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Peak values of resistivity in high-mobility quantum-Hall-effect samples

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Shubnikov–de Haas oscillations have been studied, at low temperatures, in a high-mobility quantum-Hall-effect sample. A linear increase with increasing magnetic field of the peak values of resistance is observed and is explained quantitatively by a quantum diffusion model with predominantly small-angle scattering. The results are inconsistent with recent suggestions that the peak values of conductivity, between integer-quantum-Hall-effect plateaus, are $e^2/2h$.

Despite a wealth of experimental results a detailed and quantitative understanding of the amplitude of the Shubnikiov–de Haas (SdH) oscillations in quantum Hall systems has not yet been achieved. This is particularly the case for the resistivity between integer (IQHE) and fractional quantum-Hall-effect (FQHE) plateaus. For example, there is, as yet, no explanation for the observation by Chang and Tsui[1] and Stormer *et al.*[2] that in high-mobility samples there is, for all FQHE features, an empirical relationship between the diagonal resistivity (ρ_{xx}) and the Hall resistivity (ρ_{xy}). Also, a matter of some debate at present is the suggestion by Clark *et al.*[3] and Lee *et al.*[4] that the peak value of conductivity is $e^2/2h$ between IQHE plateaus and $e^{*2}/2h$ between FQHE plateaus with $e^* = e/q$ for the p/q fractions.

The peak values of conductivity (σ_{xx}) were predicted by Ando and Uemura,[5] for two-dimensional systems with short-range scatterers, to vary as $N_L + \frac{1}{2}$ where N_L is the Landau level at the Fermi level. For a system with constant density this corresponds to peak conductivities that vary as $1/B$. In high-mobility systems $\rho_{xy} \gg \rho_{xx}$ so $\rho_{xx} \approx \sigma_{xx}\rho_{xy}^2$. Also in these systems the quantum Hall plateaus are observed, experimentally, to be narrow so ρ_{xy} is quite accurately proportional to B. The peak values of ρ_{xx} are then predicted to vary as $1/(N_L + \frac{1}{2})$, i.e., to be proportional to B. Specifically,

$$\rho_{xx}^{pk} = (g_s/\pi)(h/e^2)1/(2N_L + 1) , \tag{1}$$

where g_s, indicating the spin degeneracy, is 1 or 2 depending on whether the spin splitting is resolved or unresolved and the factor $1/\pi$ comes from the assumption of a Lorentzian shape for the Landau levels.

Experimentally, in $GaAs/Ga_{1-x}Al_xAs$-based heterojunctions, the absolute magnitudes of the peak resistivities are at least an order of magnitude smaller than these values. Figure 1 shows the low-field SdH oscillations, measured at low temperatures (~ 30 mK), in a high-mobility (280 $m^2/V\,s$) sample. This is a conventional $GaAs$-$Al_xGa_{1-x}As$ two-dimensional electron gas (2DEG), grown at NRC, with a spacer layer of 40 nm and has a density after illumination with red light of 3.4×10^{15} m^{-2}. After an initial exponential increase in amplitude [cf. Eq. (3)] the peak values of resistance increase linearly with magnetic field and decrease by a factor of 2 when spin splitting becomes resolved. This is shown more clearly in Fig. 2 where the peak values of resistivity multiplied by the filling factor ν are plotted against ν. The dotted lines, which are the result of the theoretical estimate discussed below, represent a linear increase of a factor of about 15 times smaller than the predictions of Eq. (1). The general reduction in peak amplitude was first explained by Ando and Uemura[5] as being due to the long-range nature of the scattering potential in these samples but, for long-range scatterers, their calculations do not explain the rather precise linear increase with B that is observed experimentally.

A linear increase of ρ_{xx} is also implied by the data of Chang and Tsui[1] and Stormer *et al.*,[2] who pointed out the empirical relationship that exists for high-mobility samples, i.e.,

$$\beta\rho_{xx} = B\,\partial\rho_{xy}/\partial B , \tag{2}$$

where the constant β varies from sample to sample but is

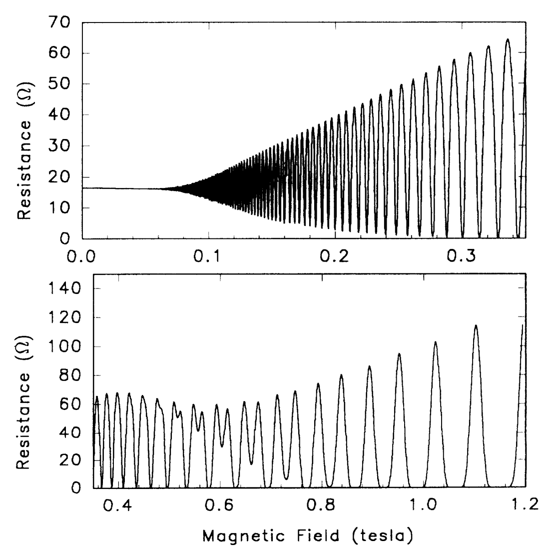

FIG. 1. Low-field Shubnikov–de Haas oscillations measured at approximately 30 mK. The density is 3.4×10^{15} m^{-2} and mobility 280 $m^2/V\,s$. The Hall bar used for the measurements had a width of 300 μm with the separation of 750 μm between the voltage probes.

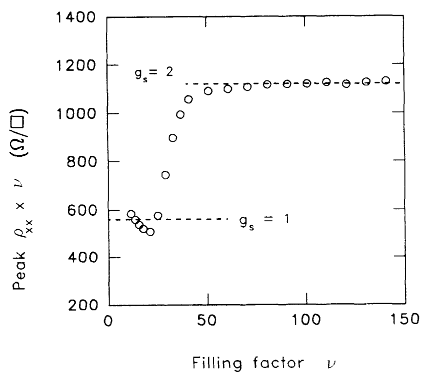

FIG. 2. Peak values of resistivity, from the data in Fig. 1, multiplied by the filling factor ν and plotted against ν. The dotted lines are theoretical estimates from Eq. (6) using the value of μ_q taken from Fig. 3 and with g_s equal to 1 or 2 according to whether the spin splitting is resolved or unresolved.

typically between 20 and 40. The emphasis in this work was placed on the way this expression reflects all the FQHE features. However, in the IQHE regime, where the plateaus are narrow, $B\partial\rho_{xy}/\partial B$ between plateaus is approximately equal to ρ_{xy} and this relationship then describes peak values of ρ_{xx} which again vary linearly with B but which are smaller, by a factor of π/β, than the predictions of Eq. (1).

In low magnetic fields, at low temperatures, the amplitude ($\Delta\rho$) of the SdH oscillations is well described by the expression[6,7]

$$\Delta\rho/\rho_0 = 4\exp(-\pi/\omega_c\tau_q) = 4\exp(-\pi/\mu_q B) , \qquad (3)$$

where ω_c is the cyclotron frequency, ρ_0 the resistivity at zero field, τ_q a quantum lifetime, and μ_q the corresponding quantum mobility. Because small-angle scattering dominates[8,9] τ_q is significantly shorter than the corresponding transport lifetime τ_{tr}. Figure 3 shows a Dingle plot for the oscillations shown in Fig. 1 and there is good agreement with Eq. (3). In particular, and in agreement with data in many lower-mobility samples,[7,8] the prefactor of 4 is obtained correctly. The quantum mobility $\mu_q = 12.3$ m^2/V s and the ratio τ_0/τ_q (with τ_0 the zero field value of τ_{tr}) is 23 confirming the small-angle nature of the scattering. At higher magnetic fields this behavior breaks down and, as described above, a linear increase with B of the resistance maxima is observed. This linear dependence is explained below, quantitatively, in terms of the same parameter τ_q which characterizes the disorder at low fields.

The usual expression for the conductivity in a magnetic field can be written in terms of a diffusion process as

$$\sigma_{xx} = ne^2\tau_{tr}/m^*(1+\omega_c^2\tau_{tr}^2) = e^2 D^* g(E_F) , \qquad (4)$$

where $g(E_F)$ is the density of states at the Fermi level. In zero magnetic field the diffusion coefficient D^* is just $\lambda^2/2\tau_0$ where λ is the mean free path and τ_0 the zero-field value of the transport scattering time τ_{tr}. In magnetic fields such that $\omega_c\tau_{tr} \gg 1$, i.e., under normal experimental conditions, $D^* = R_{cycl}^2/2\tau_{tr}$ where R_{cycl} is the cyclotron radius. This expression can be interpreted, following

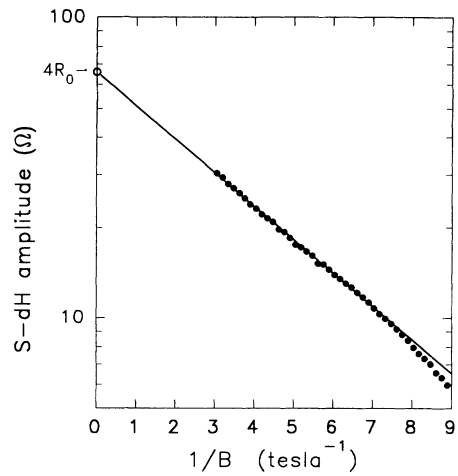

FIG. 3. Dingle plot for the low-field Shubnikov–de Haas oscillations shown in Fig. 1. The straight line is a fit to Eq. (3) with a slope corresponding to $\mu_q = 12.3$ m^2/V s. The deviation from linearity at low fields is attributed to macroscopic dephasing associated with an inhomogeneity of order 1% along the length of the Hall bar. At high fields ($1/B$ less than 3 T^{-1}) the effects of spin splitting become important.

Ando and Uemura,[5] as a process of "quantum diffusion" corresponding to hopping between orbit centers with a mean free path given by R_{cycl}. When the scattering rate $1/\tau_{tr}$ is proportional to the density of states[6,7] the conductivity (and the resistivity ρ_{xx}) are proportional to $[g(E_F)]^2$. This does not *require* that Δg, the deviation of the density of states from the zero-field value, be small but when Δg *is* small Eq. (3) can be derived from this expression. Observation experimentally of the prefactor of 4 in Eq. (3) can therefore be considered as confirmation that $1/\tau_{tr}$ is, indeed, proportional to the density of states;[6] a prefactor of 2 would result if τ_{tr} was independent of $g(E_F)$.

When Δg is not small the density of states must be determined self-consistently and then Eq. (3) may not be valid. The peak heights of separated Landau levels are inversely proportional to the width; but the width is determined by the total scattering rate which is itself dependent on the final density of states. This problem was first treated[5,10] using the self-consistent Born approximation and has recently been extended beyond that approximation, for high Landau levels in smooth, random potentials, by Raikh and Shahbazyan.[11] When the width of the Landau levels is less than the separation ($\hbar\omega_c$) they find, in agreement with earlier results,[10] that the density of states can be described by a sum of Gaussians with a width Γ given by

$$\Gamma^2 = \hbar^2\omega_c/2\pi\tau_q , \qquad (5)$$

where τ_q is defined to be consistent with Eq. (3).

Using this expression for the density of states in Eq. (4), $D^* = R_{cycl}^2/2\tau_{tr}$ and the assumption that $1/\tau_{tr}$ is proportional to $g(E_F)$, the peak values of resistivity increase linearly with B, and are given by

$$\rho_{xx}^{pk} = g_s\tfrac{1}{2}(B/ne)(\tau_q/\tau_0) . \qquad (6)$$

Here τ_q and τ_0 are both explicitly independent of magnetic field. For spin-unresolved peaks and $\tau_q=\tau_0$ this expression reduces to Eq. (1) within a factor $\pi/2$ which appears because the line shape is Gaussian rather than Lorentzian.

The values ρ_{xx}^{pk} deduced from Eq. (6), using the value of τ_q/τ_0 obtained from the low-field data (Fig. 3), are shown as dotted lines in Fig. 2. It can be seen they provide good, *quantitative,* agreement with experiment. Also, for the data in Ref. 2, it can be seen that between the IQHE plateaus $\partial\rho_{xy}/\partial B \approx \rho_{xy}/B$, i.e., $\beta\rho_{xx}^{pk}\approx\rho_{xy}$. Under the same conditions Eq. (6) can be written as $\rho_{xx}^{pk}=\frac{1}{2}(\tau_q/\tau_0)\rho_{xy}$ so the coefficient β can be identified as $2\tau_0/\tau_q$. Experimentally observed values of β between 20 and 40 are consistent with measured and calculated values for the ratio τ_0/τ_q that are in the literature[7–9,12–14] and, indeed, this provides an alternative means of estimating τ_q. It should also be noted that in Ref. 2 the IQHE "peaks" are actually distorted into extended regions where the resistivity increases linearly with magnetic field. This is entirely consistent with Eq. (6) provided the top of the Landau levels are essentially flat. The equivalence between Eqs. (2) and (6) for IQHE peaks is also trivially true on plateaus. However, a general explanation of Eq. (2), for all the FQHE features, has not been obtained although the argument could be extended, within the composite particle picture,[15] to FQHE peaks and plateaus. The essential requirement is that the width of the Landau levels and the scattering rates $1/\tau_{tr}$, respond in the same way to changes in $g(E_F)$.

The linear field dependence can be seen in other experimental data in the literature (see, e.g., Ref. 14) but it is not universally observed. To be obvious it requires samples with sufficiently low disorder that the Landau levels are well separated without localization effects being important and, indeed, a linear dependence is often more apparent at higher temperatures when the shorter inelastic mean free path suppresses localization effects.[16] The magnitude of the linear term is proportional to τ_q/τ_0; this ratio may be as small[13] as 0.01, compared to 0.04 here, which makes the linear behavior less obvious. For a linear dependence to be clearly visible the Landau levels must all have essentially the same shape; this is not the case when the two spin states are differentially scattered. Also, the linear behavior is not seen in experimental configurations such that edge state, rather than bulk, conductivity is being measured.[17]

The quantitative explanation of the peak resistivities relies on (a) a transport scattering rate proportional to the density of states and (b) Gaussian-shaped Landau levels with a width varying as $B^{1/2}$. However, this particular model for the Landau-level structure is not needed to get a linear B dependence, only to determine the exact constant of proportionality. The linear B dependence comes, not from the detailed shape of the Landau level, but from the degeneracy factor ($1/2\pi l_m^2$, where l_m is the magnetic length), in the density of states. The essential requirement is that the width of the Landau levels and the transport scattering rate ($1/\tau_{tr}$) have the same dependence on magnetic field. This is the case for long-range scattering potentials because then the integrals involved in calculating the Landau-level width and τ_{tr} can be evaluated in the asymptotic limit.[11] This limit is valid when the cyclotron radius is much larger than the correlation length for the scattering potential. For the sample used here the cyclotron radius is about 100 nm at 1 T and the correlation length, equal to twice the spacer thickness, is 80 nm and the asymptotic limit is approximately satisfied over most of the field range.

It has been observed[3] that in activation plots for the conductivity minima the intercepts are very often close to e^2/h and this has been interpreted[3,4] as evidence that the peak conductivity of the adjacent maxima is $e^2/2h$. This is, of course, inconsistent with the data and explanation presented above. Between 1 and 2 K, the sample measured here also shows activated behavior with intercepts, for ν between 12 and 24, all approximately $0.5e^2/h$. Comparison with Fig. 1 shows that for these fields (0.6–1.2 T), at low temperatures, spin splitting is only just resolved. As the temperature is raised the spin splitting is suppressed and, at 2 K, it has almost completely vanished. The temperature dependence of the conductivity, in this field and temperature range, is therefore determined as much by the spin splitting as by the basic activation process and the intercepts are only coincidentally close to e^2/h. The peak conductivities vary essentially as $1/B$ but with a factor of 2 change when the spin splitting becomes resolved.

Experimental results are presented which show peak values of resistivity in a high-mobility 2DEG that increase linearly with magnetic field. The results are explained quantitatively in terms of a quantum diffusion model using one, independently determined, experimental parameter. Although the results are consistent with Gaussian-shaped Landau levels, with widths varying as $B^{1/2}$, a less stringent requirement, that the transport relaxation time and the quantum lifetime have the same dependence on magnetic field also explains the linear increase. This is reasonable for the long-range potentials which dominate the scattering in high mobility 2DEG's. The explanation goes part way to explaining the proportionality between ρ_{xx} and $B\partial\rho_{xy}/\partial B$ that is observed in high-mobility samples but, with the experimental data, is inconsistent with universal values for the peak conductivity between integer quantum Hall plateaus.

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