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Triplet superconductive pairing in the lowest Landau level: Comparison of superconductive and charge-density-wave critical temperatures

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An analog of triplet superconducting pairing in the lowest Landau level is proposed. The pairing occurs between electrons with the same spin projection along the axis of the magnetic field. The self-consistency equation for a three-dimensional system is written in a closed form. The mean-field solution forms the Abrikosov lattice. We obtain T_c for a general nonretarded interaction and compare it with the critical temperature for a charge-density-wave transition. For certain interactions we show that the superconductive transition has a higher mean field T_c than the charge-density-wave one.

Recently, Tešanović and co-workers[1] have shown that the reentrance of superconductivity takes place at high magnetic fields. Within the mean-field theory, the critical temperature $T_c(H)$ of a superconductor in the quantum limit (QL), where all electrons are in the lowest Landau level (LLL), can be as high as T_{c0}. The self-consistent solution for $T < T_c(H)$ was found in Refs. 2 and 3, where it was shown that the order parameter forms the Abrikosov lattice.[4]

Previously the case of singlet pairing has been treated. Dukan, Andreev, and Tešanović[2] and Akera *et al.*[3] considered the case when the effective g factor, the ratio of the electron's magnetic moment to $e\hbar/m^*c$ (m^* being the effective of electron) is small. This is a good approximation in many low carrier density systems. Norman, Akera, and MacDonald[5] also considered the case where the effective g factor is close to 2. In such systems electrons in the Nth spin-up Landau level pair with electrons in the $N+1$ spin-down Landau level. In this paper we show that in the QL pairing into a triplet state should occur. It should be noted that such pairing does not necessitate any restrictions on the effective g factor. We also compare the critical temperatures for the superconductive and the charge-density-wave (CDW) transition.

We consider a three-dimensional system of interacting electrons in a magnetic field strong enough that only the lowest Landau level states need be considered. The Zeeman splitting is assumed to be of the same order as the separation between Landau levels, so only the states with one spin polarization are occupied. We further assume that the Fermi momentum of this band is small compared to the inverse magnetic length, $p_F \ll l^{-1} = (\frac{eH}{\hbar c})^{1/2}$, and that the interaction is weak compared to the Fermi energy and the cyclotron frequency. Below we put magnetic length and Planck's constant equal to unity. We will work in the Landau gauge $A_y = Hx$, $A_x = A_\zeta = 0$. The annihilation operator can be written as $\Psi(\zeta, z) = \int \frac{dp}{2\pi} \sum_k \exp(ip\zeta)\phi_k(z)a_{p,k}$, where $z = x - iy$ is the complex coordinate perpendicular to the field, ζ is the coordinate along the field, $\phi_k(z) = L^{-1/2}\exp[iky-(x+k)^2/2]$ and $a_{p,k}$ is the operator annihilating a particle in a state with quantum numbers k and p.

An important aspect of the system can be seen if we rewrite the interaction Hamiltonian in terms of $\tilde{V}(p_3, q) = \exp(-q^2/2)\int \exp(-ip_3\zeta - iq_x x - iq_y y)V(\mathbf{r})d\mathbf{r}$ (Ref. 6)

$$H = \frac{1}{2}\sum_{k_1,k_2}\int \frac{dp_1dp_2dp_3}{(2\pi)^3}\int \frac{d^2q}{(2\pi)^2}\tilde{V}(p_3,q)\exp[iq_x(k_1-k_2-q_y)]a^\dagger_{p_1,k_1}a^\dagger_{p_2,k_2}a_{p_2-p_3,k'_2}a_{p_1+p_3,k'_1} \quad (1)$$

where $k'_1 = k_1 - q_y$ and $k'_2 = k_2 + q_y$. We assume that the interaction is short ranged with the range shorter or equal to l. Because $k_F \ll l^{-1}$ for momentum transfers of order $2k_F$ we can take $\tilde{V}(p_3, q)$ to be independent of p_3 and equal to its value $\tilde{V}(q) = \tilde{V}(0, q)$ at $p_3 = 0$. If we rewrite our interaction potential as $\tilde{V}(q) = \frac{1}{2\pi}\int \exp(-i\mathbf{ql})\tilde{V}'(l)d\mathbf{l}$ where $\tilde{V}'(q) = \frac{1}{2\pi}\int \tilde{V}(p)\exp(i\mathbf{pq})d\mathbf{p}$ then after very simple transformations and changes in notation the interaction Hamiltonian can be rewritten in terms of $\tilde{V}'(q)$ as[7,8]

$$H = \frac{1}{2}\sum_{k_1,k_2}\int \frac{dp_1dp_2dp_3}{(2\pi)^3}\int \frac{d^2q}{(2\pi)^2}\tilde{V}'(q)\exp[iq_x(k_1-k_2-q_y)]a^\dagger_{p_1,k_1}a^\dagger_{p_2,k_2}a_{p_2-p_3,k'_1}a_{p_1+p_3,k'_2} \; . \quad (2)$$

Taking half the sum of Eqs. (1) and (2) and noting that both $\tilde{V}(q)$ and $\tilde{V}'(q)$ are independent of p_3 we obtain that the effective interaction potential is of the following form $\tilde{U}(q) = 1/2[\tilde{V}(q) - \tilde{V}'(q)]$. If we expand $\tilde{V}(q)$ in terms of Haldane pseudopotentials[9] $\tilde{V}(q) = 4\pi\sum_m V_m L_m(q^2)\exp(-q^2/2)$ then we can write the effective potential as $\tilde{U}(q) = 4\pi\sum_m V_{2m+1}L_{2m+1}(q^2)\exp(-q^2/2)$. Thus we see that in the limit of small k_F only odd Haldane pseudopotentials matter.[7,8] We will show that if at least one of the Haldane pseudopotentials in this expansion is negative the system is unstable with respect to formation of a superconducting state.

We now turn to treating this superconductive instability. The pairing will occur into a state which is uniform along the field. If we introduce the Fourier transform of the field operators $\Psi_p(z) = \int \exp(-ip\zeta)\Psi(\zeta, z)$ the generalized Hartree Hamiltonian is as follows:

$$H = \int \frac{dp}{2\pi} d^2 z \Psi_p^\dagger(z) \left(\frac{p^2}{2m} - \mu\right) \Psi_p(z) + \frac{1}{2}\int \frac{dp}{2\pi} d^2 z_1 d^2 z_2 [\Psi_p(z_1)\Psi_{-p}(z_2)\Delta^*(z_1, z_2) + \text{H.c.}] \tag{3}$$

where $\Delta(z_1, z_2) = \hat{P}_{\rm LLL} \int \frac{dp}{2\pi} V(|z_1 - z_2|) F_p(z_1, z_2)$ is the off-diagonal pairing potential [$F_p(z_1, z_2) = \langle \Psi_p(z_1)\Psi_{-p}(z_2)\rangle$ here is the anomalous Gor'kov average and $\hat{P}_{\rm LLL}$ is the projector to LLL for both z_1 and z_2] and $V(|z_1 - z_2|) = \int d\zeta V(r)$ is the zeroth Fourier component of the interaction between electrons along the ζ axis.

Pairing occurs into a state which forms an Abrikosov lattice in the plane perpendicular to the magnetic field. For simplicity the unit cell of this lattice is taken to be a square, its side being $a = \pi^{1/2} l$. The electronic unit cell is twice as big and is spanned by vectors $\mathbf{e}_1 = (a, 0)$ and $\mathbf{e}_2 = (0, 2a)$. If we introduce the center-of-mass coordinate $Z = X - iY = (z_1 + z_2)/2$ and relative coordinate $z = x - iy = z_1 - z_2$ of a Cooper pair the order parameter can be rewritten in the following form:

$$\Delta(Z, z) = \sum \frac{\Delta_m}{2^{m+1}\sqrt{\pi m!}} z^m \exp\left(-|z|^2/8 - ixy/4\right) \times \sum_n \exp[-(X + na)^2 + i2naY] \tag{4}$$

where Δ_m is the amplitude of the mth partial wave and the sum goes over odd m only. As a function of the center-of-mass coordinate it forms a square lattice and as a function of relative coordinate it corresponds to a superposition of states with relative angular momenta m gauge transformed to the Landau gauge. Note that the partial waves here are different from those introduced by Akera *et al.*[3] where they refer to the Landau level of relative motion rather than angular momentum.[10] In general (if one pairs electrons on different LLs) the coupling channel is characterized by the Landau level and the angular momentum of the relative motion. In our model the question of Landau level never arose because all electrons were in the LLL and, therefore, coupling was bound to occur in the LLL for relative motion.

The electronic wave functions are characterized by a momentum p along the ζ axis and an in-plane quasimomentum $\mathbf{q}$ corresponding to the magnetic translation group.[11] The eigenfunctions of magnetic translations are given by[6]

$$\psi_{p,\mathbf{q}}(z) = \exp(ip\zeta)\frac{a^{1/2}}{S^{1/2}\pi^{1/4}} \times \sum_n \exp[-\tfrac{1}{2}(x + q_y + na)^2 + iy(q_y + na) + inaq_x] \tag{5}$$

where S is the area of the system and L_ζ is its length along the field.

The Bogoliubov transformation diagonalizing the Hamiltonian (3) can be written as

$$\Psi(\zeta, z) = \sum_{\mathbf{q}} \int \frac{dp}{2\pi}[u_{p,\mathbf{q}}\hat{c}_{p,\mathbf{q}}\psi_{p,\mathbf{q}}(z) + v^*_{p,\mathbf{q}}\hat{c}^\dagger_{p,\mathbf{q}}\psi_{-p,-\mathbf{q}}(z)] \tag{6}$$

where the sum over $\mathbf{q}$ goes over the magnetic Brillouin zone. In this representation the Bogoliubov–de Gennes equations for different p and $\mathbf{q}$ decouple and reduce to

$$\begin{aligned} (\xi - E_{p,\mathbf{q}})u_{p,\mathbf{q}} + \Delta(\mathbf{q})v_{p,\mathbf{q}} &= 0, \\ \Delta^*(\mathbf{q})u_{p,\mathbf{q}} + (-\xi - E_{p,\mathbf{q}})v_{p,\mathbf{q}} &= 0, \end{aligned} \tag{7}$$

where $\xi = \frac{p^2}{2m} - \mu$, $E_{\mathbf{q}} = \sqrt{\xi^2 + \Delta(\mathbf{q})^2}$, and the matrix element $\Delta(\mathbf{q})$ is given by

$$\Delta(\mathbf{q}) = \int d^2 Z d^2 z \psi^*_{p,\mathbf{q}}(Z + z/2)\psi^*_{-p,-\mathbf{q}}(Z - z/2)\Delta(Z, z)\,. \tag{8}$$

The general structure of the anomalous average is $F(z_1, z_2) = \sum_{\mathbf{q}} \int \frac{dp}{2\pi} F_{p,\mathbf{q}}\psi_{p,\mathbf{q}}(\zeta_1, z_1)\psi_{-p,-\mathbf{q}}(\zeta_2, z_2)$, where $F_{p,\mathbf{q}} = u^*_{p,\mathbf{q}}v_{p,\mathbf{q}}[1 - 2n_f(p, \mathbf{q})]$ [$n_f(p, \mathbf{q}) = \frac{1}{1+\exp(E_{p,\mathbf{q}}/T)}$].

After a little algebra one can show that $\psi_{p,\mathbf{q}}(\zeta_1, z_1)\psi_{-p,-\mathbf{q}}(\zeta_2, z_2)$ can be decomposed into a sum of two terms, such that each term factorizes into a product of a function of Z and a function of z:

$$\begin{aligned} \psi_{p,\mathbf{q}}(\zeta_1, z_1)\psi_{-p,-\mathbf{q}}(\zeta_2, z_2) = \left(\frac{a}{S\sqrt{\pi}}\right) & \left(\sum_n \exp[2inaY - (X + na)^2]\sum_k \exp\left[iy(q_y + ka) + 2iq_x ka - \left(\frac{x}{2} + q_y + ka\right)^2\right]\right. \\ & + \sum_n \exp\{2iYa(n + 1/2) - [X + (n + 1/2)a]^2\} \\ & \left.\sum_k \exp\{iy[q_y + (k + 1/2)a] - \left[\frac{x}{2} + q_y + (k + 1/2)a\right]^2 + i2(k + 1/2)aq_x\}\right)\,. \end{aligned} \tag{9}$$

The first term of (9) is a product of a periodic function of Z with periods $(a,0)$ and $(0,a)$ and a quasiperiodic function of z with quasimomentum $\mathbf{q}$ for periods $(2a,0)$ and $(0,2a)$. The second term is a product of a quasiperiodic function of Z with quasimomentum $\mathbf{q}=(0,a)=(0,\pi/a)$ and a quasiperiodic function of z with quasimomentum $\mathbf{q}=(q_x,q_y+a/2)=(q_x,q_y+\pi/2a)$. Thus, we see that the second term changes its sign under a magnetic translation of Z on $(0,a)$ and, therefore, does not contribute to the matrix element (8). Performing the integration in (8) we obtain the following expression for $\Delta(\mathbf{q})$

$$\Delta(\mathbf{q}) = -\sum_{m,\mathrm{odd}} \sqrt{\frac{\pi}{2^{m-1}m!}}\Delta_m \sum_n H_m[\sqrt{2}(q_y+na)]\exp[-(q_y+na)^2-2iq_x na] \; . \tag{10}$$

Note that each partial wave with relative angular momentum m produces a contribution to $\Delta(\mathbf{q})$ which belongs to the mth Landau level and is antisymmetric and magnetically periodic in $\mathbf{q}$ space. The "vector potential" in $\mathbf{q}$ space is $\tilde{A}_x = Hq_y$, $\tilde{A}_y=0$. The antisymmetry of $\Delta(\mathbf{q})$ ensures that the unitarity condition of transformation (6) is satisfied. For the case of singlet pairing $\Delta(\mathbf{q})$ belongs to the lowest Landau level.

Using the amplitudes $u_{\mathbf{q}}$ and $v_{\mathbf{q}}$ from (7) and taking into account the antisymmetry of $\Delta(\mathbf{q})$, we arrive at the following expression for $\Delta(Z,z)$:

$$\Delta(Z,z) = -\hat{P}_{\mathrm{LLL}}\frac{1}{2}S\int\frac{dp}{2\pi}\int\frac{d^2q}{(2\pi)^2}\frac{\Delta(\mathbf{q})}{E_{p,\mathbf{q}}}\tanh(E_{p,\mathbf{q}}/2T)V(z)\psi_{p,\mathbf{q}}\left(Z+\frac{z}{2}\right)\psi_{-p,-\mathbf{q}}\left(Z-\frac{z}{2}\right) \tag{11}$$

where S is again the area of the system and $\Delta(Z,z)$ is given by (4) and the integration over $\mathbf{q}$ is performed over the magnetic Brillouin zone. The second term in (9) does not contribute to the integral in (11) because $\Delta(\mathbf{q})$ and $E_{\mathbf{q}}$ stay invariant under the shift $q_x \to q_x+\pi/a$, whereas the second term in (9) changes sign. Thus, we see that the Z dependences on the left-hand side and on the right-hand side of (11) are the same and can be canceled.

Now we use the following property of our effective interaction $V(z)$ in the symmetric gauge: If we multiply any function from LLL by $V(z)$ and then project it back to LLL the amplitudes of partial waves will be multiplied by the corresponding Haldane pseudopotentials. In a mathematical form this condition can be written as $\hat{P}_{\mathrm{LLL}}V(z)\sum_m a_m z^m\exp(-|z|^2/8) = \sum_m V_m a_m z^m \exp(-|z|^2/8)$. We have consequently proved that the assumed form of $\Delta(Z,z)$ (4) is indeed self-consistent. Because the wave function of a Cooper pair is extended in space as a function of the relative coordinate there is an interaction between the center of mass and relative degrees of freedom, yet this does not cause frustration of the relative angular motion due to the existence of the lattice, which is not clear *a priori.* Multiplying both sides of (11) by $2^{-m-1}(\pi m!)^{-1/2}(x+iy)^m\exp[-(x^2+y^2)/8+ixy/4]$, canceling the Z dependences and integrating over the relative coordinates we reduce the self-consistency equation (11) to the following system of equations for the amplitudes Δ_m:

$$\Delta_m = \frac{V_m\pi^{1/2}}{(2^m m!)^{1/2}}\int\frac{dpd^2q}{(2\pi)^3}\frac{1}{E_{p,\mathbf{q}}}\tanh\left(\frac{E_{p,\mathbf{q}}}{2T}\right)\Delta(\mathbf{q})\sum_k\exp[-(q_y+ka)^2+2iq_x ka]H_m[\sqrt{2}(q_y+ka)] \; . \tag{12}$$

At T_c only one Δ_m will be nonzero and small. System of equations (12) will decouple into independent equations for each m because $E_{p,\mathbf{q}}$ will be replaced by ξ and the critical temperature will be determined by the channel having the most negative Haldane pseudopotential V_{m0}. The self-consistency equation at T_c can be written as

$$1 = -\frac{V_{m0}\pi\sqrt{2}}{2^{m_0-2}m_0!}\int\frac{dpd^2q}{(2\pi)^3}\frac{1}{\xi(p)}\tanh\left(\frac{\xi(p)}{2T_c}\right)\left|\sum_k\exp[-(q_y+ka)^2+2iq_x ka]H_{m0}[\sqrt{2}(q_y+ka)]\right|^2 \; . \tag{13}$$

This corresponds to the formation of Cooper pairs with relative angular momentum m_0.

Solving (13) for the critical temperature we obtain

$$T_S = 1.14\mu\exp\left(-\frac{\pi v_F}{|V_{m0}|}\right) \tag{14}$$

where v_F is the Fermi velocity.

If the interaction contains only one odd Haldane pseudopotential in its expansion the system of equations (12) reduces to a single nonlinear equation for the amplitude of the corresponding partial wave

$$1 = -\frac{V_m\pi\sqrt{2}}{2^{m-2}m!}\int\frac{dpd^2q}{(2\pi)^3}\frac{1}{E_{p,\mathbf{q}}}\tanh\left(\frac{E_{p,\mathbf{q}}}{2T}\right)\left|\sum_k\exp[-2(q_y+ka)^2+2iq_x na]H_m[\sqrt{2}(q_y+ka)]\right|^2 \; . \tag{15}$$

Fukuyama[12] considered the CDW instability for an electron system in the same limit as we are considering now. The wave vector of the CDW has a component $2k_F$ along the ζ axis and a component Q perpendicular to it. The value of Q is determined by the condition that $\tilde{U}(Q)$ be a minimum. The critical temperature for the CDW transition is given by

$$T_{\rm CDW} = 1.14\mu \exp\left(-\frac{2\pi^2 v_F}{|\tilde{U}_{\rm min}|}\right) \tag{16}$$

where $\tilde{U}_{\rm min}$ is the minimum value of $\tilde{U}(q)$ and should be negative for the transition to take place.

From Eqs. (14) and (16) we see that the superconductive instability wins if $V_{m0} > \frac{\tilde{U}_{\rm min}}{2\pi}$. This criterion is valid for any interaction which is nonretarded, isotropic in the plane perpendicular to the ζ axis and is not long range along the ζ axis.

As an example we now consider a system with model interaction which is assumed to be of the form $\tilde{U}(q) = -4\pi|V|[L_1(q^2) - \gamma L_3(q^2)]\exp(-q^2/2)$. This represents a short-range interaction with an attractive core. The mean-field critical temperature for the superconductive transition is greater than that for the CDW formation if $0.54 < \gamma < 0.92$. The ratio of the critical temperatures is the largest for $\gamma \simeq 0.7$ and is given by $\frac{T_S}{T_{\rm CDW}} = \exp(-\frac{0.064\pi v_F}{|V|})$. In the case of weak coupling this ratio can be significant. This demonstrates that it is possible for such a superconductive transition to take place.

Another interesting feature of the system under consideration is that its excitation spectrum given by $E_{\mathbf{q}} = \sqrt{\xi^2 + |\Delta_{\mathbf{q}}|^2}$ is gapless because the order parameter has to turn to zero at a certain point in the magnetic Brillouin zone. This coincides with the result for singlet pairing[2] and leads to powerlike temperature dependences of thermodynamic quantities.

To conclude: the mean-field analysis of the low-temperature behavior of the system has been carried out. We obtained the self-consistency equation for a general nonretarded interaction and the criterion which tells which of the instabilities will win. It has been shown that for certain interactions the superconductive instability can win and in the weak-coupling limit it can have a significantly higher T_c than the CDW instability. It is therefore possible that when the fluctuations are taken into account the superconductive transition will still persist. It has also been shown that the existence of the Abrikosov lattice is consistent with the extended nature of the relative motion of Cooper pairs. A transition of this type might be found in low carrier density systems in magnetic fields that are already experimentally achievable. We argue that pairing in systems which have a sizable g factor is likely to occur into a triplet state as opposed to singlet pairing proposed earlier[1–3] because it is insensitive to the Pauli pair breaking.

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