Magnetophonon resonances in quasi-one-dimensional quantum wires

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Explicit expressions for the oscillatory behavior of the magnetoconductivity, associated with the magnetophonon resonance effect in quantum wires, are obtained in the case of polar-optical-phonon scattering, by taking the linear-response limit of the results of nonlinear-response theory [Phys. Rev. B 44, 11 328 (1991)] developed previously. The magnetoconductivity σ_{xx} consists of two types of contribution; one is related to the current carried by the electron *hopping* motion between the localized states, and the other is caused by the electron *nonhopping* motion within the localized states through the electronphonon interaction. In both cases, the dependence of the magnetoconductivity on the phonon occupation number and the coupling constant of the electron-optical-phonon interaction is investigated. The results obtained here are compared with the work of some other authors.

I. INTRODUCTION

Recently, magnetophonon resonance (MPR) effects in low-dimensional systems $^{1-3}$ received much attention from both experimental and theoretical points of views. However, so far, only a few attempts have been made to investigate the MPR effects in a quasi-one-dimensional electron gas (Q1DEG). Vasilopoulos et al.⁴ have studied MPR effects in Q1D quantum-wire structures assuming a parabolic confinement potential of frequency Ω , based on the Kubo formula⁵ and the quantum Boltzmann equation,² and their calculations revealed that the ordinary resonance condition $\omega_{\rm LO} = P \omega_c$ is modified to $\omega_{\rm LO} = P \widetilde{\omega}_c$, where P is an integer, $\omega_{\rm LO}$ and ω_c are the longitudinaloptical-phonon (LO-phonon) frequency and cyclotron frequency, respectively, and $\tilde{\omega}_c$ is the renormalized cyclotron frequency given by $\tilde{\omega}_c = (\omega_c^2 + \Omega^2)^{1/2}$. Therefore, the position of the peaks gives information about the confining frequency. It should be noted that only one contribution with respect to the magnetoconductivity is included in the expression appearing in Ref. 4, which is valid for weak confinement potentials. In 1992, Mori, Momose, and Hamaguchi⁶ presented a theory of MPR for the same model as treated by Vasilopoulos et al.,⁴ by utilizing the Kubo formula and the Green's-function method.⁷ A numerical analysis with respect to the magnetoconductivity has been performed for weak and strong confinement potentials by introducing the current-density operator due to the electron-phonon interaction and confinement potential. It should be noted that in the formulation of Mori, Momose, and Hamaguchi,⁶ an analytical calculation with respect to the MPR effects has not been made explicitly. Concerning the MPR effects in Q1D quantum-wire structures, we are not aware of theoretical work other than that of Vasilopoulos et al.⁴ and Mori, Momose, and Hamaguchi,⁶ and it is clear, in general, that the investigation of such effects is at an initial stage both experimentally and theoretically. Therefore, we are motivated to present an alternative approach to the analysis of MPR effects in Q1D quantum-wire structures. The purpose of the present work is to investigate analytically the MPR effects in Q1D semiconductor quantum-wire structures, by taking the linear-response limit of the results of nonlinear-response theory⁸ developed previously, to understand the unusual behavior of the MPR line shape (e.g., conversion of MPR maxima into minima, and MPR peak shift due to the parabolic confinement potential), and to compare our results with those of other authors obtained by the different methods.

II. MODEL FOR A QUANTUM WIRE

We consider the transport of an electron gas in a Q1D quantum-wire structure, where a static magnetic field $\mathbf{B}(\|\hat{\mathbf{z}})$ is applied to the wire. For the sake of simplicity, the confinement of electrons with respect to the quantum wire is modeled by a triangular potential well (such as that realized in heterostructures) in the z direction, which leads to electronic subbands, and by a parabolic potential well with the confining frequency Ω in the y direction. The conduction electrons are free along only one (x)direction of the wire. Here we assume electron densities such that only the lowest subband with one-electron energy E_z^0 is occupied. Applying the effective-mass approximation for conduction electrons confined in the quantum wire, the one-particle Hamiltonian (h_{e}) for such electrons together with its normalized eigenfunctions $(|\lambda\rangle)$ and eigenvalues (E_{λ}) , in the Landau gauge of vector potential $\mathbf{A} = (-By, 0, 0)$, are given, respectively, by⁴

$$h_e = (\mathbf{p} - e \mathbf{A})^2 / 2m^* + m^* \Omega^2 y^2 / 2 + H_0(z) , \qquad (1)$$

$$|\lambda\rangle \equiv |N,k_x,0\rangle = \phi_N(y-y_\lambda) \exp(ik_x x) \Psi_0(z) / \sqrt{L_x} , \qquad (2)$$

$$E_{\lambda} \equiv E_{n,k_{\perp},0} = (N+1/2)\hbar\widetilde{\omega}_{c} + \hbar^{2}k_{x}^{2}/2\widetilde{m} + E_{z}^{0} , \qquad (3)$$

where **p** is the momentum operator of a conduction electron. Here N denotes the Landau-level index, where $\tilde{\omega}_c = (\omega_c^2 + \Omega^2)^{1/2}$ and $\tilde{m} = m^* \tilde{\omega}_c^2 / \Omega^2$ denote the renormalized cyclotron frequency with respect to the cyclotron frequency $\omega_c = eB / m^*$ and the renormalized mass with respect to the effective mass m^* associated with the confining frequency Ω , respectively. Also $\phi_N(y - y_\lambda)$ represents harmonic-oscillator wave functions, centered at $y = y_\lambda = -b \tilde{l}_B^2 k_x$. Here k_x is the wave vector in the x

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direction, $\tilde{b} = \omega_c / \tilde{\omega}_c$, and $\tilde{l}_B = (\hbar/m^* \tilde{\omega}_c)^{1/2}$ is the effective radius of the ground-state electron orbit in the (x,y) plane. For $\Psi_0(z)$ we take the usual variational wave function $\Psi_0(z) = b_0^{3/2} z \exp(-b_0 z/2)/\sqrt{2}$ with average thickness $\langle L_z \rangle = 3/b_0$. The dimensions of the sample are assumed to be $V = L_x L_y L_z$.

III. MAGNETOCONDUCTIVITY ASSOCIATED WITH RELAXATION RATES

We want to calculate the static electric conductivity component σ_{xx} for the Q1DEG system, subjected to crossed electric $\mathbf{E}(\|\mathbf{\hat{x}}\|)$ and magnetic $\mathbf{B}(\|\mathbf{\hat{z}}\|)$ fields, by taking the linear-response limit of the results of nonlinearresponse theory derived in Ref. 8 and considering the following matrix elements in the representation (2):

$$\begin{split} |\langle \lambda | j_x | \lambda' \rangle|^2 &= (e \Omega^2 / \omega_c)^2 y_{\lambda}^2 \delta_{\lambda',\lambda} \\ &+ (e \tilde{l}_B \omega_c / \sqrt{2})^2 [(N+1) \delta_{\lambda',\lambda+1} + N \delta_{\lambda',\lambda-1}] , \quad (4) \end{split}$$

. .

where $j_x = -(e/m^*)(p_x + eBy)$ is the x component of a single-electron current operator and the Kronecker symbols $\delta_{\lambda',\lambda} = \delta_{N',N} \delta_{k'_{\star},k_{\star}} \delta_{0',0}$ and $\delta_{\lambda',\lambda\pm 1} = \delta_{N',N\pm 1} \delta_{k'_{\star},k_{\star}} \delta_{0',0}$ indicate the selection rules, which arise during the integration of the matrix element with respect to each direction. It should be noted that the matrix element with respect to the current operator in Eq. (4) is directly proportional to the dc magnetoconductivity, which contains two types of contribution as follows: one corresponding to the first term of the right-hand side in Eq. (4) is related to the current carried by the electron on hopping motion within the localized cyclotron orbits, and the other corresponding to the second term is caused by the current carried by electron hopping motion between the localized cyclotron orbits. Considering the selection rules of Eq. (4) above, we can see that the sum over the λ_2 state in Eq. (2.31) of Ref. 8 includes the $\lambda_2 = \lambda_1 (\equiv \lambda)$ and $\lambda_2 = \lambda \pm 1$ terms. Furthermore, separating the sum over the λ_2 state in Eq. (2.31) of Ref. 8 into two types of contribution, carrying out the Cauchy integral in the same equation, and taking the Ohmic condition $E \rightarrow 0$ in Eq. (2.29) of Ref. 8, the complex transverse conductivity formula for the Q1D version is obtained, as it appears in Eq. (2.39) of Ref. 8, as

$$\sigma_{xx}^{h} = (1/V) \lim_{E_{x} \to 0} \sum_{\lambda, \lambda'} \{ [f(E_{\lambda}) - f(E_{\lambda'})] / E_{\lambda} - E_{\lambda'} \} \\ \times j_{x\lambda\lambda'} \langle \tilde{j}_{x\lambda'\lambda}(E_{x}) \rangle_{ph} \delta_{\lambda'\lambda \pm 1} , \qquad (5)$$

$$\sigma_{xx}^{\rm nh} = (-\beta/V) \lim_{E_x \to 0} \sum_{\lambda} f(E_{\lambda}) [1 - f(E_{\lambda})] j_{x\lambda\lambda} \langle \tilde{j}_{x\lambda\lambda}(E_x) \rangle_{\rm ph} , \quad (6)$$

where $X_{\lambda\lambda'} \equiv \langle \lambda | X | \lambda' \rangle$ for any operator X, $\langle \cdots \rangle_{\rm ph}$ denotes the average over the phonon scatterings, $\beta = 1/k_B T$ with k_B being Boltzmann's constant, and $f(E_{\lambda})$ is a Fermi-Dirac distribution function associated with the state λ of Eq. (2) and the energy E_{λ} of Eq. (3). Also, $\lim_{E_x\to 0} \tilde{j}_x(E_x)$ is the Laplace transform of $j_x(t|h_e+v+H_{\rm ph})$. Here v and $H_{\rm ph}$ are the electronphonon scattering potential and the phonon Hamiltonian, respectively. Noting that the detailed derivation of $\langle \tilde{j}_x(E_x) \rangle_{\rm ph}$ is given in Sec. III of Ref. 8, we find that,

from Eqs. (3.46), (3.47), and (4.38) of Ref. 8 and Eqs. (3)-(5) and (6) of the present text, the transverse conductivity σ_{xx} for the Q1D version is given by the sums of the hopping part σ_{xx}^h and the nonhopping part σ_{xx}^{nh} , which are

$$\sigma_{xx}^{h} = \frac{(e\tilde{I}_{B}\omega_{c})^{2}}{\tilde{\omega}_{c}} \sum_{\lambda} (N+1)[f(E_{\lambda}) - f(E_{\lambda+1})]A_{\lambda+1\lambda} , \qquad (7)$$

$$\sigma_{xx}^{nh} = (\hbar\beta e^2 \Omega^4 / \omega_c^2) \sum_{\lambda} y_{\lambda}^2 f(E_{\lambda}) [1 - f(E_{\lambda})] A_{\lambda\lambda} , \qquad (8)$$

where the spectral density $A_{\mu\nu}$ is given by

$$A_{\mu\nu} = \Gamma_{0\mu\nu} / \{ [E_{\nu} - E_{\mu} - \nabla_{0\mu\nu}]^2 + [\hat{\Gamma}_{0\mu\nu}]^2 \}$$
(9)

for any localized quantum states μ and ν . It is noted that the quantities $\widetilde{\Gamma}_0$ and $\widetilde{\nabla}_0$ play a role in determining the width and the shift in the spectral line shape, respectively. In the presence of collisions, assuming⁹ $\widetilde{\Gamma}_0, \widetilde{\nabla}_0 \ll \hbar \widetilde{\omega}_c (= E_{\lambda+1} - E_{\lambda})$ and shift zero, which are usually satisfied and which is in fact the condition for observing the oscillatory behavior of MPR, the spectral densities in Eqs. (7) and (8) can then be approximated as $\widetilde{\Gamma}_{0\lambda+1\lambda}/(\hbar\widetilde{\omega}_c)^2$ and $1/\widetilde{\Gamma}_{0\lambda\lambda}$, respectively. To express the dc magnetoconductivities of Eqs. (7) and (8) in simpler forms, we assume that the f's in Eqs. (7) and (8) are replaced by the Boltzmann distribution function for nondegenerate semiconductors, i.e., $f(E_{\lambda}) = f_{N,0}(k_x)$ $\approx \exp[\beta(E_F - E_{N,k_x,0})], \text{ where } E_{N,k_x,0} = (N + \frac{1}{2})\hbar\tilde{\omega}_c + \hbar^2 k_z^2/2\tilde{m} + E_z^0 \text{ and } E_F \text{ denotes the Fermi energy.}$ Then, we can further perform the sum over N (if N is large) by writing $\sum N \exp(-\alpha N) = -(\partial/\partial \alpha) \sum \exp(-\alpha N)$ and summing the geometric series, and carry out the one summation with respect to k_x in \sum_{N,k_x} by making use of the following relation: $\sum_{k_x} \rightarrow (L_x/2\pi) \int_{-\infty}^{\infty} \cdots dk_x$. Thus, we obtain

$$\sigma_{xx}^{h} \approx [(e^{2}\tilde{b}^{2}b_{0}\tilde{l}_{B}^{2}N_{s}^{1\mathrm{D}})/8\hbar(\hbar\tilde{\omega}_{c})^{2}\Lambda]\tilde{\Gamma}_{0\lambda+1\lambda}, \qquad (10)$$

$$\sigma_{xx}^{nh} \approx [(16\pi e^2 \Omega^4 \tilde{m} \tilde{l}_B^4 N_s^{1D} \Lambda) / b_0 \tilde{\omega}_c] [\tilde{\Gamma}_{0\lambda\lambda}]^{-1} , \qquad (11)$$

where $\Lambda = b_0 / 8\pi \hbar \tilde{\omega}_c$. To derive Eqs. (10) and (11) we <u>utilized</u> the electron density⁴ given as N_s^{1D} $= \sqrt{\tilde{m}L_x^2/8\pi\hbar^2\beta} \exp[\beta(E_F - E_z^0)]/\sinh(\beta\hbar\tilde{\omega}_c/2)$ and approximated the factor $1 - f(E_{\lambda})$ in Eq. (8) by 1 (nondegenerate limit) since polar-optical phonons are dominant at high temperatures. We note that the transverse magnetoconductivity σ_{xx} is, as seen from Eqs. (10) and (11), related to the two different relaxation rates $\Gamma_{0\lambda+1\lambda}$ and $\tilde{\Gamma}_{0\lambda\lambda}$. The electronic transport properties (e.g., electronic relaxation processes, MPR effects, etc.) in the Q1D quantum-wire structures can be studied by examining the behavior of $\tilde{\Gamma}_0$ as a function of relevant physical parameters introduced in the present theory.

IV. MAGNETOPHONON RESONANCES

For the calculation of the relaxation rates $\widetilde{\Gamma}_{0\lambda+1\lambda}$ and $\widetilde{\Gamma}_{0\lambda\lambda}$ for a specific electron-LO-phonon interaction in Eqs. (10) and (11), we need the interaction potential C(q)for polar-LO-phonon scattering given by the Fröhlich interaction potential:^{4,6} $|C(q)|^2 = 4\pi \alpha \hbar (\hbar \omega_{\rm LO})^{3/2} /$ $[(2m^*)^{1/2}Vq^2] \equiv A / [V(q_1^2 + q_z^2)]$ with A and α being

the constant of the polar interaction and the dimensionless electron-phonon (polaron) coupling constant, respectively, and the following matrix elements in the representation (2):

$$|\langle \lambda| \exp(\pm i \mathbf{q} \cdot \mathbf{r}) |\lambda' \rangle|^2 = |J_{NN'}(u)|^2 [J(\pm q_z)]^2 \delta_{k'_x, k_x \mp q_x}, \qquad (12)$$

$$[J(\pm q_z)]^2 = \left| \int_{-\infty}^{\infty} \Psi_0^*(z) \exp(\pm iq_z z) \Psi_0(z) dz \right|^2, \qquad (13)$$

$$\sum_{q_z} \frac{[J(q_z)]^2}{q_\perp^2 + q_z^2} = (L_z/2\pi) \int_{-\infty}^{\infty} \cdots dq_z$$

$$= (L_z/2\pi) \frac{b_0 \pi}{q_\perp^2} \frac{r^5}{(r+1)^6} = (L_z/2\pi) \frac{b_0}{q_\perp^2} I(r) , \quad (14)$$

$$|J_{NN'}(u)|^2 = (N_n!/N_m!)e^{-u}u^{N_m-N_n}[L_{N_n}^{N_m-N_n}(u)]^2, \qquad (15)$$

where $r = b_0/q_1$ with $q_1^2 = q_x^2 + q_y^2$, $N_n = \min\{N, N'\}$, $N_m = \max\{N, N'\}$, $u = \tilde{l}_B^2(\tilde{b}^2 q_x^2 + q_y^2)/2$, and $L_N^M(u)$ is an associated Laguerre polynomial.¹⁰ We assume that the phonons are dispersionless (i.e., $\hbar \omega_q \approx \hbar \omega_{\rm LO} \approx {\rm const}$, where $\omega_{\rm LO}$ is the polar-LO-phonon frequency) and bulk (i.e., three-dimensional). Therefore, we have neglected any changes in the electron-phonon interaction brought about by the Q1D confinement of the electrons and the surface-roughness effect. The detailed derivation of the relaxation rate and its general expression in the lowestorder approximation for the weak-coupling case of an electron-phonon system can be seen in Eq. (4.39) of Ref. 8. Using Eq. (12), the Q1D version of this quantity associated with the electronic transition between the state $|\lambda_1\rangle$ and $|\lambda_2\rangle$ can be evaluated as

$$\begin{split} \tilde{\Gamma}_{0\lambda_{2},\lambda_{1}} &= \pi \sum_{\mathbf{q}} \sum_{\lambda' \neq \lambda_{2}} |C(q)|^{2} |J_{N_{2}N'}(u)|^{2} |J(q_{z})|^{2} ((N_{0}+1)\delta[(N_{1}-N')\hbar\tilde{\omega}_{c} + \{\hbar^{2}k_{1x}^{2} - \hbar^{2}(k_{2x}-q_{x})^{2}\}/2\tilde{m} - \hbar\omega_{\mathrm{LO}}] \\ &+ N_{0}\delta[(N_{1}-N')\hbar\tilde{\omega}_{c} + \{\hbar^{2}k_{1x}^{2} - \hbar^{2}(k_{2x}+q_{x})^{2}\}/2\tilde{m} + \hbar\omega_{\mathrm{LO}}]) \\ &+ \pi \sum_{\mathbf{q}} \sum_{\lambda' \neq \lambda_{1}} |C(q)|^{2} |J_{N'N_{1}}(u)|^{2} |J(q_{z})|^{2} ((N_{0}+1)\delta[(N'-N_{2})\hbar\tilde{\omega}_{c} + \{\hbar^{2}(k_{1x}-q_{x})^{2} - \hbar^{2}k_{2x}^{2}\}/2\tilde{m} + \hbar\omega_{\mathrm{LO}}] \\ &+ N_{0}\delta[(N'-N_{2})\hbar\tilde{\omega}_{c} + \{\hbar^{2}(k_{1x}+q_{x})^{2} - \hbar^{2}k_{2x}^{2}\}/2\tilde{m} - \hbar\omega_{\mathrm{LO}}]) , \end{split}$$

$$(16)$$

which is due to the inelastic scattering in the collision processes. Here N' indicates intermediate localized Landau states and N_0 is the polar-LO-phonon distribution function given by $N_q = [\exp(\beta \hbar \omega_q) - 1]^{-1}$ with $\omega_q = \omega_{LO}$. The energy-conserving δ functions in Eq. (16) imply that when the electron undergoes a collision by absorbing energy from the field, its energy can only change by an amount equal to the energy of a phonon involved in the transitions. This in fact leads to the MPR effect, whereby $\hbar \tilde{\omega}_c \gg \tilde{\Gamma}_0$ [or $\tilde{\omega}_c \tau \gg 1$] is satisfied. Proceeding as in Vasilopoulos *et al.*⁴ with respect to the summation over **q** in Eq. (16), making an approximation $N' \pm 1 \approx N'$ for very large N', and setting N' - N = -P in the emission term and N' - N = P in the absorption term, the relaxation rates $\tilde{\Gamma}_{0\lambda+1\lambda}$ and $\tilde{\Gamma}_{0\lambda\lambda}$ can be written in a simple form:

$$\bar{\Gamma}_{0\lambda+1\lambda} \approx A \Lambda I(r)(2N_0+1) \sum_{P} \left\{ \delta(P + \omega_{\rm LO}/\tilde{\omega}_c) + \delta(P - \omega_{\rm LO}/\tilde{\omega}_c) \right\} , \qquad (17)$$

$$\widetilde{\Gamma}_{0\lambda\lambda} \approx 2A\Lambda I(r)(2N_0+1) \sum_{P} \delta(P+\omega_{\rm LO}/\widetilde{\omega}_c) .$$
(18)

As seen from Eqs. (17) and (18) the transverse magnetoconductivity [(10) and (11)] shows resonant behaviors: MPR at $P\tilde{\omega}_c = \omega_{\rm LO}$. The above conditions for the MPR give the resonance magnetic fields (i.e., the MPR peak positions at) \tilde{B}_p : $\tilde{B}_p = \sqrt{B_P^2 - (m^*\Omega/e)^2}/P$, where $B_P(=m^*\omega_{\rm LO}/eP)$ is the fundamental magnetic field for the ordinary MPR (no confinement in the y direction). We see that the effect of confinement in the y direction for the Q1D quantum-wire structure is to shift the ordinary MPR peak position to lower magnetic field. Furthermore, it is very interesting to point out that, for fixed confining frequency Ω , the relative peak shift $(\Delta B)^P/B_P[=1-\{1-P^2(\Omega/\omega_{\rm LO})^2\}^{1/2}]$ increases strongly with increasing P or decreasing magnetic field, where

 $(\Delta B)^P = B_P - \tilde{B}_P$ denotes the MPR peak shift. These characteristics for the MPR peak shift are identical with those indicated by Vasilopoulos et al.⁴ The remarkable point is that the relaxation rates $\tilde{\Gamma}_{0\lambda+1\lambda}$ and $\tilde{\Gamma}_{0\lambda\lambda}$ have identical characteristics with respect to the MPR peak shift. Note that the relaxation rates diverge whenever the conditions for the MPR are satisfied. These divergencies (associated with the complete quantization of the electron energy spectrum [cf. Eq. (3)] in the presence of a magnetic field and the confining frequency) may be removed by including higher-order electron-phonon scattering terms or by inclusion of the fluctuation effects of the center of mass.¹¹ The simplest way to avoid the divergencies is by introducing a width parameter γ so that each δ function in Eqs. (17) and (18) is approximated by Lorentzians of width and shift zero. Employing this collision-broadening model^{4,9} and applying Poisson's summation formula¹² for the \sum_{P} in Eqs. (17) and (18) we then obtain

$$\widetilde{\Gamma}_{0\lambda+1\lambda} = \widetilde{\Gamma}_{0\lambda\lambda} \approx 2A\Lambda I(r)(2N_0+1)\Psi \left[\frac{\gamma}{\varkappa \widetilde{\omega}_c}, \frac{\omega_{\rm LO}}{\widetilde{\omega}_c}\right], \qquad (19)$$

where

$$\Psi(a,b) = 1 + 2 \sum_{s=1}^{\infty} e^{-2\pi s a} \cos(2\pi s b)$$

= $\frac{\sinh(2\pi a)}{\cosh(2\pi a) - \cos(2\pi b)}$ (a > 0). (20)

To obtain the width parameter γ , we assume the width parameter γ to be the same for all associated states and approximate $\tilde{\Gamma}_0$ on the left-hand side of Eq. (19) as γ . Then, considering $\Psi(\gamma/\hbar\omega_c,\omega_{\rm LO}/\tilde{\omega}_c) = \coth(\pi\gamma/\hbar\omega_c)$ for $\omega_{\rm LO} = P\tilde{\omega}_c$ and utilizing $\coth X \approx 1/X + X/3 - X^3/45$ for $X \ll 1$, the resonance width γ is given by the approximate result

$$\gamma \approx (15\{(1-\Delta)+[(1-\Delta)^2+4/5]^{1/2}\}/2\pi^2)^{1/2}\hbar\widetilde{\omega}_c$$

with

$$\Delta = \frac{12(\hbar \widetilde{\omega}_c)^2}{[Ab_0 I(r)(2N_0+1)]}.$$

Inserting Eq. (19) into Eqs. (10) and (11) we obtain the magnetoconductivity σ_{xx} given by the sum of the hopping part σ_{xx}^{h} and the nonhopping part σ_{xx}^{nh} as

$$\sigma_{xx}^{h} \approx \frac{e^2 A \tilde{b}^2 b_0 \tilde{l}_B^2}{4 \hbar (\hbar \tilde{\omega}_c)^2} (2N_0 + 1) N_s^{1D} I(r) \Psi \left[\frac{\gamma}{\hbar \tilde{\omega}_c}, \frac{\omega_{\rm LO}}{\tilde{\omega}_c} \right], \qquad (21)$$

 $\sigma_{xx}^{\rm nh} \approx (8\pi e^2 \Omega^4 \tilde{m} \tilde{l}_B^4 / Ab_0 \tilde{\omega}_c)$

$$\times N_{s}^{1\mathrm{D}}\left[(2N_{0}+1)I(r)\Psi\left[\frac{\gamma}{\hbar\tilde{\omega}_{c}},\frac{\omega_{\mathrm{LO}}}{\tilde{\omega}_{c}}\right]\right]^{-1}.$$
 (22)

It should be noted that the amplitude of the magnetoconductivity in Eq. (21) is somewhat different from the theoretical result of Vasilopoulos *et al.*⁴ with respect to the confining potential and/or temperature dependence. However, if N=0 is assumed in Eq. (7) and we take the high-temperature limit so that

$$\begin{split} f(E_{\lambda}) - f(E_{\lambda+1}) &= [1 - \exp(-\beta \hbar \widetilde{\omega}_{c})] f(E_{\lambda}) [1 - f(E_{\lambda+1})] \\ &\approx \beta \hbar \widetilde{\omega}_{c} f(E_{\lambda}) \end{split}$$

in Eq. (7), the term $[1/(\hbar \tilde{\omega}_c)^2]$ in the amplitude in Eq. (21) is replaced by $\beta/\hbar \tilde{\omega}_c$. Therefore, we can expect that Eq. (21) gives an identical result to that of Vasilopoulos *et al.*⁴ if any approximation has been made. However, it should be noted that the contribution with respect to the magnetoconductivity, which has not been considered by Vasilopoulos *et al.*,⁴ is included.

V. CONCLUDING REMARKS

In this work we have presented a theory of MPR for electron-polar-LO-phonon scattering and derived the analytical expression describing the MPR effects of Q1DEG formed in quantum-wire structures, by taking the linear-response limit of the results of nonlinearresponse theory developed previously.⁸ As seen from Eqs. (10) and (11), the transverse magnetoconductivity σ_{xx} appears in the form of two types of contribution associated with the selection rules of the current-density operator. The transverse magnetoconductivity σ_{xx}^{h} is directly pro-

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portional to the relaxation rate $\tilde{\Gamma}_{0\lambda+1\lambda}$, while the transverse magnetoconductivity σ_{xx}^{nh} is inversely proportional to the relaxation rate $\tilde{\Gamma}_{0\lambda\lambda}$. The amplitude of the relaxation rates $(\tilde{\Gamma}_{0\lambda+1\lambda}, \tilde{\Gamma}_{0\lambda\lambda})$ given in Eq. (19) is directly proportional to the coupling constant (α) of the electronphonon interaction and the phonon occupation number (N_0) and hence σ_{xx}^h is directly proportional to α and N_0 , while σ_{xx}^{nh} is inversely proportional to α and N_0 . These results agree with the theoretical results^{4,6} obtained from a different approach to the same model system. It should be noted that Vasilopoulos et al.⁴ did not consider the effect of the σ_{xx}^{nh} . The relaxation rates $(\tilde{\Gamma}_{0\lambda+1\lambda}, \tilde{\Gamma}_{0\lambda\lambda})$ for polar-LO-phonon scattering show the Q1D MPR at $P\widetilde{\omega}_c = \omega_{\text{LO}}$. Here P is an integer. Since $\omega_c < \widetilde{\omega}_c$ for $\Omega > 0$, the resonances are shifted to smaller magnetic fields. Furthermore, the dependence of the magnetocon-ductivities σ_{xx}^h and σ_{xx}^{nh} on the oscillatory term $\Psi(\gamma/\hbar\tilde{\omega}_c, \omega_{\rm LO}/\tilde{\omega}_c)$ is inversely proportional to each other. Therefore, we can expect different oscillatory behaviors for σ_{xx}^h and σ_{xx}^{nh} . It is noted that our result for the relaxation rates $(\tilde{\Gamma}_{0\lambda\lambda}, \tilde{\Gamma}_{0\lambda+1\lambda})$ and hence σ_{xx} is tied to the following approximations: negligence of any modification of the electron-phonon interaction brought about by the confinement of phonons, and of any influence due to surface roughness. In addition, another approximation has been made with respect to the summation over q of Eq. (16), as Vasilopoulos et $al.^4$ did, which is valid for the weak confinement potential. There are several important issues under continuous study, including impurity scattering, acoustic-phonon scattering, the MPR shift due to dynamical screening (plasmon-LOphonon coupling), and the MPR shift⁹ due to the intracollisional field effect at strong electric fields only. We shall consider these effects elsewhere.

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