Magnetophonon resonances of quantum wires in tilted magnetic fields

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The magnetoconductivity of parabolic quantum wires in strong tilted magnetic fields, associated with the magnetophonon resonance effect, is obtained analytically for optical-phonon scattering, by taking the linear-response limit of the results of the nonlinear-response theory developed previously. Neglecting the coupling Hamiltonian term $\sim B_x B_z xz$, since its contribution to the total electron energy is minor, the results for the magnetophonon resonance effect display two different periods of oscillation as a function of the strength and direction of the magnetic field and the confinement frequencies. In particular, it is shown that the direction of the magnetic field plays a significant role in determining the relaxation rates and the magnetoconductivity related to the magnetophonon resonances.

I. INTRODUCTION

Over the past few decades, there has been considerable interest[1-9] in understanding the magnetophonon resonance (MPR) effect in low-dimensional electron-gas systems, since it provides useful information on the relative transport properties of semiconductors, such as the carrier relaxation mechanism, damping of the oscillations due to the electron-phonon interaction, the phonon frequencies, and band structure (i.e., the effective mass m^*). So, some studies[6,7,9] have been made on the MPR effect of quasi-one-dimensional (Q1D) quantum-wire structures, but their analysis has been mainly confined to the case where the magnetic field is applied in the direction normal to the interface layer of the systems. We see that, in this case, one kind of the Landau-level index is formed and the MPR effect arises from the resonant scattering of electrons quantized in Landau levels by phonons. However, if the magnetic field is tilted with respect to the normal, it serves to add an extra confining potential to the initial confinement, gives rise to two different kinds of Landau-level indices, and causes a dramatic change in the energy spectrum, leading to so-called hybrid magnetoelectric quantization.[10] As a result, one would expect different behaviors of the magnetoconductivity of electrons in such systems. Thus, we are motivated to analyze MPR effects of Q1D quantum wires in tilted magnetic fields.

In this paper, we present a theory of the MPR of Q1D quantum wires in tilted magnetic fields, by taking the linear-response limit of the field-dependent conductivity formula[11] defined in the Ohm's-law form of the nonlinear current density and we study the physical characteristics of the MPR effects in such structures. Here we employ a simple model, as treated by Ihm *et al.*[12] for a Q1D electron gas (EG) confined in the quantum-wire structure subject to the electric field $\mathbf{E}(\|\hat{y})$ and the magnetic field $\mathbf{B}=(B_x,0,B_z)$. For the sake of simplicity, we assume that the coupling Hamiltonian term $\sim B_x B_z xz$ appearing in the model of IHM *et al.* is negligible since its contribution to the total electron energy is minor,[13] and we assume that the interaction with optical phonons is the dominant scattering mechanism. Based on this model, we will evaluate the magnetoconductivity and the relaxation rate which is closely related to the MPR effects.

The present paper is organized as follows: In Sec. II, we will describe the simple model of the system. In Sec. III, we present the σ_{yy} magnetoconductivity formula and the relaxation rate due to the collision process by taking the linear-response limit of the result of nonlinear-response theory[11] obtained previously. In Sec. IV, the relaxation rate for bulk optical-phonon scattering in the Q1D quantum-wire structure is calculated. The MPR effect is also discussed for such a system, where special attention is given to the unusual behavior of the MPR line shape, such as reduction in MPR amplitude, conversion of MPR maxima into minima or splitting of the MPR peaks, and shift of MPR peaks. Concluding remarks are given in Sec. V.

II. MODEL FOR A QUANTUM WIRE IN TILTED MAGNETIC FIELDS

We consider the transport of an electron gas in a Q1D quantum-wire structure as treated by Ihm *et al.*,[12] in which a Q1D electron gas is confined by two different parabolic potential wells with the confinement frequencies ω_1 and ω_2 in the x and z directions, respectively, and the conduction electrons are free along only one direction (y direction) of the wire. Then, in a magnetic field, the one-particle Hamiltonian (h_e) for such electrons is given as

$$h_e=[\mathbf{p}+e\,\mathbf{A}]^2/2m^*+m^*\omega_1^2x^2/2+m^*\omega_2^2z^2/2\ . \quad (1)$$

By taking into account the magnetic field in the transverse tilt direction, $\mathbf{B}=(B_x,0,B_z)$, with the Landau gauge $\mathbf{A}=(0,xB_z-zB_x,0)$ and a trial shift of the origin of coordinates, the Hamiltonian can be written as

$$h_e=\frac{p_x^2}{2m^*}+\frac{1}{2}m^*\Omega_1^2x^2+\frac{p_z^2}{2m^*}+\frac{1}{2}m^*\Omega_2^2z^2-m^*\omega_x\omega_z xz+\frac{p_y^2}{2\tilde{m}^*}\ , \quad (2)$$

where $\omega_x = eB_x/m^* = \omega_c\cos\theta$, $\omega_z = eB_z/m^* = \omega_c\sin\theta$, $\Omega_1^2 = \omega_1^2 + \omega_z^2$, $\Omega_2^2 = \omega_2^2 + \omega_x^2$, and $\tilde{m}^* = m^*(\Omega_1^2\Omega_2^2 - \omega_x^2\omega_z^2)(\omega_1^2\omega_2^2)^{-1}$. The eigenstates are of the form $\exp(ik_y y)\chi(x-x_0, z-z_0)$, where $x_0 = -p_y\omega_z(\tilde{m}^*\omega_1^2)^{-1}$ and $z_0 = p_y\omega_x(\tilde{m}^*\omega_2^2)^{-1}$. We see that Eq. (2) represents two coupled harmonic oscillators, and the last term in Eq. (2) indicates that the y-component kinetic energy has a magnetic-field- and confinement-frequency-dependent effective mass $\tilde{m}^*$. Furthermore, Eq. (2) can be easily diagonalized by an appropriate rotation of coordinates x and z as follows:

$$\begin{bmatrix} x \\ z \end{bmatrix} = \frac{1}{\sqrt{\omega_-^2 - \omega_+^2}} \begin{bmatrix} \sqrt{\omega_+^2 - \Omega_2^2} & \sqrt{\omega_-^2 - \Omega_2^2} \\ \sqrt{\omega_+^2 - \Omega_1^2} & \sqrt{\omega_-^2 - \Omega_1^2} \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix}, \quad (3)$$

where $2\omega_\pm^2 = \Omega_1^2 + \Omega_2^2 \pm [(\Omega_1^2 - \Omega_2^2)^2 + 4\omega_x^2\omega_z^2]^{1/2}$. The angle of rotation α is related to the above in that $\sin 2\alpha = 2\omega_x\omega_z(\omega_-^2 - \omega_+^2)^{-1}$. Then, the resulting Hamiltonian, and its normalized eigenfunctions and eigenvalues are, respectively, given as

$$h_e = \frac{p_X^2}{2m^*} + \frac{1}{2}m^*\omega_+^2 X^2 + \frac{p_Z^2}{2m^*} + \frac{1}{2}m^*\omega_-^2 Z^2 + \frac{p_y^2}{2\tilde{m}^*}, \quad (4)$$

$$|n,l,k_y\rangle = (1/L_y)^{1/2}\Phi_n(X)\Phi_l(Z)\exp(ik_y y), \quad (5)$$

$$E_{nl}(k_y) = (n+\tfrac{1}{2})\hbar\omega_+ + (l+\tfrac{1}{2})\hbar\omega_- + \frac{p_y^2}{2\tilde{m}^*}, \quad (6)$$

where k_y is the wave vector in the y direction, $n\,(=0,1,2,\ldots)$ and $l\,(=0,1,2,\ldots)$ are the Landau-level indices due to the tilted magnetic field, and $\Phi_n(X)$ and $\Phi_l(Z)$ represent harmonic-oscillator wave functions. Note that it is very complicated to evaluate the transverse magnetoconductivity analytically by using the eigenfunctions and eigenvalues given in Eqs. (5) and (6) since the eigenfunctions are given by the linear combination of x and z. Therefore, for the sake of simplicity, we assume that the coupling term $\sim B_x B_z xz$ in Eq. (2) is negligible[13] since its contribution to the total electron energy in these systems is minor. Then, in this case, the normalized eigenfunctions and eigenvalues are, respectively, reduced to

$$|n,l,k_y\rangle = (1/L_y)^{1/2}\phi_n(x-x_0)\phi_l(z-z_0)\exp(ik_y y), \quad (7)$$

$$E_{nl}(k_y) = (n+1/2)\hbar\Omega_1 + (l+1/2)\hbar\Omega_2 + \frac{p_y^2}{2\tilde{m}}, \quad (8)$$

where $\tilde{m} = m^*(\Omega_1^2\Omega_2^2)(\omega_1^2\omega_2^2 - \omega_x^2\omega_z^2)^{-1}$, $x_0 = -b_1 l_{B1}^2 k_y$, and $z_0 = b_2 l_{B2}^2 k_y$. Here $l_{B1} = (\hbar/m^*\Omega_1)^{1/2}$, $l_{B2} = (\hbar/m^*\Omega_2)^{1/2}$, $b_1 = \omega_z/\Omega_1$, and $b_2 = \omega_x/\Omega_2$. As shown in Eqs. (7) and (8), the electron energy spectrum in Q1D quantum wires is hybrid quantized by the confinements in the x and z directions and the tilted magnetic field, and the set of quantum numbers is designated by (n,l,k_y). The dimensions of the sample are assumed to be $V = L_x L_y L_z$. In the following, we will utilize Eqs. (7) and (8) to obtain the transverse magnetoconductivity analytically. It is clear from Eq. (8) that one of the two different Landau-level indices n and l is changed into the subband level index if the magnetic field is applied along the x or z directions of the system, i.e., $\theta = 0°$ or $90°$. Thus, a suitably directed magnetic field causes a dramatic change in the energy spectrum. It is interesting to note that the dependence of the single-electron energy spectrum in Eq. (8) on the confinement frequency, and the direction and strength of the magnetic field has an important effect on the transverse magnetoconductivity and the relaxation rates, as well as on the MPR effects for a Q1D quantum wire. A detailed discussion of these effects will be given explicitly in the next two sections.

III. MAGNETOCONDUCTIVITY ASSOCIATED WITH RELAXATION RATES

We want to evaluate the transverse magnetoconductivity $\sigma_{yy}(E)$ for the Q1DEG system, subject to crossed electric $\mathbf{E}$ ($\|\hat{\mathbf{y}}$) and magnetic $\mathbf{B}$ $[=(B_x,0,B_z)]$ fields, by taking the linear response limit, i.e., $\lim_{E\to 0}\tilde{\sigma}_{kl}(E)$ with respect to the general expression for the nonlinear dc conductivity $\tilde{\sigma}_{kl}(E)$ $(k,l = x,y,z)$ derived in Ref. 11 and considering the following matrix elements in the representation (7):

$$|\langle k_y,l,n|j_y|k_y',l',n'\rangle|^2 = \frac{e^2 m^{*2}}{\tilde{m}^2 b_1^2} x_0^2 \delta_{nn'}\delta_{ll'}\delta_{k_y k_y'} + (e\omega_z l_{B1}/\sqrt{2})^2[n\delta_{n'n-1} + (n+1)\delta_{n'n+1}]\delta_{ll'}\delta_{k_y k_y'} + (e\omega_x l_{B2}/\sqrt{2})^2[l\delta_{l'l-1} + (l+1)\delta_{l'l+1}]\delta_{nn'}\delta_{k_y k_y'}, \quad (9)$$

where $j_y = -(e/\tilde{m})p_y - e\omega_z x + e\omega_x z$ is the y component of a single-electron current operator and the Kronecker symbols $(\delta_{n'n}, \delta_{l'l}, \delta_{k_y' k_y})$ denote the selection rules, which arise during the integration of the matrix elements with respect to each direction. It should be noted that the matrix element with respect to the current operator in Eq. (9) is directly proportional to the dc magnetoconductivity, which contains two types of contributions as follows: one corresponding to the first term of the right-hand side in Eq. (9) is related to the current carried by the electron nonhopping motion within the localized cyclotron orbits, and the other corresponding to the second and third terms is caused by the current carried by the electron hopping motion between the localized cyclotron orbits. In particular, it should be noted that the current due to the electron hopping motion is determined by the contribution of two different Landau-level indices (n and l) and is closely related to the directionality of magnetic fields. If a magnetic field is applied to the z direction of the system, i.e., $\theta=90°$, the cyclotron frequency ω_x in the x direction becomes zero. In this case, the current due to the electron hopping motion is influenced by the contribution of only one Landau-level index n, while if it is applied to the x direction of the system, i.e., $\theta=0°$, the cyclotron frequency ω_z becomes zero. Then, the current due to the electron hopping motion is determined by the contribution of only one Landau-level index l. Therefore, we can see that the directionality of the magnetic field gives rise to a new contribution to the current due to the Landau-level indices n or l. As a result, we can expect new phenomena of magnetoconductivity associated with the MPR effect to arise from the directionality of the magnetic field.

For the calculation of the transverse magnetoconductivity σ_{yy} for the Q1D quantum wire, we use the general expression for the complex nonlinear dc conductivity $\tilde{\sigma}_{kl}(E)$ $(k,l=x,y,z)$ given in Ref. 11:

$$\tilde{\sigma}_{kl}(E)=-\frac{\beta}{V}\sum_{\lambda_1} f(E_{\lambda_1})[1-f(E_{\lambda_1})]\langle\lambda_1|j_l|\lambda_1\rangle\langle\langle\lambda_1|\tilde{j}_k(E)|\lambda_1\rangle\rangle_B + \frac{1}{V}\sum_{\lambda_1\neq\lambda_2}\frac{f(E_{\lambda_1})-f(E_{\lambda_2})}{E_{\lambda_1}-E_{\lambda_2}}\langle\lambda_1|j_l|\lambda_2\rangle\langle\langle\lambda_2|\tilde{j}_k(E)|\lambda\rangle\rangle_B , \quad (10)$$

$$\langle\langle\lambda_2|\tilde{j}_k(E)|\lambda_1\rangle\rangle_B=\frac{\hbar}{i}\lim_{s\to 0}\frac{\langle\lambda_2|j_k|\lambda_1\rangle}{-i\hbar s-E_{\lambda_2}+E_{\lambda_1}-\tilde{\nabla}_{\lambda_2\lambda_1}(E)+i\tilde{\Gamma}_{\lambda_2\lambda_1}(E)} , \quad (11)$$

where $\beta=1/k_BT$ with k_B being the Boltzmann constant and T temperature. Also, λ indicates the quantum state, $\langle\cdots\rangle_B$ means the average over the background (phonon or impurity) configuration, and $f(E_\lambda)$ is a Fermi-Dirac distribution function associated with the state λ. Note that Eq. (10) reduces to the Kubo formula[14] for the complex dc conductivity tensor when the electric-field-dependent dc conductivity [Eq. (10)] defined in the Ohm's-law form of the nonlinear electric current density is taken as $\lim_{E\to 0}\tilde{\sigma}_{kl}(E)$. Furthermore, the dc linear conductivity we want to obtain is given by taking the real part of the complex dc conductivity, i.e., $\lim_{E\to 0}\text{Re}\{\tilde{\sigma}_{kl}(E)\}\equiv\sigma_{kl}$. Then, the quantities $\tilde{\Gamma}$ and $\tilde{\nabla}$ given in Eq. (11), which appear in terms of the collision broadening due to the electron-background (impurity and/or phonon) interaction, play the role of the width and the shift in the spectral line shape, respectively.

Now, we apply Eqs. (10) and (11) to the quantum wire modeled in Sec. II by using the selection rules of Eq. (9), replacing the λ_1 and λ_2 states of Eqs. (10) and (11) by the representation (7), and taking the real part of Eqs. (10) and (11). Then, σ_{yy} can be easily obtained by the sum of the hopping part σ_{yy}^h and the nonhopping part σ_{yy}^{nh}, which are

$$\sigma_{yy}^{nh}=\frac{\hbar\beta e^2 m^{*2}}{\bar{m}^2 b_1^2 V}\sum_{n,l,k_y} x_0^2 f[E_{nl}(k_y)]\{1-f[E_{nl}(k_y)]\}/\tilde{\Gamma}(n,l,k_y;n,l,k_y) , \quad (12a)$$

$$\sigma_{yy}^{h}=\frac{e^2 b_1^2}{m^*\Omega_1^2 V}\sum_{n,l,k_y}(n+1)\{f[E_{nl}(k_y)]-f[E_{n+1l}(k_y)]\}\tilde{\Gamma}(n+1,l,k_y;n,l,k_y) + \frac{e^2 b_2^2}{m^*\Omega_2^2 V}\sum_{n,l,k_y}(l+1)\{f[E_{nl}(k_y)]-f[E_{nl+1}(k_y)]\}\tilde{\Gamma}(n,l+1,k_y;n,l,k_y) , \quad (12b)$$

where $f[E_{nl}(k_y)]$ is a Fermi-Dirac distribution function associated with the eigenstate $|n,l,k_y\rangle$ of Eq. (7) and the energy $E_{nl}(k_y)$ of Eq. (8). We assumed $\tilde{\Gamma}$, $\tilde{\nabla}\ll\hbar\Omega_1[=E_{n+1l}(k_y)-E_{nl}(k_y)]$ and $\hbar\Omega_2[=E_{nl+1}(k_y)-E_{nl}(k_y)]$, and shift zero ($\tilde{\nabla}\approx 0$) to observe the oscillatory behavior of the MPR effect as some other authors did.[3,6,8] Note that the first term of Eq. (12b) is the magnetoconductivity given in terms of the electron hopping motion between the Landau-level states n and $n+1$, while the second term of Eq. (12b) is the magnetoconductivity given in terms of the electron hopping motion between the Landau-level states l and $l+1$. To express the dc magnetoconductivity of Eq. (12) in simpler forms, we assume that the f's in Eq. (12) are replaced by the Boltzmann distribution function for nondegenerate semiconductors, i.e., $f(E_{n,l}(k_y)\approx\exp[\beta(E_F-E_{nl}(k_y)]$, where E_F denotes the Fermi energy. Then, we can further perform the sum over n (or l) [if n (or l) is large] by writing $\sum n\exp(-\alpha n)=-(\partial/\partial\alpha)\sum\exp(-\alpha n)$, summing the geometric series, and carrying out the one summation with respect to k_y in $\sum_{n,l,k_y}$ in terms of the following relation:[3,6] $\sum_{k_y}(\cdots)\to(L_y/2\pi)\int_{-\infty}^{\infty}dk_y(\cdots)$. Thus, we obtain

$$\sigma_{yy}^{nh}\approx\frac{16\hbar e^2 N_s^{1D}}{\sqrt{\bar{m}\Omega_1^2 V}}\tilde{\Gamma}^{-1}(n,l,k_y;n,l,k_y) , \quad (13a)$$

$$\sigma_{yy}^{h} \approx \frac{4\sqrt{\tilde{m}}\, e^2 b_1^2 N_s^{1D}}{m^*\Omega_1^2 V}\tilde{\Gamma}(n+1,l,k_y;n,l,k_y) + \frac{4\sqrt{\tilde{m}}\, e^2 b_2^2 N_s^{1D}}{m^*\Omega_2^2 V}\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)\ , \quad (13b)$$

where we utilized the electron density[6] given as

$$N_s^{1D} = \frac{\sqrt{k_B T L_y^2/32\pi\hbar^2}\exp[\beta E_F]}{\sinh(\beta\hbar\omega_1/2)\sinh(\beta\hbar\omega_2/2)}$$

and approximated the factor $1-f[E_{nl}(k_y)]$ in Eq. (12a) by 1 (nondegenerate limit) since optical phonons are dominant at high temperatures. As seen from Eq. (13), the transverse magnetoconductivity σ_{yy} is closely related to the three different relaxation rates $\tilde{\Gamma}(n,l,k_y;n,l,k_y)$, $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$, and $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$. Especially the magnetoconductivity σ_{yy}^{h} is directly proportional to the relaxation rates $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$ and $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$ for the electron hopping motion, while the magnetoconductivity σ_{yy}^{nh} is inversely proportional to the relaxation rate $\tilde{\Gamma}(n,l,k_y;n,l,k_y)$ for the electron nonhopping motion. However, if the magnetic field is applied along the z direction of the system, i.e., $\theta=90°$, the second term of Eq. (13b) does not appear. In this case, the relationship between the relaxation rates and the magnetoconductivity is identical to the previous results[9] obtained for the quantum wire modeled by Vasilopoulos *et al.*[6] Thus, the electronic transport properties (e.g., electronic relaxation processes, magnetophonon resonances, etc.) in the Q1D quantum-wire structures can be studied by examining the behavior of $\tilde{\Gamma}$ as a function of the relevant physical parameters introduced in the theory.

IV. MAGNETOPHONON RESONANCES IN TILTED MAGNETIC FIELDS

For the evaluation of the relaxation rates $\tilde{\Gamma}$ for a specific electron-phonon interaction in Eq. (13), we need the Fourier component of the interaction potential[3] for optical-phonon scattering given by $|C(q)|^2=D'/V$ with $D'=\hbar D^2/2\rho\omega_L\approx$const, D being a constant, and ρ being the density, where the assumption that the phonons are dispersionless (i.e., $\hbar\omega_{\mathbf{q}}\approx\hbar\omega_L\approx$const, where ω_L is the optical phonon frequency) and bulk (i.e., three-dimensional) was made. We also need the following matrix element in the representation (7):

$$|\langle k_y,l,n|\exp(\pm i\mathbf{q}\cdot\mathbf{r})|n',l',k_y'\rangle|^2 = |J_{nn'}(u_1)|^2|J_{ll'}(u_2)|^2\delta_{k_y',k_y\mp q_y}\ , \quad (14)$$

$$|J_{nn'}(u)|^2 = \frac{n_<!}{n_>!}e^{-u}u^{\Delta n}[L_{n_>}^{\Delta n}(u)]^2\ , \quad (15)$$

where $n_<=\min\{n,n'\}$, $n_>=\max\{n,n'\}$, $u_1=l_{B1}^2(q_x^2+b_1^2q_y^2)/2$, $u_2=l_{B2}^2(q_z^2+b_2^2q_y^2)/2$, and $L_{n_<}^{\Delta n}(u)$ is an associated Laguerre polynomial[15] with $\Delta n=n_>-n_<$.

The detailed derivation of the relaxation rate is given in Ref. 11 and its general expression in the lowest-order approximation for the weak-coupling case of an electron-phonon system can be seen in Eq. (4.39) of Ref. 11. Using the representation given by Eq. (7), the Q1D version of this quantity associated with the electronic transition between the states $|n',l',k_y'\rangle$ and $|n,l,k_y\rangle$ can be evaluated as

$$\begin{aligned}\tilde{\Gamma}(n',l',k_y';n,l,k_y) = {} & \pi\sum_{\mathbf{q}}\sum_{k_y}\sum_{(n_1,l_1)\neq(n',l')}|C(q)|^2|J_{n'n_1}(u_1)|^2|J_{l'l_1}(u_2)|^2 \\ & \times\{(N_0+1)\delta[(n-n_1)\hbar\Omega_1+(l-l_1)\hbar\Omega_2+S^-(q_y)-\hbar\omega_L] \\ & +N_0\delta[(n-n_1)\hbar\Omega_1+(l-l_1)\hbar\Omega_2-S^+(q_y)+\hbar\omega_L]\} \\ & +\pi\sum_{\mathbf{q}}\sum_{k_y}\sum_{(n_1,l_1)\neq(n,l)}|C(q)|^2|J_{n_1n}(u_1)|^2|J_{l_1l}(u_2)|^2 \\ & \times\{(N_0+1)\delta[(n_1-n')\hbar\Omega_1+(l_1-l')\hbar\Omega_2-S^-(q_y)+\hbar\omega_L] \\ & +N_0\delta[(n_1-n')\hbar\Omega_1+(l_1-l')\hbar\Omega_2+S^+(q_y)-\hbar\omega_L]\}\ , \end{aligned} \quad (16)$$

where $S^{\mp}(q_y)=\hbar^2(2k_yq_y\mp q_y^2)/2\tilde{m}$, n_1 and l_1 indicate the intermediate localized Landau-level indices, and N_0 is the optical-phonon distribution function given by $N_{\mathbf{q}}=[\exp(\beta\hbar\omega_{\mathbf{q}})-1]^{-1}$ with $\omega_{\mathbf{q}}=\omega_L$. It should be noted that the Landau-level indices n' and l' given in Eq. (16) are, respectively, replaced by n and l for the electron nonhopping motion, while they are also replaced by $n+1$, l or n, $l+1$ for the electron hopping motion, depending on the type of the transitions associated with the Landau-level indices. The δ functions in Eq. (16) express the law of energy conservation in one-phonon collision (absorption and emission) processes. The strict energy-conserving δ functions in Eq. (16) imply that when the electron undergoes a collision by absorbing energy from the field, its energy can only change by an amount equal to the energy of a phonon involved in the transitions. This in fact leads to magnetophonon resonance effects due to the Landau levels. The remarkable thing is that, unlike the case where a static magnetic field is applied in a specific direction ($\theta=0°$ or $90°$) to the wire, two different kinds of MPR effects in terms of two different Landau-level indices n and l take place. We can see these effects from the condition $(n_1,l_1)\neq(n,l)$ in the summation of Eq. (16), which contains three types of contributions: (1) $n_1\neq n$, $l_1\neq l$, (2) $n_1\neq n$, $l_1=l$, and (3) $n_1=n$, $l_1\neq l$. Here the first and second conditions give the case where the resonant transition takes place in terms of the Landau-level index n, i.e., the MPR effect being due to the Landau level n, whereby $\hbar\Omega_1 \gg \tilde{\Gamma}$ is satisfied. Furthermore, the first and third conditions also give the case where the resonant transition takes place in terms of the Landau-level index l, i.e., the MPR effect being due to the Landau level l, whereby $\hbar\Omega_2 \gg \tilde{\Gamma}$ is satisfied, since the MPR in the Q1D quantum-wire structure is due essentially to the inter-Landau-level (inelastic resonant phonon) scattering. Thus, the summations of Eq. (16) over the Landau level can be, respectively, divided into two cases: (1) $\sum_{n_1\neq n'}\sum_{l_1}$ and $\sum_{n_1\neq n}\sum_{l_1}$ (2) $\sum_{n_1}\sum_{l_1\neq l'}$ and $\sum_{n_1}\sum_{l_1\neq l}$. Note that, as discussed earlier, the MPR effect due to the Landau level n or l does not appear if a static magnetic field is applied in a specific direction to the wire (i.e., $\theta=0°$ or $90°$) since the renormalized frequency Ω_1 or Ω_2 in Eq. (8) becomes the confinement frequency ω_1 or ω_2 and the Landau-level index n or l changes to the subband level index.

Let us now calculate the relaxation rate $\tilde{\Gamma}$ associated with the MPR effect due to the Landau-level index n. At first, transforming the sum over $\mathbf{q}$ in Eq. (16) into an integral form in the usual way, considering the interaction potential for optical phonon scattering, and taking into account $S^{\pm}(q_y)=0$ as an approximation as Vasilopoulos *et al.* did,[6] the relaxation rate $\tilde{\Gamma}$ in Eq. (16) involves integrations with respect to q_x, q_y, and q_z in Cartesian coordinates. The integral over q_x, q_y, and q_z is very difficult to evaluate analytically since it must be done separately for each n and n'. So, to simplify the calculations, we consider that $l=l'=0$ and $u_1+u_2\approx u_1+\frac{1}{2}l_{B2}^2q_z^2$ under the assumption $l_{B1}b_1 \gg l_{B2}b_2$. With these approximations we can do the integral over q_x, q_y, and q_z. In particular, the integrals over q_x and q_y can be reduced to integrals with respect to θ and u_1 in cylindrical coordinates, where the θ integration gives 2π. To get the relaxation rate given in a simple form, we further assume[8] that n' is very large, so we can, then, approximate $n'\pm1\approx n'$. Setting $n'-n=-P$ in the emission term and $n'-n=P$ in the absorption term, and noting[3,15] that $\int_0^\infty |J_{nn'}(u_1)|^2du_1=1$, we obtain for the electron nonhopping and hopping motion as

$$\tilde{\Gamma}(n,l,k_y;n,l,k_y)=\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)\approx\Lambda_1\sum_P(2N_0+1)\delta[P-\omega_L/\Omega_1] , \tag{17a}$$

$$\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)\approx(\Lambda_1/2)\sum_P(2N_0+1)\{\delta[P-(\omega_L+\Omega_2)/\Omega_1]+\delta[P-(\omega_L-\Omega_2)/\Omega_1]\} , \tag{17b}$$

where $\Lambda_1=D'/(\sqrt{2\pi}\hbar b_1 l_{B1}^2 l_{B2}\Omega_1)$. We see from Eq. (17) that the magnetoconductivity (13) associated with the relaxation rates shows the resonant behaviors: magnetophonon resonances at $P\Omega_1=\omega_L$ for $\tilde{\Gamma}(n,l,k_y;n,l,k_y)$ and $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$, and at $P\Omega_1=\omega_L\pm\Omega_2$ for $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$ (P is an integer). The above conditions for MPR give the resonance magnetic fields (i.e., the MPR peak positions at) B_P, B_P^+, and B_P^-:

$$B_P=\sqrt{(B_0/P)^2-(m^*\omega_1/e)^2}/\sin\theta , \tag{18a}$$

$$P\sqrt{\omega_1^2+(e\sin\theta/m^*)^2B_P^{\pm2}} =\omega_L\pm\sqrt{\omega_2^2+(e\cos\theta/m^*)^2B_P^{\pm2}} , \tag{18b}$$

where $B_0(=m^*\omega_L/e)$ is the fundamental field for the ordinary MPR. It is shown that additional MPR peaks (subsidiary peaks) appear at $B_P^{\pm}$ on both sides of the MPR peaks at B_P. The origin of the appearance of the subsidiary peaks in the Q1DEG system is mainly due to the relaxation rate $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$, i.e., the directionality of magnetic fields. It is very interesting to point out that the MPR peak positions are closely related to the confinement frequencies (ω_1,ω_2) and the direction of the magnetic field (B). If the direction of the magnetic field B is taken as the z direction of the system, i.e., $\theta=90°$, Eq. (18a) is identical to the result of Vasilopoulos *et al.*,[6] for Q1D quantum-wire structure modeled by a triangular potential well and a parabolic potential well. In that case, the effect of confinement due to the confinement frequency (ω_1) is to change the ordinary MPR peak positions to lower magnetic-field values. Similarly, to get the relaxation rate associated with the MPR effect due to the Landau-level index l, we assume that $n=n'=0$ and $u_1+u_2\approx u_2+(1/2)l_{B1}^2q_x^2$ under the assumption $l_{B2}b_2 \gg l_{B1}b_1$, and also assume that l' is very large. We can, then, approximate $l'\pm1\approx l'$. Setting $l'-l=-P$ in the emission term and $l'-l=P$ in the absorption term, and noting[3,15] that $\int_0^\infty |J_{ll'}(u_2)|^2du_2=1$, we obtain for the electron nonhopping and hopping motion as

$$\tilde{\Gamma}(n,l,k_y;n,l,k_y)=\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)\Lambda_2\sum_P(2N_0+1)\delta[P-\omega_L/\Omega_2] , \tag{19a}$$

$$\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)\approx(\Lambda_2/2)\sum_P(N_0+1)\left\{\delta\left[\frac{\omega_L+\Omega_1}{\Omega_2}\right]+\delta\left[P-\frac{\omega_L-\Omega_1}{\Omega_2}\right]\right\} , \tag{19b}$$

where $\Lambda_2 = D'/(\sqrt{2\pi}\hbar b_2 l_{B1} l_{B2}^2 \Omega_2)$. It should be noted that if the magnetic field is taken in the z direction of the system, i.e., $\theta=90°$, Eq. (19) associated with MPR effect does not arise since the renormalized frequency Ω_2 reduces to the confinement frequency ω_2. We see that the magnetoconductivity (13) associated with the relaxation rate shows the resonant behaviors: magnetophonon resonances at $P\Omega_2=\omega_L$ for $\tilde{\Gamma}(n,l,k_y;n,l,k_y)$ and $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$, and at $P\Omega_2=\omega_L \pm \Omega_1$ for $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$ (P is an integer). The above conditions for the MPR give the resonance magnetic fields (i.e., the MPR peak positions at) B_p, B_p^+, and B_p^-:

$$B_P = \sqrt{(B_0/P)^2 - (m^*\omega_2/e)^2}/\cos\theta \ , \tag{20a}$$

$$P\sqrt{\omega_2^2 + (e\cos\theta/m^*)^2 B_P^{\pm 2}} = \omega_L \pm \sqrt{\omega_1^2 + (e\sin\theta/m^*) B_P^{\pm 2}} \ . \tag{20b}$$

It should be noted that the relaxation rate for optical-phonon scattering diverges whenever the above conditions are satisfied. These divergences may be removed by including higher-order electron-phonon scattering terms or by inclusion of the fluctuation effects of the center of mass.[16] The simplest way to avoid the divergences is to replace each δ function in Eqs. (17) and (19) by Lorentzians with a width parameter γ. Employing this collision-broadening model,[3,8] applying Poisson's summation formula[17] for the $\sum_p$ in Eqs. (17) and (19), and taking into account the following property:[8,15]

$$\Psi(a,b) = 1 + 2\sum_{s=1}^{\infty} e^{-2\pi s a}\cos(2\pi s b) = \frac{\sinh(2\pi a)}{\cosh(2\pi a) - \cos(2\pi b)} \ , \quad (a>0) \tag{21}$$

we then obtain

$$\tilde{\Gamma}(n,l,k_y;n,l,k_y) = \tilde{\Gamma}(n+1,l,k_y;n,l,k_y) \approx \Lambda_1(2N_0+1)\Psi\left[\frac{\gamma}{\hbar\Omega_1}, x_1\right] \ , \tag{22a}$$

$$\tilde{\Gamma}(n,l+1,k_y;n,l,k_y) \approx (\Lambda_1/2)(2N_0+1) \times \left\{ \Psi\left[\frac{\gamma}{\hbar\Omega_1}, x_1(1+y_1)\right] + \Psi\left[\frac{\gamma}{\hbar\Omega_1}, x_1(1-y_1)\right]\right\} \tag{22b}$$

for the MPR effect due to the Landau-level index n and

$$\tilde{\Gamma}(n,l,k_y;n,l,k_y) = \tilde{\Gamma}(n,l+1,k_y;n,l,k_y) \approx \Lambda_2(2N_0+1)\Psi\left[\frac{\gamma'}{\hbar\Omega_2}, x_2\right] \ , \tag{23a}$$

$$\tilde{\Gamma}(n+1,l,k_y;n,l,k_y) \approx (\Lambda_2/2)(2N_0+1) \times \left\{ \Psi\left[\frac{\gamma'}{\hbar\Omega_2}, x_2(1+y_2)\right] + \Psi\left[\frac{\gamma'}{\hbar\Omega_2}, x_2(1-y_2)\right]\right\} \tag{23b}$$

for the MPR effect due to the Landau-level index l, where $x_1=\omega_L/\Omega_1$, $y_1=\Omega_2/\omega_L$, $x_2=\omega_L/\Omega_2$, and $y_2=\Omega_1/\omega_L$. For simplicity, we assumed[8] that $\gamma_i=\gamma$ ($i=1$, 2, and 3) for the collision damping terms of Eq. (22) and $\gamma_i=\gamma'$ ($i=4$, 5, and 6) for the collision damping terms of Eq. (23). To obtain the width parameter γ of Eq. (22) explicitly, we assume the width parameter γ to be the same for all associated states and approximate $\tilde{\Gamma}$ on the left-hand side of Eq. (22a) as γ. Then, considering

$$\Psi(\gamma/\hbar\Omega_1, \omega_L/\Omega_1) = \coth(\pi\gamma/\hbar\Omega_1) \quad \text{for } \omega_L = P\Omega_1$$

and utilizing

$$\coth X \approx 1/X + X/3 - X^3/45 \quad \text{for } X \ll 1 \ ,$$

the resonance width γ is given by the approximate result

$$\gamma \approx (15\{(1-\Delta) + [(1-\Delta)^2 + \tfrac{4}{5}]^{1/2}\}/2\pi^2)^{1/2}\hbar\Omega_1 \ ,$$

with $\Delta = 3\hbar\Omega_1/[\Lambda_1\pi(2N_0+1)]$. Similarly, the resonance width γ' of Eq. (23) is given by

$$\gamma' \approx (15\{(1-\Delta') + [(1-\Delta')^2 + \tfrac{4}{5}]^{1/2}\}/2\pi^2)^{1/2}\hbar\Omega_2 \ ,$$

with $\Delta' = 3\hbar\Omega_2/[\Lambda_2\pi(2N_0+1)]$. Equations (22) and (23) give a general description of magnetophonon oscillations in the Q1D quantum-wire structure for the MPR effect due to the Landau-level index n and the Landau-level index l, respectively. For the MPR effect due to the Landau-level index n, Eq. (22a) shows that the period of the oscillation is given under the condition of $\omega_L/\Omega_1=P$ and is determined by the direction of the magnetic field and by the confinement frequency ω_1. Obviously, if the broadening is not included, i.e., $\gamma \to 0$ in Eq. (22a), $\tilde{\Gamma}$ and hence $1/\tau$ diverge at the resonance. In the case of $\theta=90°$ [i.e., $\mathbf{B}=(0,0,B_z)$], the oscillatory behavior of Eq. (22a), except for the amplitude of the relaxation rate, is in agreement with the previous result[9] obtained by the model of Vasilopoulos *et al.*[6] Furthermore, the relaxation rate $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$ in Eq. (22a) gives the same oscillatory behavior as that obtained from the result of Vasilopoulos *et al.*[6] We see that Eq. (22b) exhibits additional complexity of oscillations, the subsidiary (MPR) peaks appear at $P\Omega_1=\omega_L \pm \Omega_2$, and the position of these subsidiary peaks and the period of additional oscillations in Eq. (22b) are sensitive to the direction of the magnetic field and the confinement frequencies (ω_1,ω_2). It is shown that the oscillation in the relaxation rate is damped by the direction and strength of the magnetic field and the confinement frequency since these parameters give a direct influence on the effect of collision damping. Unlike the MPR effect due to the Landau-level index n, the relaxation rates of Eq. (23) associated with the MPR effect due to the Landau-level index l have another oscillatory period $P=\omega_L/\Omega_2$ and the subsidiary (MPR) peaks appear at $P\Omega_2=\omega_L \pm \Omega_1$. It should be noted that the MPR effects of Eqs. (22b) and (23) take place in the case where the magnetic field given in the tilt direction is applied to the Q1D quantum-wire system. If the direction of the magnetic field is taken in the z direction of the system ($\theta=90°$), these effects do not occur.

V. CONCLUDING REMARKS

In this paper, we have presented a theory of MPR and investigated the physical characteristics of the MPR effects in the Q1D quantum-wire structure, where a Q1DEG confined by two different parabolic potential wells in the x and z direction is subjected to crossed electric ($\mathbf{E}\|\hat{y}$) and magnetic fields $\mathbf{B}=(B_x,0,B_z)$. The origin of this formalism[11] dates back to the discovery of the theory of nonlinear static conductivity. On the basis of the linear-response limit of this formalism, the relaxation rate for the weak-coupling case has been utilized with respect to the electron–optical-phonon interaction and its behavior (relative transport processes) has been discussed in connection with the MPR effect. The problem of the δ-function singularities in the relaxation rate $\tilde{\Gamma}$ has been removed[3,8] by introducing a resonance width parameter γ, which gives contributions to the collisional damping of the MPR effect.

It is shown from Eqs. (9) and (13) that the magnetoconductivity σ_{yy} appears in the form of three types of contribution associated with the selection rules of the current density operator. The magnetoconductivity σ_{yy}^{h} is directly proportional to the relaxation rates $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$ and $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$ for the electron hopping motion, while the magnetoconductivity σ_{yy}^{nh} is inversely proportional to the relaxation rates $\tilde{\Gamma}(n,l,k_y;n,l,k_y)$ for the electron nonhopping motion. It should be noted that the relaxation rates for the electron hopping motion are closely related to the directionality of the magnetic field. If magnetic fields are applied to the x or z direction, one of two relaxation rates for the electron hopping motion disappears. Then, the dependence of the magnetoconductivity on the relaxation rates gives the same results[9] obtained from the model of Vasilopoulos *et al.*[6] Due to the directionality of magnetic fields, the relaxation rates and hence the magnetoconductivity for the electron hopping motion have two different properties. For the MPR effect due to the Landau-level index n, the relaxation rates $\tilde{\Gamma}(n,l,k_y;n,l,k_y)$ and $\tilde{\Gamma}(n+1,l,k_y;n,l,k_y)$ show that the period of the oscillation is given under the condition $P=\omega_L/\Omega_1$, while the relaxation rate $\tilde{\Gamma}(n,l+1,k_y;n,l,k_y)$ exhibits additional complexity of oscillations, the subsidiary MPR peaks appearing at $P\Omega_1=\omega_L\pm\Omega_2$. Here P is an integer. Note that if the magnetic field is applied to the z direction of the system, the subsidiary MPR peaks disappear since the matrix element of the current operator in Eq. (9) due to the electron hopping term of the Landau-level index l becomes zero. In that case, the oscillatory behavior of the relaxation rates is identical with the previous result[9] obtained for the model of Vasilopoulos *et al.*[6] The relaxation rates associated with the MPR effect due to the Landau-level index l have another oscillatory period $P=\omega_L/\Omega_2$ and the subsidiary MPR peaks appear at $P\Omega_2=\omega_L\pm\Omega_1$. Therefore, the MPR peak positions are closely related to the direction and strength of the magnetic fields and the confinement frequency. Furthermore, it is shown that the oscillation in the relaxation rates are damped by these parameters since these parameters have a direct influence on the effect of collision damping. It is noted that our result for the relaxation rate and the magnetoconductivity is valid when the coupling term $\sim B_xB_zxz$ in Eq. (2) is negligible[13] since its contribution to the total electron energy is minor, and is far from being rigorous since our result is tied to the approximations: the $S^{\pm}(q_y)$ terms of Eq. (16) have been neglected, as Vasilopoulos *et al.* did,[6] and another approximation has been made by putting $u_1+u_2\approx u_1+\frac{1}{2}l_{B2}^2q_z^2$ for $l_{B1}b_1\gg l_{B2}b_2$ and $u_1+u_2\approx u_2+\frac{1}{2}l_{B1}^2q_x^2$ for $l_{B1}b_1\ll l_{B2}b_2$, in order to get analytical expressions within the integration over $\mathbf{q}$ of Eq. (16). Furthermore, we have not taken into account any modification of the electron-phonon interaction brought about by the confinement of phonons (we used the interaction for bulk phonons). However, we can expect that our result makes it possible to understand qualitatively the physical characteristics on the MPR effect of the Q1D quantum-wire structure in a tilted magnetic field. Unfortunately, we are not aware of any relevant experimental work to compare our theory with. Therefore, to test the validity of this prediction, new experiments are needed.

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