Tilt-wave instability of the flux-line lattice in anisotropic superconductors

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The stability of the Abrikosov flux-line lattice for uniaxially anisotropic superconductors has been studied in the London approximation as a function of the anisotropic mass ratio and the angle θ between the **B** field and crystal c axis. For anisotropic mass ratio M_c/M_{ab} larger than $(3+\sqrt{8})$ and for a range of angles $\theta_{c1} < \theta < \theta_{c2}$, the straight flux lines become unstable and a tilt-wave instability occurs, irrespective of the magnitude of the **B** field. We show that this instability occurs for a variety of cutoff schemes (which are needed to make London theory finite). The instability is probably related to the formation of a "combined" lattice of flux lines in which two species of vortices coexist simultaneously but run in different directions.

I. INTRODUCTION

The discovery of high-temperature superconductivity has recreated interest in type-II superconductors. High-T_c superconductors are strongly anisotropic materials. In the presence of a sufficiently strong magnetic field type-II materials change from the Meissner superconducting state, in which all magnetic flux is expelled, to a mixed state with a flux-line lattice. For uniaxially anisotropic superconductors, Campbell, Doria, and Kogan[1] and Petzinger and Warren[2] have predicted that this lattice, at large **B** field, is a *deformed* triangular lattice and the spacings between the flux lines vary in a simple way with the anisotropic mass ratio M_Z/M and the **B** field orientation with respect to the crystal c axis. Here, $M_Z \equiv M_c$ and $M \equiv M_{ab}$ are the masses along the crystal c axis and the ab plane respectively. The stability of this lattice against elastic deformations has been a subject of intensive investigation. The further study of this question is the main aim of this paper.

Recently Sudbø and Brandt[3] have reported that for the lattice configuration mentioned above,[1,2] the energy associated with the transverse mode corresponding to a pure *shearing* deformation of the flux lattice can become negative for extremely anisotropic superconductors ($M_Z/M \gg 1$) in the very low induction regime when $b = B/B_{c2} \ll 1$, where B_{c2} is the upper critical field. This signals a structural instability of the flux lattice. Daemen, Campbell, and Kogan[4] have proposed an ordered state in which at low induction the flux lattice is still a deformed triangular lattice, but where the spacings between the flux lines do not scale uniformly in all directions as $B^{-1/2}$. In addition, these spacings are different functions of M_Z/M and θ (the angle between the **B** field and the c axis) than in the state of Refs. 1 and 2.

In this paper we investigate a quite different type of instability of the flux lattice. We show that an instability at finite values of k_z (a *tilt*-wave instability) develops, regardless of the values of the induction b, in an angular range $\theta_{c1} < \theta < \theta_{c2}$ where both critical oblique angles depend on M_Z/M. (Here k_z is the z component of the wave vector along the **B** field). This suggests that the ordered state for $\theta_{c1} < \theta < \theta_{c2}$ may not belong to a configuration of straight flux lines. However, the experimental results of Ref. 5 suggest[6] that it is more likely that the ordered state of the flux lattice will be a "combined" lattice of flux lines running parallel and perpendicular to the c axis or alternatively[7] a "combined" lattice in which some flux lines lie parallel to the c axis while others lie at an angle θ with respect to the c axis. We shall be discussing these possibilities later in more detail.

We shall use throughout anisotropic London theory. This theory suffers a serious inability to account for variations of the order parameter near the core region as it assumes that the order parameter is a constant everywhere. Unfortunately, this gives rise to an unphysical divergence in the calculations of the free energy and sums over the wave vectors have to be cut off to make the free energy finite. Several cutoff schemes have been proposed. We show that the occurrence of a tilt-wave instability is independent of the cutoff scheme used. For illustrative purposes, we shall produce numerical results using a Gaussian cutoff.

The possibility of a tilt-wave instability was first investigated by Koyama and Tachiki.[8] Their analysis was restricted to the case of the **B** field along the crystal c axis. They found that a single vortex line developed a spiral vortex instability. They later discovered that their work was in error and that a straight vortex line had a lower free energy than a spiral vortex line.[9] A more systematic study of this problem has been recently carried out by Carneiro, Doria, and de Andrade.[10] Their procedure was again to use anisotropic London theory accompanied by some regularization method. They concluded that with the **B** field along the crystal c axis an instability could develop at finite values of k_z, depending on the cutoff procedure and the value of λ_{ab} (ξ_{ab}) and λ_c (ξ_c), the penetration depths (coherence lengths) along the ab plane and the crystal c axis respectively. We find that there is no tilt-wave instability when the vortex lines are parallel to the c axis and we believe that only unphysical cutoff procedures can produce instability. We argue for the validity of our cutoff technique in Sec. II.

The outline of this paper is as follows. In Sec. II we

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present a brief review of nonlocal elasticity theory for anisotropic superconductors within the London regime. In Sec. III we evaluate (numerically) the normal modes (eigenvalues) of the elastic matrix. We also find conditions under which a tilt-wave instability may set in. This condition is related exclusively to the anisotropic mass ratio M_Z/M and the oblique angle θ, and not to other parameters like b. In Sec. IV we discuss possible implications of these results as well as the limitations on their validity.

II. NONLOCAL ELASTICITY THEORY

Type-II superconductors are conveniently described by the (linear) phenomenological London equation. Anisotropy is allowed for by replacing the square penetration depth λ^2 in the isotropic version of this equation by a tensor λ_{ij}^2. We shall study only uniaxial compounds where $\lambda_{XX} = \lambda_{YY} = \lambda_{ab} \neq \lambda_{ZZ} = \lambda_c$. The coherence length, which is present only in the full Ginzburg-Landau theory, is also a tensor. One has $\lambda_c/\lambda_{ab} = \xi_{ab}/\xi_c = \sqrt{M_Z/M}$. This is the so-called *effective mass model.* In general the anisotropic superconductors are *layered* materials. However, at not too low temperatures the interlayer spacing may be much smaller than ξ_c when the layered structure becomes irrelevant and the *continuum* version of the effective mass model may be applicable; otherwise the Lawrence-Doniach[11] type of model has to be used. The continuum limit shall be used in the present work. Within this context, the London free energy is given by

$$F = \frac{1}{8\pi}\int d^3r\left[\mathbf{H}^2 + \left(\frac{\Phi_0}{2\pi}\nabla\varphi - \mathbf{A}\right)\cdot \overset{\leftrightarrow}{\Lambda}^{-1}\cdot\left(\frac{\Phi_0}{2\pi}\nabla\varphi - \mathbf{A}\right)\right], \tag{1}$$

where $\mathbf{H}$ is the magnetic field and is related to the vector potential $\mathbf{A}$ by $\mathbf{H} = \nabla\times\mathbf{A}$, Φ_0 is the quantum flux, φ is the phase of the order parameter, and $\mathbf{B} = \langle\mathbf{H}\rangle = B\hat{\mathbf{z}}$. Here $\overset{\leftrightarrow}{\Lambda}$ is the square penetration depth tensor and is given by $\Lambda_{XX} = \Lambda_{XX} = \lambda_{ab}^2$ and $\Lambda_{ZZ} = \lambda_c^2$. The $\mathbf{B}$ field lies in the XZ plane and is tilted away from the crystal c axis by an angle θ (see Fig. 1). It is then convenient to rotate the crystal frame XYZ by the same angle around the Y axis. In the new (vortex) frame xyz the square penetration depth tensor is given by $\Lambda_{\alpha\beta} = \Lambda_1\delta_{\alpha\beta} + \Lambda_2 c_\alpha c_\beta$ with $(\alpha,\beta) = (x,y,z)$ where $\Lambda_1 = \lambda_{ab}^2$ and $\Lambda_2 = \lambda_c^2 - \lambda_{ab}^2$. Here c_α denotes the α component of the unit vector $\mathbf{c}$ in the vortex frame.

Next we minimize (1) with respect to $\mathbf{A}$ to find

$$\overset{\leftrightarrow}{\Lambda}\cdot\nabla\times\mathbf{H} - \left(\frac{\Phi_0}{2\pi}\nabla\varphi - \mathbf{A}\right) = \mathbf{0}\,. \tag{2}$$

The phase is such that $\nabla\times\nabla\varphi = 2\pi\sum_i \delta_2(\mathbf{r} - \mathbf{r}_i(z))d\mathbf{r}_i(z)/dz$, where $\mathbf{r}_i(z) = (x_i(z), y_i(z), z)$ is the position of the ith flux line at height z in an ensemble of arbitrarily distorted flux lines; the function $\delta_2(\mathbf{r} - \mathbf{r}_i(z)) \equiv \delta(x - x_i(z))\delta(y - y_i(z))$. By taking the curl of both sides of (2) we then obtain the London equation

$$\nabla\times[\overset{\leftrightarrow}{\Lambda}\cdot\nabla\times\mathbf{H}] + \mathbf{H} = \Phi_0\sum_i \delta_2(\mathbf{r} - \mathbf{r}_i(z))\frac{d\mathbf{r}_i(z)}{dz}\,. \tag{3}$$

This equation can also be obtained from the second Ginzburg-Landau equation by taking the order parameter as a constant. Since it is linear in the magnetic field, its solution is straightforward. One has

$$H_\alpha(\mathbf{r}) = \Phi_0\sum_i\int dr_i^\beta(z)V_{\alpha\beta}(\mathbf{r} - \mathbf{r}_i(z))\,, \tag{4}$$

where the Fourier transform of the London (tensor) potential $V_{\alpha\beta}(\mathbf{r})$ is defined by

$$\tilde{V}_{\alpha\beta}(\mathbf{k}) = \frac{1}{1+\Lambda_1 k^2}\left[\delta_{\alpha\beta} - \frac{\Lambda_2 q_\alpha q_\beta}{1+\Lambda_1 k^2 + \Lambda_2 q^2}\right], \tag{5}$$

where $\mathbf{q} = \mathbf{k}\times\hat{\mathbf{c}}$.[12] By inserting (2) into (1) and using (3) and (4) we obtain for the free energy

$$F = \frac{\Phi_0^2}{8\pi}\sum_{i,j}\int\int dr_i^\alpha dr_j^\beta\, V_{\alpha\beta}(\mathbf{r}_i - \mathbf{r}_j)\,. \tag{6}$$

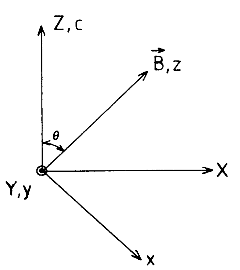

FIG. 1. The axis X, Y, Z corresponds to the crystal frame and x, y, z to to the vortex frame. The latter can be obtained from the former by rotating the crystal frame by an angle θ around the Y axis.

Here we have used the convention that sums are made over repeated indices. The minimum mean-field free energy, in the high field regime, is predicted to be a periodic arrangement of straight flux lines $[dr_i^\alpha/dz = \hat{\mathbf{z}}$ in (6)] with hexagonal symmetry.[1,2] The basis lattice vectors are $\mathbf{R}_1 = a\gamma\hat{\mathbf{x}}$, $\mathbf{R}_2 = a(\gamma\hat{\mathbf{x}} + \sqrt{3}\hat{\mathbf{y}}/\gamma)/2$, where $\gamma^4 = \cos^2\theta + M\sin^2\theta/M_Z$ and $a^2 = 2\Phi_0/\sqrt{3}B$. The corresponding basis vectors of the reciprocal lattice are $\mathbf{Q}_1 = 2\pi(\sqrt{3}\hat{\mathbf{x}} - \gamma^2\hat{\mathbf{y}})/\sqrt{3}a\gamma$, $\mathbf{Q}_2 = 4\pi\hat{\mathbf{y}}/\sqrt{3}a\gamma$. The excess free energy due to small displacements $\mathbf{s}(\mathbf{R}_i(z))$ of the flux lines from their equilibrium positions in the ordered state $\mathbf{R}_i(z) \equiv n\mathbf{R}_1 + m\mathbf{R}_2$ has been derived in Refs. 13 and 14; m and n are integers. Here $\mathbf{s}$ is a two dimensional vector, since displacements along the flux lines have no physical meaning. Upon keeping only terms up to second order in these displacements we have

$$\Delta F = \frac{1}{2}\int d^3k\, s_\alpha(-\mathbf{k})\Phi_{\alpha\beta}(\mathbf{k})s_\beta(\mathbf{k})\ , \qquad (7)$$

where the integration over $\mathbf{k}_\perp = (k_x, k_y)$ runs over the first Brillouin zone and over k_z on the interval $(-\infty,\infty)$. The elasticity matrix is given by

$$\Phi_{\alpha\beta}(\mathbf{k}) = \frac{B^2}{4\pi}\sum_{\mathbf{Q}}[f_{\alpha\beta}(\mathbf{k}+\mathbf{Q}) - f_{\alpha\beta}(\mathbf{Q})]\ , \qquad (8)$$

$$f_{\alpha\beta}(\mathbf{k}) = k_z^2\tilde{V}_{\alpha\beta}(\mathbf{k}) + k_\alpha k_\beta \tilde{V}_{zz}(\mathbf{k}) - 2k_z k_\beta \tilde{V}_{z\alpha}(\mathbf{k})\ . \qquad (9)$$

In the derivation of (7) no specific equilibrium lattice is required. However, in what follows we consider an ordered state with hexagonal symmetry $\mathbf{Q} \equiv \mathbf{Q}_{mn} = n\mathbf{Q}_1 + m\mathbf{Q}_2$, where the basis vectors of the reciprocal lattice are as above.

Equations (6)–(9) deserve some comments. As one can see from (5), the function $\tilde{V}_{\alpha\beta}(\mathbf{k})$ falls off as $1/k^2$ as $k\to\infty$, which implies a singular behavior of $V_{\alpha\beta}(\mathbf{r})$ at $\mathbf{r}=\mathbf{0}$. This means that the integrals in (6) over all vortex segments dr_i^α, dr_j^β need a cutoff for the self-energy contributions $i=j$. Similarly the lattice sums are also divergent in (8). As we have already emphasized before, these divergencies stem from the fact that in London theory, variations of the order parameter near the cores are neglected; i.e., the cores are considered as δ function singularities. A convenient way to cure this deficiency of London theory is to replace the δ function $\delta_2(\mathbf{r}-\mathbf{r}_i(z))$ in the right-hand side of the London equation (3) by a short range function $S(\mathbf{r}-\mathbf{r}_i(z))$ to account for the variation of the order parameter perpendicular to the cores.[15] This modification of the London model does not change the form of (7)–(9), except by the fact that now the Fourier transform of the London potential of (5) is replaced by

$$\tilde{V}_{\alpha\beta}(\mathbf{k}) = \frac{\tilde{S}(\mathbf{k}_\perp)}{1+\Lambda_1 k^2}\left[\delta_{\alpha\beta} - \frac{\Lambda_2 q_\alpha q_\beta}{1+\Lambda_1 k^2 + \Lambda_2 q^2}\right]\ , \qquad (10)$$

where $\tilde{S}(\mathbf{k}_\perp)\to 1$ for $k_\perp \ll 1/\xi_{ab}$.

In this work we shall use mostly an elliptic Gaussian cutoff $\tilde{S}(\mathbf{k}_\perp) = e^{-2g(\mathbf{k}_\perp)}$, where $g(\mathbf{k}_\perp) = \xi_{ab}^2(\mathbf{k}_\perp\times\hat{\mathbf{c}})^2 + \xi_c^2(\mathbf{k}_\perp\cdot\hat{\mathbf{c}})^2$.[16] Notice that the cutoff function S depends on $\mathbf{k}_\perp$ and not on $\mathbf{k}$. The reason for this choice is that the spatial variation of the order parameter is in the plane perpendicular to the vortex direction for straight vortices and, therefore, the cutoff (in Fourier space) should not involve the z component of the wave vector. The authors of Refs. 16 and 10 employed a cutoff procedure involving k_z, rather than just $\mathbf{k}_\perp$, which always increases the tendency towards instability, and explains why they found an instability even when $\mathbf{B}\parallel c$. We must emphasize that once the two-dimensional δ function in the London equation is replaced by the short range function S, the derivation of (10) is exact. We believe that all cutoff functions $\tilde{S}$ which depend only on $\mathbf{k}_\perp$ will give qualitatively similar results.

III. TILT-WAVE INSTABILITY

Let us now move our discussion to the stability of the lattice. Our analysis is based on the normal modes (eigenvalues) of the elasticity matrix (8) using the above mentioned Gaussian cutoff and the form of $\tilde{V}_{\alpha\beta}(\mathbf{k})$ given by (10). There are two normal modes of excitation, one transverse (Ω_-) and one longitudinal (Ω_+). The longitudinal mode remains always hard and we do not consider it further.

In what follows lengths are measured in units of a and wave vectors in units of $1/a$. In Fig. 2 we show a plot of the transverse mode as a function of k_z for $\mathbf{k}_\perp = \mathbf{0}$, $\kappa = 50$, $M_Z/M = 3600$, $b = 0.001$, and several values of θ. From this figure it can be seen that for $\theta = 3\pi/8$ the transverse eigenvalue becomes negative, indicating a tilt-wave instability. For $\theta = \pi/4$ we can also see a tendency towards an instability. During the numerical work we noticed that the limit $k_z\to\infty$ always corresponded to the most unstable situation of all. Hence, to investigate the instability it is convenient to take the limit $k_z \gg \xi_{ab}^{-1}$ in (8). Since the summand in this equation is cut at values ξ_{ab}^{-1} we obtain in this limit $\Phi_{xy}(0,0,\infty) = \Phi_{yx}(0,0,\infty) = 0$ and

$$\Omega_-(0,0,\infty) = \Phi_{yy}(0,0,\infty) = \frac{B^2}{4\pi}\sum_{\mathbf{Q}}\left[\frac{\tilde{S}(\mathbf{Q})}{\Lambda_1+\Lambda_2\sin^2\theta} - Q_y^2\tilde{V}_{zz}(\mathbf{Q})\right]\ , \qquad (11)$$

where the first terms in the right-hand side of both equations come from the diagonal terms of the London potential of (5).

In Fig. 3 we plot the asymptotic limit of the soft mode, $\Omega_-(0,0,\infty)$ in units of $B^2/4\pi a^2$ and normalized to its value at $\theta=0$, as a function of θ for several values of

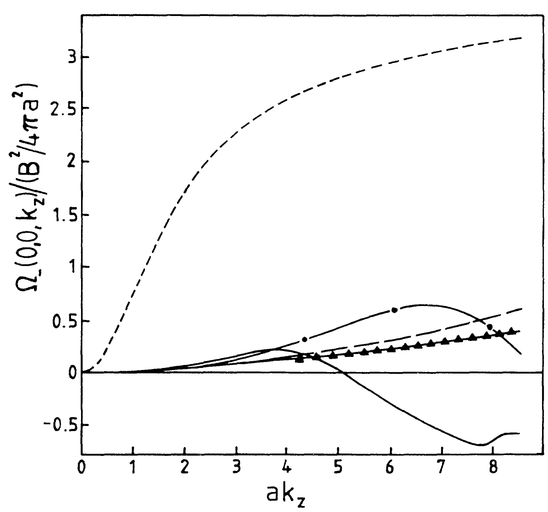

FIG. 2. Transverse mode for $\mathbf{k}_\perp = 0$ as a function of k_z at $\theta = 0$ (short-dashed line), $\pi/8$ (long-dashed line), $\pi/4$ (dotted line), $3\pi/8$ (solid line), and $\pi/2$ (triangle marked line), with $\kappa = 50$, $M_Z/M = 3600$, and $b = 0.001$. The normal mode is measured in units of $B^2/4\pi a^2$ and k_z in units of $1/a$.

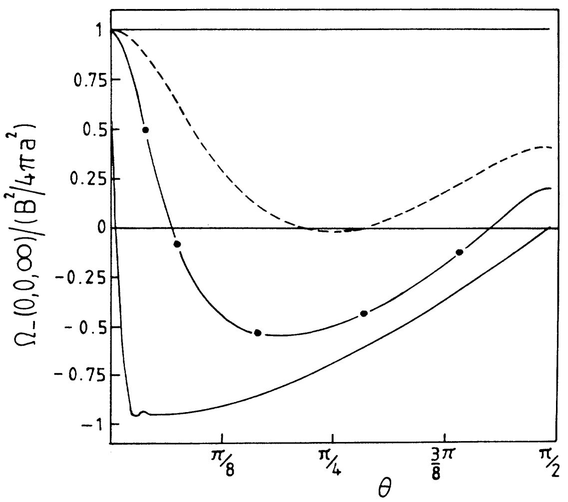

FIG. 3. Transverse mode as a function of θ for $\mathbf{k}_\perp = 0$ and $k_z = \infty$ with $\kappa = 50$, $M_Z/M = 1$ (constant line), 6 (dashed line), 25 (dotted line), and 3600 (solid line), and $b = 0.001$. The normal mode is measured in units of $B^2/4\pi a^2$ and it is normalized to its value at $\theta = 0$.

M_Z/M and for $\kappa = 50$, $b = 0.001$. It can be seen that for a certain value of the anisotropy larger than a certain critical value ($M_Z/M = 3 + \sqrt{8} \approx 5.45$), the transverse mode changes its sign at $\theta = \theta_{c1}$ and $\theta = \theta_{c2}$, indicating a tilt-wave instability for $\theta_{c1} < \theta < \theta_{c2}$. Stability is assured for the $\mathbf{B}$ field along the c axis and along the ab plane, no matter what M_Z/M is. [We also evaluated the sum (11) by taking $\tilde{S}(\mathbf{Q}) = 1$ and using a sharp cutoff in which the sum is truncated at all values of $Q > 1/\xi_{ab}$, the length scale which London theory starts to break down. No significant difference is produced in the behavior of the soft mode as a function of θ.] Another important feature of (11) is that the asymptotic limit of the soft mode is very insensitive to variations of b in our approximation scheme. (However, at extremely low induction our results may become less accurate since the free energy is dominated by the core energy and the cutoff scheme might break down.) The tilt wave instability is determined only by the anisotropy and the angle θ, which is remarkably different from the case of the shearing instability found by Sudbø and Brandt[3] which only occurred at extremely low induction.

Before we close this section let us present an analytical derivation of the critical angles mentioned above. The sum in (11) can be replaced by an integral over $\mathbf{Q}$ as it is dominated by the large values of $\mathbf{Q}$. Similarly, for large $\mathbf{Q}$ one has

$$\tilde{V}_{zz}(\mathbf{Q}) = \frac{1 + \lambda_c^2\gamma^4 Q^2}{(1 + \lambda_{ab}^2 Q^2)[1 + \lambda_c^2(\gamma^4 Q_x^2 + Q_y^2)]}\tilde{S}(\mathbf{Q}) \approx \frac{\gamma^4}{\lambda_{ab}^2(\gamma^4 Q_x^2 + Q_y^2)}\tilde{S}(\mathbf{Q}) \;. \tag{12}$$

Simple algebra yields

$$\Omega_-(0,0,\infty) = \left(\frac{1}{\gamma_1^4} - \frac{\gamma^4}{2}\right)\frac{B^2 A}{4\pi\lambda_{ab}^2}\int \frac{dQ'}{2\pi}\, Q'\tilde{S}(Q') \;, \tag{13}$$

where γ has been defined in the previous section and $\gamma_1^4 = \cos^2\theta + M_Z \sin^2\theta/M$; $A = \Phi_0/B$ is the area of a unit cell. In order to obtain the last equation we have made the following change of variable $\mathbf{Q}' = (\gamma Q_x, Q_y/\gamma)$. This rescaling keeps the Jacobian unchanged. Furthermore, it removes any angular dependence of the cutoff function. We can see this if we rewrite $g(\mathbf{Q})$ as $g(\mathbf{Q}) = \xi_{ab}^2(\gamma^4 Q_x^2 + Q_y^2) = \xi_{ab}^2\gamma^2(Q')^2$. This allows us to write the soft mode as in (13).

The soft mode eigenvalue Ω_- then vanishes identically if

$$\frac{1}{\gamma_1^4} - \frac{\gamma^4}{2} = 0 \;. \tag{14}$$

A straightforward manipulation of this equation produces

$$\sin^2\theta_i = \frac{1}{2}\left[1 \pm \sqrt{1 - \frac{4\frac{M_Z}{M}}{(\frac{M_Z}{M} - 1)^2}}\right] \;, \tag{15}$$

where $i = c1$ for the minus sign and $i = c2$ for the plus sign.

In order to satisfy the inequality $-1 \le \sin\theta_i \le 1$ and that the roots of (14) be real one has to fulfill the condition $M_Z/M \ge 3 + \sqrt{8}$. Note that the prescription for a tilt-wave instability, Eq. (14), does not depend on the definition of the cutoff function $\tilde{S}(\mathbf{Q})$. The only restriction to this statement is that the cutoff function can be brought into a form which depends on Q but not on $\mathbf{Q}$, by using the rescaling specified above, which holds within all effective mass models.

IV. DISCUSSION

In summary we have investigated the stability of the flux-line lattice with respect to tilt deformation by using linear nonlocal elasticity theory. Our results indicate that a tilt-wave instability may develop for sufficient large anisotropy and within a certain angular range. In addition, we have determined a critical value for the anisotropy mass ratio beyond which a tilt-wave instability may become favorable.

Several questions arise from these results. One obvious question is to ask whether this tilt-wave instability is a real effect. Two approximations have been used in this work, namely, London theory and the continuum model rather than a Lawrence-Doniach model which is usually more appropriate for an anisotropic superconductor. Although the present results indicate that the tilt-wave instability is insensitive to the cutoff model used for the function $\tilde{S}(\mathbf{k}_\perp)$, it would be desirable to check if this effect can also be detected by giving a more rigorous treatment to the cores with the use of Ginzburg-Landau theory. If Ginzburg-Landau theory is not capable of removing this tilt-wave instability, it is also important to check if the Lawrence-Doniach model for the superconducting layers also reproduces it. Therefore we cannot rule out the possibility of a tilt-wave instability being an artifact of either London theory or the continuum model. Notice that the instability (see Figs. 2 and 3) occurs at such large values of k_z (say of order $1/\xi_c$) that such an instability might be claimed to be taking place in a regime where the London equations could not be expected to have any validity and so the instability discussed might be without physical validity. However, we would point out that the value of k_z at which the instability actually occurs can often be quite small.[17] There is also a question of principle. The London equation (3) is widely studied. The correct solution in the angle range $\theta_{c1} < \theta < \theta_{c2}$ is not the straight line solution. If one dismisses the instability as an artifact, then one should find a way of changing (3), or (3) with a cutoff, to prevent such an instability taking place. Until that is done, no reliable conclusions can be drawn from a study of the London equation.

If one admits the possibility of a tilt-wave instability being a real effect, one would expect a new type of the ordered state in the angle range $\theta_{c1} < \theta < \theta_{c2}$. One possibility is a state of the form $\mathbf{R}_i(z) + \mathbf{u}(z)$ where $\mathbf{u}(z)$ is a periodic function of z. As we mentioned above, the most unstable situation corresponds to large k_z. We expect then that the periodicity of $\mathbf{u}(z)$ is very small, perhaps of order ξ_{ab}. The determination of this function which gives the absolute minimum of the free energy is not a trivial task, but a start has been reported by Ivlev *et al.*[18]

If the tilt-wave instability is a real effect, then one would expect physical quantities such as magnetization and resistivity to have a singularity as θ passes through θ_{c1} and θ_{c2}. We are unaware of data showing these features.

However, we would like to point out a possible connection between our results and what has been experimentally observed by Bolle *et al.*[5] as interpreted by Huse[6] and Daemen *et al.*[7] Their experimental observations on Bi-Sr-Ca-Cu-O ($\sqrt{M_Z/M} \approx 55$) show the presence of two distinct "species" of vortices coexisting simultaneously but running in different directions. A qualitative interpretation of this experiment has been given by Huse.[6] He proposes that the ordered state of the flux lattice is a "combined" lattice of flux lines running parallel and perpendicular to the c axis. An alternative interpretation of the experiment of Bolle *et al.*[5] has been given by Daemen *et al.*[7] They show that a "combined" lattice in which some flux lines lie parallel to the c axis and others lie at an angle θ with respect to the c axis lowers the free energy with respect to a simple deformed triangular lattice tilted away from the c axis. However, it is not clear if the formation of this "combined" lattice occurs above a certain critical value of anisotropic mass ratio M_Z/M and in an angular range $\theta_{c1} < \theta < \theta_{c2}$ as in the present work. Given the observations, we consider that some version of a "combined" lattice is more likely than a lattice of spiraling or staircase vortices as an explanation of the state which forms in the region where the tilt-wave instability rules out the conventional lattice.

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