Peak effect and anomalous flow behavior of a flux-line lattice

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Dynamics of a "quasi-two-dimensional" flux-line lattice in the layered superconductor 2H-NbSe₂ for field normal to the layers is investigated at high fields. Comparisons with numerical simulations imply the presence of plastic flow for soft lattices. Dimensionality effects are more pronounced in the disorder-dominated regime, where a power-law scaling form for the *I*-*V* curve yields the conductivity exponent above the percolation threshold.

The magnetic flux-line lattice (FLL) in a type-II superconductor has become a subject of renewed interest along two specific avenues of inquiry. First, in the context of high- T_c superconductors,¹ new equilibrium phases have been predicted² and new regimes of "thermal" dynamics³ have been postulated. Experimental evidence for these scenarios, or the lack thereof, has generated varying degrees of controversy. Second, the FLL is also a prototype of "driven" nonlinear dynamics of disordered many-body systems where randomness and interaction compete; in this case the focus is on nonequilibrium issues such as a depinning transition and the dynamics in the moving state. Transport studies often require a careful disentanglement of these two distinct issues. Indeed, experiments showing a change of mobility of an FLL have been variously described as equilibrium phenomena such as melting or nonequilibrium ones such as depinning without a resolution of the controversy as yet.¹

In order to understand this complex issue, we have focused on the remarkable phenomenon of the "peak effect," observed in conventional superconductors where the pinning force density reaches a pronounced peak slightly below H_{c2} .⁴ The effect is attributed to the rapid softening of the FLL (Ref. 5) and it occurs in the same part of the (H-T) phase diagram where phase transitions in the FLL are likely to occur. In spite of spawning important theoretical ideas such as collective pinning,⁶ it has eluded a comprehensive explanation so far. It was recently shown⁷ that the I-V curve changes drastically in the peak regime for a three-dimensional (3D) FLL, caused by a crossover of the dynamics from an interaction-dominated regime to a disorder-dominated one. The onset of motion was interpreted as a plastic flow of the FLL in the latter case, in a qualitative departure from descriptions in terms of an elastically distorted FLL.

In this paper we report new results on the dynamics of a highly coherent and weakly pinned quasi-2D FLL for the magnetic field normal to the layers⁸ in 2*H*-NbSe₂. The purpose of this study is to, first, understand the role of the effective dimensionality on the dynamics and, second, compare experimental results with theory and/or simulations. Extensive numerical simulations are available for the 2D FLL,^{9,10} in contrast with the 3D FLL.⁷ Our results show that the effective dimensionality has a stronger effect in the disorder-dominated regime and provide a testing ground for, e.g., some of the scenarios of FLL dynamics envisaged for the high- T_c superconductors.¹⁻³

Measurements were made on high-quality singlecrystal samples which show a marked peak effect. The relevant parameters¹¹ are $T_c = 7.2$ K, the in-plane Ginzburg-Landau parameter $\kappa \sim 9$, and the anisotropy in the upper critical field ~ 3.2 . Typical sample dimensions are $(1 \text{ mm} \times 2 \text{ mm} \times 25 \ \mu\text{m})$ and the normal-state resistivity above T_c is $\sim 5 \ \mu\Omega$ cm.

Figure 1 summarizes the observed behavior near the peak effect: the *H* dependence of the resistance R_s of the sample at two values of *T*, measured at a constant ac (100 Hz) current by the standard lock-in detection, shows a dramatic minimum in R_s , corresponding to a conductance peak (i.e., the "peak effect") at a field slightly below H_{c2} . The minimum occurs approximately at the reduced field $b (\sim H/H_{c2}) \sim 0.9$, for all values of *T*. The same effect is also observed when *T* is varied at a fixed value of *H*, as shown in the lower inset, i.e., the peak effect has a unique locus in the (H, T) space. The upper inset shows the strong nonlinearity of R_s : a twofold increase in *I* results in an order of magnitude increase in R_s at the



FIG. 1. Peak effect in 2H-NbSe for H perpendicular to the layers. The minimum in the resistance occurs slightly below H at both temperatures. The upper inset shows the nonlinearity of the effect through the strong current dependence of the resistance. The lower inset shows the same effect obtained at a fixed field and varying temperature.

minimum. This implies that the effect is related to an unusual H dependence in pinning.

From the *I-V* curves measured at various *H*, we can obtain J_c , the critical current density for the onset of flow (given by a voltage criterion of 100 nV). The resulting *H* dependences of $F_p(=|\mathbf{J}_c \times \mathbf{B}|)$ are shown in Fig. 2; the lower inset shows the variation of J_c , which is proportional to the pinning force per flux line. Both reach a pronounced peak at $b_p \sim 0.9$ and decrease rapidly to zero as H_{c2} is approached. The upper inset shows that the data for two temperatures can be collapsed to a single plot by normalizing F_p by $H_{c2}^{3/2}$, ¹² and the magnetic field by H_{c2} .

The first qualitative explanation of the peak effect is due to Pippard⁵ who argued that the shear modulus of the FLL softens faster than the depletion of the pinning interaction, as H approaches H_{c2} . Thus the competition between the pinning interaction and the elasticity of the FLL, results in an enhancement of the threshold pinning force. A more elaborate explanation was provided by Larkin and Ovchinnikov (LO) within the collective pinning model.⁶ Here we briefly described the essential elements of this model. The presence of random pins destroys the true long-range order of the FLL but it remains ordered over a correlation volume $V_c = R_c^2$, L_c , where R_c and L_c are the transverse and longitudinal correlation lengths, respectively. The pinning force and the correlation lengths are given by $F_p = |\mathbf{J}_c \times \mathbf{B}| = (W/V_c)^{1/2}$, $R_c = (C_{44}^{1/2}C_{66}^{3/2}r_f^2)/W$, and $L_c = (C_{44}/C_{66})^{1/2}R_c$, where $W = n_p \langle f^2 \rangle$; f is the elementary pinning interaction and n_p is the volume density of pins. C_{44} and C_{66} are the tilt and shear moduli, respectively, and r_f is the interaction range between the pinning center and the FLL. In the LO scenario, $F_p(H)$ is given by the field dependences of fand, more nontrivially, of the elastic moduli. This also shows that a softer FLL will be more strongly pinned as envisaged in the Pippard scenario. Note also that the critical current density is nearly six orders of magnitude smaller than typical superconductors. We obtain an or-



FIG. 2. The field dependence of the pinning force density F_p at two values of temperature. The field-independent regime at low fields is followed by the sharp peak and the rapid decrease to zero near H_{c2} . The upper inset shows that the data can be collapsed to a single curve by normalizing F_p by $H_{c2}^{3/2}$ plotted against the reduced field. The lower inset shows the peak effect in J_c , the critical current density. Note the extremely small value of J_c . See the text for discussions.

der of magnitude estimate, at, e.g., T=4.2 K and H=1 T, for the correlation length: $L_c \sim 400 \ \mu$ m, which is much greater than the sample thickness and thus the system is in the quasi-2D regime. In this regime the expression for R_c is different: $R_c \sim r_f C_{66} \{8\pi d / [W \ln(w/R_c)]\}^{1/2}$, where w is the width, and d the thickness of the sample. Using this expression we obtain an estimate of $R_c/a_0 \sim 400$, a_0 being the lattice constant, and the description of the system as a highly correlated FLL of rigid flux lines is well satisfied, in agreement with earlier work.⁸

We have measured the H dependence of the I-V curves. A systematic study near the peak regime is complicated by the presence of severe thermal instabilities. As a result, reliable data can be obtained only if the system is submerged in normal or superfluid ⁴He. In Fig. 3 we show the evolution of behavior in a narrow field regime, between 1.9 and 2.1 T, for T=4.2 K, of the differential resistance (dV/dI) versus I, which is more illustrative than the *I-V* curve. The evolution of the shape follows the same trend observed earlier⁷ for the 3D case: at fields below the peak, dV/dI grows monotonically, corresponding to a concave upwards V-versus-I curve, as is commonly seen in FLL's. In the peak regime, however, the shape changes to a peak in dV/dI slightly above the onset of motion. For H = 1.975 T, for example, this peak value is greater than the normal resistance and is thus a nonlinear effect and not an asymptotic flux flow resistance. At even larger fields, it returns to the earlier shape as is obvious in Fig. 3(a).

Simulations of flux flow for the 2D case have been performed by Jensen and co-workers⁹ and recently by Shi and Berlinsky (SB). ¹⁰ These results have striking similarities with our data. Figure 3(b) shows results of a numerical simulation of a 2D FLL by Shi and Berlinsky who



FIG. 3. (a) The current dependence of the differential resistance obtained from dc *I-V* curves at different fields near and in the peak regime. Note the rapid crossover from the conventional behavior for H=1.9 T to a peaked structure at H=2.0 T. (b) Current dependence of the differential resistance obtained from the 2D simulation of plastic flow in Ref. 10. Note the close similarity with data in (a) for H=1.975 T. The simulations show that the dynamically generated defects heal above the peak in dV/dI. See the text for discussions.

evaluated the *I-V* curve for one value of the quenched disorder. Given that the simulation is identical to our data for, say, H=1.975 T, there is little doubt that the essential physics of these experiments is captured in the simulations.

These authors^{9,10} point out that a purely elastic medium description of the FLL is not adequate. In their simulations they find that the interaction of the FLL with quenched disorder generates defects in the FLL and the onset of motion is due to the so-called plastic flow of the FLL in channels, rather than a coherent motion of an elastic medium. We note that the importance of dynamically generated defects ("phase slips") and plastic flow has been emphasized recently by Coppersmith¹³ in the case of the charge-density wave (CDW) conductors. SB (Ref. 10) have also obtained a measure of the dynamically generated defects from their simulations and find that they heal at large drives where pinning is very small. We find that in experiments, this corresponds to an inflection point in the *I-V* curves, i.e., a peak in dV/dI as in Fig. 3(a) (for, say, H = 2.0 T); the same result is obtained in the simulations.

Therefore we conclude that the dynamics of a disordered flux lattice in the quasi-2D regime is qualitative similar to the 3D case. Below the peak regime (e.g., in the *H*-independent part of F_p in Fig. 2) one obtains a coherent depinning of a largely defect-free elastic medium. In the peak regime the rapid increase in F_p is due to the appearance of plastic deformations in the FLL (Ref. 14) and the onset of motion is due to the filaments of connected paths through which the FLL moves.^{9,15} Above the peak regime the defective flow dominates the dynamics. In what follows we focus on the new insights on the role of disorder that may be obtained by a comparison of the effects of dimensionality on the FLL dynamics.

In order to accomplish this, it is essential that the I-Vcurves presented here be understood in some detail and be compared and contrasted with results obtained in 3D FLL's. Since little has been done in this area for the FLL's, we follow the analogous system of the CDW conductors which have been explored in much greater detail both theoretically and experimentally. Most of the CDW work starts with the basic premise of a dynamical critical phenomenon describing the depinning transition first proposed by Fisher.¹⁶ In this scenario a power-law scaling form is supposed to describe the *I*-*V* curve: $V \sim (I - I_c)^{\beta}$. Figure 4 represents typical results of these fits. We show two curves at T=4.2 K below and above the peak regime with $\beta \sim 1.2$ and 1.3, respectively. A power-law fit to the data is poor in the peak regime. For a comparison, a 3D FLL yields⁷ $\beta \sim 1.2$ and 1.8, respectively.

In order to understand the dimensionality effects in the interaction-dominated regime of flux flow we consider the simulations of the elastic model by Sibani and Littlewood (SL) (Ref. 17) for the CDW case. They yield an apparent exponent slightly greater than unity, nearly 1.2 for reduced forces as small as $\sim 10^{-2}$, typical for CDW experiments as well as what we report here. Although it appears not to be the true critical exponent, ¹⁶ one may view the power-law form as a useful parametrization. SL find that in this reduced force regime the apparent exponents



FIG. 4. Typical scaling plots for the *I-V* curves at fields below the peak regime (H=1.5 T) and above the peak regime (H=2.1 T) at T=4.2 K. For clarity, the latter data set is shifted up by one and a half decade in the reduced current.

vary only slightly, $\sim 15\%$ between 3D and 2D, which is not outside the uncertainties of our fitting procedure. The situation for the elastic regime for the FLL appears to be similar. The difference in the apparent exponents, if any, is small and not qualitatively different from what is expected for the CDW.

The situation is different and more interesting for the regime above the peak where disorder dominates and the FLL motion resembles a fluid flow.¹⁸ Since the density of pins typically exceeds that of the flux lines, the latter are likely to be individually pinned. In this situation the onset of voltage corresponds to the formation of the first end-to-end continuous path for the vortices to flow along. This path or filament then represents the path along which the maximum local pinning barrier has the smallest value of all possible paths. Upon further increase of the driving force, the flow along this path increases and at the same time other paths connect. Note that the scenario described above represents the conductance σ in a percolation process, also given by a power-law form: $\sigma \sim (p - p_c)^{\mu}$, where p_c is the percolation threshold and $\mu = 1.3$ in d = 2.¹⁹ Our measured exponent of 1.3 is in surprisingly good agreement with this naive analogy. Even in the 3D case the measured exponent of 1.8 is close to the expected value of 2.¹⁹ While the 2D case may reasonably be a simple percolation problem, the 3D case could be different due to processes such as vortex cutting and/or entanglement, etc., and the resulting topology of defects and of connectivity. Although more work is needed to understand if the agreement is fortuitous or not, the results described here suggest a new way to view the FLL dynamics in the disorder-dominated "fluid flow" regime.¹⁸

To conclude, we have observed anomalous I-V curves in the peak regime in a clean and extremely weak-pinning quasi-2D FLL. The results are in excellent agreement with numerical simulations^{9,10} that take into account dynamically generated defects of plastically distorted FLL's. We also find that the nonequilibrium phase diagram is similar to the 3D case.⁷ The flow behavior in the interaction-dominated regime is weakly dependent on dimensionality in the reduced force regime studied in this work. This is not inconsistent with simulations of elastic media. But in the disorder-dominated plastic flow regime, the difference is large. Interestingly, we find that the apparent scaling exponents are close to the conductivity exponents for percolation. More work is needed to explore this interesting possibility. These results exemplify the importance of dynamically generated defects on the flux flow problem in particular and in driven nonlinear many-body systems in general.^{9,10,13}

Note added. It has been brought to our attention^{20,21} that an essentially identical peak effect occurs in highquality single crystals of the high- T_c superconductor YBa₂Cu₃O_{7- δ}. Moreover, anomalous *I-V* curves also obtain.²¹ These results imply that the FLL dynamics is

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similar in fundamental ways in the high- T_c systems as well. We thank Xinsheng Ling and W. K. Kwok for bringing these results to our attention.

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