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Influence of a dc field on polaritons confined in magnetic wires

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This paper presents a complete study on the influence of a dc external magnetic field on polaritons confined in uniaxial antiferromagnetic cylinders. Using different values for the cylinder radius and the strength of the external field, we study the effect of these external parameters on the dispersion relation of confined mixed modes. It is shown that, for any finite value of the static magnetic field, only modes with their rf magnetic field perpendicular to the anisotropy axis can be found confined in the sample. Numerical results are presented for the uniaxial Heisenberg antiferromagnet MnF_2.

Electromagnetic waves propagating in a material may excite internal degrees of freedom of the medium, and as a consequence, the resulting traveling wave consists of oscillating electric and magnetic fields, mediated by the electric and/or magnetic polarization induced in the medium by the electromagnetic wave itself. In other words, when electromagnetic waves propagate through a material, in a frequency region where either the magnetic or electric dynamical response of the medium exhibits resonances in their frequency-dependent refractive index, they incorporate detailed information about the characteristics of the specimen. This is one of the motivations to study these mixed modes, since from knowledge of their behavior much information on the internal structure of the material can be extracted. On the other hand, some features of these modes suggest a great variety of interesting practical applications. As an example, a few years ago it was suggested that nonreciprocal behavior of magnetostatic surface waves could be used in the development of microwave-signal-processing devices.[1]

For a long time, it has been known that long-wavelength surface spin waves, propagating on the surface of a semi-infinite ferromagnetic specimen which has its magnetization parallel to the surface, have striking and unusual properties.[2] In the beginning of the 1980s, Camley[3] showed that on the surface of a uniaxial antiferromagnet with sublattice magnetization parallel to the surface, long-wavelength magnetostatic spin waves with properties similar to those of ferromagnetic samples should be observed. Shortly after this work and as its extension, the influence of an external dc magnetic field applied parallel to the surface and along the anisotropy axis on the spectrum of magnetic polaritons was investigated.[4] In Ref. 4 the nonreciprocal behavior of surface modes induced by an external field was observed. In other words, in the presence of a dc magnetic field, if a surface mode has a wave vector $\mathbf{k}_{\parallel}$ and frequency ω, it is no longer true that the mode with wave vector $-\mathbf{k}_{\parallel}$ has the same frequency (a good discussion of the nonreciprocal behavior of surface modes is presented in Ref. 1). A great number of authors have dealt with the properties of these modes in thin films and superlattices, in the presence (or not) of a dc field. For a good review, see Ref. 5 and references therein.

The study of surface nonradiative electromagnetic modes in circularly cylindrical interfaces has also motivated many authors.[6,7] They have considered long cylinders of radius a, optically described by an isotropic dielectric function $\epsilon(\omega)$, embedded in a cladding medium with different dielectric functions. The main interest of these works has been the study of properties of bulk and surface polaritons formed at frequencies in the vicinity of resonances of the dielectric function $\epsilon(\omega)$. One of the features discussed in Ref. 6 is that, except for modes without angular dependence, this geometry does not allow separation of pure magnetic waves (TM modes) from pure electric waves (TE modes), and so the eigenmodes in the bulk are superpositions of these waves.

More recently, interest in spatially confined modes was renewed by the successful use of cylindrical optical fiber for practical purposes.[8] The formal transcendental equation, whose solution defines the polariton-dispersion relation, has been known for many years for specimens described by the isotropic dielectric function. Nevertheless, only recently have these coupled modes been studied in materials with uniaxial anisotropy.[9,10] In Ref. 9, Zhu and Cao have studied propagation of magnetic polaritons in a rectangular waveguide with multilayer structure, taking into account not only the anisotropy of the material, but also the geometric periodicity. In Ref. 10 an implicit dispersion relation was obtained which has a general application for any cylindrical specimen, and the authors specialize in studying uniaxial Heisenberg antiferromagnets. Despite the anisotropy, introduced via magnetic permeability, the results preserve the same general features of the isotropic dielectric medium.

In this work we extend the calculation carried out in Ref. 10 to include the effect of an external dc magnetic field applied parallel to the cylinder axis (defined as the z direction), which is also parallel to the anisotropy direction of the specimen. It should be remarked that, under such conditions, the dynamical response of the system (magnetic susceptibility) is not diagonal and the presence of off-diagonal terms is the main reason for changes in the properties of the modes studied in Ref. 10. It should be remarked that ferromagnetic cylinder polaritons were studied, both theoretically and experimentally,[11] in the later 1960s and a detailed analysis of the spectrum of electromagnetic waves in longitudinally magnetized ferrite rods has been done. The system studied in this paper is rather similar to that investigated in Ref. 11, and so we will use the same nomenclature used there.

We assume a cylindrical specimen described by the permeability and dielectric tensors $\overleftrightarrow{\mu}(\omega)$ and $\overleftrightarrow{\epsilon}(\omega)$, which, in Cartesian coordinates, may be put into a very useful form:

$$\overleftrightarrow{\mu}(\omega)=\begin{bmatrix}\mu_1 & i\mu_2 & 0\\ -i\mu_2 & \mu_1 & 0\\ 0 & 0 & \mu_0\end{bmatrix} \tag{1a}$$

and

$$\overleftrightarrow{\epsilon}(\omega)=\begin{bmatrix}\epsilon_1 & 0 & 0\\ 0 & \epsilon_1 & 0\\ 0 & 0 & \epsilon_0\end{bmatrix} . \tag{1b}$$

For uniaxial Heisenberg antiferromagnets described by the two-sublattice model and in the presence of an external field $\mathbf{H}=H_0\hat{\mathbf{z}}$, $\mu_0=1$, and μ_1 and μ_2 are given by[12]

$$\mu_1=1+4\pi\Omega_s^2\left[\frac{1}{\Omega_0^2-(\Omega-\gamma H_0)^2}+\frac{1}{\Omega_0^2-(\Omega+\gamma H_0)^2}\right] , \tag{2a}$$

$$\mu_2=4\pi\Omega_s^2\left[\frac{1}{\Omega_0^2-(\Omega-\gamma H_0)^2}-\frac{1}{\Omega_0^2-(\Omega+\gamma H_0)^2}\right] . \tag{2b}$$

Here Ω_0 is the antiferromagnetic resonance frequency in zero magnetic field, which depends on the anisotropy and exchange interactions. Namely, we have $\Omega_0=\gamma\sqrt{(2H_aH_e+H_a^2)}$, where γ is the gyromagnetic ratio and H_a and H_e denote the strength of the anisotropy and exchange fields, respectively.[13] Also, Ω_s^2 is given by $\gamma^2H_aM_s/8\pi$, with M_s denoting the saturation magnetization of each sublattice. For practical purposes we use a dielectric tensor independent of the frequency.

Assuming the dependence of the components of the electromagnetic field as

$$\mathbf{E}=[E_r(r),E_\theta(r),E_z(r)]\exp[i(kz+n\theta-\Omega t)] \tag{3a}$$

and

$$\mathbf{H}=[H_r(r),H_\theta(r),H_z(r)]\exp[i(kz+n\theta-\Omega t)] , \tag{3b}$$

Maxwell's equations can be written as

$$\frac{1}{r}(nE_z-krE_\theta)=\frac{\Omega}{c}(\mu_1H_r+i\mu_2H_\theta) , \tag{4a}$$

$$\frac{dE_z}{dr}-ikE_r=-\frac{i\Omega}{c}(\mu_1H_\theta-i\mu_2H_r) , \tag{4b}$$

$$\frac{d}{dr}(rE_\theta)-inE_r=i\frac{\Omega r}{c}\mu_0H_z , \tag{4c}$$

$$\frac{\Omega r}{c}\epsilon_1E_r=krH_\theta-nH_z , \tag{4d}$$

$$i\frac{\Omega}{c}\epsilon_1E_\theta=\frac{dH_z}{dr}-ikH_r , \tag{4e}$$

$$i\frac{\Omega r}{c}\epsilon_0E_z=inH_r-\frac{d}{dr}(rH_\theta) , \tag{4f}$$

$$\frac{d}{dr}[r(\mu_1H_r+i\mu_2H_\theta)]+in(\mu_1H_\theta-i\mu_2H_r)+ikr\mu_0H_z=0 , \tag{4g}$$

and

$$\frac{d}{dr}(\epsilon_1rE_r)+in\epsilon_1E_\theta+ikr\epsilon_0E_z=0 . \tag{4h}$$

It is an easy task to find out that, if $E_z=0$ (TE modes), then H_z satisfies the differential equation

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dH_z}{dr}\right]\left[\frac{\mu_0}{\mu_1}(k_0^2-k^2)-\frac{n^2}{r^2}\right]H_z=0 , \tag{5}$$

where $k_0^2=(\Omega^2/c^2)\mu_1\epsilon_1$. On the other hand, if we consider $H_z=0$ (TM modes), a bit of algebra shows that $(d/dr)(rE_r)=-inE_\theta$ and the substitution of this result in Eq. (4h) shows that only a trivial solution ($E_z=0$) survives. It should be remarked that this result is true only if $\mu_2\neq0$, i.e., for an uniaxial Heisenberg antiferromagnet, when a nonzero dc external magnetic field is present. This is the main difference between the results shown in Ref. 10 and the ones here. In that paper it was shown that polaritons confined in cylindrical uniaxial antiferromagnetic samples have characteristics similar to those in dielectric specimens in the same geometry, not allowing to separate eigenmodes with angular dependence in regions where $\nabla\cdot\mathbf{E}=0$. In the dielectric cases, as well as in the physical system studied in Ref. 10, solutions of Maxwell's equations are superpositions of electric and magnetic waves, except for modes without angular dependence. In the case studied here, we find that the presence of an external field polarizes the polaritons propagating inside the sample. From Eq. (5) we find their amplitude given by the Bessel function of order n, $J_n(k_{\text{in}}r)$ (if k_{in} is real) or $I_n(k_{\text{in}}r)$ (if k_{in} is imaginary), with k_{in} defined by

$$k_{\text{in}}^2=\frac{\mu_0}{\mu_1}(k_0^2-k^2) . \tag{6}$$

Since we are interested in polaritons confined in the cylindrical specimen, solutions outside of the sample must have $\Omega^2/c-k^2<0$, and so their amplitudes are proportional to the Hankel functions of order n, $H_n(k_{\text{out}}r)$, with k_{out} defined by the square root of $(k^2-\Omega^2/c)$. With these results on hand, an implicit dispersion relation for these polaritons can be obtained by using the usual boundary conditions. A straightforward algebra gives the expression

$$\frac{n}{k_{\rm in}a}\left[\frac{(q^2-\omega^2)(q^2-\omega^2\mu_1\epsilon_1)-(q^2-\omega^2\mu_1\epsilon_1)^2+\omega^4\epsilon_1^2\mu_1^2-(\omega^2-q^2)\omega^2\mu_1\epsilon_1}{\omega^2\mu_2\epsilon_1(\omega^2-q^2)}\right]\zeta_n(k_{\rm in}a)=\zeta_{n+1}(k_{\rm in}a)\ , \tag{7}$$

where $q=kc/\Omega_0$, $\omega=\Omega/\Omega_0$, and $\zeta_n(k_{\rm in}a)$ is equal to $J_n(k_{\rm in}a)$ if $k_{\rm in}$ is real (bulklike modes) or $I_n(k_{\rm in}a)$ if $k_{\rm in}$ is imaginary (surfacelike modes). From Eq. (7) we can see that, for modes without angular dependence ($n=0$), only bulk polaritons are allowed, since $I_1(x)$ has no zero for any finite value of x. Moreover, these bulk polaritons have a simple dispersion relation which can be written as a function of the roots of the Bessel function $J_1(k_{\rm in}a)$. If we name the mth zero of $J_1(x)$ by x_{1m}, we find the dispersion relations of these modes obeying

$$(ka)^2=\mu_1\left[\frac{x_{1m}}{\mu_3}-\frac{\Omega^2}{c^2}\epsilon_1\right]. \tag{8}$$

Another interesting feature that should be stressed in Eq. (7) is the reciprocal character of these modes. Unlike the case of a semi-infinite specimen in the presence of a static field, the quadratic dependence of the dispersion relation on the wave vector assures that modes confined in these cylinder wires (bulk and surface), propagating from the left to the right or vice versa, have the same frequency if the absolute value of their wave vectors have the same value.

We have solved Eq. (7) numerically, using the physical parameters of a MnF_2 specimen. They have the following values: $\epsilon_1=4$, $\epsilon_0=8$, $H_a=7.85$ kOe, $H_e=550.0$ kOe, $M_s=0.6$ kG, and $\gamma=1.87\times10^7$ rad/G. These values give an antiferromagnetic resonance frequency of about 260 GHz. In Figs. 1(a) and 1(b) we show dispersion-relation curves for $n=1$, considering $H_0=300$ G, and diameters equal to 1 and 2 mm, respectively. In Figs. 2(a) and 2(b) we use the same values for the diameters and a

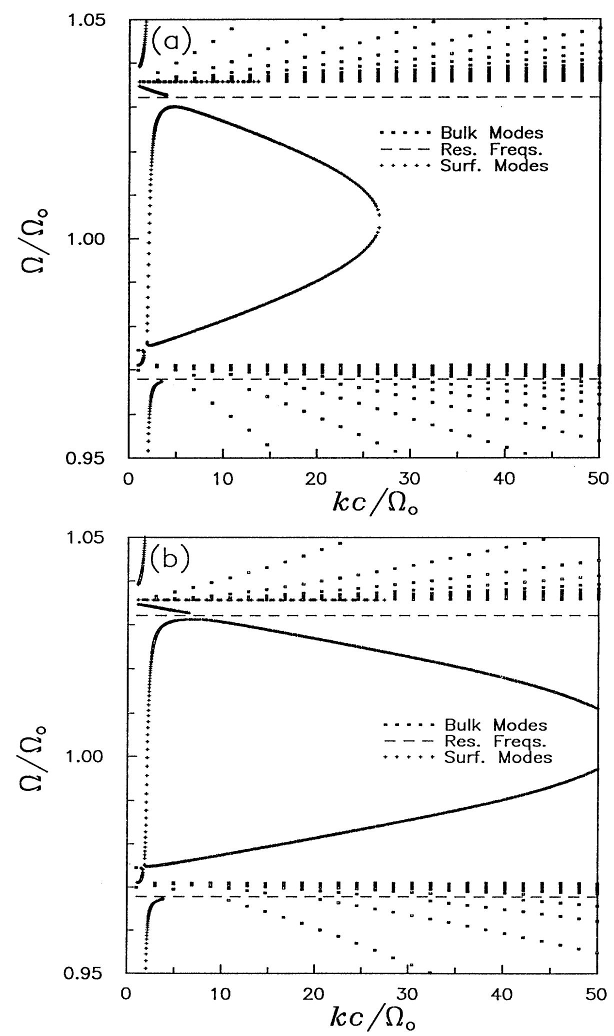

FIG. 1. Dispersion-relation curves for $n=1$, $H_0=300$ G, and cylinder diameter equal to (a) 2 mm and (b) 1 mm.

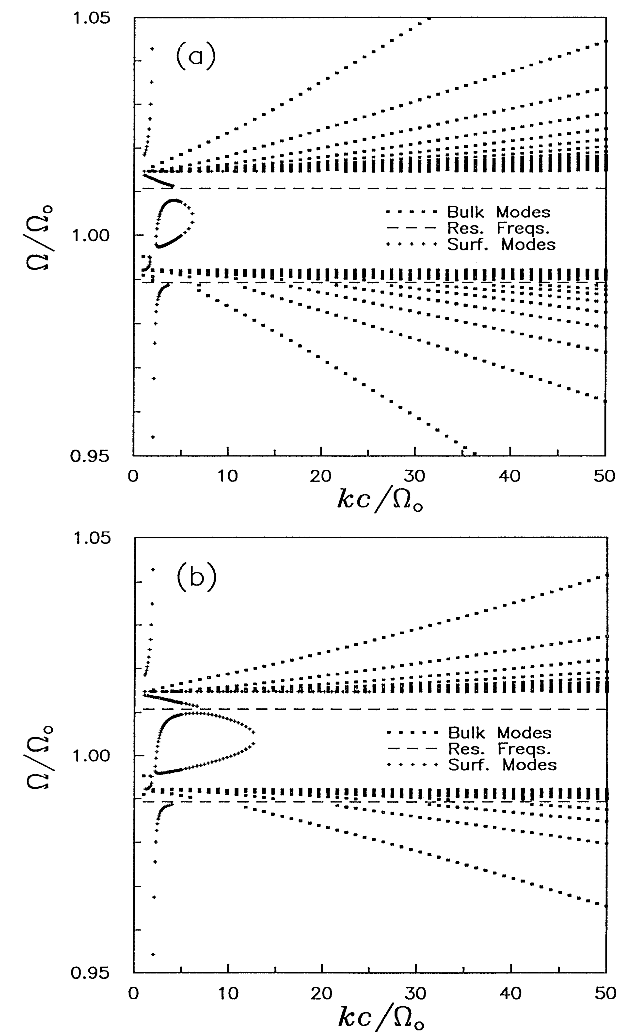

FIG. 2. Dispersion-relation curves for $n=1$, $H_0=100$ G, and cylinder diameter equal to (a) 2 mm and (b) 1 mm.

static field $H_0 = 100$ G. It is clear from these results that bulk modes appear in branches localized above $\Omega_r + \gamma H_0$ and also around $\Omega_r - \gamma H_0$. It can be observed that when the diameter of the sample is increased, the spacing between branches decreases in such a way that in wide cylinders, for a given value of the wave vector, these modes exist in a continuum region of frequency. As usual, in other geometries, in these systems surface modes also exist in regions where bulk modes do not. Their main characteristic is that for any finite value of the diameter, they exist only for finite value of k. These results tell us that there is no magnetostatic surface mode confined in the sample. The diameter of the sample determines the region in k space where surface modes can be found. For large diameters, surface modes are restricted to small values of k. When the diameter is decreased, the domain of k where surface modes can be found is amplified. The influence of the strength of the static field can be easily observed when we compare Figs. 1(a) with 2(a) and Figs. 1(b) with 2(b). As can be seen there, when H_0 is decreased, the branches of the bulk modes have no noticeable modification in their behavior. However, for any value of the diameter, the surface dispersion-relation curves are compressed not only in k space, but also in the frequency domain. For values of H_0 close to zero, the distance between dashed lines (equal to $2\gamma H_0$) goes to zero and all surface modes in that frequency region disappear. In this limit the resulting dispersion-relation curves reproduce results previously shown in Ref. 10.

Finally, we must say that the behavior of modes studied in this work would be very different if a cylindrical shell had been used instead of a full cylinder or if the cladding medium were also optically active. We intend to study these systems in future works.

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