Orbital spins of the collective excitations in Hall liquids

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We show that the Chern-Simons-Ginzburg-Landau theory of the quantum Hall effect needs a modification, because the order parameter in that theory carries an intrinsic orbital angular momentum. This quantum number contains additional information about the topological order of the Hall liquids. We propose to measure this angular momentum by circular-polarized Raman scattering.

It is now well known that the states of a two-dimensional electron gas that exhibit quantized Hall conductivity σ_{xy} are incompressible quantum liquids (the Hall liquids). It turns out that σ_{xy} alone is *insufficient* to specify a hall liquid. In order to do that one also needs to know the hierarchy scheme[1,2] by which the Hall liquid is constructed. Loosely speaking, a hierarchy is specified by a matrix[3] and σ_{xy} is only the (1,1) element of its inverse. The purpose of this paper is to answer the question "what kind of experiments bare information about the rest of the matrix?" We demonstrate that a quantum number called "orbital spin"[4] carried by all elementary excitations of a Hall liquid contains such information. In particular, we purpose to measure the orbital spins of the collective excitations by circular-polarized Raman scattering.

Before going into details, we first summarize our main results. (i) We show that the Chern-Simons-Ginzburg-Landau theory[5] of the quantum hall effect needs a modification because the order parameter[6] in that theory carries an intrinsic orbital angular momentum. This angular momentum (the orbital spin) is constrained to be locally perpendicular to the two-dimensional surface. There are some physical consequences due to the existence of the orbital spin. Some of them (e.g., the Berry phase[7] caused by the motion of orbital spins) are only effective on *curved* two-dimensional surfaces, and others (e.g., the selection rule) are applicable on *flat* surfaces as well. (ii) We calculate the values of the orbital spin carried by the elementary excitations of Hall liquids, and use those to explain features in the numerical spectra. To measure the orbital spin, (iii) we propose and predict the outcome of circular-polarized Raman scattering using the orbital spin selection rule.

To get a physical feeling for the orbital spin, let us think of the motion of an electron in magnetic field as the superposition of the fast cyclotron and slow guiding-center motions. If we concentrate on the slow degrees of freedom, we may think of the cyclotron motion as giving rise to an orbital spin to the guiding center. The value of the orbital spin is determined by the Landau-level index of the cyclotron motion.

It turns out that the orbital spin has direct consequences on the spectra of Hall liquids. In Fig. 1 we present the excitation spectra of the $\nu=\frac{1}{3}$ and $\frac{2}{5}$ Hall liquids. These results are obtained by diagonalizing the Hamiltonian describing a finite number of electrons on a sphere with Coulomb interaction. A careful examination of Fig. 1 reveals several differences between these two cases. (a) The minimum angular momenta of the collective mode are 2 in the $\frac{2}{5}$ liquid, and 1 in the $\frac{1}{3}$ liquid. (b) The collective mode of the $\frac{1}{3}$ liquid merges with the continuum at small angular momenta, while that associated with the $\frac{2}{5}$ liquid remains separated from the continuum. (c) At small angular momenta, the $\nu=\frac{1}{3}$ collective mode disperses downward while the $\nu=\frac{2}{5}$ mode disperses upward.[8]

First we discuss the origin of orbital spins. Following Ref. 5 we consider the following Chern-Simons-Ginzburg-Landau (CSGL) action for an ν (the filling factor)$=1/m$ (m is an odd integer) Hall liquid.

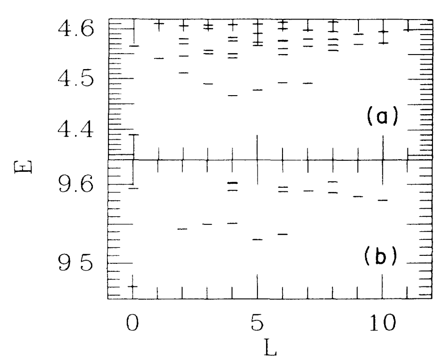

FIG. 1. The low-energy spectrums of (a) $N_e=7$ electrons at $\nu=\frac{1}{3}$ (i.e., $N_\phi=18$), and (b) $N_e=10$ electrons at $\nu=\frac{2}{5}$ (i.e., $N_\phi=21$) on a sphere (first 17 states). The interparticle interaction is Coulombic and the energy is measured in units of $e^2/\epsilon l_B$.

$$\mathcal{L}=\Phi^\dagger(\partial_0-iA_0+ia_0)\Phi-\frac{1}{2m_e}\Phi^\dagger(\partial_j-iA_j+ia_j)^2\Phi+\tfrac{1}{2}(\Phi^\dagger\Phi-\rho)V(\Phi^\dagger\Phi-\rho)-\frac{i}{4\pi m}\epsilon a\partial a\ , \qquad (1)$$

where Φ is the Girvin-MacDonald order parameter (and hence is a Bose field), V is a two-body potential (including a hard-core contribution), ρ is the average electron density, A_μ is the external electromagnetic gauge field, and a_μ is the statistical gauge field. In (1) and the rest of the paper $\epsilon\alpha\partial\beta=\epsilon^{\mu\nu\lambda}\alpha_\mu\partial_\nu\beta_\lambda$. The last term in (1), the Chern-Simons term, ensures that m flux quanta of the statistical gauge field are attached to each Φ boson (the condensate boson) and hence transmutes it back into the original electron. Let $J_\mu=\sum_k j_{k\mu}$ [where $j_{k0}=\delta(\mathbf{x}-\mathbf{x}_k)$, and $\mathbf{j}_k=\dot{\mathbf{x}}_k\delta(\mathbf{x}-\mathbf{x}_k)$] be the Bose three-current, then the term

$$\mathcal{L}_{\rm link}=\frac{i}{2}(2\pi m)\sum_{k\neq k'}\epsilon j_k\frac{1}{\partial^2}\partial j_{k'}+\frac{i}{2}(2\pi m)\sum_k\epsilon j_k\frac{1}{\partial^2}\partial j_k \qquad (2)$$

is generated after integrating out a_μ under the gauge $\partial\cdot a=\partial^\mu a_\mu=0$. In (2) $1/\partial^2=1/\partial^\mu\partial_\mu$. Equation (2) contains two terms. The first, mutual-interaction term gives a phase factor $\exp(-im\pi)$ whenever two bosons braid around each other. As the result, the statistics of the Φ boson is transmuted back to that of a fermion. The second, self-interaction term causes the coupling of the electronic motion to curvature upon using the following identity[9]

$$2\pi\epsilon j_k\frac{1}{\partial^2}\partial j_k=j_k\cdot\omega\ . \qquad (3)$$

In the above $\omega_0=0$ and $\epsilon_{ij}\partial_i\omega_j$ is the curvature of space.[4] In order to remove this self-interaction effect, a counter term must be added to (1) so as to cancel the second term in Eq. (2).

The new CSGL action which includes this counter term is given by

$$\mathcal{L}=\Phi^\dagger\left[\partial_0-iA_0+ia_0-i\frac{m}{2}\omega_0\right]\Phi-\frac{1}{2m_e}\Phi^\dagger\left[\partial_j-iA_j+ia_j-i\frac{m}{2}\omega_j\right]^2\Phi+\tfrac{1}{2}(\Phi^\dagger\Phi-\rho)V(\Phi^\dagger\Phi-\rho)-\frac{i}{4\pi m}\epsilon a\partial a\ . \qquad (4)$$

Since the (two-dimensional) space curvature is given by $\partial_1\omega_2-\partial_2\omega_1$, in flat space ω_μ is a trivial gauge and has no effect. In a curved space (e.g., a sphere), local curvature simulates the effect of external magnetic-flux density and causes a shift in the magnetic-flux density.

A straightforward generalization of the duality transformation[10] to allow for the additional flux due to the space curvature gives the following effective theory for the $\nu=1/m$ quantum Hall liquid in curved space:[4]

$$\mathcal{L}=\frac{i}{4\pi}[m\epsilon b\partial b-\epsilon(2A+m\omega)\partial b]-i\mathcal{J}\cdot b\ . \qquad (5)$$

In the above, b_μ is related to the particle three current J_μ via $J^\mu=(1/2\pi)\epsilon^{\mu\nu\lambda}\partial_\nu b_\lambda$, and, $\mathcal{J}_\mu$ is the vortex three current. If the space is a two-sphere one can show that by setting $\partial\mathcal{L}/\partial b_0=0$, when $N_\phi+m=mN_e$ (where N_e is the total particle number N_e, and N_ϕ is the total magnetic flux-quantum number), $\mathcal{J}_\mu=0$ and hence the system is incompressible. Therefore as a result of the coupling to the space curvature, a shift[4] $\Delta N_\phi=m$ in N_ϕ for the appearance of a $\nu=1/m$ Hall liquid is resulted.[4]

Using (5) we can also determine orbital-spins of the quasiparticles and quasiholes. By integrating out b_μ in (5) under the gauge $\partial\cdot b=0$ we obtain the following quasiparticle-quasihole effective action:

$$\mathcal{L}=-\frac{i}{4\pi m}\epsilon A\partial A-\frac{i}{2m}(2\pi)\epsilon\mathcal{J}\frac{1}{\partial^2}\partial\mathcal{J}-\frac{i}{m}\mathcal{J}\cdot A-\frac{i}{2}\mathcal{J}\cdot\omega\ . \qquad (6)$$

By substituting $\mathcal{J}_\mu=qj_\mu$ [where $j_\mu\equiv(\delta(\mathbf{x}-\mathbf{x}_0),\dot{\mathbf{x}}\delta(\mathbf{x}-\mathbf{x}_0))$] into (6) and use (3) we obtain the quasiparticle ($q=1$) and quasihole ($q=-1$) orbital spins as $S_{\rm qp}=\frac{1}{2}+1/2m$, and $S_{\rm qh}=-\frac{1}{2}+1/2m$.

When $m=1$ (i.e., $\nu=1$) $S_{\rm qp}=1$ and $S_{qh}=0$. Therefore, a quasihole may not be viewed as the antiparticle of a quasielectron. This is due to the fact that while a quasihole is a missing electron in the *first* Landau level, a quasielectron is an excess electron in the *second* Landau level. The extra orbital spin carried by the quasielectrons reflects the fact that the guiding-center basis for the second Landau level are that of the first Landau level multiplied by $\bar{z}-\bar{z}_g$ (where z_g is the complex coordinate of the guiding center). Similarly, we can calculate the minimum orbital spin associated with a pair of quasielectrons (quasiholes) by letting $\mathcal{J}_\mu=2j_\mu$ ($\mathcal{J}_\mu=-2j_\mu$) in (6). The result is $S'_{\rm qp}=1+2/m=3$ and $S'_{\rm qh}=-1+2/m=1$. The fact that $S'_{\rm qp,qh}\neq2S_{\rm qp,qh}$ reflects the fact that when two quasielectrons (quasiholes) are put into the second (first) Landau level the Pauli exclusion principle forces them to have a minimum relative angular momentum 1 (-1). Therefore after adding the orbital spin quantum number, the effective theory is enriched to contain the Landau level structure and exclusion principle.

We now consider a $\nu=2/(2m-1)$ (m is odd) Hall liquid. Let us first calculate ΔN_ϕ. As a second-level hierarchical state, the $\nu=2/(2m-1)$ Hall liquid is a Laughlin condensate of the $1/m$ quasiparticles on top of the $1/m$ QH state. After adding the appropriate counter terms at both levels of the hierarchy, it is straightforward to generalize the derivation of (5) to obtain[4]

$$\mathcal{L}=\frac{i}{4\pi}[K_{n'n}\epsilon b_{n'}\partial b_n-\epsilon(2t_nA+2S_n\omega)\partial b_n]-i\mathcal{J}_n\cdot b_n\ , \qquad (7)$$

where n is the level index, and $K_{n'n}$, t_n, and S_n are given by

$$K=\begin{bmatrix} m & -1\\ -1 & 2\end{bmatrix},\quad t=\begin{bmatrix}1\\0\end{bmatrix}\quad\text{and } S=\begin{bmatrix}m/2\\1\end{bmatrix}.$$

By setting $\partial\mathcal{L}/\partial b_{n0}=0$, we obtain $(\mathcal{J}_n)_\mu=0$ and hence an incompressible ground state when $N_\phi + m + 1 = (m - \frac{1}{2})N_e$. Therefore $\Delta N_\phi = m + 1$.

We now calculate the orbital spins of quasiparticle and quasihole. By integrating out $(b_n)_\mu$ in (7) under the gauge $\partial \cdot b_n = 0$, we obtain the following quasiparticle-quasihole effective action:

$$\mathcal{L} = -\frac{i}{4\pi}\frac{2}{2m-1}\epsilon A \partial A - i t'_n \mathcal{J}_n \cdot A - i S'_n \mathcal{J}_n \cdot \omega - i(2\pi) K'_{mn} \epsilon \mathcal{J}_m \frac{1}{\partial^2} \partial \mathcal{J}_n \ . \tag{8}$$

In (8), K'_{mn}, t'_n, and S'_n are given by

$$K' = K^{-1} = \frac{1}{2m-1}\begin{bmatrix} 2 & 1 \\ 1 & m \end{bmatrix}, \quad t' = \frac{1}{2m-1}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

and

$$S' = \frac{1}{2m-1}\begin{bmatrix} m+1 \\ 3m/2 \end{bmatrix} .$$

Using (3) we calculate the orbital spins of the quasiparticle-quasihole created by letting $((\mathcal{J}_1)_\mu, (\mathcal{J}_2)_\mu) = (qj_\mu, 0)$ (with $q = \pm 1$), and the result is $S_{1,\text{qp}} = (m+2)/(2m-1)$ and $S_{1,\text{qh}} = -m/(2m-1)$. Similarly we obtain the orbital spins of the quasiparticle-quasihole created by letting $((\mathcal{J}_1)_\mu, (\mathcal{J}_2)_\mu) = (0, qj_\mu)$ to be $S_{2,\text{qp}} = 2m/(2m-1)$ and $S_{2,\text{qh}} = -m/(2m-1)$. These two types of quasihole-quasiparticle are induced by inserting a unit vortex-antivortex in the first- and second-level condensate, respectively.

The collective excitations in the $\nu = 2/(2m-1)$ Hall liquid are in general made up of q_1 level-1 and q_2 level-2 vortices, respectively. Unlike in a primary Hall liquid where spatial-coinciding vortex and antivortex annihilate, in a hierarchy Hall liquid vortex and antivortex from different levels can overlap without annihilating. In particular, by letting $((\mathcal{J}_1)_\mu, (\mathcal{J}_2)_\mu) = (q_1 j_\mu, q_2 j_\mu)$ (therefore q_1 level-1 and q_2 level-2 vortices are sitting on top of each other), and using (3), we obtain the following charge and the orbital spin:

$$Q(q_1, q_2) = \frac{1}{2m-1}(2q_1 + q_2) ,$$
$$S(q_1, q_2) = \frac{1}{2m-1}[(m+1)q_1 + 3mq_2 + q_1^2 + mq_2^2 + q_1 q_2] . \tag{9}$$

To understand the physical meaning of the different excitations labeled by (q_1, q_2), let us consider the case where $m = 1$ (i.e., $\nu = 2$). The excitations labeled by $(q_1, q_2) = (0, -1)$ and $(-1, 1)$ carry charge $Q = -1$ and orbital spin $S = -1$ and 0, therefore they correspond to a hole in the second and the first Landau level, respectively. Similarly, the excitation $(q_1, q_2) = (0, 1)$ carries charge $Q = 1$ and orbital spin $S = 2$, hence corresponds to an electron in the third Landau level. Finally, the excitation $(q_1, q_2) = (1, -1)$ with $Q = 1$ and $S = 1$ has the quantum numbers of an electron in the second Landau level. However, due to the Pauli principle, such an excitation is not allowed since in the ground state the second Landau level is already full. This discussion taught us an important lesson, namely, not all (q_1, q_2) correspond to allowed excitations. Indeed, when using the effective theory to describe excitations in a Hall liquid, we have to add a supplementary rule that when an excitation contains vortices that have already condensed, it should be disallowed (due, in integer QH effect, to the Pauli exclusion principle, and, in fraction QH effect, to energetic considerations). We need to keep this point close in mind when using the effective theory to describe excitations in Hall liquids.

For general m, among the various (q_1, q_2) pairs, neutral excitations ($Q = 0$) arise when $q_2 = -2q_1$. The orbital spin associated with these $(q_1, q_2) = (n, -2n)$ excitation is $S = n(n-1)$. Among the allowed excitations (i.e., $n < 0$ according to the supplementary rule), the excitation$(q_1, q_2) = (-1, 2) = (-1, 1) + (0, 1)$ carries the minimum orbital spin $S = 2$. In the $m = 1$ (i.e., $\nu = 2$) case, this excitation corresponds to a hole in the first and an electron in the third Landau level. In the $m = 3$ (i.e., $\nu = \frac{2}{5}$) case, we identify it with the collective mode in the gap of the $\nu = \frac{2}{5}$ spectrum. Finally, in the effective theory the low-momentum cyclotron mode is described by the Gaussian fluctuation in the $b_{1,\mu}$ gauge field.

Since the $(-1, 2)$ excitation can be viewed as an overlapping $(-1, 1)$ quasihole and $(0,1)$ quasiparticle, a natural question is what happens when we pull the $(-1, 1)$ and $(0,1)$ vortices apart. Let $\mathbf{d}$ be the relative displacement, it is simple to show that for small $|\mathbf{d}|$ the following change in the Lagrangian is induced:

$$\Delta\mathcal{L} = \tfrac{1}{2}K|\mathbf{d}|^2 + iQH_0 \epsilon_{ij} d_i \dot{R}_j \ . \tag{10}$$

In (10), $\mathbf{R}$ is the position vector associated with the center of mass of the dipole, $Q = 1/(2m-1)$ is the charge of the $(0,1)$ quasiparticle, H_0 is the strength of the external magnetic-flux density, and K is an effective spring constant. The first term in (10) describes the change in Coulomb energy caused by the relative displacement, and the second term is the associated Aharonov-Bohm phase. To calculate the partition function we integrate over both $\mathbf{R}$ and $\mathbf{d}$. After $\mathbf{d}$ is integrated over a term of the form $\mathcal{L} = \frac{1}{2}[(QH_0)^2/K]|\dot{\mathbf{R}}|^2$ is generated, as the result the total action describing the center-of-mass motion of the dipole is

$$\mathcal{L} = \frac{M}{2}|\dot{\mathbf{R}}|^2 - iS\dot{\mathbf{R}} \cdot \omega , \tag{11}$$

where $M = (QH_0)^2/K$, and $S = 2$ is the orbital spin of the $(-1, 2)$ excitation. The second term describes the coupling between the orbital spin and the curvature. On a sphere where $\int d^2x(\partial_1\omega_2 - \partial_2\omega_1) = 2$, (11) describes a particle with effective mass M and charge $S(=2)$ moving under the influence of a monopole carrying two Dirac flux quanta sitting at the spherical center. The effective magnetic field seen by the particle is $H_{\text{eff}} = 1/R^2$. The spectrum associated with (11) has $E = \frac{1}{2}\hbar\omega_c[L(L+1) - 4]/2$, where $L \geq S = 2$ and $\omega_c = 2H_{\text{eff}}/M$ is the effective cyclotron frequency. This behavior is consistent with the low-L behavior of the collective mode shown in Fig. 1(b). The maximum angular momentum of the $(-1,2)$ branch is realized when the $(-1,1)$ quasihole and the $(0,1)$ quasiparticle are diametrically opposite to each other. In that case the total Berry's phase seen by this largest dipole upon rotation is given by

$$\frac{1}{2m-1}N_\phi+2(S(0,1)-S(1,1))$$
$$=\frac{2}{2m-1}\left[\frac{2m-1}{2}N_e-(m+1)\right]$$
$$+2\left[\frac{2m}{2m-1}-\frac{m-1}{2m-1}\right]=N_e\ .$$

As the result, $L_{\max}=N_e/2$. Therefore if this mode were to persist for the entire range of angular momenta, it should end at $L=N_e2$. The fact that $L_{\max}=N_e/2+1$ in Fig. 1(b) indicates that near $L_{\max}$ the collective mode is derived from the $(0,1)$ quasiparticle and $(0,-1)$ quasihole pair, whose maximum angular momentum is precisely $N_e/2+1$. Combining these facts, we conclude that at sufficiently large angular momenta, the $(-1,2)$ mode crosses with the collective mode made up of the level-2 vortex-antivortex multipole,[11] at which point the energy disperses downward as L further increases and eventually reaches the roton minimum.

Similar calculations can be done for the $\nu=\frac{2}{7}$ QH liquid. To summarize the results we find $\Delta N_\phi=2$, $S_{1,\mathrm{qp}}=\frac{3}{7}$, $S_{1,\mathrm{qh}}=-\frac{1}{7}$, $S_{2,\mathrm{qp}}=\frac{3}{7}$, $S_{2,\mathrm{qh}}=-\frac{6}{7}$. The neutral excitations made up of spatial coinciding vortices have $(q_1,q_2)=(n,-2n)$ with the associated orbital spin given by $S=-n(n+1)$. given the fact the $\nu=\frac{2}{7}$ Hall liquid is a condensate of the $\nu=\frac{1}{3}$ quasiholes, the supplementary rule implies that the allowed excitations corresponds to $n>0$. Among them, the minimum orbital-spin excitation has $(q_1,q_2)=(1,-2)$ and $S=-2$. In principle, the $(1,-2)$ excitation, like the $(-1,2)$ excitation in the $\nu=\frac{2}{5}$ liquid, is allowed. However, given the fact that the $\frac{1}{3}$ quasiparticle usually has much higher creation energy than the $\frac{1}{3}$ quasihole, and the fact that the $(1,-2)$ excitation contains a $\frac{1}{3}$ quasiparticle, we suspect it to have an excitation energy larger than twice the roton gap, and hence lies inside the continuum. Indeed, the numerical spectrum of the $\frac{2}{7}$ liquid shows no traces of this excitation in the gap.

The orbital spin of the $(-1,2)$ collective mode in the $\frac{2}{5}$ state can, in principle, be detected by circular-polarized Raman scattering. In this experiment under proper orientation of the magnetic field, a right-hand polarized light beam incident normal to the two-dimensional electron gas excites the $(-1,2)$ collective mode. As the result, the polarization of photon changes to left handed for forward scattering, and right handed for backward scattering. To be specific, the differential probability dP of an incident photon gets scattered into a solid angle $d\Omega$ is given by

$$dP=\gamma^2\alpha^2(l_Bk)^2|(\epsilon_{\mathrm{in},x}+i\epsilon_{\mathrm{in},y})(\epsilon^*_{\mathrm{out},x}+i\epsilon^*_{\mathrm{out},y})|^2d\Omega\ , \tag{12}$$

where k is the norm of the wave vector of light, ϵ_{in} (ϵ_{out}), the polarization vector of the incoming (outgoing) beam, $\alpha=\frac{1}{137}$ is the fine-structure constant, andγ is a dimensionless constant of order unity. In deriving (12) we have assumed that the energy of the photon is much larger than the excitation energy of the collective mode.

The $\nu=\frac{3}{5}$ state also has a well-defined collective mode near $k=0$ due to the particle-hole symmetry. However, the orbital spin of this mode is opposite to that of the $\frac{2}{5}$ mode, as one can show from direct calculation or from particle-hole conjugation. The $\frac{3}{5}$ mode will have an opposite effect on the Raman scattering, e.g., for the same orientation of the magnetic field, the $\frac{3}{5}$ mode will change the left-hand polarized light to the right-hand one in the forward scattering.

Finally, an interesting lesson about Jain's[12] mapping from the integer to the fractional QH effect can be learned from this work. According to this mapping, the ground-state wave function at $\nu=\frac{2}{5}$ is equal to that at $\nu=2$ operated upon by the Landau-level projection operator. If one generalizes this idea to include the excited states, one finds the following puzzling situation. Under such mapping, the $(-1,2)$ collective mode corresponds to the first to third (1–3) Landau-level particle-hole excitation of the $\nu=2$ liquid. From the Landau-level point of view it is very unnatural to see the 1–3 (instead of 2–3) excitation to have the lowest energy. The resolution of this puzzle comes with the realization of the effects of Landau-level projection. The projection, in addition to entirely annihilates the 2–3 excitation at $L=1$,[13] modifies its properties qualitatively at small angular momenta. This constitutes an example in which the straightforward correspondence between the integer and fractional quantum Hall effect is spoiled by the Landau-level projection.

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[1]F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983); B. I. Halperin, *ibid.* **52**, 1583 (1984).

[2]S. Girvin, Phys. Rev. B **29**, 6012 (1984); A. H. MacDonald and D. B. Murray, *ibid.* **32**, 2707 (1985); D-H. Lee and M. P. A. Fisher, Phys. Rev. Lett. **63**, 903 (1989); J. K. Jain, *ibid.* **63**, 199 (1989); Phys. Rev. B **41**, 7653 (1991).

[3]B. Blok and X. G. Wen, Phys. Rev. B **42**, 8133 (1990); **42**, 8145 (1990); N. Read, Phys. Rev. Lett. **65**, 1502 (1990); J. Fröhlich and A. Zee, Nucl. Phys. B **364**, 517 (1991); Z. F. Ezawa and A. Iwazaki, Phys. Rev. B **43**, 2637 (1991); X. G. Wen and A. Zee, *ibid.* **46**, 2290 (1992).

[4]X. G. Wen and A. Zee, Phys. Rev. Lett. **69**, 953 (1992).

[5]S. C. Zhang, T. H. Hansson, and S. Kivelson, Phys. Rev. Lett. **62**, 82 (1989).

[6]S. M. Girvin and A. H. MacDonald, Phys. Rev. Lett. **58**, 1252 (1987).

[7]M. V. Berry, Proc. R. Soc. London Ser. A **392**, 45 (1984).

[8]The spectrum of the $\frac{2}{5}$ state on a torus was obtained by S. P. Su and F. Y. Wu, Phys. Rev. B **36**, 7565 (1987). Their results give a collective mode dispersion that shows double minima and disperses downward at small momenta. We do not understand the origin of the spectral difference between sphere and torus at present.

[9]A. M. Polyakov, Mod. Phys. Lett. A **3**, 325 (1988); F. Wilczek and A. Zee, Phys. Rev. Lett. **51**, 2250 (1983).

[10]D-H. Lee and M. P. A. Fisher, Int. J. Mod. Phys. B **5**, 2675, 1991.

[11]D-H. Lee and S-C. Zhang, Phys. Rev. Lett. **66**, 1220 (1991).

[12]J. K. Jain (Ref. 2).

[13]G. Dev and J. K. Jain (unpublished). A qualitative similar conclusion was also reached in B. I. Halperin, P. A. Lee, and N. Read (unpublished); A. Lopez and E. Fradkin (unpublished).