , the mobility  $\mu_1$  should be more sensitive to the scattering from charge acceptors than those of  $\mu_0$  and  $\mu_2$ .

Theoretical values of  $\mu_0$  are in the same range as those given by experiments,  $26,28-30,32,33$  whereas the calculate values of  $\mu_2$  which do not exceed 2600 cm<sup>2</sup>/V s are smaller than the highest experimental values by a factor of about 2.

It seems that the experimental data can better be explained if we assume some broadening of the doping profile. In that case  $\mu_0$  and  $\mu_2$  should increase, whereas  $\mu_1$  should decrease. Also, as is shown in Ref. 34, multiple-scattering effects are important when multiple-scattering effects are important when  $n_{\text{2DEG}} < 10^{12} \text{ cm}^{-2}$ .

#### VI. CONCLUSION

Summarizing, we have calculated the low-temperature 2DEG mobility in  $\delta$ -doped and in addition, a weakly ptype GaAs, solving the Boltzmann equation in the relaxation-time approximation. We have shown that in contrast to the TF method and to the approach in which the screening by 2DEG is ignored, the RPA describes quite well the screening of the scattering potential.

Theoretical mobilities are in semiquantitative agreement with experimental data, so that further theoretical studies are needed, especially those which take into account the scattering from. charged acceptors as well as the broadening of the doping profile.

Calculated low-temperature mobility exhibits a drop when the Fermi level coincides with the bottom of the second excited subband. Such drop is negligible when the first excited subband becomes occupied. Detailed experimental investigations in order to verify these predictions would be very interesting.

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