Intrasubband plasmons and optical transmission in random-layer-thickness *n-i-p-i* semiconductor superlattices

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We investigate theoretically the intrasubband collective plasmon modes and the optical transmission in a specially constructed semiconductor superlattice consisting of *n*- and *p*-type doped semiconductors separated by an undoped intrinsic (*i*) semiconductor. The thicknesses of the constituent layers are randomly distributed in accordance with a designed probability, and all of them are assumed to be sufficiently large so that the quantum-size effect can be ignored. The materials of the layers are characterized by frequency-dependent dielectric functions. The discontinuity at the interfaces is dealt with by using the transfer-matrix method. The calculation of dispersion relation and optical transmission shows that in varying the degree of thickness randomness, the frequencies of the plasmon modes only exhibit small shifts, but the transmission coefficients are changed. A particular random structure is found in which some modes of electromagnetic waves with special frequencies are completely unscattered by the randomness, whereas the other modes rapidly decay. This provides a possibility of building a high-quality optical filter.

I. INTRODUCTION

In the past decade, the physics of the artificial semiconducting heterostructures and superlattices has attracted a great deal of interest.[1] The electronic, transport, and optical properties of periodic semiconductor superlattices have been systematically studied in both experimental and theoretical works.[2-4] The motivations are in their possible applications as useful new devices. It has been shown that such materials have band structures, different from their parent ones, and possess unusual electronic and optical properties.[5] In recent years, the artificially constructed aperiodic layered structures have also attracted much attention in investigations. These include the quasiperiodic systems and the systems with randomly distributed layer thicknesses. For a given wave-vector component parallel to the layers, the propagation of waves along the perpendicular direction is just like the motion in a one-dimensional (1D) aperiodic system. It is found that the states in a 1D quasiperiodic system have exotic Cantor-set-like features.[6] At the same time, in a 1D disordered system, most of the states are localized, as indicated by scaling theory,[7] but for special disorder types, there exist particular extended states.[8,9] The random-thickness layered structure provides another possibility of producing new devices. In recent experiments, several interesting optical properties have been observed in semiconductor superlattices with random layer thicknesses.[10,11]

The knowledge of collective excitations is of fundamental importance to the understanding of the electronic and optical properties in layered structures. In recent years, the collective plasmon polaritons in periodic semiconducting superlattices have been studied extensively.[12-14] The magnetoplasmon polaritons in a binary superlattice were studied by Wallis, Szenics, Quinn, and Giuliani.[15] Farias, Auto, and Albuquerque considered the propagation of bulk and surface modes in this structure with carriers strictly confined to the interfaces.[16] Johnson and Camley have generalized the theory of magnetoplasmon polaritons in semi-infinite superlattices to include propagation in arbitrary directions with respect to a magnetic field parallel to the interfaces, and to present results for attenuated-total-reflection probes of these excitations.[17] At the same time, the modulationally doped semiconductor superlattices, the *n-i-p-i* ones, were included in the original proposal.[1] Many works on the electronic and optical properties of these materials have been pursued by Döhler and Ploog.[4] Recently, the intrasubband plasmon modes in semi-infinite *n-i-p-i* superlattice were also studied by Kushwaha.[18]

There have been several theoretical works on the collective excitations in random-layered structures. The acoustic properties in such media were investigated by Gilbert,[19] and Levine and Willemsen.[20] Johnson and Camley have presented numerical examples demonstrating the effect of varying the thickness of a single film on the plasmon excitations in a finite binary superlattice.[21] The aim of the present work is to investigate theoretically the changes in the dispersion relations and the propagation of the plasmon polaritons due to introducing the randomness of layer thicknesses in a *n-i-p-i* superlattice. The layer thicknesses are random variables satisfying a designed probability distribution, but all of them are assumed to be large enough so that the quantum-size effect can be neglected and the constituent layers can be de-

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scribed by macroscopic dielectric functions. By the use of the transfer-matrix method, the dispersion relations and optical transmission are calculated for finite samples with different degrees of randomness. It is found that for a specially designed random structure, several modes with particular frequencies are completely unscattered by the randomness, whereas the other modes rapidly decay. This situation is similar to the existence of a small portion of extended electronic states in special 1D disordered systems,[8,9] and makes it possible to build new kinds of high-quality optical filters.

In Sec. II, we describe the structure of *n*-*i*-*p*-*i* superlattices with randomly distributed layer thicknesses and the transfer-matrix formalism used to obtain the dispersion relations, and the propagation of plasmon modes in them. In Sec. III, we present numerical results of dispersion relations for several finite samples with different degrees of randomness. In Sec. IV, the results of optical transmission are presented, and a special random structure for realizing the high-quality filter is proposed. Finally, Sec. V is devoted to summarizing and discussing the obtained results.

II. STRUCTURE AND FORMALISM

The superlattice structure considered in this paper is depicted in Fig. 1. Every unit includes four material layers: *A*, *B*, *C*, and *D*, which are, respectively, *n* doped, intrinsic, *p* doped, and intrinsic semiconductors. We denote the thickness of a layer by L_{iJ}, where i $(=1,2,\ldots,N)$ is the unit index (N is the total number of the units), and J $(=A,B,C,D)$ is the species index in a unit. L_{iJ} is a random variable; the system is thus a superlattice with randomly distributed layer thicknesses. In this paper, we assume that the layer thicknesses of species *A*, *B*, *C*, and *D* satisfy, respectively, four uniform probability distributions,

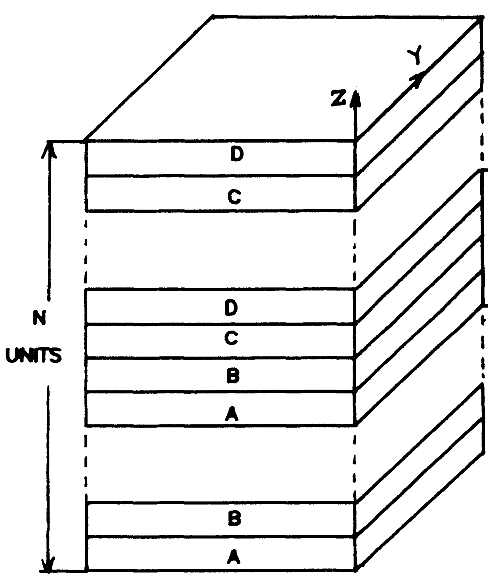

FIG. 1. Geometry of *n*-*i*-*p*-*i* semiconductor superlattice with randomly distributed layer thicknesses.

$$P(L_{iJ})=\begin{cases}1/(Y_{J1}-Y_{J2}) & \text{for } Y_{J1}\geq L_{iJ}\geq Y_{J2}\,,\\ 0 & \text{otherwise}\,,\end{cases} \tag{1}$$

with $J=A,B,C,D$. A special thickness L_{iJ} is then generated from a random-number generator according to these distributions. The average layer thickness of species J is $(Y_{J1}+Y_{J2})/2$, and the degree of randomness is controlled by the value of $J_{J1}-Y_{J2}$.

We assume all the thicknesses are large enough so that the quantum-size effect can be neglected. Thus, the four material layers $(J=A,B,C,D)$ can be described by frequency-dependent dielectric functions ε_J $(J=A,B,C,D)$, respectively. In the absence of the applied magnetic field, the general wave-field equation can be written in terms of the macroscopic electric field **E**,

$$\nabla\times(\nabla\times\mathbf{E})-q_0^2\varepsilon\mathbf{E}=0\,, \tag{2}$$

where $q_0=\omega/c$ is the vacuum wave vector (ω is the frequency and c is light velocity) and ε is the scalar dielectric function for the medium. The plasma modes are assumed to propagate in the y-z plane with wave-vector components q_y and q_z, where z is the direction perpendicular to the layers. Here, the dielectric function $\varepsilon(\omega)$ is simplified by the symmetry requirements such that $\varepsilon_{xx}=\varepsilon_{yy}=\varepsilon_{zz}=\varepsilon$ and $\varepsilon_{xy}=\varepsilon_{yz}=\varepsilon_{yx}=\varepsilon_{zy}=\varepsilon_{xz}=\varepsilon_{zx}=0$. In this situation, Eq. (2) can be rewritten as

$$\begin{bmatrix} q_0^2\varepsilon-q_y^2-q_z^2 & 0 & 0\\ 0 & q_0^2\varepsilon-q_z^2 & q_yq_z\\ 0 & q_yq_z & q_0^2\varepsilon-q_y^2\end{bmatrix}\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}. \tag{3}$$

The dielectric function for the present medium is defined as

$$\varepsilon(\omega)=\varepsilon_l\left[1-\frac{\omega_p^2}{\omega^2}\right], \tag{4}$$

where ε_l is the background dielectric constant and ω_p is the plasma frequency of the medium concerned. From the condition of existence of nontrivial solution of Eq. (3), we have

$$-q_z^2\equiv\alpha^2=q_y^2-q_0^2\varepsilon(\omega)\,. \tag{5}$$

The field solutions within layer J of the ith unit can be expressed as

$$E(r,t)=[E_{1J}^{(i)}\exp(\alpha_Jz)+E_{2J}^{(i)}\exp(-\alpha_Jz)]\times\exp[i(q_yy-\omega t)]\,, \tag{6}$$

where $J=A,B,C,D$. Analogous field solutions can be written for the magnetic field in the respective layers. The standard electromagnetic connections at the interfaces between different layers are the continuity of the tangential electric- and magnetic-field components E_x, E_y, B_x, and B_y. In the absence of the external magnetic field, only two conditions for these components are sufficient. As the procedure in Ref. 18 demonstrates, we match the field components E_y and B_x at the interfaces. We can further reduce the number of unknown amplitudes by expressing B_x in terms of E_y. Then, we write the connection conditions at the four interfaces of the ith unit as

$$\underline{M}_A(i)|A_i\rangle=\underline{N}_B(i)|B_i\rangle \quad \text{at } z=d_i+L_{iA} , \tag{7}$$

$$\underline{M}_B(i)|B_i\rangle=\underline{N}_C(i)|C_i\rangle \quad \text{at } z=d_i+L_{iA}+L_{iB} , \tag{8}$$

$$\underline{M}_C(i)|C_i\rangle=\underline{N}_D(i)|D_i\rangle \quad \text{at } z=d_i+L_{iA}+L_{iB}+L_{iC} , \tag{9}$$

$$\underline{M}_D(i)|D_i\rangle=\underline{N}_A(i)|A_{i+1}\rangle \quad \text{at } z=d_{i+1} , \tag{10}$$

where d_i is the z coordinate of the starting point of the ith unit, $|J_i\rangle$, where $J\equiv A,B,C,D$, are column vectors defined as

$$|J_i\rangle=\begin{bmatrix} E_{y1J}^{(i)} \\ E_{y2J}^{(i)} \end{bmatrix} , \tag{11}$$

and $\underline{M}_J(i)$ and $\underline{N}_J(i)$ are 2×2 matrices,

$$\underline{M}_J(i)=\begin{bmatrix} \exp(\alpha_J L_{iJ}) & \exp(-\alpha_J L_{iJ}) \\ n_J\exp(\alpha_J L_{iJ}) & -n_J\exp(-\alpha_J L_{iJ}) \end{bmatrix} , \tag{12}$$

$$\underline{N}_J(i)=\begin{bmatrix} 1 & 1 \\ n_J & -n_J \end{bmatrix} , \tag{13}$$

where

$$n_J=\varepsilon_J/\alpha_J , \quad J=A,B,C,D , \quad i=1,2,\ldots,N . \tag{14}$$

From Eqs. (7)–(10), we can see that

$$|A_{i+1}\rangle=\underline{T}_i|A_i\rangle , \tag{15}$$

where $\underline{T}_i$ is a transfer matrix defined by

$$\underline{T}_i=\underline{N}_A^{-1}(i)\underline{M}_D(i)\underline{N}_D^{-1}(i)\underline{M}_C(i) \times\underline{N}_C^{-1}(i)\underline{M}_B(i)\underline{N}_B^{-1}(i)\underline{M}_A(i) . \tag{16}$$

As the layer thicknesses are randomly distributed, there is no longer a periodic symmetry for the system. If a finite system, with $\underline{N}$ units, is attached to a single layer of species A at its last unit, then we can regard this layer as its $(N+1)$th A-material layer, and the amplitudes at the two ends are related as

$$|A_{N+1}\rangle=\underline{T}|A_1\rangle , \tag{17}$$

where

$$\underline{T}=\underline{T}_N\underline{T}_{N-1}\cdots\underline{T}_1 . \tag{18}$$

In order to obtain the dispersion relations of bulk modes for such a finite system, we look for frequencies where corresponding solutions $|A_i\rangle$ do not grow exponentially. This allows an imposed Bloch's ansatz to its ends such as[22]

$$|A_{N+1}\rangle=\exp(iQ)|A_1\rangle , \tag{19}$$

where Q is the phase difference of the two ends. If the total number of layers is large enough, such imposed periodic boundary conditions have only a little influence on the results, as have been indicated in the calculations for the quasiperiodic systems.[6] From Eqs. (17) and (19), we obtain the condition for existence of nontrivial field solutions as

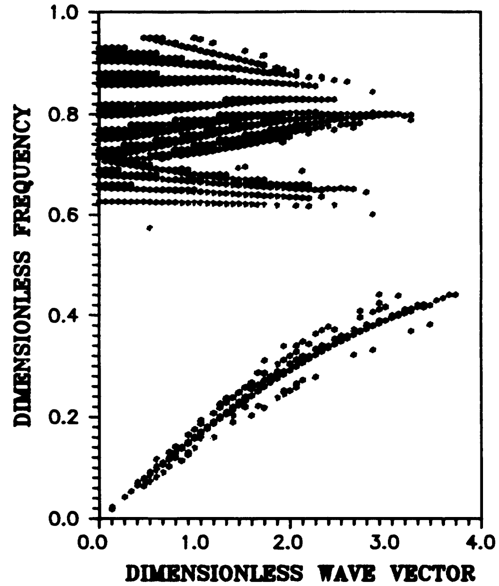

FIG. 2. Dispersion relation of plasma modes for sample with $Y_{1J}=Y_{2J}=0.25$ ($J\equiv A,B,C$), $Y_{1D}=0.45$, $Y_{2D}=0.05$, and $N=12$. Layer thicknesses, frequency, and wave vector are dimensionless as defined in the text.

$$|\mathrm{Tr}(\underline{T})|\le 2 . \tag{20}$$

Since $\underline{T}$ is a function of frequency and wave vector, from Eq. (20) we can calculate the dispersion relation of plasma modes.

III. NUMERICAL RESULTS OF DISPERSION RELATIONS

In this section we present the calculated dispersion relations for samples with different degrees of randomness.

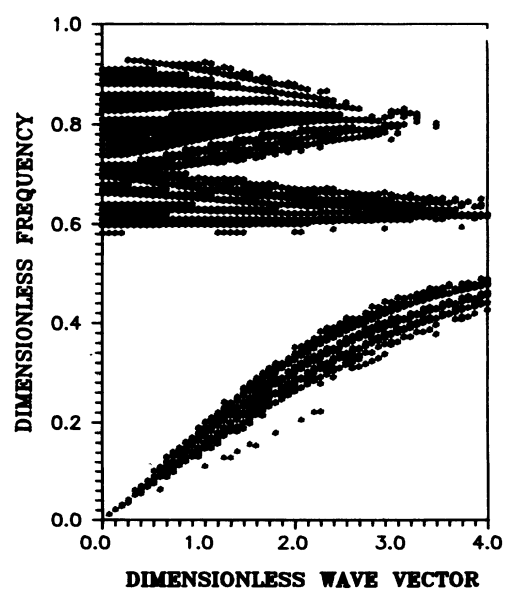

FIG. 3. Dispersion relation of plasmon modes for sample with $Y_{1J}=0.3$, $Y_{2J}=0.2$ ($J\equiv A,B,C,D$), and $N=12$.

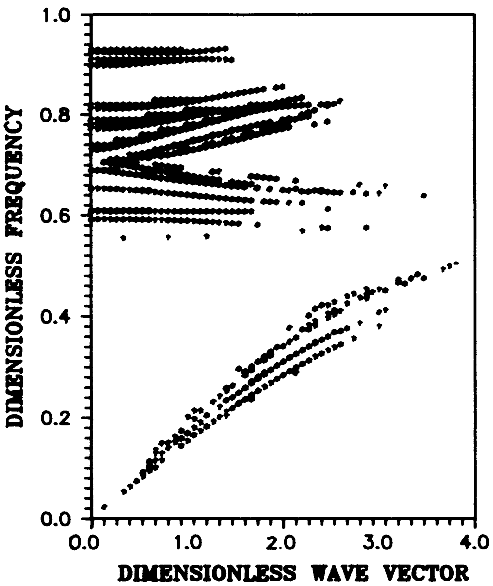

FIG. 4. Dispersion relation of plasma modes for sample with $Y_{1J}=0.45$, $Y_{2J}=0.05$ ($J\equiv A,B,C,D$), and $N=12$.

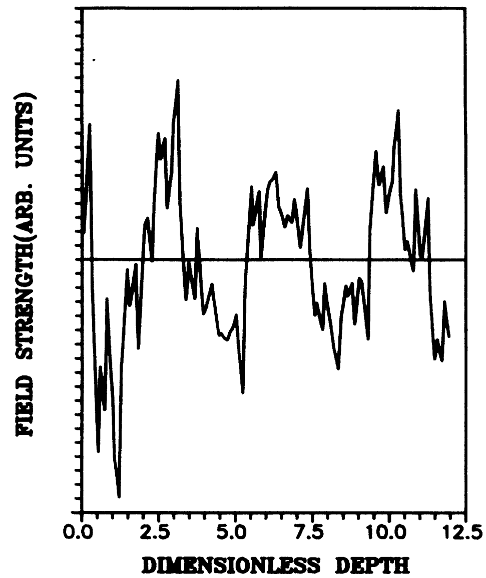

FIG. 6. Field strength of electromagnetic wave at frequency $\xi=0.9$ and wave vector $\zeta=1$ as a function of depth for the sample used in Fig. 3.

The mechanisms of the interplay between the fields and the materials are sketchily described by the macroscopic parameters ω_p and ε_l in Eq. (4). The parameter values used here are similar to those used in Refs. 18. The dielectric constants for layers A,B,C,D are ε_l ($=\varepsilon_{lA}=\varepsilon_{lB}=\varepsilon_{lC}=\varepsilon_{lD}$)$=13.13$, at the same time its value in vacuum is 1.0. The hole density in layer A and the electron density in layer B are the same, the effective mass of the hole is twice that of the effective mass of the electron, so that $\omega_{pC}=\omega_{pA}/\sqrt{2}$. The value of plasmon frequency in the intrinsic layers is zero: $\omega_{pB}=\omega_{pD}=0$. The results are plotted in terms of the dimensionless frequency $\xi\equiv\omega/\omega_{pA}$, the dimensionless wave vector $\zeta\equiv cq_y/\omega_{pA}$, and the dimensionless layer thicknesses $\delta_{iJ}\equiv\omega_{pA}L_{iJ}/c$.

In Figs. 2–4, we plot the calculated dispersion relations of the bulk modes of the plasmons for finite samples with different degrees of randomness in layer thicknesses. The result in Fig. 2 is for a sample with regular thicknesses of layer species A, B, and C, and thicknesses of intrinsic layers D randomly distributed between 0.05 and 0.45 (in dimensionless units). The results in Figs. 3 and 4 are for samples with thicknesses of all the layer species randomly distributed between 0.2 and 0.3, and be-

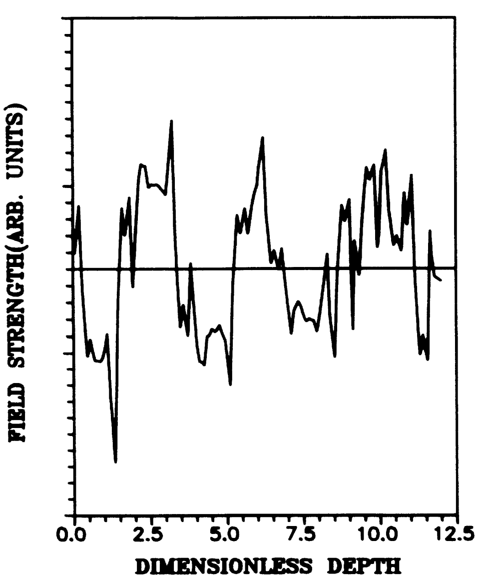

FIG. 5. Field strength of electromagnetic wave at frequency $\xi=0.9$ and wave vector $\zeta=1$ as a function of depth for the sample used in Fig. 2.

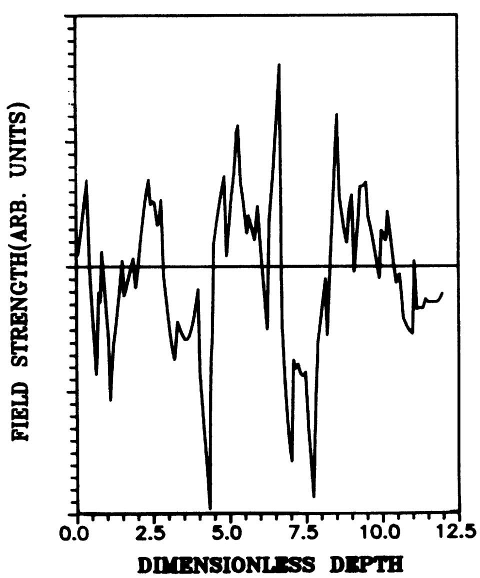

FIG. 7. Field strength of electromagnetic wave at frequency $\xi=0.9$ and wave vector $\zeta=1$ as a function of depth for the sample used in Fig. 4.

tween 0.05 and 0.45, respectively. The systems have the same average values of layer thicknesses, but different degrees of randomness. Meanwhile, the total number of layers are also the same. Because the periodic boundary condition at the two surfaces of the system is used, the obtained bulk bands represent the band structure of an infinite system whose period is just the total thickness of the finite sample. Thus, the total number of minibands is almost the same for these examples, but they are gathered together to form several separated groups. It can be seen from the figures that in varying the degree of randomness, the frequencies of the minibands are only slightly changed. This is consistent with the result reported by Johnson and Camley, in which the plasmon dispersion relation is insensitive to thickness variation of a single layer for a finite structure.[21] In the present case, however, some gaps between the miniband groups are certainly small, so the slight shift in frequencies of the minibands due to variation of randomness may cause an evident change in the structure of the gaps. Generally speaking, when the randomness increases, the gaps between the miniband groups become wider, and some isolated modes appear. To further illustrate the effect of randomness, we plot in Figs. 5–7 the field strength of a mode of electromagnetic wave at the frequency $\xi=0.9$ as a function of the depth for the three samples. It can be seen that the effect of randomness on the distribution of wave amplitudes is not very evident in such finite systems; the localization peaks appear only when the degree of randomness is very large (the third sample).

IV. OPTICAL TRANSMISSIONS

In this section we present some numerical and analytical results of the transmissions of electromagnetic waves through such a random structure. We consider the finite sample mentioned above imbedded in an infinite uniform medium of species A (n-doped semiconductor). This medium is divided by the sample into two semi-infinite parts, denoted as part 1 and part 2. The wave vector of the incident waves is still assumed to be in the y-z plane. Thus, the field component in these two parts can be written as

$$\mathbf{E}_y(\mathbf{r},t)=[\exp(iq_z z)+r\exp(-iq_z z)]\exp[i(q_y y-\omega t)] \quad \text{for } z\le d_1 ,$$
$$\mathbf{E}_y(\mathbf{r},t)=t\exp(iq_z z)\exp[i(q_y y-\omega t)] \quad \text{for } z\ge d_{N+1} , \tag{21}$$

where d_1 and d_{N+1} are the two ends of the sample, $q\equiv(0,q_y,q_z)$ is the wave vector, q_z is given by Eq. (5), and r and t are the amplitudes of reflective and transmissive waves, respectively. The amplitude of the incident wave is assumed to be unitary. Similarly to the above, the relations between these amplitudes can be obtained by using the continuity conditions at the interfaces. As the uniform medium is of the same species as layers A, there is no interface between the sample and the first part of the medium. We only need to consider $4N$ interfaces, including $(4N-1)$ interfaces within the sample and an interface between the sample and the second part of the medium. Since the continuity conditions for these interfaces are described by the transfer matrix $\underline{T}$, we have the relation

$$\begin{bmatrix} t \\ 0 \end{bmatrix} = \underline{T} \begin{bmatrix} 1 \\ r \end{bmatrix} . \tag{22}$$

From Eq. (22) and the fact that $\underline{T}$ is a unimodular 2×2 matrix, we obtain the transmission coefficient through this sample as

$$|t|^2=1/|T_{22}|^2 , \tag{23}$$

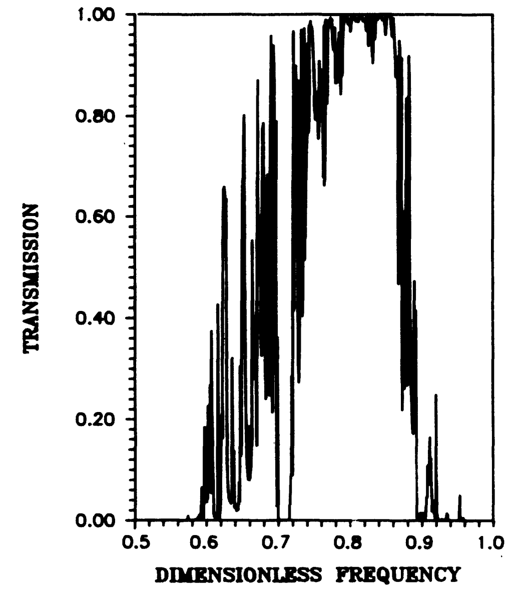

FIG. 8. Transmission of electromagnetic waves through a finite sample with $Y_{1J}=Y_{2J}=0.25$ ($J=A,B,C,D$), and $N=12$.

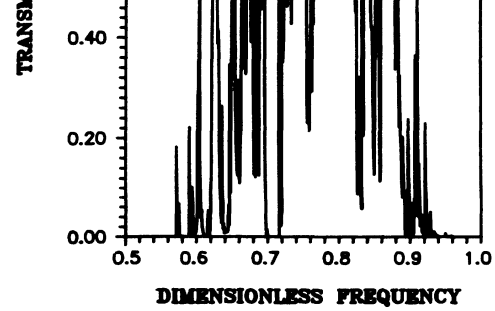

FIG. 9. Transmission of electromagnetic waves through a finite sample with $Y_{1J}=Y_{2J}=0.25$ ($J=A,B,C$), $Y_{1D}=0.45$, $Y_{2D}=0.05$, and $N=12$.

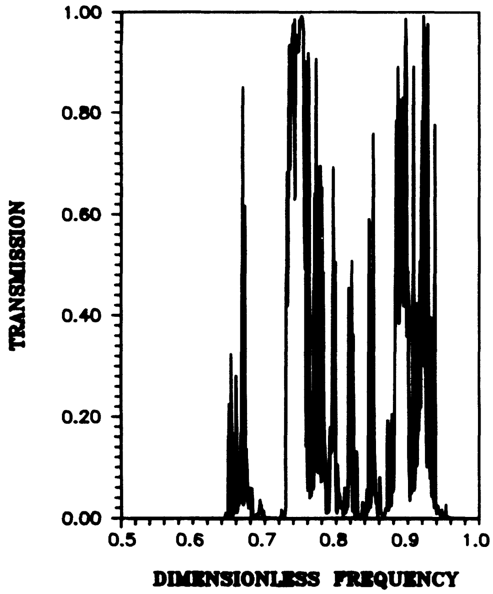

FIG. 10. Transmission of electromagnetic waves through a finite sample with $Y_{1J}=0.45$, $Y_{2J}=0.05$ $(J=A,B,C,D)$, and $N=12$.

where T_{22} is an element of matrix $\underline{T}$.

In Figs. 8–13 we plotted numerical result of the transmissions for several samples with different degrees of randomness and different numbers of total layers. The result shown in Fig. 8 is the transmission for a finite regular sample. As in usual periodic systems, it exhibits a band feature, the structures in the bands come from the finiteness of the system. The results in Figs. 9–11 are for samples where the randomness in layer thicknesses is introduced. It can be seen that the "transmission band" of the periodic sample is broken by the randomness. When

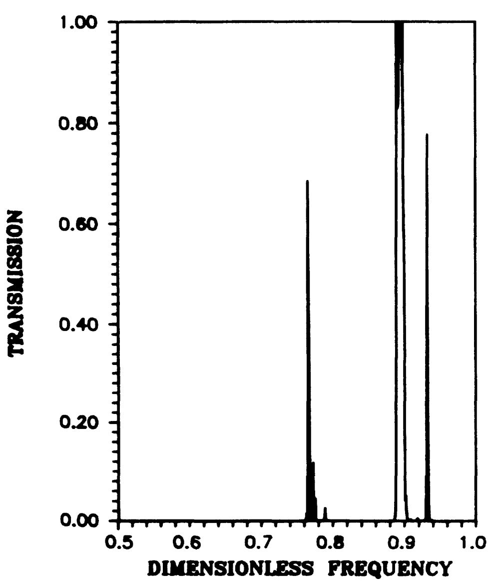

FIG. 11. Transmission of electromagnetic waves through a finite sample with $Y_{1J}=0.14$, $Y_{2J}=0.05$ $(J=A,B,C,D)$, and $N=12$.

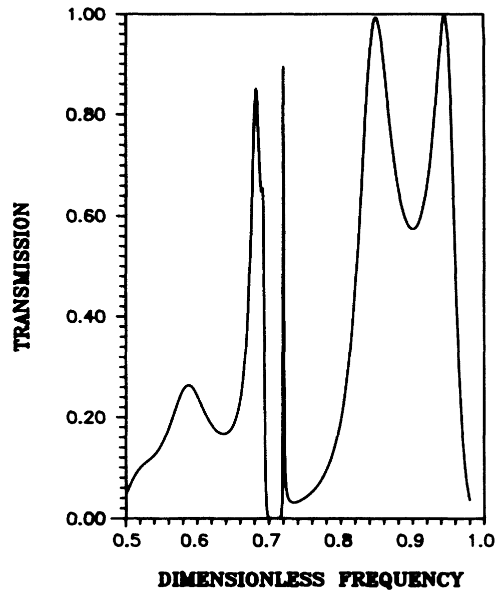

FIG. 12. Transmission of electromagnetic waves through a finite sample with $Y_{1J}=0.45$, $Y_{2J}=0.05$ $(J=A,B,C,D)$, and $N=4$.

the degree of the randomness increases, the effect is enlarged. Figure 11 shows that the peaks are narrower for samples with smaller average layer thicknesses. There are still some modes that are nearly unscattered by a large degree of randomness. This situation coincides with the conclusion for the existence of extended states in 1D disordered systems.[8,9] In the following, we will discuss it in detail in an analytic way. Figures 12 and 13 show the results for random samples with smaller numbers of total layers. It can be seen that curves become smooth when the size of the system is reduced. The re-

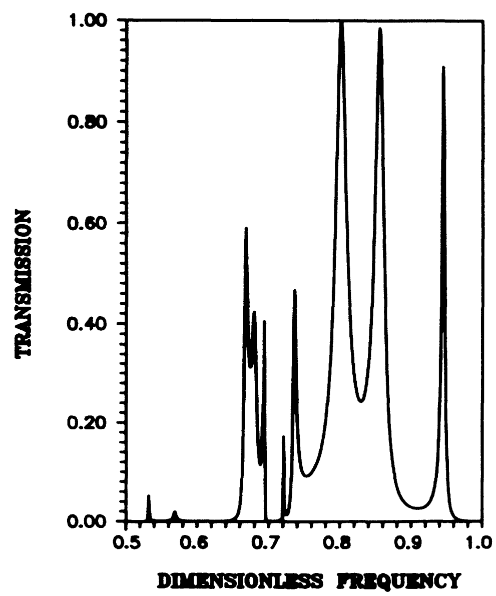

FIG. 13. Transmission of electromagnetic waves through a finite sample with $Y_{1J}=0.45$, $Y_{2J}=0.05$ $(J=A,B,C,D)$, and $N=8$.

sults plotted here are for the case where the uniform medium outside the sample is of species A. Similar results can be obtained if the medium is of other species.

Now we investigate analytically the transmission of electromagnetic waves through a special random structure, which is formed by randomly inserting a number of identical units into a uniform infinite medium of a species, namely, the intrinsic semiconductor. All the interfaces are parallel. One of the units consists of m repeated periods, and every period has four layers of D, A, B, and C, to form an i-n-i-p series. Then, the transfer matrix from layer D of a period to the i semiconductor adjacent to its other end is

$$\underline{T}^{(1)}=\underline{N}_D\underline{M}_C\underline{N}_C\underline{M}_B\underline{N}_B\underline{M}_A\underline{N}_A\underline{M}_D \ , \tag{24}$$

where $\underline{M}_J$ and $\underline{N}_J$ ($J\equiv A,B,C,D$) are given by Eqs. (12) and (13) with definite layer thicknesses L_J, respectively. The transfer matrix for a unit of m repeated periods is

$$\underline{T}^{(m)}=[\underline{T}^{(1)}]^m \ . \tag{25}$$

Because $\underline{T}^{(1)}$ is a unimodular 2×2 matrix, from a theorem illustrated in Ref. 23, we have

$$\underline{T}^{(m)}=u_{m-1}(x)\underline{T}^{(1)}-u_{m-2}(x)\underline{I} \ , \tag{26}$$

where $\underline{I}$ is a 2×2 unit matrix, $x\equiv\mathrm{Tr}[\underline{T}^{(1)}]/2$, and $u_m(x)$ is the mth Chebyshev polynomial of the second kind. When $|x|\le1$, it can be expressed as

$$u_{m-1}=\frac{\sin[m\cos^{-1}(x)]}{\sin[\cos^{-1}(x)]} \ . \tag{27}$$

If M such units are randomly inserted into a uniform intrinsic semiconductor, the transfer matrix through this random structure can be written as

$$\underline{T}_{\mathrm{whole}}=\underline{T}^{(m)}\underline{T}_1\underline{T}^{(m)}\underline{T}_2\underline{T}^{(m)}\cdots\underline{T}_{M-2}\underline{T}^{(m)}\underline{T}_{M-1}\underline{T}^{(m)} \ , \tag{28}$$

where $\underline{T}_i$ ($i\equiv1,2,\ldots,M-1$) is the intrinsic layer between the ith and the $(i+1)$th units, which has random layer thickness. If $m\ge2$ and there are some frequencies (denoted by $\omega_1,\omega_2,\ldots,\omega_n$) satisfying

$$x=\cos(l\pi/m) \ \text{ with } l=1,2,\ldots,m-1 \ , \tag{29}$$

from Eqs. (26)–(28) we have

$$\underline{T}_{\mathrm{whole}}=\pm\prod_{i=1}^{M-1}\underline{T}_i \ . \tag{30}$$

This means that the transmissions for these frequencies are just the transmission of the modes through a uniform intrinsic semiconductor. They are unity if the modes are within the bulk band of this uniform material. On the other hand, the frequencies satisfying condition (29) can always be found within the bulk bands of periodic n-i-p-i superlattice.[18] Thus, we can argue that for such a random structure there exist some plasmon modes that are completely unscattered. As the other modes are usually scattered by the randomness, we may use such a structure to produce high-quality optical filters.

V. SUMMARY

In this paper we have investigated the characteristics of the collective plasmon excitations of n-i-p-i semiconductor superlattices with randomly distributed layer thicknesses. The calculation of dispersion relation and optical transmission in finite samples shows that in varying the degree of thickness randomness, the frequencies of the plasmon modes exhibit a small shift, but the transmission coefficients are largely changed. Although the localization of the waves becomes visual only when the degree of randomness is very large, the numerical results of the optical transmissions show the strong scattering of the randomness for most of the modes of the electromagnetic waves. This means that the fluctuations in layer thicknesses in a finite system have only a small effect on the distribution of the wave amplitudes, but may cause large incoherence in the phases at the interface scattering. However, in special random structures, there is still a small portion of the modes which is nearly unscattered by the randomness. An interesting situation is analytically described in which the unscattered modes can be definitely found. This may be interesting for further investigations or possible applications.

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