RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 48, NUMBER 13 1 OCTOBER 1993-I

Minimum-energy vortex configurations in anisotropic superconductors

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(Received 4 June 1993)

In a type-II superconductor, the optimal vortex configuration is the one that minimizes the free energy for a given value of **B**, the spatial average of the magnetic field. Using anisotropic London theory, we compare the free energy of two very different configurations: (I) a lattice of vortex lines parallel to **B** and (II) a combination of a lattice of lines parallel to $\hat{\mathbf{c}}$ and a lattice of lines parallel to $\hat{\mathbf{a}}$, producing the same **B** as in (I). For large enough anisotropy, configuration (II) can have the lower free energy, for a wide range of **B**.

I. INTRODUCTION

The structure of the vortex lattice in highly anisotropic type-II superconductors is a topic that has received much theoretical attention in recent years, chiefly due to interest in the high-T_c oxide compounds. When the average magnetic field **B** is not parallel to one of the crystal symmetry axes, finding the configuration of vortices that minimizes the free energy can be a nontrivial task. For example, at low B vortex "chain" phenomena turn out to be important.[1–4]

Several recent papers have explored the idea that a lattice of vortices, with each vortex line parallel to **B**, is not necessarily the optimal configuration. Daemen *et al.*,[5] using London theory, show that when the sample is a finite slab, a mixture of two types of vortices, neither one parallel to **B**, can have a lower free energy. Bulaevskii, Ledvij, and Kogan,[6] using the Lawrence Doniach model, claim that when **B** is almost parallel to the *a*-*b* plane, a "combined lattice" consisting of vortices parallel to $\hat{\mathbf{c}}$ and vortices parallel to $\hat{\mathbf{a}}$ can be optimal. Sardella and Moore[7] perform a stability analysis on a lattice of vortex lines parallel to **B**, and find that in some cases an unstable mode exists.

In this paper we use the London theory in an infinite medium; thus none of our results depend on boundary effects or on additional physics added by the Lawrence Doniach model. Our goal is to minimize the free energy at fixed **B**. We compare two possibilities.

Case I. An array of vortex lines parallel to **B**, arranged in the Bravais lattice shown in Fig. 1.

Case II. A lattice of vortex lines parallel to $\hat{\mathbf{c}}$ plus a lattice of vortex lines parallel to $\hat{\mathbf{a}}$, producing the same **B** as in case I.

We find that for large enough anisotropy case II can have a lower free energy for a whole range of angles and field strengths. Of course there are many other interesting states to consider besides cases I and II. In this paper we simply want to make clear that case I is not always the best, even in a simple London theory calculation with no boundary effects.

II. LONDON THEORY

To do our calculation we use the London formalism as described in our previous paper.[4] We treat an anisotropic superconductor, with mutually perpendicular symmetry axes $\hat{\mathbf{a}}$, $\hat{\mathbf{b}}$, and $\hat{\mathbf{c}}$; the effective mass tensor is given by

$$M_{ij}=M_c\hat{c}_i\hat{c}_j+M_a(\hat{a}_i\hat{a}_j+\hat{b}_i\hat{b}_j)\ , \tag{1}$$

normalized so that det $(M)=M_cM_a^2=1$. The average penetration depth is λ, and an anisotropy parameter γ is defined by

$$\gamma=(M_c/M_a)^{1/2}\ . \tag{2}$$

In this paper we are only interested in the case $\gamma>1$.

The free energy is given by

$$F=\frac{1}{8\pi}\int d^3x[h^2+\lambda^2(\nabla\times\mathbf{h})\overleftrightarrow{M}(\nabla\times\mathbf{h})]\ . \tag{3}$$

One crucial result, which can easily be seen using (3), is that the interaction energy between a vortex line parallel to $\hat{\mathbf{c}}$ and a vortex line parallel to $\hat{\mathbf{a}}$ is zero. Thus when we calculate the free energy for case II, we can simply add together the energy of each vortex lattice. So for either case I or case II we need the energy density of a lattice of vortex lines, parallel to an arbitrary axis which we denote by $\hat{\mathbf{z}}$. We take $\hat{\mathbf{z}}$ to be at an angle θ with respect to the $\hat{\mathbf{c}}$

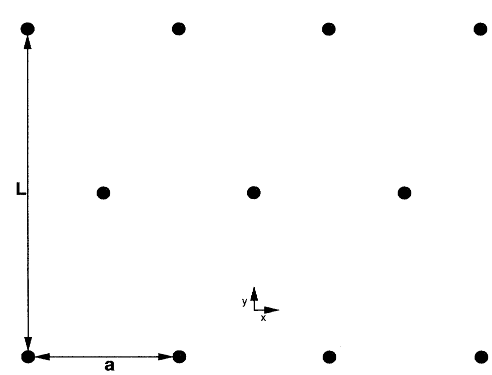

FIG. 1. The vortex line Bravais lattice in the *xy* plane.

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axis:

$$\hat{\mathbf{z}}=\hat{\mathbf{c}}\cos\theta+\hat{\mathbf{a}}\sin\theta\ , \quad (4)$$

$$\hat{\mathbf{x}}=-\hat{\mathbf{c}}\sin\theta+\hat{\mathbf{a}}\cos\theta\ , \quad (5)$$

$$\hat{\mathbf{y}}=\hat{\mathbf{b}}\ . \quad (6)$$

For case II we need to combine the answers for $\theta=0°$ and $\theta=90°$.

The vortices form a Bravais lattice, with a two-point basis, in the xy plane. The lattice vectors are given by

$$\mathbf{R}=n_1 a\hat{\mathbf{x}}+n_2 L\hat{\mathbf{y}}\ ; \quad (7)$$

vortices are located at the point $\mathbf{R}\pm\beta$, where β is given by

$$\beta=\frac{a}{4}\hat{\mathbf{x}}+\frac{L}{4}\hat{\mathbf{y}}\ . \quad (8)$$

The free energy density is then given by[4]

$$F(a,L,\theta)=\frac{\Phi_0^2}{2\pi a^2L^2}\sum_{\{\mathbf{G}\}}\phi(\mathbf{G})\frac{[1+\lambda_1^2G^2(1+\varepsilon\cos^2\theta)]\cos^2(\mathbf{G}\cdot\beta)}{[1+\lambda_1^2G^2][1+\lambda_1^2G_y^2(1+\varepsilon)+\lambda_1^2G_x^2(1+\varepsilon\cos^2\theta)]}\ . \quad (9)$$

Here, the reciprocal lattice vectors $\mathbf{G}$ are given by

$$\mathbf{G}=n_1\frac{2\pi}{a}\hat{\mathbf{x}}+n_2\frac{2\pi}{L}\hat{\mathbf{y}}\ . \quad (10)$$

The average magnetic field is parallel to $\hat{\mathbf{z}}$, and given by

$$\frac{1}{aL}\int\int_{\text{cell}}dx\,dy\,h_z=2\Phi_0\frac{1}{aL}\ . \quad (11)$$

We have introduced a cutoff function $\phi(\mathbf{G})$, since otherwise the sum over $\mathbf{G}$ diverges at large $|\mathbf{G}|$. This reflects the breakdown of London theory at short length scales. We follow the suggestion of Brandt,[8] and use an anisotropic Gaussian

$$\phi(\mathbf{G})=\exp\{-\alpha[G_x^2\xi_x^2+G_y^2\xi_y^2]\}\ . \quad (12)$$

Here α is a numerical parameter, of the order of unity, and the coherence lengths are defined by

$$\frac{\xi_x^2}{\xi_y^2}=\cos^2\theta+\frac{1}{\gamma^2}\sin^2\theta \quad (13)$$

and

$$\xi_y^2=\gamma^{2/3}\frac{\lambda^2}{\kappa^2}\ . \quad (14)$$

Here κ is an averaged Ginzburg-Landau parameter. We will usually take $\kappa=60$ and $\alpha=0.02$.

We define $\mathcal{F}(B,\theta)$ in the following way:

$$\mathcal{F}(B,\theta)=\min_{a,L,aL=2\Phi_0/B}F(a,L,\theta)\ . \quad (15)$$

That is, we adjust a and L to minimize (9), subject to the condition that $B=2\Phi_0/aL$.

III. RESULTS

We now express the energy densities for case I and case II in terms of the $\mathcal{F}(B,\theta)$ of Eq. (15). We take the mag-

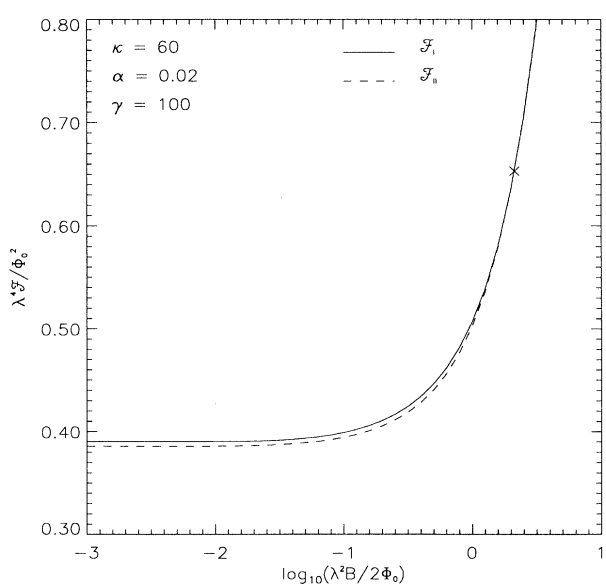

FIG. 2. Free energy as a function of B, for case I and case II. The magnetic field tilting angle is $\beta=60°$. Parameter values are $\kappa=60$, $\alpha=0.02$, and $\gamma=100$. The two curves intersect at $\log_{10}(\lambda^2B/2\Phi_0)\approx0.03$, which is indicated by the cross.

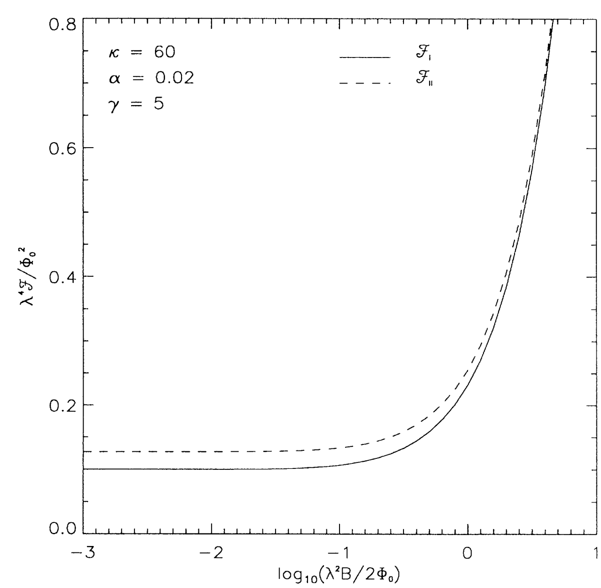

FIG. 3. Free energy as a function of B, for case I and case II. The magnetic field angle is $\beta=60°$. Parameter values are $\kappa=60$, $\alpha=0.02$, and $\gamma=5$.

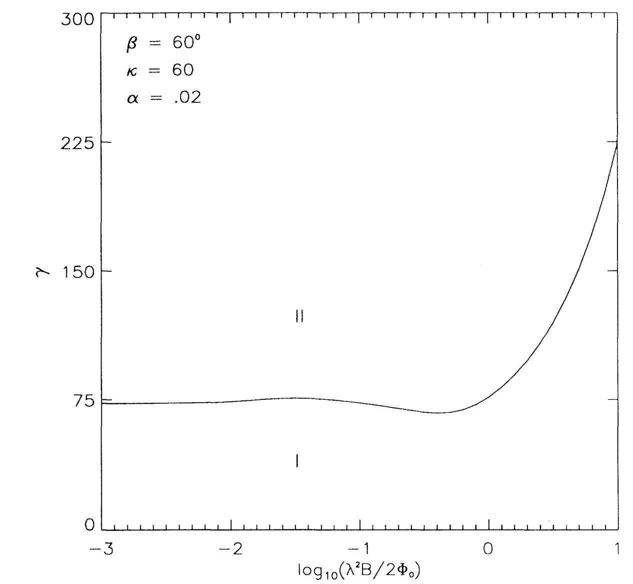

FIG. 4. Phase diagram in the γ-B space. In the region labeled II, configuration II has the lower free energy; in the region labeled I, configuration I has the lower free energy. The magnetic field angle is $\beta=60°$, while the other parameters are given by $\alpha=0.02$ and $\kappa=60$.

netic field $\mathbf{B}$ to be

$$\mathbf{B}=B_a\hat{\mathbf{a}}+B_c\hat{\mathbf{c}}=B\sin\beta\hat{\mathbf{a}}+B\cos\beta\hat{\mathbf{c}} \ , \tag{16}$$

$$B=\sqrt{B_a^2+B_c^2} \ . \tag{17}$$

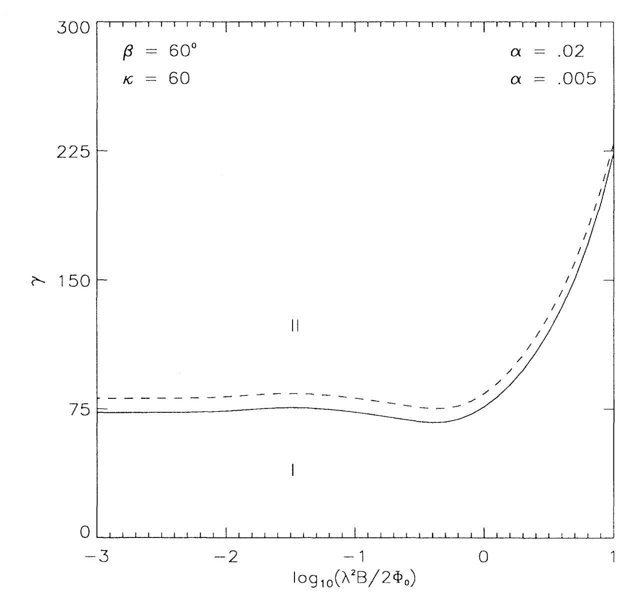

FIG. 5. This figure illustrates the effect of changing the cutoff parameter α on the γ-B phase diagram. Two phase boundaries are shown, one for $\alpha=0.02$ and one for $\alpha=0.005$. Other parameter values are $\beta=60°$ and $\kappa=60$.

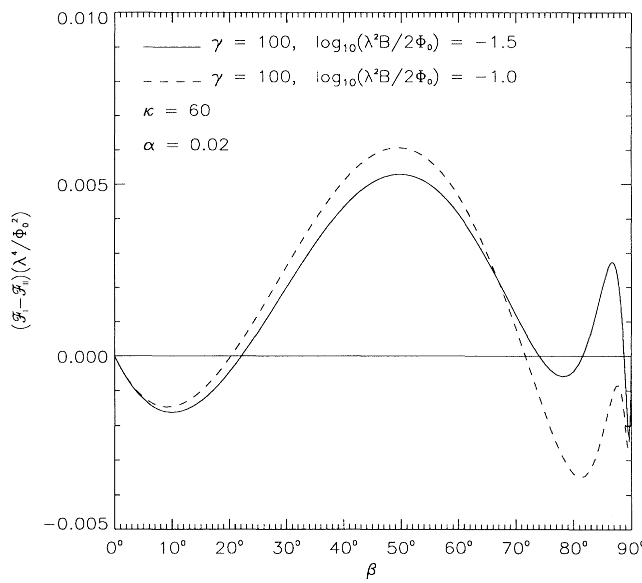

FIG. 6. The free energy difference, $\mathcal{F}_{\rm I}-\mathcal{F}_{\rm II}$, as a function of the angle β. Curves for two different values of B are shown. Parameter values are $\kappa=60$ and $\alpha=0.02$.

For case I, then, we have

$$\mathcal{F}_{\rm I}(B,\beta)=\mathcal{F}(B,\beta) \ . \tag{18}$$

For case II we have

$$\mathcal{F}_{\rm II}(B,\beta)=\mathcal{F}(B\cos(\beta),\theta=0°)+\mathcal{F}(B\sin(\beta) \ ,\theta=90°) \ . \tag{19}$$

So the expression in (19) is the sum of two terms: a vor-

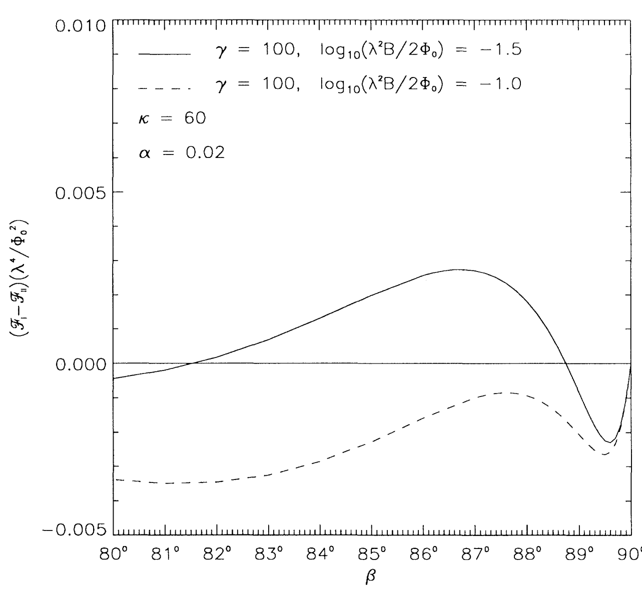

FIG. 7. Magnified version of Fig. 6, showing the region from $\beta=80°$ to $\beta=90°$ in more detail. For both curves the free energy difference reaches zero at $\beta=90°$, as it should.

tex lattice parallel to the $\hat{\mathbf{c}}$ axis with magnetic field $B\cos(\beta)$, and a vortex lattice parallel to the $\hat{\mathbf{a}}$ axis with magnetic field $B\sin(\beta)$.

In Fig. 2 we show a plot of $\mathcal{F}_{\rm I}$ and $\mathcal{F}_{\rm II}$ as functions of B, for $\beta=60°$ and $\gamma=100$. The noteworthy feature is that $\mathcal{F}_{\rm II}$ is actually slightly lower than $\mathcal{F}_{\rm I}$ up to $\log_{10}(\lambda^2 B/2\Phi_0)\approx 0.03$. Figure 3 shows the same plot for $\gamma=5$. At this much lower value for the anisotropy parameter, $\mathcal{F}_{\rm I}$ is always lower. Finally in Fig. 4 w show the phase diagram in the γ-B space, with $\beta=60°$. There is a large region, at higher γ and lower B, where $\mathcal{F}_{\rm II}$ is lower than $\mathcal{F}_{\rm I}$. We thus find, perhaps surprisingly, that the combination of a 0° vortex lattice and a 90° vortex lattice can have the lower free energy, at fixed **B**.

We can test the sensitivity of our result to the cutoff parameter. In Fig. 5, we show the phase diagram, for $\beta=60°$, for two different values of the cutoff parameter α. Changing α by a factor of 4 simply causes a small shift in the phase diagram.

It is also of interest to investigate the range of angle β over which $\mathcal{F}_{\rm II}$ is lower than $\mathcal{F}_{\rm I}$. In Fig. 6 we plot $\mathcal{F}_{\rm I}-F_{\rm II}$ as a function of β, for fixed γ and $|\mathbf{B}|$. We see that $\mathcal{F}_{\rm II}$ is lower over a wide range of angles, for the two values of $|\mathbf{B}|$ chosen. We also note that the two curves display interesting structure near 90°; this structure is shown in more detail on the expanded plot of Fig. 7.

IV. DISCUSSION

Our results show that, in the framwork of a well-defined London theory with a cutoff, a combination of two perpendicular vortex lattices can have a lower free energy than a single vortex lattice with the same **B**. This provides another example of the surprising features arising from vortices in anisotropic superconductors.

As can be seen from Fig. 1, at $\gamma=100$ case II is lower in free energy than case I by only a small amount. It is therefore important to go beyond London theory, and treat the vortex energetics with a more accurate theory; in particular, an approach which handles vortex core effects more carefully, such as the Ginzburg-Landau theory, should be applied. We hope to turn to this work in the future.

ACKNOWLEDGMENTS

This work was supported by the National Science Foundation under Grant No. DMR88-09854 through the Science and Technology Center for Superconductivity, and by DOE Grant No. DE-FG02-90ER45427 through the Midwest Superconductivity Consortium.

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