

Transverse thermomagnetic effects in the mixed state and lower critical field of high- T_c superconductors

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(Received 27 April 1993; revised manuscript received 11 June 1993)

Transverse thermomagnetic effects (Ettingshausen, Nernst effects) are discussed for a variety of phenomenological models of high- T_c and other layered superconductors. The use of the temperature-dependent vortex-line energy in determining the transport entropy is stressed, leading to predictions and possibilities for additional experiments. The dynamics of both Abrikosov and Josephson vortices is considered.

In this paper I discuss transverse thermomagnetic effects in a variety of both strongly and weakly superconducting systems. In the present context a weakly superconducting system refers to one in which Josephson tunneling occurs and there are two (or more) magnetic penetration depths. Examples of such systems are provided by layered organic superconductors, the high- T_c cuprates and bismuthates, and multilayer structures, where the penetration depth for currents along the c direction (perpendicular to the layers) can be very large compared to the in-plane penetration depth, due to weak Josephson coupling.

In this paper I consider the dynamics of both Abrikosov and Josephson vortices and their associated transport entropies. One result of this discussion is the possibility of the observation of a Nernst voltage due to the motion of Josephson vortices. In principle, a dimensional crossover in thermomagnetic measurements may also be observable. Illustrations are given on how to employ the static vortex mobility in the description of transverse thermomagnetic effects.

After some background on the thermomagnetic effects to be considered, I discuss some implications for different superconducting systems. At least one of these implications leads to a prediction for the transport entropy S_φ in contrast to earlier work and invites further experiments. Throughout I assume that S_φ is to be viewed as a local difference in entropy density between the vortex structure and the superconductor. The transport entropy is given in terms of the first derivative of a thermodynamic potential, as is the intrinsic thermodynamic entropy.

Thermomagnetic quantities that have been measured for the high-transition temperature superconductors include the thermopower, Nernst coefficient, and transport line energy.¹⁻⁹ The compounds studied include $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (Y 1:2:3),^{1-4,9} $\text{Bi}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$,⁵ $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$,⁸ $\text{Tl}_2\text{Sr}_2\text{Ca}_2\text{Cu}_3\text{O}_{10}$,⁵ and $\text{Tl}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$.^{6,7} The thermopower and Nernst effect may be thought of as thermal analogs of the (longitudinal) electrical resistivity and Hall effect, the heat current substituting for the electrical current. However, the thermopower and Hall effect are the results of "particle-hole asymmetry" whereas the Nernst effect and the resistivity

are not.¹⁰ In this paper I concentrate on the transverse effects, where the electric field is transverse to the temperature gradient, for low and intermediate magnetic fields. That is, I do not discuss the Hall effect in the mixed state here. In addition to general Ref. 11, Refs. 12 and 13 describe thermomagnetic and thermoelectric effects in terms of a proper choice of conjugate fluxes and forces.

A simple illustration of thermomagnetic effects is provided by considering a geometry with vortices along the z direction and a temperature gradient along the x direction. The Nernst coefficient Q_N is found from the relation $E_y = -Q_N B \nabla_x T$, ($B = B_z$). By making use of the dc vortex mobility $\tilde{\mu}_v(\omega=0)$, which may include weak pinning and flux-creep effects,^{14,15} we have for the (driving) thermal force $f_{\text{th}} = -S_\varphi \nabla_x T = v_x / \tilde{\mu}_v$, where \mathbf{v} is the vortex velocity. By using the Josephson relation for the electric field we then have

$$Q_N = \tilde{\mu}_v(B, T) S_\varphi(B, T). \quad (1)$$

The isothermal Nernst coefficient is obtained subject to the boundary conditions $J_x = J_y = \nabla_y T = 0$, where \mathbf{J} is the electrical current density.¹⁰ A small correction to Eq. (1) proportional to the Hall angle is neglected here.

A convenient expression for the (real) dc mobility is that of Ambegaokar and Halperin,¹⁴

$$\tilde{\mu}_v(B, T) = \frac{1}{\eta} \frac{1}{I_0^2(\nu)}, \quad \nu \equiv U(B, T) / 2k_B T, \quad (2)$$

where η is the viscous drag coefficient, $U(B, T)$ is the barrier height of the periodic pinning potential, and I_0 is the zero-order modified Bessel function of the first kind. Recall that $U(B, T)$ vanishes at T_{c2} , the field-dependent transition temperature; in the high-temperature and/or field limit, $\tilde{\mu}_v \rightarrow 1/\eta$. At intermediate temperatures and fields where $\nu \sim 1$, Eq. (2) models the effect of flux creep; $1/I_0^2(\nu)$ is the flux-creep factor.¹⁶ Due to the relatively low activation energies in the high- T_c superconductors, thermally assisted flux motion can be important.² A model for the barrier height is¹⁷ $U(B, T) = U_0(1 - T/T_{c2})^{3/2}/B$, where U_0 is a constant, on the order of 0.01–0.1 eV T for Bi or Tl compounds.⁷ The

model given by Eqs. (1) and (2) extends that of other treatments, e.g., Ref. 7. As usual U should be thought of as an average or effective barrier height as several types of analyses suggest a distribution of heights.

The Ettingshausen coefficient C_E is found from the thermal and electrical current densities U_y and J_x from the relation $C_E = U_y / BKJ_x$, where K is the thermal conductivity. The boundary conditions are $U_x = J_y = \nabla_x T = 0$. The Ettingshausen effect is inverse to the Nernst effect: Flux motion produces a temperature difference in the superconductor. The thermal current density is given by $\mathbf{U} = nTS_\varphi \mathbf{v}$, where the vortex areal density is $n = B/\phi_0$. Assuming that the Lorentz force is the sole driving force and that the same dc mobility as above may be employed, we have

$$C_E = \frac{1}{K} T \bar{\mu}_v(B, T) S_\varphi(B, T). \quad (3)$$

The product TS_φ in Eq. (3) is the transport energy per unit length of vortex. It is seen that Eqs. (1) and (3) satisfy the Bridgman relation $TQ_N = KC_E$ of irreversible thermodynamics.¹⁸

The qualitative temperature dependence of the transport entropy is that it vanishes at absolute zero and the transition temperature and has a maximum in between.¹⁹ While S_φ vanishes at T_c in the absence of fluctuation effects, these can be pronounced in high- T_c materials. That S_φ vanishes at $T=0$ is due to the third law of thermodynamics. In the very low-field limit, where the interaction between vortices can be ignored, one has for the transport entropy per unit length from considering the free energy²⁰

$$S_\varphi = -\frac{\phi_0}{4\pi} \frac{\partial H_{c1}}{\partial T}, \quad H_{c1} < H \lesssim 2H_{c1}. \quad (4)$$

Additional previous calculations of the low-field transport entropy subject to various approximations include Refs. 21–23. For intermediate fields intervortex interactions need to be taken into account and this leads to an additional factor in Eq. (4):²⁰

$$S_\varphi \approx -\frac{\phi_0}{4\pi} \frac{\partial H_{c1}}{\partial T} \frac{\ln(H_{c2}/B)}{\ln \kappa}, \quad 2H_{c1} \lesssim H \ll H_{c2}, \quad (5)$$

where H_{c2} is the upper critical field and κ is the Ginzburg-Landau (GL) parameter. In writing Eq. (5) temperature dependence in these quantities has been ignored.

Equations (4) and (5) are derived from an equilibrium free energy which is not rigorously justified.²² Generally one needs to adopt a dynamical description which includes the thermal current in the energy flow. Recently it was shown by Troy and Dorsey that throughout the mixed state the transport entropy is proportional to the equilibrium magnetization.¹⁰ This result was obtained on the basis of time dependent Ginzburg-Landau (TDGL) theory and my approach may be complementary. This recent theory extends the Maki high-temperature result.²⁴ Near H_{c1} the TDGL theory^{21,10} gives $S_\varphi \approx -\phi_0 H_{c1} / 4\pi T$, which does not vanish at $T=0$. This defect is probably an artifact of the TDGL approach and

Eq. (4) may provide a suitable substitute.

The transport entropy is largest in the low-field region.¹¹ As a function of temperature S_φ reaches a maximum at roughly $T_c/2$. The theory presented here is valid for such intermediate and lower temperatures. At higher temperatures ($T \lesssim T_{c2}$) Eqs. (4) and (5) need to be modified so that S_φ decreases to zero at T_{c2} .¹⁹ The temperature behavior of the lower critical field H_{c1} seems to be consistent with that of S_φ when it “levels off” at very low temperature, becoming independent of T there. This is a point to which I return later.

I consider various consequences of Eqs. (4) and (5), which relate a static property, the vortex-line energy, to a dynamic one, the transport entropy per unit length. In my view, the temperature dependence of the lower critical field, which mainly occurs through that of the superconductor penetration depths, leads to thermal diffusion of vortices and entropy flow. Let us apply Eq. (4) [or Eq. (5) for higher fields] to a layered high- T_c superconductor, with external magnetic field parallel to the layers.

We can model a high- T_c superconductor as an anisotropic, continuous superconductor using Ginzburg-Landau (GL) theory with an effective-mass tensor,²⁵ at temperatures above a crossover temperature T^* .^{26–28} Below T^* the superconductor discreteness becomes manifest for the vortex orientation we are considering and a Lawrence-Doniach or similar model^{29,27,28,30} is appropriate. This leads to a crossover in the temperature behavior of the lower critical field:^{28,31}

$$H_{c1a}(T) = \frac{\phi_0 \sqrt{m_a}}{4\pi \lambda^2(T)} \left[\ln \left[\frac{\kappa}{\sqrt{m_a}} \right] + 0.5 \right], \quad T > T^*, \quad (\kappa/\sqrt{m_a} \gg 1), \quad (6)$$

$$H_{c1a}(T) = \frac{\phi_0 \sqrt{m_a}}{4\pi \lambda^2(T)} \left[\ln \left[\frac{\lambda(T) \sqrt{m_b}}{s} \right] + C_{\text{core}} \right], \quad T < T^*. \quad (7)$$

Here the vortex is taken to lie along the principal a axis. (Vortices tilted away from a principal-axis direction will not be considered here.) In Eqs. (6) and (7), λ is the geometric mean penetration depth, m_a and m_b are m -plane effective masses, s is the stacking periodicity, and C_{core} is a constant of order unity. This constant arises from the contribution of the Josephson vortex core; for details on the core structure in the infinite superconductor-insulator-superconductor (SIS) multilayer model, see Refs. 27 and 30. The core boundary for a Josephson vortex is where the tunneling current attains its maximum. Thus the core is specified in terms of the gauge-invariant phase difference between superconducting layers in contrast to an Abrikosov vortex, whose core is specified in terms of the amplitude of the superconducting order parameter.

The lower-temperature expression (7) is the appropriate one to use with the (unmodified) Eq. (4). Various models of $\lambda(T)$ can be taken and the temperature dependence of S_φ determined. For instance, if in the expres-

$$\frac{4\pi}{\phi_0} \frac{1}{\sqrt{m_a}} \frac{\partial H_{c1a}}{\partial T} = \left\{ -2 \left[\ln \left(\frac{\lambda(T)\sqrt{m_b}}{s} \right) + C_{\text{core}} \right] + 1 \right\} \frac{1}{\lambda^3(T)} \frac{\partial \lambda}{\partial T}, \quad T < T^* \quad (8)$$

we use $\lambda(T) = \lambda(0)/\sqrt{1-t^\alpha}$, $t \equiv T/T_c$, ($\alpha > 0$), we have for the dominant temperature dependent factor

$$\frac{2}{\lambda^3(T)} \frac{\partial \lambda}{\partial T} = \frac{\alpha}{\lambda^2(0)T_c} t^{\alpha-1}. \quad (9)$$

The value $\alpha=4$ corresponds to the two-fluid model,²⁰ while $\alpha=2$ seems to provide a consistent fit to recent penetration depth data for Y 1:2:3.³²⁻³⁴ Of course the results of BCS or other microscopic theories can also be used in Eqs. (4) or (5). Equations (1), (4), and (7) indicate that Josephson vortices could significantly contribute to the Nernst voltage and electric field. A typical value for S_φ due to Josephson vortices from Eqs. (4), (8), and (9) is 10^{-12} J/K m. Although the contributions of Abrikosov and Josephson vortices may be qualitatively similar, a sufficiently quantitative measurement might be able to distinguish between the two and thereby allow the observation of a dimensional crossover. Furthermore, from Eq. (9) we see that the transport entropy can be sensitive to the detailed temperature dependence of the penetration depth. In particular, in the two-fluid model Eq. (9) gives a cubic temperature dependence but only a linear dependence in the recently used $\alpha=2$ model. In this way, sufficiently detailed thermomagnetic measurements could even give information on the nature of the superconducting state, for the $\alpha=2$ model indicates a state (node) in the superconducting gap and that the superconducting density of states $N_s(E)$ increases as E^2 near $E=0$.³³

My emphasis on the possible contribution of moving Josephson vortices to the transport entropy [through Eqs. (4) and (5)] is in contrast to some previous work.^{4,9} These authors state that motion of Josephson vortices does not lead to a Nernst voltage. It is probable that Josephson and Abrikosov vortices experience different thermal forces from an applied temperature gradient.³⁵ Although Josephson vortices may not experience the same thermal force as Abrikosov vortices, due to the lack of a normal metal-like core, this does not necessarily imply that they cannot transport entropy. These positions evidently invite further experiments, and a possible reexamination of earlier data where the vortices may have been aligned parallel to the planes in high- T_c materials. *A*-axis aligned films of Y 1:2:3 would be a possible medium for a test of the Josephson vortex contribution. The crossover temperature, defined by the condition that the coherence length $\xi_c(T)$ becomes comparable to $\sqrt{2}s$, can be easily estimated as^{26,28} $T^* = T_c [1 - \xi_c^2(0)/2s^2]$. Even for Y 1:2:3 the crossover temperature is quite high, $T^* \approx 0.9T_c$, so Eq. (8) has a wide temperature range of validity. In applying Eqs. (1), (2), and (3) for the Nernst and Ettingshausen coefficients for Josephson vortices, an

appropriate model for the viscous drag coefficient η is that given by Ref. 27.

Other superconducting multilayer structures can be considered for transverse thermomagnetic effects. Suppose we consider an infinite superconductor-normal-metal-superconductor (SNS) system, which has also been used to model high- T_c materials.³⁶⁻³⁸ In these systems the proximity effect occurs, becoming pronounced at low temperature. The result can be a drastic increase in the lower critical field for vortices either parallel or perpendicular to the layers at low temperature.³⁶⁻³⁸ There is some experimental evidence of this phenomenon (e.g., Ref. 39), but I consider it to be controversial. As mentioned above the third law of thermodynamics applied to S_φ puts a constraint on the very low-temperature behavior of H_{c1} , via Eq. (4). In many numerical results for the lower critical field³⁶⁻³⁸ the negative slope and positive curvature of H_{c1} versus T appears to persist to absolute zero. This behavior appears to be unphysical; perhaps there is a mechanism to insure that it is cut off. Additional experiments on multilayer SNS systems to compare Nernst effect data with Eqs. (4) and (5) would be of interest.

An experimentally more accessible way to test the Josephson vortex contribution to the transport entropy may be to study single Josephson SIS junctions. Here the lower critical field is given by⁴⁰

$$H_{c1J}(T) = \frac{2\phi_0}{\pi^2 \lambda_J(T) d(T)}, \quad (10)$$

where the Josephson penetration depth is $\lambda_J = (c\phi_0/8\pi^2 dJ_0)^{1/2}$, the magnetic thickness is $d(T) = \lambda_1(T) + \lambda_2(T) + t_i$, and t_i is the insulator thickness. When the thicknesses d_1 and d_2 of the superconducting films are not large compared to the London penetration depths λ_1 and λ_2 the magnetic thickness is modified to⁴⁰

$$d(T) = \lambda_1 \tanh(d_1/2\lambda_1) + \lambda_2 \tanh(d_2/2\lambda_2) + t_i.$$

The temperature dependence of the maximum tunneling current density J_0 may be modeled in BCS theory with the Ambegaokar and Baratoff expression⁴¹

$$J_0(T) = \frac{\pi \Delta(T)}{2eR_n} \tanh \left[\frac{\Delta(T)}{2k_B T} \right], \quad (11)$$

where $\Delta(T)$ is the superconducting energy gap and R_n is the tunneling resistance per unit area of the junction when both metals are in the normal state. When Eqs. (9) and (10) are used in Eqs. (4) or (5), a prediction for vortex entropy transport in a SIS junction is obtained.

When the insulating layer is replaced with a metallic layer, a SNS junction is obtained. As a good approximation the expression (10) can be taken for the lower critical field of a SNS junction, but with a modified tunneling current density,⁴² due to the proximity effect. Although there is some literature on SNS junctions in a temperature gradient³⁵ definitive experimental studies on entropy transport do not appear to have been performed.

General scaling relations for the reversible magnetiza-

tion and entropy change in terms of the reduced field $h = H/H_{c2}(\theta, \phi)$, $H \gg H_{c1}$ are obtained from the Gibbs free-energy difference between the normal and superconducting states.⁴³ These relations are of interest because the high-temperature transport entropy follows the behavior of the magnetization.⁴⁴

In this paper I considered the transverse thermomagnetic effects in the mixed state, mainly in the low-field regime where the transport entropy is large. I discussed the dynamics of both Abrikosov and Josephson vortices in a number of strongly and weakly superconducting systems. The expressions (1) and (3) for the Nernst and Ettingshausen coefficients in terms of the static vortex mobility enable a continuous description from small-signal pinning-dominated dynamics to flux-flow-dominated dynamics with a large Nernst voltage or Ettingshausen temperature difference. Thermomagnetic data may be able to give not only information on superconducting parameters such as the GL parameter, upper critical-field slope, and coherence length, but on the superconducting state itself. The results presented here may be complementary to those found on the basis of time-dependent Ginzburg-Landau theory, especially in the low-field regime.

By using the lower critical field in the thermodynamic relations, Eqs. (4) and (5), for a variety of layered superconducting systems, predictions were given for the transport entropy per unit length of vortex. The layered systems include infinite SIS and SNS Josephson stacks which have been commonly used to model high- T_c superconductors. Of particular interest is the thermal diffusion of

Josephson vortices, either in stacks or single junctions. The Nernst voltage signal here would indicate entropy transport by these vortices, in contrast to some earlier treatments.

In this paper I concentrated on the low and intermediate temperature regime for low and intermediate fields, where Eqs. (4) and (5) are valid. A last topic which I touch on is the role of fluctuations near the transition temperature. This role can be very significant for high- T_c superconductors and its quantitative description leads to various scaling relations.⁴⁴ Fluctuation effects have been considered in two magnetic-field regimes. Fluctuations of both the amplitude and phase of the superconducting order parameter were considered in Ref. 44, the theory holding for fields near the upper critical field H_{c2} . This theory used time-dependent GL equations to find both transverse and longitudinal transport properties of a layered superconductor in a magnetic field. The results for the transport coefficients can be written in terms of scaling variables and scaling functions.⁴⁴ A theory for phase fluctuations (vortex positions),⁴⁵ valid for $H \ll H_{c2}$, predicts the temperature at which the magnetization is independent of field and could also be used in the analysis of thermomagnetic effect data. Both of these theories can be used to perform data collapse of the magnetization or transport entropy at temperatures near the mean-field transition temperature $T_{c2}(H)$.

I thank J. Rice, V. Kogan, M. Ledvij, and A. Simonov for useful discussions.

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