PHYSICAL REVIEW B VOLUME 48, NUMBER 12 15 SEPTEMBER 1993-II

Current-driven plasma instabilities in modulated lower-dimensional semiconductor systems

K. Kempa, P. Bakshi, and H. Xie
Department of Physics, Boston College, Chestnut Hill, Massachusetts 02167-3811

(Received 1 June 1993; revised manuscript received 23 July 1993)

High threshold drift velocities (exceeding the Fermi velocity) required in semiconductor systems to generate current-driven plasma instabilities are the primary obstacle in their experimental verification. We demonstrate the feasibility of a current-driven plasma instability in *modulated* lower-dimensional systems at much lower driving electric fields. We also discuss possible experimental systems where this instability could be observed.

Current-driven plasma instabilities are known to occur in gaseous plasmas.[1,2] We have shown[3-9] the feasibility of generating analogous instabilities in a variety of layered solid-state systems. So far current-driven plasma instabilities have not been directly observed experimentally in such systems. However, current-driven infrared emission from layered semiconductors, due to thermal plasmon generation[10] or Smith-Purcell effect,[11,12] have already been detected. A recent experiment[13] in a current-driven superconductor system provides a preliminary indication of energy transfer from the driving current into the carrier plasma in broad agreement with our predictions.[5]

The basic physical mechanism of the current-driven plasma instabilities is the transfer of energy from the current to the growing plasma waves of a given system. With a suitable coupling arrangement such as a grating, this energy can be further converted to electromagnetic radiation.[14] Such systems can then serve as radiation sources or amplifiers, with potential device applications. In semiconductor systems the typical threshold drift velocity for a plasma instability is greater than the Fermi velocity, and therefore very difficult to achieve experimentally. In this paper we show that by employing *density modulated* lower-dimensional systems, plasma instabilities can be generated at significantly lower driving electric fields.

A setup consisting of periodically distributed electrode strips or electrode grids over a uniform two-dimensional electron gas (2DEG) provides the experimental arrangement for generating a periodic density modulation of the electron gas, in one or two directions.[15] By varying the bias voltage, the amplitude of this modulation can be varied and density-modulated systems such as one-direction modulated 2DEG, two-direction modulated 2DEG [so-called lateral surface superlattice (LSSL) (Ref. 16)], and modulated unconnected strips or wires (i.e., modulated 1DEG), can be realized.[15] Density-modulated systems can also be obtained by etching a periodic pattern on the surface of the sample just above the 2DEG. Charges trapped in surface states then modulate the underlying electron gas.[11,12]

Due to periodicity of the density modulation, miniband structure formation occurs in modulated systems. In dimensionally restricted systems (e.g., modulated 1DEG or LSSL) under certain conditions, carrier transport caused by an external field can occur only through the movement of electrons in the uppermost, partially filled miniband. For a sufficiently strong electric field this group of electrons can climb up on one branch of the miniband, opening a gap of allowed energy states below the displaced electron distribution. As a result of this population inversion, massive downward single-electron transitions become possible, which generate plasmons. Basically the same physical mechanism leads to current-driven plasma instabilities in unmodulated systems, but instead of displacing the population of a single miniband, the entire population of electrons must be displaced, thus requiring a large drift velocity, exceeding twice the Fermi velocity.[8]

We develop a formalism to study the current-driven response of modulated electron-gas systems. In the random-phase approximation, the Fourier transformed total suscptibility is

$$\chi(\mathbf{q},\mathbf{q}';\omega)=\chi_0(\mathbf{q},\mathbf{q}';\omega)+\sum_{\mathbf{q}''}\chi_0(\mathbf{q},\mathbf{q}'';\omega)v(\mathbf{q}'')\chi(\mathbf{q}'',\mathbf{q}';\omega)\ , \tag{1}$$

where χ_0 is the single-electron susceptibility given by

$$\chi_0(\mathbf{r},\mathbf{r}';\omega)=\frac{2}{A}\sum_{\mathbf{k}}\sum_{\mathbf{k}'}\frac{f_{\mathbf{k}}-f_{\mathbf{k}'}}{\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k}'}+\hbar\omega}\psi_{\mathbf{k}}(\mathbf{r})\psi_{\mathbf{k}'}(\mathbf{r})\psi_{\mathbf{k}}(\mathbf{r}')\psi_{\mathbf{k}'}(\mathbf{r}') \tag{2}$$

where A is a volume factor and $v(\mathbf{q})$ is the Fourier transform of the Coulomb potential.

In the presence of a periodic modulation in one direction (e.g., z direction), Fourier transforms of χ_0 and χ acquire a discrete spectrum for z components of wave vectors (q_z), which become quantized in terms of the reciprocal lattice vector $g_0=2\pi/a$ (a is the period of the modulation). Taking into account this quantization, and assuming that the system is uniform in the remaining directions (x and y), Eq. (2) can be rewritten in the form

$$\chi_0(\mathbf{Q},n,n')=(4\pi/A)\sum_{k_z}\sum_{m}A_m^*A_{m+n-n'}\times\sum_{\mathbf{K}}\frac{f_{\mathbf{K}+\mathbf{k}_z}-f_{\mathbf{K}'+\mathbf{k}_z'}}{\varepsilon_{\mathbf{K}+\mathbf{k}_z}-\varepsilon_{\mathbf{K}'+\mathbf{k}_z'}+\hbar\omega}\ , \tag{3}$$

where $\mathbf{K}'=\mathbf{K}+\mathbf{Q}$; $k_z'=k_z-\dot{q}-(m+n)g_0$; $\mathbf{Q}$ and $\mathbf{K}$ are the x-y components of the photon and electron momenta, respectively; $q+$ is the reduced photon (plasmon) momentum (in z direction); m,n,n' are integers; A_m is defined as follows:

$$u_{k_z}(z)^* u_{k_z'}(z) = \sum_m A_m(k_z, k_z')\exp(img_0 z) , \tag{4}$$

where the periodic functions u are defined through $\psi_{k_z}(z) = u_{k_z}(z)\exp(ik_z z)$.

To demonstrate the main phenomenon, we will now consider the case of a modulated one-dimensional electron gas (1DEG). This is an experimentally realizable system, since superlattices of quantum wires (1DEG's) can now be fabricated,[15] and modulated as discussed above.

To simplify analysis we consider here a system with only two minibands. The energy is then given in the extended-zone scheme by

$$\begin{aligned} \varepsilon(k_z) &= (\hbar^2 z_k^2/2m^*) + UF(k_z), \quad 0 < |k_z| < g_0 , \\ F(k_z) &= \{B - \mathrm{sgn}(B)\sqrt{B^2+U^2}\}/U , \\ B &= (\hbar^2/4m) g_0 (g_0 - 2|k_z|) , \end{aligned} \tag{5}$$

where U is the first component of the Fourier expansion of the periodic potential. For the model total potential $V\cos(g_0 z)$, $U = V/2$. Then the single-particle susceptibility reduces to

$$\chi_0(n,n';\omega) = \frac{1}{\pi} \sum_{m=-1}^{1} A_m^* A_{m+n-n'} \int dk_z \frac{f_{k_z} - f_{k_z'}}{\varepsilon_{k_z} - \varepsilon_{k_z'} + \hbar\omega} \tag{6}$$

as the only nonzero A_m are

$$\begin{aligned} &A_0 = [1 + F(k_z)F(k_z')]R, \quad A_1 = F(k_z)R , \\ &A_{-1} = F(k_z')R, \quad R^{-2} = [1 + F(k_z)^2][1 + F(k_z')^2] . \end{aligned} \tag{7}$$

The integration is restricted to the range where both k_z and k_z' remain in the extended zone $-g_0$ to g_0. The total density response function is given by a modified Eq. (1),

$$\chi(n,n';\omega) = \chi_0(n,n';\omega) + \sum_{n''} \chi_0(n,n'';\omega) v(n'') \chi(n'',n';\omega) , \tag{8}$$

where the Coulomb factor is[17] $v(n) = (2e^2/\varepsilon)K_0(q_n d)$, d being the width of the wire, $q_n = q + ng_0$, and ε is the dielectric constant of the semiconductor. In matrix notation, χ can be expressed in terms of the single-particle susceptibility χ_0 as $\chi = \chi_0(1 - v\chi_0)^{-1}$.

We now turn to finding the modifications of the electron distribution function under the influence of the current-driving electric field. The problem of transport in lower-dimensional systems (including modulated systems) has been explored with a variety of approaches. However, to illustrate the main features of the phenomenon studied here, the exact knowledge of the distribution function is not necessary. It suffices to realize, that in this dimensionally restricted case at $T=0$, electrons in the occupied miniband *cannot* contribute to conductivity, and their distribution function effectively remains unchanged. In contrast, the distribution function of the higher partially occupied miniband generally shifts as individual electrons acquire extra momenta towards the field, and distorts due to various scattering effects. The key feature, here, is that as a result of the applied electric field, a *gap* of allowed states opens (in k_z) below the shifted distribution. We model this by assuming $f_{k_z} = \theta(\varepsilon_F - \varepsilon_{k_z})\theta(g_0/2 - |k_z|)$ in the lower miniband, and $f_{k_z} = \theta(\varepsilon_F - \varepsilon_{k'})\theta(|k_z| - g_0/2)$ in the upper miniband, with $k' = k_z - k_d$ for $k_z > g_0/2 + k_d$ or $k_z < -g_0/2$ and $k' = k_d - k_z + g_0$ for $g_0/2 < k_z < g_0/2 + k_d$. The momentum shift $k_d = eE\tau/\hbar$, where τ is the effective collision time.

The system becomes unstable when the amplitude of the induced charge diverges, which is a result of diverging $\chi(n,n';\omega)$. We have obtained the dispersions of unstable modes by searching for complex solutions [ω vs q, with $\mathrm{Im}(\omega) > 0$] of

$$\det(1 - v\chi_0) = 0 , \tag{9}$$

which makes χ diverge.

Figure 1 depicts the dispersion relation [$\mathrm{Re}(\omega)$ vs q] and the growth rate [$\mathrm{Im}(\omega)$ vs q] obtained from Eq. (9) for various energy gap sizes, for $k_{dr} = 0.35k_F$ and $g_0 = 1.8k_F$, where k_F defines the wave-number scale in terms of the (average) linear density n through $k_F = (\pi/2)n$. The frequency and energy scales are defined in terms of $\omega_F = \hbar k_F^2/2m^*$. The Fermi energy for the system is $\varepsilon_F(U, g_0) = \varepsilon(k_z = k_F)$ from Eq. (8). The most striking, and essential, feature of these results is that the instability occurs at much lower drifts than in all previously studied cases of unmodulated semiconductor systems. The mode is acoustic, with phase velocity $\omega/q \gtrsim v_F$, almost independent of U. The growth rate attains a maximum value at $q = q_{\max} \approx k_{dr}$, and drops back to zero near $q_0 = k_{dr} + (k_F - g_0/2)$, which represents the largest q jump a particle from the drifted distribution can make. For larger U the growth rate develops a two-peaked structure, with the peak strength diminishing as U is increased. For typical parameters, near $q_{\max}$, $\mathrm{Re}(\omega) \approx 10^{12}$–$10^{13}$ sec^{-1} and $\mathrm{Im}(\omega) \approx 10^{11}$–$10^{12}$ sec^{-1}, indicating a strong instability. The onset of this instability occurs for k_{dr} as low as $0.22k_F$ for $U/\hbar\omega_F = 0.1$, and even smaller values for smaller U (e.g., $0.12k_F$ for $U/\hbar\omega_F = 0.01$); these values are almost a factor of 10 below the threshold for an unmodulated system.[6]

We can expect to understand the above results, at least to zeroth order, by considering the case of a *uniform* 1DEG with a simulated distribu-

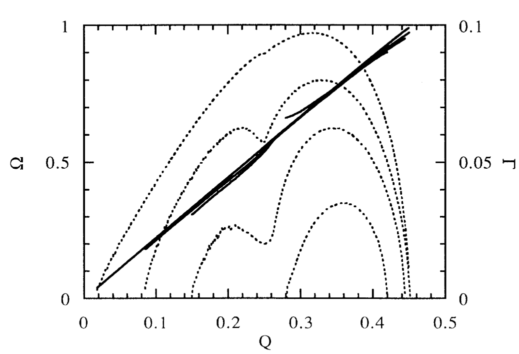

FIG. 1. Mode frequency $\Omega = \mathrm{Re}\omega/\omega_F$ (solid lines) and growth rate $\Gamma = \mathrm{Im}\omega/\omega_F$ (dashed lines) vs $Q = q/k_F$ for $U/\hbar\omega_F = 0.01$, 0.05, 0.08, and 0.12. Material parameters are $m^*/m_e = 0.0665$, $\varepsilon = 13.1$, and $d = 200$ Å.

tion function $f_{k_z}=\theta(g_0/2-|k_z|)+\theta(k_z-k_{dr}+k_F-g_0)-\theta(k_z-k_{dr}-k_F)$. For drifts $k_d>k_F-g_0/2$, this faithfully represents the physical distribution function of the modulated 1DEG system with a small U, under the influence of the applied constant electric field. The velocity gap that opens up due to the drift assures the existence of a plasma instability for the simulated distribution function as a corollary of the Penrose criterion.[2] In fact this is an illustration of a *velocity gap instability* which we have discussed earlier in another context.[18] The susceptibility can be evaluated analytically and the resulting mode structure and the growth rate agree very well with the results for the modulated system for small U in Fig. 1.

The plasmon growth rates, shown in Fig. 1, represent a balance between generation of plasmons due to the downward single-particle transitions (from the upper miniband), and absorption of plasmons due to the upward transitions. For larger U, the flattening of the minibands leads to an increased density of states near the Brillouin zone, and a corresponding increase in interminiband absorption transitions. Both normal and umklapp transitions occur with increasing strength, and counter the basic velocity-gap instability, represented by the upper $(U\approx 0)$ curve. The difference between the various U curves quantitatively represents this effect of increased absorption. The "dip," which remains near $q_{\min}=k_{dr}-(k_F-g_0/2)$, is caused by the strong absorption due to upward transitions from the top of the lower miniband to the top of the velocity gap. For a sufficiently large U the overall balance shifts in favor of absorption, destroying the instability for all q.

The simple interpretation of this plasma instability as a velocity-gap instability gives us a general insight, which can be extended to modulated systems with several minibands, and in more than one dimension. The physical velocity of a group of Bloch electrons of momentum $\approx\mathbf{k}$ in a given miniband n is given by $v_{n\mathbf{k}}=(1/\hbar)d\varepsilon_{n\mathbf{k}}/d\mathbf{k}$. Each miniband has point of maximum slope defining the characteristic maximum velocity the electrons in this band can achieve along the direction of the periodicity. In the one-dimensional system, when an external field is applied, the distribution merely shifts by a constant amount k_{dr}, but due to the periodicity in k space, there is no change at all in the overall momentum distribution of the completely filled minibands (at $T=0$), since $f_k=1$ in the entire Brillouin zone. The electrons in the partially filled uppermost miniband, on the other hand, do respond to the external field and in the process acquire large velocities: $v(k)\rightarrow v(k+k_{dr})$. This is illustrated in Fig. 2 for the one-dimensional two-miniband system, where the velocity space distribution functions,

$$f(v)=\sum_n f_n(v),\quad f_n(v)=\sum_j \frac{f_{nk}}{|\partial v_{nk}/\partial k|}\Bigg|_{k=k_j},\quad v=v_{nkj}, \tag{10}$$

are displayed with and without drift. The formation of the velocity gap is quite evident.

For small q, and small U (and taking the perpendicular $Q=0$), it can be shown that the only significant component $(n=n'=0)$ of the susceptibility, Eq. (5) reduces to

$$\chi_0(q,\omega)\sim\int dk_z\frac{q(\partial f/\partial k_z)}{q(\partial\varepsilon/\partial k_z)-\hbar\omega}\sim\int dv\frac{q(\partial f/\partial v)}{qv-\omega} \tag{11}$$

in terms of the effective velocity distribution function $f(v)$ which now includes integration over the perpendicular momenta. This expression for the susceptibility is in the standard form as obtained in classical plasma theory,[2] and properties of $f(v)$ thus (approximately) provide the instability criteria for the modulated system.

It should be noted that the key to achieve this instability is to partially fill the uppermost miniband in such a way that the applied field will significantly increase the velocity of these particles. This necessarily requires that the Fermi level be well below the point of maximum velocity of that miniband. Thus the filling factor $k_1/(\pi/a)$, where $\varepsilon(k_1)=\varepsilon_F$, has to be just slightly greater than an integer; this condition will be described as "filling factor resonance."

This instability should occur in modulated 1DEG, and LSSL. It might also occur in the 2DEG modulated in one direction, even though interminiband transitions would fill up part of the velocity gap. This filling may not be substantial under some conditions, and a population inversion in velocity space may still exist, leading to an instability through inverse Landau damping.

In a recent experiment[16] negative dynamic resistance has been observed in a LSSL. In a high-electron-mobility transistor (HEMT) structure ($GaAs/Al_xGa_{1-x}As$ modulation doped heterostructure) the gate was made of a la-

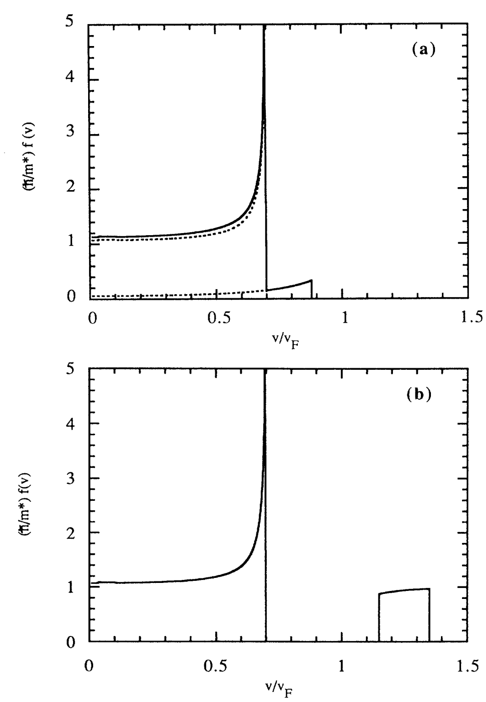

FIG. 2. Velocity space distribution functions for $U/\hbar\omega_F=0.1$, $g_0=1.8k_F$. (a) $k_{dr}=0$. Individual miniband populations (dashed lines); total (solid lines). Note $f(-v)=f(v)$. (b) $k_{dr}=0.35k_F$. Now the minibands have no overlap. Lower is same as (a), upper has all particles near $v\approx 1.25v_F$.

teral metallic grid of period of 0.2 μm. When polarized, this grid "imprints" a density modulation of the same period in the 2DEG in the HEMT channel. It was estimated that the potential modulation can produce multiminiband occupation. At low temperatures the negative dynamic resistance was observed in a narrow-gate voltage domain. An explanation in terms of sequential tunneling was proposed. The current-driven plasma instability due to a velocity gap, as discussed above, could occur in such a system for appropriate parameters. A large enough k_{dr} can be achieved in this experiment within their range of applied voltages. The filling-factor resonance condition can be met by tuning the gate voltage, which changes the Fermi energy. One possible test of the occurrence of such a plasma instability would be emission of sharply peaked radiation by applying a grating coupler with wave number $q_{\max}$.

Another experiment[12] could also provide a system where the existence of such plasma instability could be explored. In this experiment broadband far-infrared emission induced by a weak periodic potential was observed in high-mobility GaAs/Al_xGa_{1-x}As heterostructures. This radiation was ascribed to the Smith-Purcell-type of emission. For a sufficiently strong potential modulation, and by varying the density to achieve the filling-factor resonance, the conditions for the existence of this instability might be realized. Far-infrared emission can then be obtained by employing a grating coupler with a period equal to $2\pi/q_{\max}$, where $q_{\max}$ is the wave number corresponding to the maximum growth rate of the plasma waves. The frequency of emission will be $\omega = q_{\max} v_{\rm ph}$, where $v_{\rm ph}$ is the phase velocity of the plasma mode slightly higher than the Fermi velocity of the sample. This emission would be more efficient since it is coherent radiation due to a collective mode, to be contrasted with the individual particle origin of the broadband Smith-Purcell-type of emission.

We have shown the feasibility of a current-driven plasma instability in a modulated one-dimensional system, with a much lower driving field than is required in unmodulated systems. For samples of high mobility ($\sim 10^6$ cm^2/V sec) and with Fermi velocity $v_F \sim 10^7$ cm/sec, electric fields of the order of a few V/cm would suffice instead of the much larger several tens of V/cm for an unmodulated system. Typical threshold current densities are of the order of 10^4 A/cm^2 and the current is $\sim 10^7$ A. Unlike the unmodulated systems, the instability here is achieved in the domain of essentially cold electron transport, since the (frozen) lower minibands are unchanged and the heating of the upper miniband electrons is only of the order of $(k_{dr}/k_F)^2 \varepsilon_F \approx (0.1)\varepsilon_F$. If the instability occurs, it can be detected and exploited in many ways: it can be detected through the emission of coherent electromagnetic radiation as already mentioned, or by Raman spectroscopy, and it can be employed to develop radiation sources in submillimeter wave range or to achieve amplification of radiation in that range.

We now comment on a number of relevant points. (i) The modulation strength U has to exceed the thermal broadening, as well as the change in energy across one period under the influence of the applied field. The latter condition is easily met for high-mobility systems. In addition, we note that the effective field in the interior will be significantly screened.[19] (ii) In higher-dimensional systems U has to be strong enough to moderate the effects of scatterings which tend to redistribute the miniband populations. (iii) The calculated growth rate should exceed the inherent electron-phonon or impurity-electron collision frequency to achieve instability.[9] (iv) The velocity-gap plasma instability studied here must not be confused with the Bloch oscillations,[15,20] which can develop in high-mobility modulated lower-dimensional systems subjected to an external electric field sufficiently strong to displace electrons across the entire Brillouin zone.

This work was supported by a U.S. Army Research Office grant.

[1] A. B. Mikhailovskii, *Theory of Plasma Instabilities* (Consultant Bureau, New York, 1974), Vol. 1.

[2] N. Krall and A. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973).

[3] P. Bakshi *et al.*, J. Appl. Phys. **64**, 2243 (1988).

[4] J. Cen *et al.*, Phys. Rev. B **38**, 10 051 (1988).

[5] K. Kempa *et al.*, Phys. Rev. B **39**, 2852 (1989).

[6] P. Bakshi *et al.*, Solid State Commun. **76**, 835 (1990).

[7] J. Cen *et al.*, Solid State Commun. **78**, 433 (1991).

[8] K. Kempa *et al.*, Phys. Rev. B **43**, 9273 (1991).

[9] H. Xie *et al.*, J. Appl. Phys. **72**, 4767 (1992).

[10] D. C. Tsui *et al.*, Solid State Commun. **35**, 875 (1980).

[11] E. Gornik *et al.*, Solid State Electron. **31**, 751 (1988).

[12] C. Wirner *et al.*, Phys. Rev. Lett. **70**, 2609 (1993).

[13] F. Dong *et al.*, Solid State Commun. **84**, 785 (1992); M. J. Graf *et al.*, Physica B (to be published).

[14] K. Kempa *et al.*, Phys. Rev. B **47**, 4532 (1993).

[15] See, for example, a review paper by W. Hansen *et al.*, in *Semiconductors and Semimetals,* edited by M. A. Reed (Academic, New York, 1990), Chap. 6, pp. 1–97, and references therein.

[16] K. Ismail *et al.*, Appl. Phys. Lett. **54**, 460 (1989).

[17] S. Das Sarma and W. Lai, Phys. Rev. B **32**, 1401 (1985).

[18] P. Bakshi and K. Kempa, Phys. Rev. B **40**, 3433 (1989).

[19] D. A. Broido *et al.*, Solid State Commun. **76**, 613 (1990).

[20] G. Bernstein and D. K. Ferry, Z. Phys. B **67**, 449 (1987).