# Thermally activated depinning of flux lines from columnar defects

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(Received 15 March 1993)

The dynamics of a fluctuating vortex in the presence of a columnar defect is described with a functional Fokker-Planck equation. Within this picture thermally activated depinning due to an external electric current perpendicular to the defect axis corresponds to a nonzero probability current over a saddle point $\mathbf{u}_1$ of the energy in configuration space. If the energy barrier is large enough this probability current may be calculated summing over all fluctuations orthogonal to the direction of steepest descent at $\mathbf{u}_1$. This gives the prefactor in the Arrhenius law to be proportional to $j^{5/2}T^{-1/2}$ for the escape rate of a pinned vortex, and to $j^{3/2}T^{-1/2}$ for the resistivity $\rho$.

Pinning of vortices by long columnar defects as produced by the irradiation of heavy ions is regarded as an effective mechanism to achieve maximum loss-free currents in high-$T_c$ superconductors.[1] Such channels are able to generate large pinning forces and therefore large critical currents due to the electromagnetic and core interaction.[2] As pointed out by Brandt[3] in this situation not only the pinning force but also the pinning energy per unit length $V_p$ plays an important role, since the latter enters into the activation energy $U$ for thermal activated depinning of the flux lines (FL's). This leads to a finite resistivity $\rho \sim \exp(-\beta U)$ where $\beta = 1/k_B T$ is the inverse temperature.

The activation energy $U$ in the presence of an external current $J$ perpendicular to the vortices has been calculated within the frame of anisotropic London theory in the case of a uniaxial superconductor when columnar defects and vortices are aligned to the $c$ axis.[3] $U = (2/3)\Delta V_p$ is the energy of a critical FL configuration (Fig. 1) with a depinned parabolic section of width $\Delta \sim 1/J$ and height $h \sim 1/J$ provided that the distance to the neighboring pinning channels $a$ is sufficient large, $a > h$. In the present paper this situation shall be analyzed in more detail including also vortex fluctuations to calculate the prefactor $\omega_a$ in the Arrhenius law $R = \omega_a \exp(-\beta U)$ for the escape rate $R$ of a pinned vortex. $\omega_a$ is the attempt frequency of the formation of the critical "bubble" as shown in Fig. 1.

The method used here has some analogies to the treatment of quantum tunneling problems with path integrals.[4]

In what follows we consider a single FL pinned by a single columnar defect directed along the $c$ axis which shall be represented by a two-dimensional (2D) pinning potential $V(u)$, $u = |\mathbf{u}|$, $\mathbf{u} = (x, y)$. Vortex-vortex interactions shall be neglected, and other pinning channels are assumed to be far away ($h < a$). Then, within the linear elasticity approximation the energy $F$ of a FL with distortion $\mathbf{u}(z)$ in the presence of an external current $J = (c/\Phi_0)j$, and pinning potential $V(u)$ may be written[5,6]

$$F[\mathbf{u}(z)] = \int_{-L/2}^{L/2} \left[ \frac{m}{2} \left( \frac{d\mathbf{u}}{dz} \right)^2 + V(u) - j \cdot x \right] dz. \quad (1)$$

$\Phi_0$ is the flux quantum, $c$ the velocity of light, $L$ the sam-

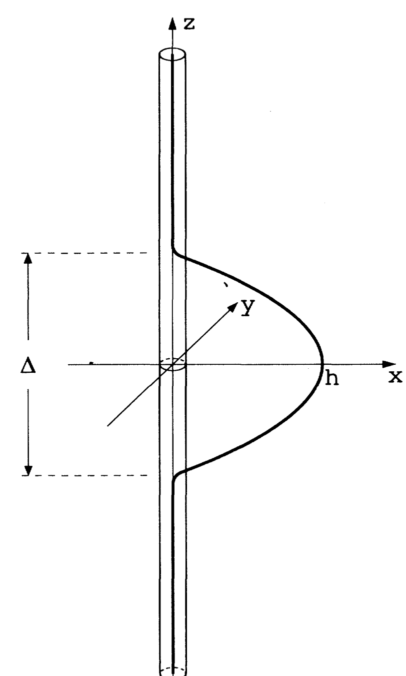


FIG. 1. Flux line $\mathbf{u}(z)$, $\mathbf{u} = (x, y)$ pinned by a columnar defect which inflates into a critical parabolic configuration of width $\Delta$ and height $h$ under the influence of an external current in $y$ direction. Once formed the bubble will grow and the vortex depins.

 

ple thickness, and $m = \frac{\Phi_0^2(\ln\kappa+0.5)}{16\pi^2\lambda_c^2}$ where $\kappa = \lambda_{ab}/\xi_{ab} = \lambda_c/\xi_c$. $\lambda_{ab(c)}$ is the London penetration depth, and $\xi_{ab(c)}$ the coherence length in the direction of the $ab(c)$ axis.

The pinning potential $V(u)$ shall not be specified in detail except of the following properties: $V(u)$ has its minimum at $u = 0$ with $V(0) = -V_p = -\frac{\Phi_0^2\epsilon}{16\pi^2\lambda_{ab}^2}$, $\epsilon \lesssim 1$, and curvature $d^2V(0)/du^2 = 2V_p/\sigma^2$ where $\sigma \gtrsim \xi_{ab}$ is approximately the radius of the pinning channel. $V(u)$ decays rapidly to zero for $u > \sigma$.

At first we will study the structure of the energy functional $F[\mathbf{u}(z)]$ from (1) in FL configuration space. $F$ may formally be regarded as the Euclidian action of a particle of mass $m$ in the potential $V(u) - j \cdot x$ with imaginary time $z$.[5,7] The boundary conditions can be taken as $\mathbf{u}(\pm\frac{L}{2}) = \mathbf{0}$ if the vortex is assumed to be pinned at its ends. The entirely pinned vortex $\mathbf{u}_0 = \mathbf{0}$ and the critical configuration $\mathbf{u}_1 = (x_1, 0)$ from Fig. 1 extremize the energy functional $F[\mathbf{u}]$, and therefore are solutions of the Euclidian equation of motion

$$\frac{\delta F}{\delta \mathbf{u}(z)} = -m\frac{d^2\mathbf{u}}{dz^2} + \boldsymbol{\nabla} V - \mathbf{j}\cdot\mathbf{e}_x = 0. \qquad (2)$$

Actually there are also other FL configurations which extremize $F$ (e.g., two or more bubble configurations) but these have much larger activation energies, and therefore may be disregarded. For $j \ll j_0 \equiv 2V_p/\sigma$ with $\Delta = 2\sqrt{2mV_p}/j$ we have

$$x_1(z) = \frac{V_p}{j} - \frac{j}{2m}z^2 \text{ for } |z| \lesssim \frac{\Delta}{2}, \qquad (3)$$

and exponential behavior $x_1(z) \sim \exp(-\omega_0|z|)$ for $|z| \gtrsim \Delta/2$ where $m\omega_0^2 \equiv d^2V(0)/du^2$. For $L \to \infty$ this asymptotically fulfills the boundary conditions.

Throughout this paper it shall be assumed $L \gg \Delta$ so that one can deal with the limit $L \to \infty$. For finite $L$ that means $j/j_0 \gg (L\omega_0)^{-1}$ which requires $L \gg \omega_0^{-1}$.

A functional expansion of $F$ at $\mathbf{u}_0$, and $\mathbf{u}_1$ up to second order gives

$$F[\mathbf{u}_\alpha + \delta\mathbf{u}] = F[\mathbf{u}_\alpha] + \frac{1}{2}\langle\delta\mathbf{u}|\mathcal{O}_\alpha|\delta\mathbf{u}\rangle + \cdots, \qquad (4)$$

with $\alpha \in \{0, 1\}$, and

$$\mathcal{O}_\alpha = \begin{pmatrix} \mathcal{O}_\alpha^x & 0 \\ 0 & \mathcal{O}_\alpha^y \end{pmatrix} = \begin{pmatrix} -m\frac{d^2}{dz^2} + V''(x_\alpha) & 0 \\ 0 & -m\frac{d^2}{dz^2} + \frac{V'(x_\alpha)}{x_\alpha} \end{pmatrix}, \qquad (5)$$

where the scalar product is defined by $\langle\mathbf{v}|\mathbf{w}\rangle \equiv \int_{-L/2}^{L/2} \mathbf{v}\cdot\mathbf{w}dz$. $F[\mathbf{u}_0]$ shall be set equal to zero from now on (which only means the addition of an irrelevant constant to $F$).

The vanishing of the nondiagonal elements of $\mathcal{O}_\alpha$ is due to the fact that the pinning potential has rotational symmetry and that $y_\alpha = 0$. Under consideration of the above boundary conditions the eigenvalues of the Schrödinger operators $\mathcal{O}_0^x$, $\mathcal{O}_0^y$, and $\mathcal{O}_1^y$ are all positive but $\mathcal{O}_1^x$ possesses one negative and one zero eigenvalue in the limit $L \to \infty$. The existence of the zero eigenvalue is easily shown by differentiation of the equation of motion (2). Since the corresponding eigenvector $\dot{\mathbf{u}}_1$ ($\equiv \partial\mathbf{u}_1/\partial z$) has one node at $z = 0$ there is one lower eigenvalue $\lambda_{-1} < 0$.

This shows that $\mathbf{u}_1$ corresponds to a saddle point of $F$ in the space $\mathcal{H}$ of all FL configurations $\mathbf{u}(z)$ whereby $\mathbf{u}_0$ corresponds to an absolute minimum. This structure is schematically shown in Fig. 2. The existence of the zero eigenvalue reflects translational invariance; i.e., the critical bubble $\mathbf{u}_1$ may be shifted up and down without changing the energy.

To introduce thermal fluctuations we now consider the FL being under the influence of stochastic forces due to a heat bath with inverse temperature $\beta$. Thus, our starting point is the Langevin equation

$$\Gamma^{-1}\frac{\partial\mathbf{u}}{\partial t} = -\frac{\delta F}{\delta\mathbf{u}(z)} + \mathbf{f}(z,t), \qquad (6)$$

with viscosity $\Gamma^{-1}$ and Gaussian stochastic forces $\mathbf{f}$ which obey $\langle\mathbf{f}\rangle = 0$ and $\langle f_i(z,t)f_j(z,t)\rangle = 2\beta^{-1}\Gamma^{-1}\delta_{ij}\delta(z - z')\delta(t-t')$. Note that $\Gamma^{-1}$ can be related to the ideal flow resistivity $\rho_0$, $\Gamma^{-1} = \frac{B\Phi_0}{c^2\rho_0}$. The corresponding functional Fokker-Planck equation for the probability $W[\mathbf{u}(z)]$ can be written[8]

$$\frac{\partial}{\partial t}W[\mathbf{u}] = -\int dz' \int dz\delta(z - z')\frac{\delta}{\delta\mathbf{u}(z')}\mathbf{S}(z|\mathbf{u}),$$

with the probability current

$$\mathbf{S}(z|\mathbf{u}(z)) = -\frac{\Gamma}{\beta}e^{-\beta F[\mathbf{u}]}\frac{\delta}{\delta\mathbf{u}(z)}\left\{e^{\beta F[\mathbf{u}]}W[\mathbf{u}]\right\}. \qquad (7)$$

If the energy at the saddle point is sufficient large, i.e., $\beta F[\mathbf{u}_1] \gg 1$, two assumptions can be made: (1) $\partial W/\partial t$ is small so that an entirely pinned vortex is almost in thermal equilibrium, i.e., $W[\mathbf{u}] = \exp(-\beta F[\mathbf{u}])W[\mathbf{u}_0]$ near $\mathbf{u}_0$, and (2) the total probability current $\mathcal{S}$ out of the pinned state is essentially nonzero only near the saddle point of $F$ at $\mathbf{u}_1$ which suggests using the expansion (4). Then we can proceed in analogy to Kramers' calcula-

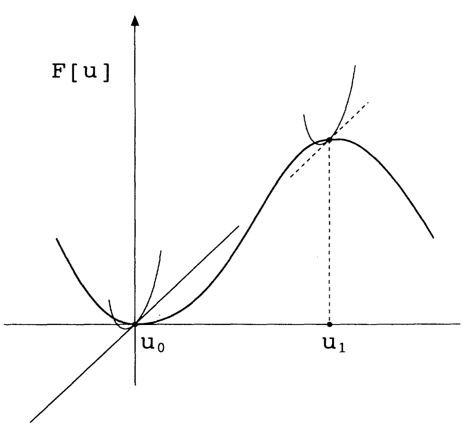


FIG. 2. Structure of the energy functional $F[\mathbf{u}(z)]$ (schematic). At $\mathbf{u}_1$ there is a saddle point with maximum negative curvature $|\lambda_{-1}|$. The direction with zero curvature has not been included in the figure.

tion of the escape rate over a potential barrier in one dimension.[9] The total probability current $\mathcal{S}$ is found integrating (7) within a saddle-point approximation to be

$$\mathcal{S} = \frac{\Gamma}{\beta}\sqrt{\frac{|\lambda_{-1}|\beta}{2\pi}} \oint_{\mathcal{A}} \mathcal{D}\mathbf{u}(z) e^{-\beta F[\mathbf{u}]} W[\mathbf{u}_0], \tag{8}$$

where $\oint_{\mathcal{A}} \mathcal{D}\mathbf{u}(z)$ means a functional integration over a hypersurface $\mathcal{A} \subset \mathcal{H}$ which contains all bounce solutions $\mathbf{u}_1(z+\mu), |\mu| < L/2$, and is perpendicular to the direction of the steepest descent, i.e., maximum negative curvature of $F$.

The total probability of the pinned state $\mathcal{W}$ can be written

$$\mathcal{W} = W[\mathbf{u}_0] \int_{\mathcal{V}} \mathcal{D}\mathbf{u}(z) e^{-\beta F[\mathbf{u}]},$$

where $\int_{\mathcal{V}} \mathcal{D}\mathbf{u}(z)$ stands for a functional integration over all pinned FL configurations.

Now all functional integrations can again be calculated within a saddle-point approximation whereby the mode belonging to the zero eigenvalue of $\mathcal{O}_1^x$ has to be treated separately. The latter gives simply a factor $\| \dot{\mathbf{u}}_1 \| L$ which is the curve length of the line $\{\mathbf{u}_1(z+\mu), |\mu| < L/2\} \subset \mathcal{H}$. Hence, the vortex escape rate is $R \equiv \mathcal{S}/\mathcal{W} = \omega_a \exp(-\beta U)$ with the activation energy $U = F[\mathbf{u}_1] = \frac{2}{3}\Delta V_p$ and attempt frequency

$$\omega_a = \frac{\Gamma}{\beta}\sqrt{\frac{|\lambda_{-1}|\beta}{2\pi}} \frac{\int \mathcal{D}''\mathbf{u}(z) \exp\{-\frac{\beta}{2}\langle \mathbf{u}|\mathcal{O}_1|\mathbf{u}\rangle\}}{\int \mathcal{D}\mathbf{u}(z) \exp\{-\frac{\beta}{2}\langle \mathbf{u}|\mathcal{O}_0|\mathbf{u}\rangle\}} \| \dot{\mathbf{u}}_1 \| L, \tag{9}$$

where the double dash means the exclusion of the directions with negative and zero eigenvalues of $\mathcal{O}_1^x$.

The Gaussian path integrals from (9) may now be expressed in terms of determinants of the operators $\mathcal{O}_\alpha$:

$$\frac{\int \mathcal{D}''\mathbf{u}(z) \exp\{-\frac{\beta}{2}\langle \mathbf{u}|\mathcal{O}_1|\mathbf{u}\rangle\}}{\int \mathcal{D}\mathbf{u}(z) \exp\{-\frac{\beta}{2}\langle \mathbf{u}|\mathcal{O}_0|\mathbf{u}\rangle\}} = \left(\frac{\det \mathcal{O}_0^x}{|\det \mathcal{O}_1^x|}\right)^{\frac{1}{2}} \left(\frac{\det \mathcal{O}_0^y}{\det \mathcal{O}_1^y}\right)^{\frac{1}{2}} \sqrt{\lambda_0}\sqrt{|\lambda_{-1}|}\frac{\beta}{2\pi}.$$

$\lambda_0$ is the zero eigenvalue in the limit $L \to \infty$. For finite $L$, but $L \gg \Delta$, it has an exponential small value which is obtained perturbatively,[4] $\lambda_0 = \frac{4m\omega_0^3\sigma^2}{\|\dot{\mathbf{u}}_1\|^2} e^{-\omega_0(L-\Delta)}$. The quotients of the fluctuation determinants can be calculated using the Gelfand-Yaglom formula[4] that gives the renormalized determinant $D(L/2) \sim \det \mathcal{O}$ as the solution of the homogenous differential equation $\mathcal{O}D(z) = 0$ with initial conditions $D(-L/2) = 0$, and $D'(-L/2) = 1$. This yields for $L \gg \Delta \gg \omega_0^{-1}$, i.e., for small $j$, $\det \mathcal{O}_0^x / |\det \mathcal{O}_1^x| = \frac{1}{2}e^{\omega_0 L}$, and $\det \mathcal{O}_0^y / \det \mathcal{O}_1^y = \frac{2}{\omega_0 \Delta} e^{\omega_0 \Delta}$. The negative eigenvalue $\lambda_{-1}$ of $\mathcal{O}_1^x$ can be calculated in the limit of small currents $j$ considering the behavior of the corresponding symmetric eigenfunction $\psi_{-1}(z)$ for $z > 0$. For $j \to 0$ we must have $\psi_{-1}(z) \to \dot{x}_1$; thus $\lambda_{-1} \to 0$. Since then $V''(z) \approx m\omega_0^2 \gg \lambda_{-1}$ for $z \gtrsim \frac{\Delta}{2}$ one may write $\psi_{-1}(z \gtrsim \frac{\Delta}{2}) = \dot{x}_1 + O(j)$, and on the other hand $\psi_{-1}(z \lesssim \frac{\Delta}{2}) \sim \cosh\left(\sqrt{-\lambda_{-1}/m}\, z\right)$. Matching both together gives $\lambda_{-1} = -m\zeta^2(2/\Delta)^2 + O(j^3)$ where $\zeta = 1.1997$ is the root of $\zeta \tanh \zeta - 1$.

Finally gathering all terms we obtain

$$\omega_a = \frac{\Gamma V_p}{\sigma^2} \frac{4\zeta^2}{(2\pi)^{3/2}} \sqrt{\beta\omega_0 V_p} \left(\frac{j}{j_0}\right)^{\frac{5}{2}} \exp\left(2\frac{j_0}{j}\right) L, \qquad j \ll j_0. \tag{10}$$

It shall be mentioned that the arguments of all exponentials have only been calculated up to order $1/j$. There are also corrections of order unity depending on the detailed shape of the pinning potential which have to be considered in order to determine the exact numerical prefactor in Eq. (10).

The escape rate $R$ may be written $R = \Omega_0 \times \exp(-\beta U_{\text{eff}})$ with

$$\Omega_0 = \frac{\Gamma V_p}{\sigma^2} \frac{\sqrt{3}\zeta^2}{\pi^{3/2}} L\omega_0 \left(\frac{j}{j_0}\right)^{\frac{5}{2}} \left(\frac{T}{T_0}\right)^{-\frac{1}{2}} \tag{11}$$

and an effective activation energy

$$U_{\text{eff}} = 2k_B T_0 \left(1 - \frac{T}{T_0}\right) \left(\frac{j}{j_0}\right)^{-1}, \tag{12}$$

where $k_B T_0 = (2/3)V_p/\omega_0$. $U_{\text{eff}}$ can be regarded as the difference of the free energies $F_1$, $F_0$ of a pinned vortex segment of length $\Delta$, $F_0 = k_B T \omega_0 \Delta$, and an unpinned one with $F_1 = (2/3)V_p\Delta$. The present theory is valid only if $(F_1 - F_0)/k_B T \gg 1$. Since $\Delta \sim j^{-1}$ this requires $T < T_0$. For $T > T_0$ there exists no pinned state of the FL anymore. Such a renormalization of the pinning energy of a columnar defect due to thermal fluctuations has also been calculated by Nelson[5] who considers a square well pinning potential in contrast to the harmonic oscillator approximation used here.

Having calculated the escape rate of a vortex pinned by a columnar defect we look now how this is connected to the resistivity $\rho$. If the vortices are not pinned they will move transverse to the applied current with velocity $v_0 = \Gamma j$ which gives rise to a finite resistivity $\rho_0 \sim v_0/j$. Consider random distributed columnar defects of density $n_c$, and density of vortex lines $n_v = B/\Phi_0$ where $B$ is the magnetic field. We will assume that $n_c > n_v$ and that each vortex spends most of its time in the pinned state so that vortex-vortex interactions may be neglected. Then the FL's move with average velocity $v = Ra^* \ll v_0$ where $a^*$ [guessed to be $\sim (n_c - n_v)^{-1/2}$] is the mean free path along which a depinned vortex or vortex segment moves before it is pinned again. Thus, from Eq. (10) the resistivity reads

$$\rho \sim \rho_0 \frac{a^*}{\sigma} L\omega_0 \left(\frac{T}{T_0}\right)^{-\frac{1}{2}} \left(\frac{j}{j_0}\right)^{\frac{3}{2}} e^{-\beta U_{\text{eff}}}, \tag{13}$$

whereby $v \ll v_0$ requires $\rho/\rho_0 \ll 1$.

Let us now apply the preceding results to $YBa_2Cu_3O_{7-\delta}$ (YBCO) (taking $\xi_{ab} = 20$ Å, $\gamma = \lambda_c/\lambda_{ab} = 5$, $\kappa = 50$) and Bi-Sr-Ca-Cu-O (BSCCO) ($\xi_{ab} = 20$ Å, $\gamma = 60$, $\kappa = 95$). For BSSCO a FL consists of a stack of 2D point vortices[10] (pancakes) since $\xi_c$ is much smaller than the distance $d$ of superconducting CuO planes. This is in contrast to YBCO where $\xi_c > d$ (3D case). However, as shown by Bulaevski *et al.*[11] also in the 2D case anisotropic London theory is applicable as long as the horizontal distance between the pancakes in neighboring layers is smaller than the Josephson length $\lambda_J = \gamma d$. Here, this means $|\ \dot{\mathbf{u}}\ | \lesssim \gamma$, which also is the condition for validity of the linear elasticity approximation entering in Eq. (1) as recently discussed by Brandt.[3]

The largest slope of the vortex appears at the ends of the critical bubble ($z = \pm\Delta/2$). In both cases, YBCO and BSCCO, one gets $|\ \dot{\mathbf{u}}\ (\pm\Delta/2)| \lesssim \gamma/4$ showing that the above condition is fulfilled. Insertion of the parameters with $\epsilon \approx 1$, and $\sigma \approx \xi_{ab}$ then yields $\omega_0^{-1} \approx 2\xi_c$, and $T_0$ =660 K (16 K) for YBCO (BSCCO). From this it is seen that pinning by columnar defects is effective for YBCO ($T_0 \gg T_C = 87.2$ K) but not for BSCCO ($T_0 < T_C = 87$ K) at higher temperatures. At last we will check the condition $j/j_0 \gg (L\omega_0)^{-1}$: If a typical sample thickness is taken to be $10^4$ Å then $j/j_0 \gg 10^{-3}(10^{-4})$ which shows the applicability down to rather small currents.

In conclusion we have calculated the escape rate of a vortex pinned by a columnar defect in the presence of a transversal current. The activation energy $\sim 1/j$ is renormalized due to thermal fluctuations which lead to an enhanced free energy in the pinned state. The prefactor in the Arrhenius law has been calculated to be proportional to $j^{5/2}T^{-1/2}$ for the escape rate, and to $j^{3/2}T^{-1/2}$ for the resistivity.

We thank K. D. Schotte for helpful discussions.

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* On leave of absence from Institute of Physics, Belgrade University, Belgrade, Yugoslavia.

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