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# Thermally activated flux avalanches in single crystals of high-$T_c$ superconductors

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We have experimentally investigated long-time (> 10 h) magnetic relaxation in single crystals of $Bi_2Sr_2CaCu_2O_x$ and $YBa_2Cu_8O_x$ at wide ranges of temperature and field. We have found that the flux motion in single crystals of high-$T_c$ superconductors exhibits two distinctly different behaviors at a wide regime of driving force. At large driving force, the flux motion is dominated by thermally activated avalanches, which can be described by a so-called self-organized criticality theory. At small driving force, however, the avalanche effect is significantly reduced, and the flux motion is characterized by slow creep which can be described by the Anderson-Kim model. Therefore, there is a dynamic crossover between the flux avalanches and a pure thermally activated regime in high-$T_c$ superconductors.

## I. INTRODUCTION

In studying the vortex property of high-$T_c$ superconductors, various models including the thermally activated flux-flow (TAFF) model[1] and the collective-creep (CC) model[2] have been proposed to explain the flux-motion behavior such as nonlogarithmic magnetic relaxation. These models, however, consider only the thermal effects on flux motion, and the depinned flux lines are treated with no temporal correlation. Furthermore, both TAFF and CC models are applicable only for small driving force [i.e., $j \ll j_c(T,H)$].

To describe flux-motion behavior in a wide range of driving force, particularly near the critical state, an alternative interpretation has been proposed by several researchers[3–5] based on a fundamentally different physical model, the so-called self-organized criticality (SOC) introduced by Bak, Tang, and Wiesenfeld.[6,7] The original idea of SOC is that an open dissipative system far from equilibrium state can self-organize into a new critical state. Ling, Shi, and Budnick[3] applied this SOC picture to the current decay from the Bean critical state.[8] They proposed that the transport of depinning magnetic flux lines in the critical state is similar to the avalanches of a sandpile under constant perturbation.[6] An important point here is that, according to the SOC concept, the occurrence of flux motion does not have to be thermally activated, and external mechanical vibration can be the single source responsible for initiating flux motion. Ling, Shi, and Budnick have also pointed out that the current decay in type-II superconductors can be best described by a stretched exponential function of time, which is associated only with avalanche dynamics.

However, a pure avalanche mechanism may not account for the entire flux-motion process since the thermal effects are much more pronounced in high-$T_c$ systems. Recently, Tang[9] argued that flux motion is originally thermally activated but the subsequent hopping process is dominated by avalanches, particularly at large driving force. He has proposed an equation of flux motion by taking into account both thermal activation and avalanchelike dynamics. Tang has pointed out that flux-motion velocity is determined by two factors, namely, the flux hopping rate and the flux avalanche size. The initial depinning process is mainly associated with thermal effects, while the subsequent hopping process of depinned flux lines is governed by the spatially and temporally correlated avalanches. This physical picture is particularly relevant at large driving forces as a result of violent flux motion caused by a high magnetic pressure.

In this paper, we report that results of a systematic magnetic relaxation measurement on single crystals of both $Bi_2Sr_2CaCu_2O_x$ (Bi 2:2:1:2) and $YBa_2Cu_3O_x$ (Y 1:2:3) over wide ranges of temperature and magnetic field. Our experimental results have shown good agreement with theoretical predictions by Tang.[9] We show that flux motion always has a thermally activated origin, but at large driving force (large $j$ achieved at low temperature, low magnetic field, and short relaxation time), the flux motion is dominated by avalanche dynamics. We also show that a low driving force (small $j$ achieved at high temperature, high magnetic field, and long relaxation time), the thermally activated flux avalanche effects are significantly reduced and the flux-motion characteristics change to slow creep. Therefore, a dynamic crossover has been found to exist.

## II. EXPERIMENTAL PROCEDURE

Magnetization measurements were carried out on a Quantum Design superconducting quantum interference device (SQUID) magnetometer. The samples were first zero-field cooled to a desired temperature $T$ below the transition temperature $T_c$. A magnetic field $H$ was then applied to the $c$ axis of the single crystals. The magnetization $M$ of the samples was measured as a function of time $t$. The initial data points of the magnetization were taken after $t_0 = 180$ s (using standard procedures with the commercial SQUID, we cannot obtain relaxation data earlier than that time after the field is settled). The travel length of the sample in each scan was 3 cm to avoid field inhomogeneity.

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To be able to study the dynamical behavior for both flux entry and exit, we have also carried out measurements during the reverse cycle of the magnetic field. The sample was first cooled in zero magnetic field to a desired temperature below $T_c$. The field was then increased to a value (5 T) much greater than the field (2 T) at which the magnetic relaxation data were taken. The subsequent procedure is the same as described above. We note that the reverse cycle field measurement gives a relaxation process most similar to a sandpile (i.e., the initial flux profile has the shape of a rooflike triangle with the slope gradually decreasing).

### III. RESULTS AND DISCUSSION

Figure 1 shows magnetic relaxation data taken from single crystals of both Bi 2:2:1:2 and Y 1:2:3 on a Quantum Design SQUID at the temperatures and fields indicated. The relaxation period is 22 h for Bi 2:2:1:2 and 10 h for Y 1:2:3. As shown in this figure, magnetic relaxation of both systems exhibit nonlogarithmic decay over the entire time period at all temperatures and fields tested in this experiment. This nonlogarithmic behavior has been observed in other type-II superconductors and interpreted by previously proposed models.[10] Here, we interpret our relaxation data based on a concept of self-organized criticality (SOC),[6,7,9] which, we believe, offers a much better physical picture of flux motion.

From the classical Anderson-Kim theory,[11] the flux hopping rate $\omega$, with which the flux lines (or bundles) jump over the pinning barrier, can be described by an Arrhenius-type expression

$$\omega=\omega_0 \exp(-U/kT) , \tag{1}$$

where $\omega_0$ is an attempt frequency and $U$ is an effective activation energy. From the shape of the individual pinning potential, Beasley, Labusch, and Webb[12] concluded that the activation energy $U$ has a power-law dependence,

$$U=U_0(1-|\nabla p|/|\nabla p|_c)^{\beta} , \tag{2}$$

where $|\nabla p|$ is the magnetic pressure gradient, $|\nabla p|_c$ is some constant slightly less than the maximum pinning force,[12,13] $U_0$ is the characteristic energy, and $\beta$ is the exponent constant. Beasley, Labusch, and Webb argued that $\beta$ should change slightly with the shape of the pinning barrier.

Tang[9] proposed that the flux motion in the vortex state is governed by thermally activated avalanches (i.e., a flux line is initially thermally activated and the subsequent hopping is accomplished through flux avalanches). He suggested that the flux-motion velocity $v$ is determined by multiplication of the thermal activation rate (time scale) $\omega$ and the avalanche size (length scale) $s$. He defined the avalanche size $s$ as the total flux-line displacement involved in each avalanche. The characteristic size for an SOC-like system can be expressed as[7,9]

$$s \sim (|\nabla p|_c-|\nabla p|)^{-\alpha} ,$$
$$=s_c(1-|\nabla p|/|\nabla p|_c)^{-\alpha} , \tag{3}$$

where $\alpha$ is a critical exponent and $s_c$ is the characteristic avalanche size. Therefore,[9,14]

$$v=s\omega . \tag{4}$$

Equation (4) illustrates a fundamentally different physical flux-motion process compared with previously proposed models. In the Anderson-Kim model, once the flux lines

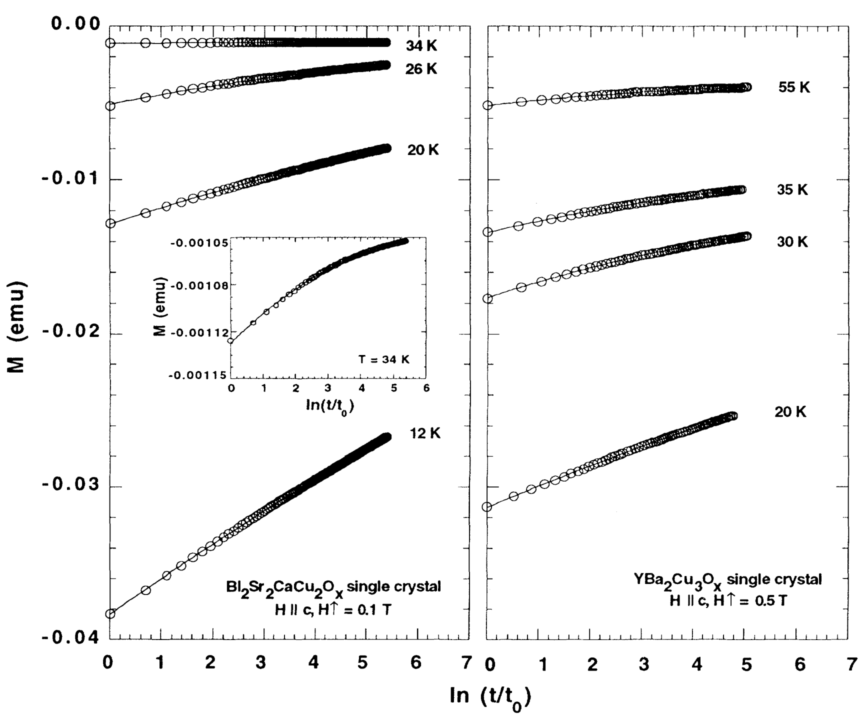


FIG. 1. Magnetization vs $\ln(t/t_0)$ at 0.1 T applied parallel to the $c$ axis of single crystals of $Bi_2Sr_2CaCu_2O_x$ and $YBa_2Cu_3O_x$ at temperatures indicated. The solid lines are the fits with $\ln(t/t_0)+b\,\ln^2(t/t_0)$ indicating nonlogarithmic relaxation. The inset shows the pronounced nonlogarithmic behavior for Bi 2:2:1:2 at 34 K.

are depinned, the subsequent hopping process has no spatial and temporal correlation. Thus, flux creep in the vortex state is an individual and independent event and, therefore, the flux motion is characterized by an averaged hopping distance. However, in a real situation, correlation between flux lines during hopping must be important, especially at large driving force (high magnetic pressure). In a zero-field cooled relaxation measurement, the critical state is reached from an initial state $dj/dx \sim \infty$, where $j \gg j_c(T=0)$. Since $U=0$ at the critical state, the initial flux entry is violent, which is similar to avalanches as observed in sandpiles. Once a flux line is thermally depinned, instead of slowly hopping into an adjacent barrier valley as considered in the Anderson-Kim model, avalanche effects will cause it to move a significantly longer distance. Therefore, the effective flux-motion distance is characterized by avalanche size which is much longer than the average hopping distance in the Anderson-Kim model.[11] The effective flux-motion distance is determined by flux-motion velocity $v$ (Ref. 9) and Anderson-Kim[11] flux hopping rate $\omega$.

According to the Bean critical-state model, $|\nabla p|$ is proportional to the critical current density $j$ (Ref. 13). From Eqs. (1)–(3), we can rewrite the velocity[9] in Eq. (4) as

$$v=\omega_0 s_c(1-|\nabla p|/|\nabla p|_c)^{-\alpha}\exp(-U/kT)$$
$$=\omega_0 s_c(1-j/j_c)^{-\alpha}\exp[-U_0(1-j/j_c)^{\beta}/kT]\ . \quad (5)$$

The flux conservation equation proposed by Beasley, Labusch, and Webb[12] can be written in one-dimensional form as

$$dB/dt=-\nabla D=-\nabla(Bv)\ , \quad (6)$$

where $D$ is the flux-flow density. By considering a slab of thickness $d$ and integrating Eq. (6) over a sample volume, one obtains

$$d(j/j_c)/d(t/\tau)=-(4\tau B\omega_0 s_c/\mu_0 d^2 jc)(1-j/j_c)^{-\alpha}$$
$$\times\exp[-U_0(1-j/j_c)^{\beta}/kT]$$
$$=-(1-j/j_c)^{-\alpha}$$
$$\times\exp[-U_0(1-j/j_c)^{\beta}/kT]\ , \quad (7)$$

where the critical current density $j_c$ is a function of the temperature and magnetic field, and $\tau=B_c d/2B\varepsilon s_c\omega_0$ [where $B_c=\mu_0(d/2)j_c$]. The term $(1-j/j_c)^{-\alpha}$ is associated with avalanches, while the term $\exp[-U_0(1-j/j_c)^{\beta}/kT]$ is derived from pure thermal effects. We therefore expect thermal effects to be a dominating factor at small driving force, a situation that requires $\alpha$ to be zero. This situation implies that the flux lines jump from the pinning barriers independently (i.e., no spatial and temporal correlation). According to Anderson and Kim, $U=U_0(1-j/j_c)$ (Ref. 11). The decay rate will then approach the rate of $\exp[-U_0(1-j/j_c)^{\beta}/kT]$ with $\beta=1$. We have found that the relation $\alpha=(\beta-1)$ has produced a good fit to the magnetic relaxation data for both Bi 2:2:1:2 and Y 1:2:3.

In Fig. 2, we plot the flux-motion rate $[d(j/j_c)/d(t/\tau)]$ vs normalized critical current density $[j/j_c(T,H)]$ for Bi 2:2:1:2 at 12 K. Here, $j_c$ is a function of both temperature and field. If we assume that the shape of the pinning barrier has a functional form of $\frac{1}{2}$ $Up\cos(\pi Z/Z_p)$, where $Z$ is the position, then according to (Ref. 12), $\beta=1.5$. As shown in Fig. 2, the $[d(j/j_c)/d(t/\tau)]$ vs $[j/j_c(T,H)]$ curve is well fitted with Eq. (7) with $U_0$ (=0.027 eV) being the only fitting parameter. It should be pointed out that Beasley, Labusch, and Webb[12] showed that $\beta$ may change slightly with any smooth function instead of $\cos(\pi Z/Zp)$ for the pinning barrier.

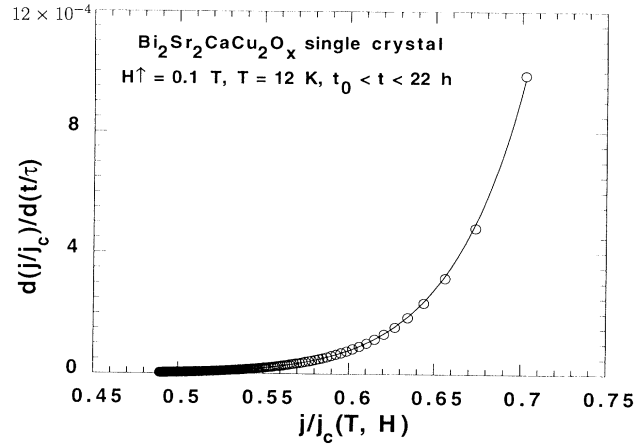


FIG. 2. $d(j/j_c)/d(t/\tau)$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=12$ K. The solid line is the fit to Eq. (7).

As the temperature increases, we found that the relaxation data can be well described by Eq. (7) only at early times. For instance, at 20 K as shown in Fig. 3(a), the

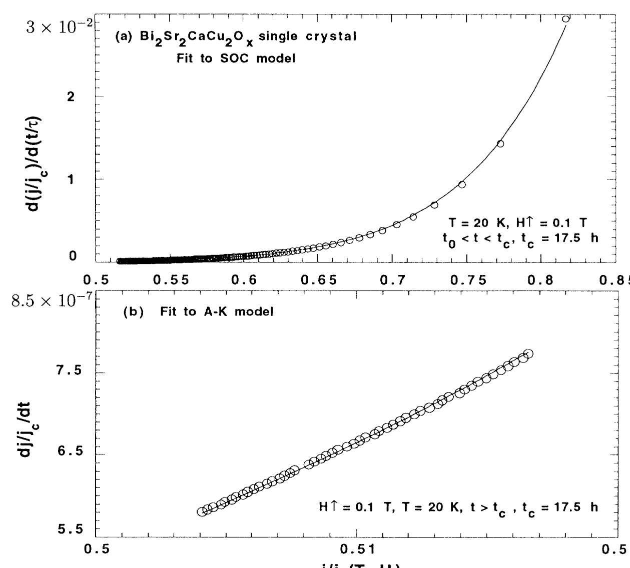


FIG. 3. (a) $d(j/j_c)/d(t/\tau)$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=20$ K, where the solid line is the fit to Eq. (7). (b) $d(j/j_c)/dt/$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=20$ K for $t>t_c$, where $t_c$ is the crossover time and the solid line is the fit to the Anderson-Kim model.

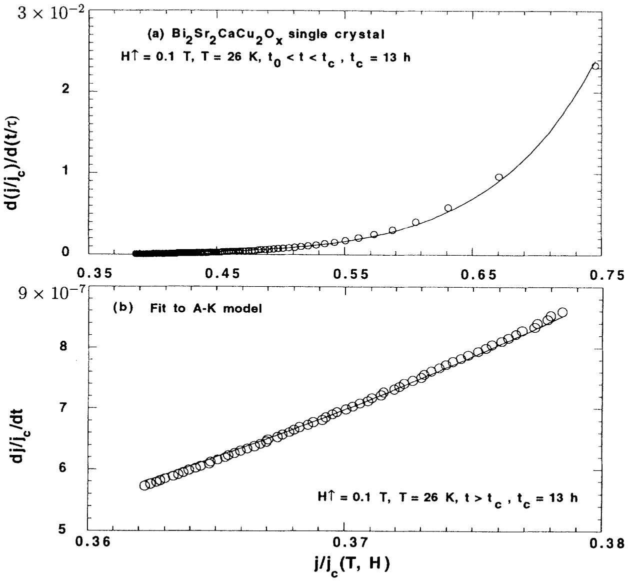


FIG. 4. (a) $d(j/j_c)/d(t/\tau)$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=26$ K, where the solid line is the fit to Eq. (7). (b) $d(j/j_c)/dt/$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=26$ K for $t>t_c$, where the solid line is fit to the Anderson-Kim model.

$[d(j/j_c)/d(t/\tau)]$ vs $[j/j_c(T,H)]$ curve follows Eq. (7) up to 17.5 h, but the later data begin to deviate from such a dependence. Thus, a crossover time $t_c$ is defined as the time at which Eq. (7) fails and the avalanche effects are unobservably small. After $t_c$, as shown in Fig. 3(b), the relaxation data can be well described by the Anderson-Kim model. Similar behavior has been found at even higher temperatures such as 26 and 34 K, as shown in Figs. 4(b) and 5, respectively, except that Eq. (7) is valid only within shorter time windows at high temperatures. It should be noted that avalanches causing the system to deviate from the Anderson-Kim regime have also been reported by Pan and Doniach.[5] We note that the dynamics of the avalanche process have a slowing down characteristic. The avalanche time is small at the beginning and becomes longer as the driving force decreases. From

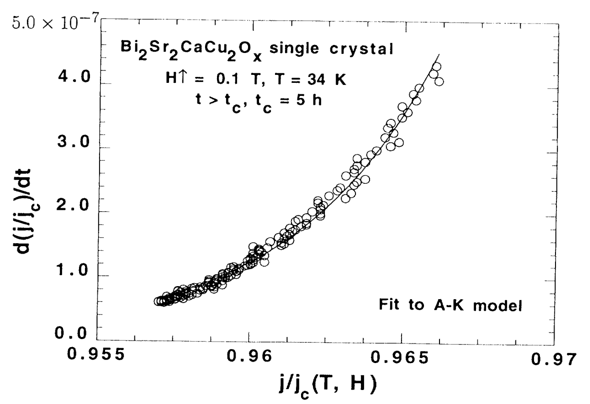


FIG. 5. $d(j/j_c)/dt/$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=34$ K for $t>t_c$, where the solid line is fit to the Anderson-Kim model.

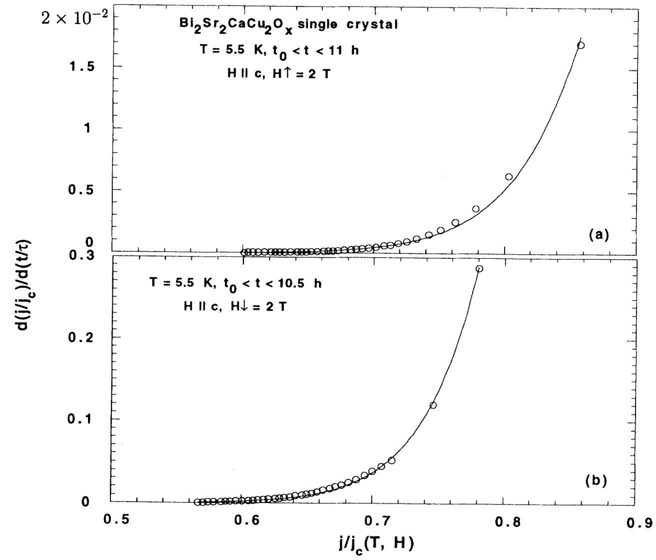


FIG. 6. (a) $d(j/j_c)/d(t/\tau)$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=5.5$ K for decreasing field $H\downarrow=2$ T. (b) $d(j/j_c)/d(t/\tau)$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=5.5$ K for increasing field $H\uparrow=2$ T, where the solid line is the fit to Eq. (7).

Figs. 2–5, we can see that two mechanisms operate the flux-motion process: a flux line is always thermally activated regardless of the level of driving force, but the avalanche effect is pronounced and dominating only at large driving force.

We have also carried out relaxation measurements at lower temperatures and high field (2 T) for both increasing field ($H\uparrow$) and decreasing field ($H\downarrow$, or reverse cycle field). Figures 6 and 7 show that the dynamical behavior

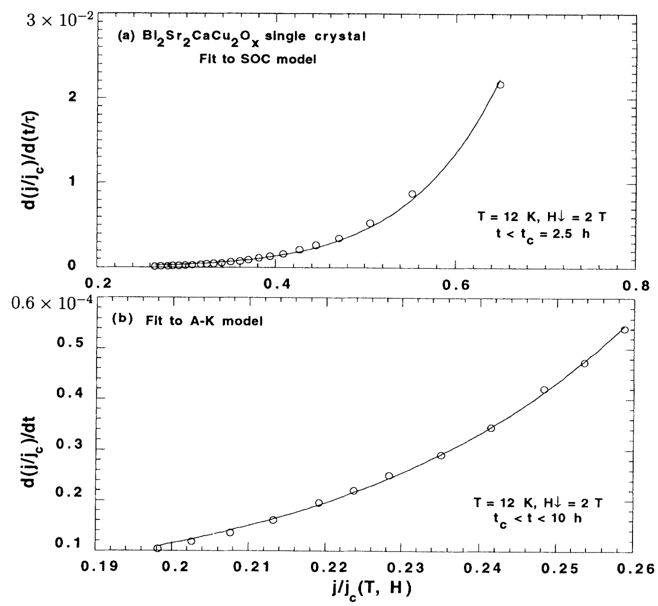


FIG. 7. (a) $d(j/j_c)/d(t/\tau)$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=12$ K for decreasing field $H\downarrow=2$ T, where the solid line is the fit to Eq. (7). (b) $d(j/j_c)/d(t/\tau)$ vs $j/j_c$ for a single crystal of Bi 2:2:1:2 at $T=12$ K for $t>t_c$, where the solid line is the fit to the Anderson-Kim model.

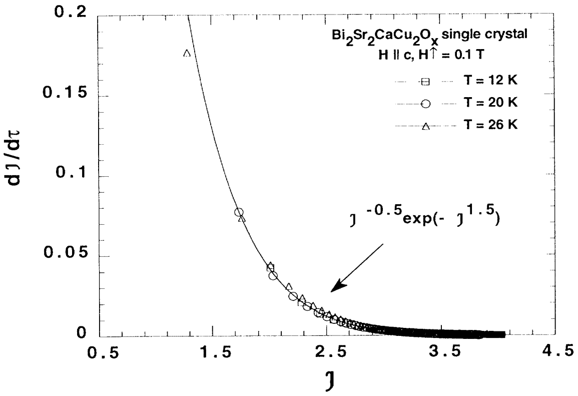


FIG. 8. The universal behavior of the flux-creep rate for a single crystal of Bi 2:2:1:2 normalized by temperature, where the scaled curve is fitted with $\vartheta^{-(\beta-1)}\exp(-\vartheta^{\beta})$.

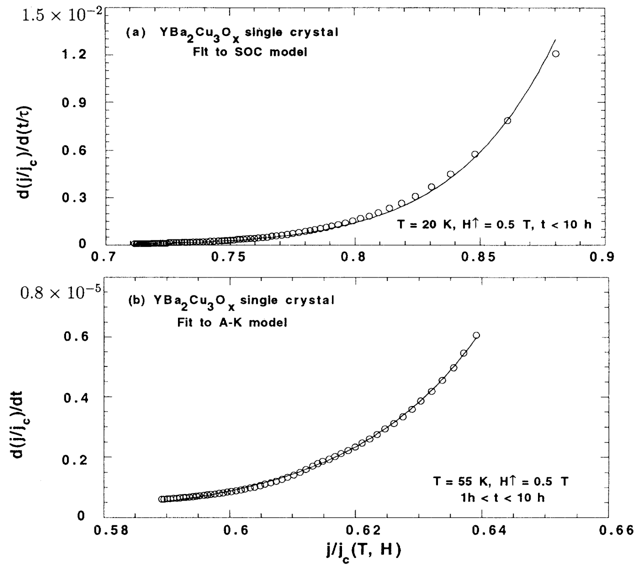


FIG. 9. (a) $d(j/j_c)/d(t/\tau)$ vs $j/j_c$ for a single crystal of Y 1:2:3 at $T=20$ K, where the solid line is fit to Eq. (7). (b) $d(j/j_c)/dt/$ vs $j/j_c$ for a single crystal of Y 1:2:3 at $T=55$ K for $t>t_c$, where the solid line is the fit to the Anderson-Kim model.

at low temperatures and high field is quite similar to behavior observed at high temperatures (Figs. 2–5). At 5.5 K and 2 T as a result of large driving force, the avalanche effects are pronounced within the entire measuring time and, therefore, Eq. (7) fits the relaxation data well for both increasing and decreasing field. At a high temperature of 12 K and 2 T, the reduced driving force results in a much shorter crossover time of $t_c=2.5$ h. It should be noted that the fitting parameter $U_0$ used for data presented in Figs. 2–7 has been found to have only small changes (on the order of 0.03 eV). The result indicates that the characteristic energy is related to the intrinsic property of the material.

By rescaling Eq. (7) with temperature $T$ and substituting $\alpha$ with $\beta-1$, one can define a unitless parameter $\vartheta$ related to current density, which can be expressed as

$$d\vartheta/d(t/\tau)=\vartheta^{-(\beta-1)}\exp(-\vartheta^{\beta}) , \qquad (8)$$

where $\varepsilon=U_0/kT, \vartheta=\varepsilon^{1/\beta}(1-j/j_c)$, $\tau=B_c d/2B\varepsilon s_c\omega_0$, and $B_c=\mu_0(d/2)j_c$, which is the lowest field required for full flux penetration.

In Fig. 8, we show $d\vartheta/d(t/\tau)$ vs $\vartheta$ for different temperatures indicated. As can be seen in this figure, the magnetic relaxation has the same dependence in a wide temperature regime, and, therefore, Eq. (7) serves as an excellent description of flux motion in the vortex state for Bi 2:2:1:2.

In addition to measuring the flux motion of Bi 2:2:1:2, we have done the same measurements on another high-$T_c$ oxide, Y 1:2:3. As shown in Fig. 9, exactly the same behavior has been observed in this superconducting system. These results clearly show that thermally activated flux avalanche is a universal behavior in the vortex state of high-$T_c$ superconductors. We believe that similar flux-motion behavior also exists in conventional type-II systems.

## IV. CONCLUSION

We have shown that magnetic relaxation in single crystals of $Bi_2Sr_2CaCu_2O_x$ and $YBa_2Cu_3O_x$ can be well described by a physical model, namely, self-organized criticality. We conclude that flux motion in the vortex state is initially thermally activated but that the subsequent hopping process is dominated by avalanchelike dynamics at large driving force. Similar to the dynamical behavior of a sandpile, the avalanches are slowed down as the driving force is decreased and become unobservably small at high temperature and field (or longer time). In other words, the avalanche size is significantly reduced at small driving force. We have found a dynamical crossover time at which the flux-motion characteristics change fundamentally from temporally correlated avalanches to slow creep.

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[1]P. H. Kes, J. Aarto, J. van den Berg, C. J. van der Beek, and J. A. Mydosh, Supercon. Sci. Technol. **1**, 242 (1989).
[2]V. Feigel'man, V. B. Geshkenbein, and V. M. Vinokur, Phys. Rev. Lett. **63**, 2303 (1989).
[3]X. S. Ling, D. Shi, and J. I. Budnick, Physica C **185-189**, 2181 (1991).

[4]O. Pla and F. Nori, Phys. Rev. Lett. **67**, 919 (1991).
[5]W. Pan and S. Doniach (unpublished).
[6]P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987).
[7]C. Tang and P. Bak, Phys. Rev. Lett. **60**, 2347 (1988).
[8]C. P. Bean, Phys. Rev. Lett. **8**, 250 (1962).
[9]C. Tang, Physica A **194**, 315 (1993).
[10]D. Shi and M. Xu, Phys. Rev. B **44**, 4548 (1991); S. Sengupta, Donglu Shi, Zuning Wang, M. E. Smith, and P. J. McGinn, *ibid.* **47**, 5165 (1993).
[11]P. W. Anderson, Phys. Rev. Lett. **9**, 309 (1962). P. W. Anderson and Y. B. Kim, Rev. Mod. Phys. **36**, 39 (1964).
[12]M. R. Beasley, R. Labusch, and W. W. Webb, Phys. Rev. **181**, 682 (1969).
[13]P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966), p. 83.
[14]J. M. Carlson, J. T. Chayes, E. R. Grannan, and G. H. Swindle, Phys. Rev. Lett. **65**, 2547 (1990).