Monte Carlo studies of two-dimensional random-anisotropy magnets

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We have carried out a systematic set of Monte Carlo simulations of the Harris-Plischke-Zuckermann lattice model of random magnetic anisotropy on a two-dimensional square lattice, using the classical Metropolis algorithm. We have considered varying temperature T, external magnetic field H (both in the reproducible and irreproducible limits), time scale of the simulation τ in Monte Carlo steps and anisotropy ratio D/J. In the absence of randomness this model reduces to the XY model in two dimensions, which possesses the familiar Kosterlitz-Thouless low-temperature phase with algebraic but no long-range order. In the presence of random anisotropy we find evidence of a low-temperature phase with some disordered features, which might be identified with a spin-glass phase. The low-temperature Kosterlitz-Thouless phase survives at intermediate temperatures for low randomness, but is no longer present for large D/J. We have also studied the high-H approach to perfect order, for which there are theoretical predictions due to Chudnovsky.

I. INTRODUCTION

Over the last few years there has been considerable interest in the magnetic behavior of amorphous alloys of rare-earth metals.[1] Important anomalous properties of these materials include the presence, in the low-temperature regime, of history-dependent magnetization; more specifically, the low-temperature magnetization depends on whether the system has been cooled in that field *(field cooled)* or not *(zero-field cooled)*.[2] The onset of such behavior defines some kind of field-dependent glass transition temperature $T_g(H)$.[3] There also tends to be a magnetic susceptibility maximum close to T_g, though this does not seem to have the features usually associated with thermodynamic phase transitions. There are also some anomalous features associated with the magnetic part of the structure factor which indicate an algebraic falloff in the short-range magnetic correlations in the low-temperature phase.[4]

The clue to a theoretical understanding of these materials seems to lie in the orbital angular momentum due to the electrons in the partly filled $4f$ shell; this feature is common to all rare-earth metals. In crystalline rare earths, this angular momentum couples with the crystal field due to the nearest-neighbor atoms, causing a magnetocrystalline anisotropy with the symmetry of the underlying lattice. Once the lattice is amorphous, and in principle this may occur in pure materials as well as alloys, the leading-order effect of the anisotropy at each site is to create a term in the Hamiltonian which couples to $\mathcal{J}_z^2$ (with $\mathcal{J}$ the angular momentum operator), but with a special direction which changes from site to site. This is the physical idea behind the random-anisotropy magnet (RAM).

A model which supposes that this effect dominates all other randomness in governing the low-temperature magnetic properties is due to Harris, Plischke, and Zuckermann (HPZ).[5] In the HPZ model, magnetic spins are arranged on a regular lattice, with energies dictated by local exchange coupling between neighboring spins, coupling with external fields, and magnetoelastic effects, as well as the random anisotropy on each spin. The resulting Hamiltonian is

$$\mathcal{H}=-J\sum_{\{i,j\}}\mathbf{S}_i\cdot\mathbf{S}_j-D\sum_i(\mathbf{S}_i\cdot\mathbf{n}_i)^2 -D_0\sum_i(\mathbf{S}_i\cdot\mathbf{n}_0)^2-\sum_i\mathbf{H}\cdot\mathbf{S}_i\ , \tag{1}$$

where the sums over pairs $\{i,j\}$ are taken over nearest-neighbor sites on the regular lattice and sums over sites are taken over all sites of the lattice. The vector spins are characterized by n components; normally we would study $n=2$ (the XY model) or $n=3$ (the Heisenberg model).

We remind the reader that in addition to the exchange-coupling term, the random-anisotropy term, characterized by the random axis $\mathbf{n}_i$, and the external field term, there is also a coherent anisotropy term associated with a global easy axis in the direction $\mathbf{n}_0$, whose strength is characterized by the ratio D_0/J. The temperature scale of phenomena within this model is determined by the exchange coupling J, and thus the effect of the randomness is measured by the ratio D/J. In this paper T, D, etc. are measured in energy units, and where there is ambiguity, the unit of energy is J.

The HPZ model has been the subject of many studies, both analytic and computational, and we shall give a brief review of some of the work relevant to our own study in the next section. At this stage we merely point out that, as with all such statistical mechanical models, a number of features might be expected to affect its properties. Apart from the explicit model parameters, these in-

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clude the dimension d, the number of independent spin components n, and the number of easy directions p. Although $p=2$ in Eq. (1) as written, we may easily imagine the random-anisotropy potential as $-D\cos[p(\vartheta-\vartheta_i)]$, with ϑ_i one easy direction. This form reduces to the anisotropy term in Eq. (1) if $p=2$.

In this paper we study, using Monte Carlo simulations, the two-dimensional, $n=2$, $p=2$, HPZ model. Although the primary motivation for our study is that this model (at least for $d=3, n=3$) represents magnetic behavior, it is worth noting that other workers have found direct application of this limit to the adsorption of superfluid helium on bumpy interfaces.[6]

The paper is arranged as follows. In Sec. II we shall give a brief overview of other work relevant to our study. In Sec. III we present a brief overview of the relevant theory. Our simulation results are in Secs. IV and V. In the final section we shall present some brief conclusions. A preliminary version of this work has been presented elsewhere.[7,8]

II. REVIEW OF PREVIOUS WORK

We shall confine the scope of this brief review to properties of the HPZ model itself. We shall not concern ourselves in any detail with the question as to what extent the HPZ model actually describes magnetic behavior in amorphous rare-earth models. We have carried out a rather more comprehensive review of the theoretical literature elsewhere.[8]

A first crucial question concerns the nature of the low-temperature phase in the HPZ model. Early speculations and calculations suggested that perhaps this phase was also magnetic, at least for sufficiently low D (assuming that $D_0=0$).[9,10] Subsequent calculations, both analytic and computational, have suggested rather that, even at low randomness, the low-temperature magnetic state is destroyed,[11–16] and replaced by a random state that is sometimes identified with a spin glass.

An important tool in the understanding of the way randomness affects the magnetic state was introduced by Chudnovsky and collaborators,[17–21] based on earlier arguments due to Imry and Ma on random-field magnetic systems.[22] The low-temperature phase may be a correlated spin glass (CSG), with a magnetic correlation length which increases with decreasing D. For higher D, it may be a speromagnet (SM), in which the local magnetic moments point, more or less, along a local easy direction. In the high-field limit, deviations from perfect magnetization have a typical correlation length which decreases as H is increased; this phase is known as a ferromagnet with wandering axes (FWA). However, these qualitative arguments do not address the question of whether the low-temperature phase is a true spin glass;[23] in the sense of nonergodicity, with free-energy barriers between competing ground states increasing with size and strong dynamic effects. Rather they merely assert that the low-temperature phase will simply be magnetically inhomogeneous.

The subject of our study is the two-dimensional HPZ model with $n=2$. This particular limit presents one simplifying feature from a computational point of view, namely, that it has a low dimension and a low number of spin components. It therefore should, in principle, be possible to obtain computational data on the statistical mechanical properties relatively easily. Unfortunately, as is well known, however, this limit presents other problems from a statistical mechanical viewpoint. In the $D\rightarrow 0$ limit we are left with the two-dimensional XY model. The low-temperature phase was elucidated by Kosterlitz and Thouless.[24] There is no long-range magnetic order, but magnetic correlations fall off algebraically rather than exponentially at long distances. One can no longer inquire whether randomness destroys the magnetic phase, as there is no longer a magnetic phase to destroy. Rather we must ask whether the low-temperature Kosterlitz-Thouless phase ("almost magnetism") survives the addition of the random-anisotropy term. This phase also presents pitfalls to the simulator; a line of critical points everywhere in the low-temperature phase ensures that equilibration times remain long.

A number of authors[6,25–28] have carried out analytical studies of this model. Some studies seek to examine the stability of the XY behavior against perturbation by the random anisotropy.[6,25,27] Others, by contrast, seek to examine the limit $D/J\rightarrow\infty$.[26,28] The general argument in this latter case is that renormalization-group flows should take the system toward this limit in any case, so that for global properties of the phase diagram, taking this limit should not make any difference. In fact, this argument applies (or not) equally for any n and d. Cardy and Ostlund[6] find, using a replica space method, that if the number of anisotropy directions p is greater than $2\sqrt{2}$ (essentially 3 in a real system), the Kosterlitz-Thouless low-temperature phase survives the addition of randomness. The low-temperature phase, however, is a glassy phase; the Kosterlitz-Thouless phase is now an intermediate-temperature phase which disappears at the same temperature as it would in the classical XY model. However, in the model that we study in this paper ($p=2$), they assert that there will be no phase transition. As we shall see, the results obtained in our work seem to controvert this conclusion.

There is also recent computational work on the two-dimensional $n=2$ HPZ model. Dieny and Barbara[29] have examined hysteresis curves, but at zero temperature, in the context of an investigation of the topological structure of the magnetization process. They discovered somewhat erratic and irreproducible hysteresis curves. We shall return to this point below. Reed[30] has studied the $p=6$ case and discovered a low-T XY phase, in partial agreement with Cardy and Ostlund. Finally Dickman and Chudnovsky[31] have studied the one-dimensional $n=2$ HPZ model at $T=0$. They found some results consistent with Imry-Ma-Chudnovsky scaling, but noted that there are strong nonequilibrium effects.

III. THEORY

A. General

Much of the interpretation of our data will be carried out using the Imry-Ma-Chudnovsky schema.[17–22] The crucial idea of this model is that the low-temperature phase involves a balance between magnetic alignment of spins, favored by the exchange interaction, and alignment along a local easy axis, favored by the random magnetic anisotropy. In general, for dimension $d<4$ this involves a compromise in which there is ferromagneticlike alignment over a coherence length ξ_m. Of course, such ideas do not take into account the complex topological excitations known to be important in two dimensions, but they may nevertheless give useful insight into the simulation results.

These arguments have been extensively explored elsewhere,[8,17-21] and we shall go into detail only where our considerations add to those in the literature. In the absence of a field, the coherence length (in units of the lattice size a) is given by

$$\xi_m \sim D^{-2/(4-d)} . \tag{2}$$

Thus, in the case $d=2$, we expect

$$\xi_m \sim D^{-1} . \tag{3}$$

This result should only be true for D not too large. For large D, the spin direction will be governed by the local-spin anisotropy, and $\xi_m \sim 1$. Thus we expect a changeover for some $D \sim 1$.

B. High-field regime

We have carried out many simulations in the high-field regime. In the limit of high H, the magnetization per site M approaches unity, and it is of interest to examine the process whereby it does so.

We present here a modified Imry-Ma-Chudnovsky argument which describes this phenomenon. At high fields the spins are almost aligned in the FWA state. The fluctuations from perfect alignment are supposed to be of typical magnitude ϑ, and the correlation length over which each fluctuation is coherent is ξ. The interesting quantity is $\delta m = 1-\langle M \rangle$. Then

$$\delta m = 1 - M = 1 - \langle \cos\vartheta \rangle \sim \vartheta^2 . \tag{4}$$

The free energy per site $\mathscr{f}$ is the sum of the energy and entropy terms,

$$\mathscr{f} = \varepsilon - T\mathscr{s} , \tag{5}$$

with

$$\varepsilon = \varepsilon_{\text{exch}} + \varepsilon_{\text{anis}} + \varepsilon_{\text{field}} . \tag{6}$$

The usual Imry-Ma-Chudnovsky arguments[8,17-21] then give rise to

$$\varepsilon \sim J(\vartheta/\xi)^2 - D(\vartheta/\xi) + H\vartheta^2 , \tag{7}$$

where terms independent of ϑ and ξ have been ignored. At $T=0$ it suffices to minimize ε. At the minimum, if possible, all terms will be of the same order of magnitude.

For simplicity, in what follows we measure quantities in units of J. Let us first consider the CSG, in which $D \lesssim 1$. It is straightforward to show that $\xi \sim H^{-1/2}$, $\vartheta \sim DH^{-1/2}$, and $\delta m \sim (D^2/H)$. However, this regime only applies so long as $\xi \lesssim 1$, which is the case so long as $H \lesssim 1$, and so long as $\vartheta \lesssim 1$, which demands that $H \gtrsim D^2$. At lower fields we are no longer in the FWA regime. At higher fields, $\xi \sim 1$, and the balance is only between $\varepsilon_{\text{anis}}$ and $\varepsilon_{\text{field}}$. Now $\vartheta \sim D/H$, and thus $\delta m \sim (D/H)^2$. This result clearly holds so long as $\delta m \lesssim 1$, or so long as $H \gtrsim D$ if $D > J$.

These results are modified at finite temperature. At zero temperature the departure from perfect alignment is caused by the randomness-induced disorder. At higher temperatures the disorder may be entropy-induced. We may suppose that $\mathscr{s} \sim k_B \ln\vartheta$. In the high-field limit $T\ln\vartheta$ must balance $H\vartheta^2$, yielding $\delta m \sim \vartheta^2 \sim (T/H) \sim H^{-1}$. The zero-$T$ behavior will dominate when the anisotropy term is larger than the entropy term; this will be the case if $(T/H) \sim (D/H)^2$. The crossover thus takes place at $H \sim D^2/T$.

These results can be summarized as follows. In the CSG ($D \lesssim J$) we have four regimes: (a) Low field; $H < H_1 \cong D^2$. True CSG behavior with spins in clusters of size $\xi_m \sim D^{-1}$ and with clusters pointing essentially in random directions; (b) $H_1 < H < H_2 \cong J$. $\delta m \sim D^2/H$, with cluster size $\xi \sim H^{-1/2}$; (c) $H_2 < H < H_3 \cong D^2/T$. Now $\xi \sim 1$, and $\delta m \sim (D/H)^2$; (d) very high-field regime $H > H_3$. $\xi \sim 1$ as in (c), but $\delta m \sim T/H$.

It is interesting to observe that there are two regimes with $\delta m \sim H^{-1}$, but with different coefficients, separated by a region with $\delta m \sim H^{-2}$. However, for $T \gtrsim D^2$, the intermediate regime disappears, and we presumably only expect H^{-1} behavior.

This behavior is modified for a speromagnet, because then the cluster size is not a factor. Regimes (c) and (d) survive; the lower limit for (c) becomes $H \cong D$, below which speromagnetic behavior essentially survives.

IV. ZERO-FIELD SIMULATIONS

A. Simulation method and computational detail

The feature of amorphous magnets most in need of explanation, apart from magnetic inhomogeneity is irreversibility. This property is often associated with glassy behavior. In our simulations we have used the classical Monte Carlo algorithm of Metropolis *et al.*[32] In the investigation of glassy behavior this procedure has been criticized. Fisch[13,33] and other workers have been concerned that the single spin-flip Monte Carlo method was not adequately sampling the system ensemble, and hence not reaching thermodynamic equilibrium. While not wishing to minimize this problem, our point of view has been more empirical. We simply make the observation that if the Monte Carlo method does not reach equilibrium quickly, then this may (and often does) reflect a real analogous problem for the physical system being modeled. In this case the amorphous magnets under consideration do indeed exhibit irreversibility, so difficulties in reaching equilibrium, if they exist, should be considered simply as observable phenomena rather than as problems.

In our study we have not considered the effect of coherent anisotropy [the D_0 term in Eq. (1)]. All energies in our calculation are measured in terms of the exchange parameter J. We have considered the effect of changing temperature T, randomness D, and magnetic field H. In order to be more sure of the reliability of our conclusions, we have also simulated system sizes between 8^2 and 128^2, and have examined results of simulations with variable run times between 500 and 10^6 cycles, where one cycle consists of one Monte Carlo move per particle. Among physical variables monitored were hysteresis curves with associated coercive fields and remanant magnetizations, magnetic susceptibility, specific heat, and spin-spin correlations, both as a function of space and Monte Carlo time.

The simulations were run on a Meiko system with a parallel architecture, with 32 T-800 transputers contributing to the simulation. Each transputer carries 256 kbytes memory and contributes roughly one Mflop. Most of our production runs were made on systems of 64^2 lattice sites, though after one of the transputers developed a hardware fault this was reduced to 62^2.

B. Qualitative observations

It is helpful, at this stage, to recall some results from the $D=0$ case, i.e., the $d=2$ XY model. This model has no equilibrium bulk magnetization in the thermodynamic limit. However, simulations of systems of finite size L do show a spin M per site, with $M \sim L^{-\eta/2}$. Thus a plot of the temperature dependence of the apparent magnetization does give some useful information, though not, of course, the equilibrium magnetization. A long-time plot of a Monte Carlo simulation magnetization shows a drifting of the magnetization direction, with the drifting becoming slower as the system size increases. However, the Kosterlitz-Thouless transition can be identified by usual finite-size scaling techniques[34] and occurs at approximately $T=1.0$.

We first present some qualitative results which reinforce the idea of a glassy low-temperature phase. In Fig. 1 we show the results of simulations with $L=64$, lasting for 10^6 cycles, with $D=0.4$, for various different temperatures. We start the simulation with all spins aligned in the x direction and simply observe the total magnetization direction ϑ (in radians with respect to the x axis). The information is presented both explicitly, as a function of Monte Carlo time, and (inset) as a probability distribution.

At $T=0$ [Fig. 1(a)] the system seeks a nearby easy direction. That this direction is not unique, however, is shown at $T=0.1$ [Fig. 1(b)], in which the system has found another easy direction. By $T=0.4$ [Fig. 1(c)], it finds yet another, at $\vartheta \cong 1.2$ radians. In order to arrive at this minimum, however, the magnetization has undergone a series of sudden jumps which show the extremely long relaxation times ($\sim 10^5$ cycles) which appear to occur. From here up to about $T=0.7$ [Fig. 1(d)], the system now reaches this easy axis, although sometimes it finishes up pointing in the opposite direction. However, as temperature increases, the fluctuations of the mean direction from its equilibrium value seem to get larger, until at about $T=0.8$, the system starts to jump between the opposite directions along the same easy axis. Finally above about $T=1.0$ [Fig. 1(e)], the system fluctuates constantly in direction, though still showing a slight preference towards the easy axis.

Of course these results are at best only indicative. Too many uncertainties remain. How is angular drift affected by system size? What would happen if we were to further increase the time scale of our runs? Are we finding equilibrium or nonequilibrium behavior? How would the results have been affected if we had been able to study a large set of different, but statistically similar, random systems. Nevertheless these results do set the scene and suggest that something interesting, not within the scope of the normal XY model, should be occurring in the low-temperature regime.

C. Spin correlations

In this subsection we report the results of more controlled calculations. We have carried out measurements of the magnetic spin-spin correlation function,

$$g(r)=\langle \mathbf{m}(\mathbf{0})\cdot\mathbf{m}(\mathbf{r})\rangle \ . \tag{8}$$

The samples in this case are annealed (or cooled, in the figure captions). The simulations were carried out starting at approximately $T=1.3$, and the temperature was progressively reduced in steps of $0.03J$, with 50 000 Monte Carlo time steps performed at each temperature.

In order to analyze the correlation functions, we have fitted $g(r)$ to the form $Ar^{-\eta}\exp(-r/\xi)$. In fact the fitting procedure has to be carried out rather carefully, only considering $r/L \lesssim 0.3$, in order to avoid distortions due to the finite size of the samples, and in addition, the data are rather noisy. We recall that the Kosterlitz-Thouless theory[24] predicts that in the low-temperature phase $\xi=\infty$, and η increases progressively from 0 at $T=0$ to $\eta=0.25$ at the transition temperature T_{KT}, which we find to occur at approximately $T=1.0$. At higher temperatures ξ is finite and $\eta=1$. At $D=0$ our simulations indeed confirm this picture, apart from some scatter in the apparent value of η just above the transition temperature, which we ascribe to finite-size effects.

We have carried out simulations at a number of different nonzero values of D ranging up to $D=2$. We find that for low values of D, $g(r)$ can be fitted to the Kosterlitz-Thouless form over an intermediate range of temperatures, and that over this range of temperatures (and above) the behavior of $g(r)$ is identical to the $D=0$ case. However at low temperatures, the effective η flattens out, and now ξ^{-1} is no longer zero. In fact, the hypothesized form for $g(r)$ does not provide a very good fit in this regime; in the analogous case for $d=1$ Dickman and Chudnovsky[31] have been able to suggest a more plausible form on analytical grounds, but we have not found a two-dimensional analog. The temperature at which ξ^{-1} disappears increases as D is increased, thus more or less identifying a low-temperature "glassy" phase. We place the quotation marks around the term glassy because there is no evidence of glassiness, as understood in a technical sense by spin-glass theorists.[23] Finally we observe that above $D \cong 1$, the glassy phase overwhelms the Kosterlitz-Thouless phase.

Whenever the possibility of irreversible behavior arises, it is possible that the rate of cooling is a relevant variable for quasithermodynamic behavior. We have therefore carried out studies of $g(r)$ on quenched samples, i.e., zero temperature was applied to a sample which was initially in a random high-temperature state, and then the sample was progressively warmed at the same rate that the annealed sample was cooled. The conclusion of this set of simulations is that the low-temperature phase is reasonably well identified; it does have anomalous properties, but the detailed form of $g(r)$ is different from that in the annealed sample. This seems to derive from a reluctance of the system to approach true equilibrium in this case, and this is consistent with the studies of the previous sub-

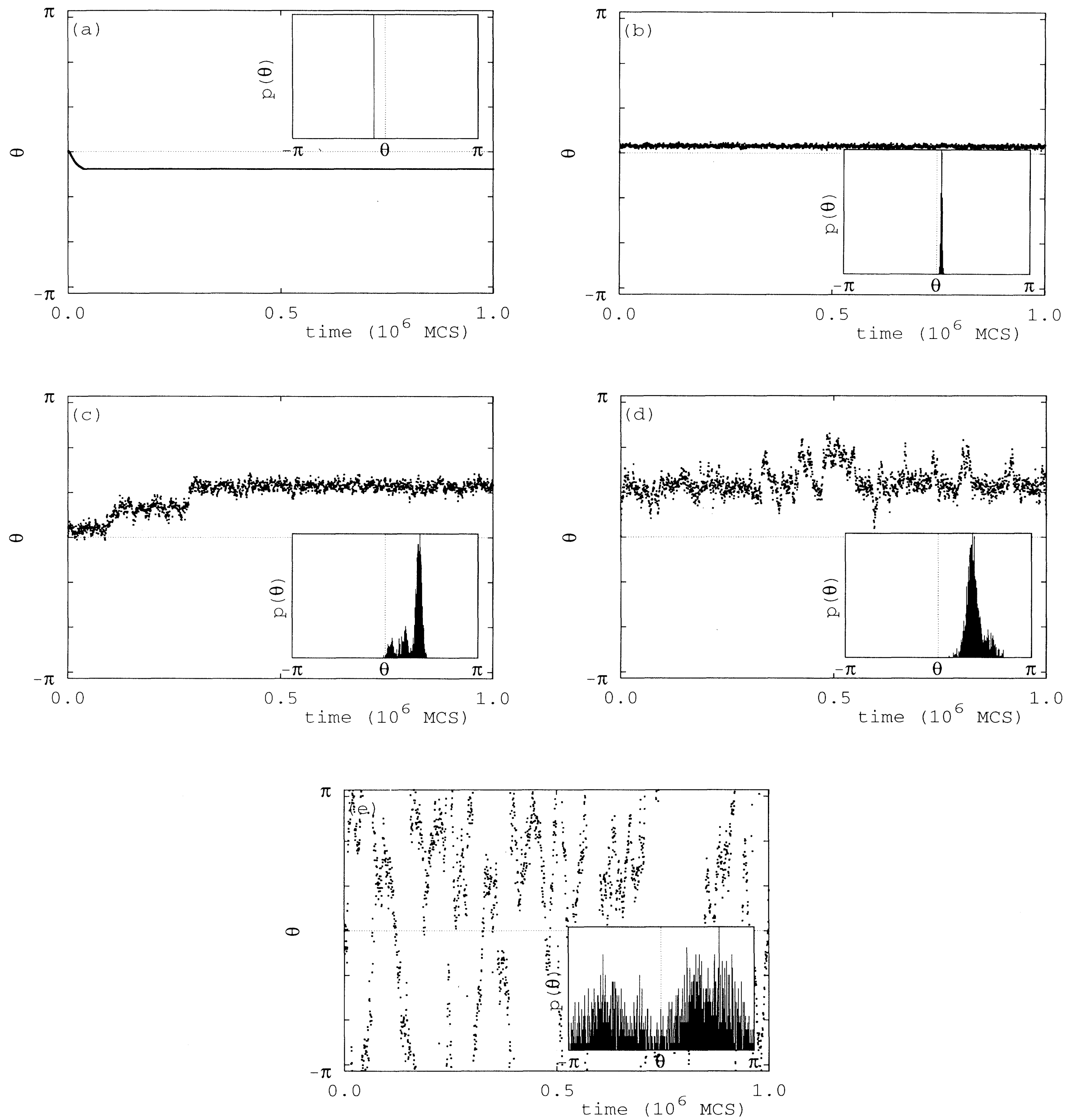

FIG. 1. Mean sample direction ϑ as a function of Monte Carlo time; $D=0.4$ with (inset) the probability distribution (see text). (a) $T=0$, (b) $T=0.1$, (c) $T=0.4$, (d) $T=0.7$, (e) $T=1.0$.

section. However, it is not possible to judge what the effect of larger systems or longer runs would have been.

A summary of these results is shown in Figs. 2 and 3, in which we show the fitted values of $\eta(T)$ and $\xi^{-1}(T)$ for different values of D. These graphs create two measures of the phase-transition temperature between the glassy phase and the Kosterlitz-Thouless phase.

D. Magnetization

In the simulations described in the last section we have also measured the magnetization as a function of temperature. This quantity is strictly zero in the two-dimensional XY model in the thermodynamic limit, and we must suppose that in the finite D HPZ model, which is inherently less magnetic, this will also be the case. However, because of the long-range algebraic order, finite simulations give rise to an observable magnetization $M(L)$, which only decreases as $L^{-\eta/2}$ as system size is increased; this caused much confusion in early simulation studies. A low-temperature glassy phase would presumably give rise to a different size dependence of the magnetization.

In Fig. 4 we show a plot of $|M|^2$ as a function of temperature for different values of D, for the annealed simulations described in the last section. We find the behavior consistent with that expected by measuring correlations. At (not too great) finite D, the magnetization follows that of the XY model down to some temperature (which increases with D); below that the magnetization is depressed, and more or less static. Once again the picture is of an intermediate-temperature Kosterlitz-Thouless phase. It is interesting that for sufficiently high D, the algebraic order phase seems lost, as before. The simulations which involve quenching and subsequent warming give rise to a similar picture, although now the system has difficulty in reaching low-temperature equilibrium, and thus the magnetization actually rises at intermediate temperatures. Presumably, at least roughly speaking, the glassy phase is to be identified with the region of seriously depressed magnetization.

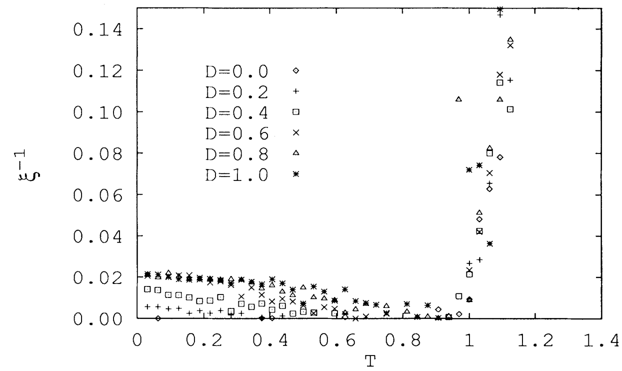

FIG. 3. Effective inverse correlation length $\xi^{-1}(T)$ for simulations of Fig. 2.

E. Specific heat

The specific heat can be measured in two ways. One can either numerically differentiate the mean energy $\langle E \rangle$ with respect to temperature, or one can use the fluctuation-dissipation formula for the specific heat C per spin, i.e.,

$$C=(NT^2)^{-1}(\langle E \rangle^2-\langle E \rangle^2) , \tag{9}$$

where N is the number of spins in the system, and E is the total energy of the system. In systems which do not reach equilibrium, the two different routes might lead to different results.

In practice, however, it turns out that the specific heat is the most stable of all the quantities that we have measured. We obtain essentially the same results for shortish runs (5×10^4 cycles), as for long runs (10^6); it seems not to matter whether the system is being heated, cooled, or started from a fixed configuration, nor, indeed, which route is being used in the calculation.

The most striking characteristic of the results is that for $D \lesssim 1$, the specific-heat curves seem essentially independent of D, and identical to the pure XY results. There is a broad peak around $T=1$, marking the onset of

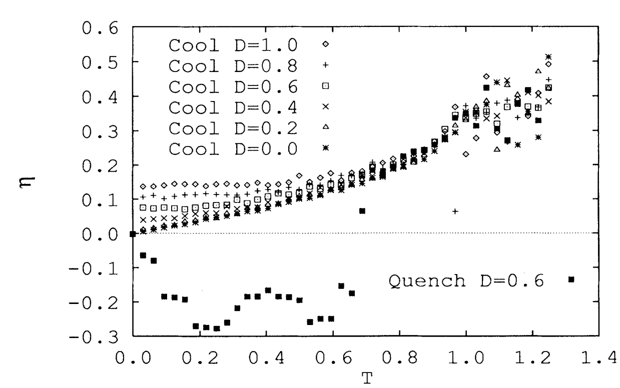

FIG. 2. Effective exponent $\eta(T)$, for various different values of D ($L=64$), for annealed and quenched simulations described in the text.

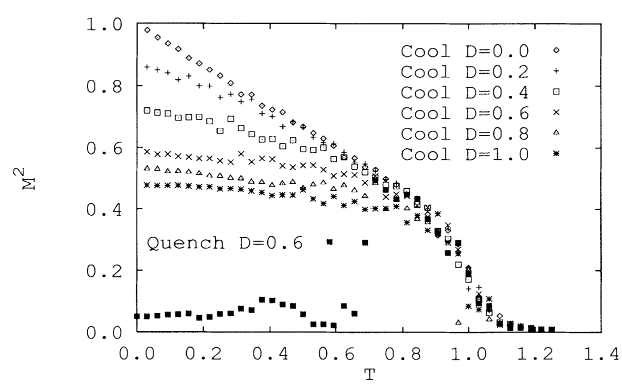

FIG. 4. Plot of $|M|^2$ as a function of T for various different D for annealed and quenched simulations of Fig. 3, showing departures from pure XY behavior at low temperatures.

algebraic order. There is no glimmer of recognition of the onset of any possible low-temperature phase, thus perhaps arguing against its existence. However, at higher D, the specific heat is depressed, and the peak moves to higher T, reaching $T \cong 1.3$ for $D = \infty$, where there are previous simulations.[15] By about $D = 10$, the large D limit has been reached. The specific-heat results are summarized in Fig. 5. It is necessary to add a factor of 0.5 to the result for $D = \infty$ for comparison, because the $D = \infty$ limit simulations are carried out in the so-called Ising limit, in which the spins are actually constrained to lie along the local easy axis. By contrast finite D simulations allow each spin to perform small oscillations around its local easy direction. Such oscillations contribute one degree of freedom per site, and hence their contribution to C is given by equipartition of energy and is 0.5 per site. It will be seen that there remain some differences between the $D = 5.5$ and $D = \infty$ data. Unfortunately, we do not have a systematic theory which describes the approach to infinite D.

Clearly no definitive conclusions can be derived from the specific-heat data. There is no signature of the onset of the glassy low-temperature phase which the magnetization and correlation data seem to imply. On the other hand, glassy phases sometimes have purely dynamical signatures, so perhaps no thermodynamic signature is required. This low-temperature phase, however, is apparently identified by anomalous static correlations. We return to this point in Sec. VI. However, what we can say is that because there is apparent identity between the specific heat at nonzero D and that of the XY model, we should not be surprised if the low-temperature Kosterlitz-Thouless phase survives the addition of randomness. On the other hand, at higher values of D, the specific heat is noticeably different, and we should not, therefore, be surprised if that were to accompany the destruction of this phase that seems to be implied more explicitly by other data.

We have also examined $C(D)$ at constant T. In the low-temperature regime there is a relatively shallow peak at $D \sim 4$; above $T = 1$ this peak broadens and approximately follows $D = 5T$. We can identify the peaks in the slices at constant T as a tentative delineation between a low-D correlated spin-glass regime, and a high-D speromagnetic regime. We shall return to this point below in Sec. IV H.

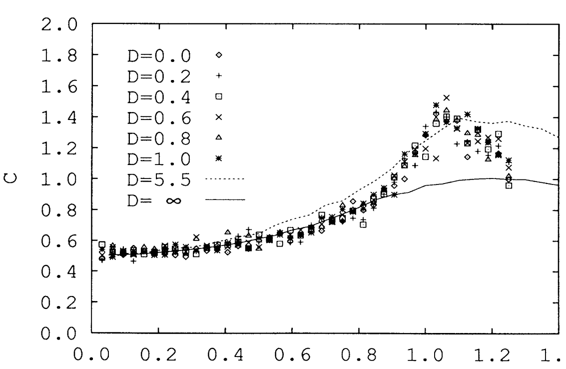

FIG. 5. Specific heat as a function of temperature for different values of D (with correction factor applied for $D = \infty$, as discussed in text).

F. Finite-size studies

We have examined carefully the onset of glassy behavior using the correlation function data, in order to check whether this might be a finite-size effect. Using these data, this effect seems relatively independent of system size. However, as we have already mentioned above, these data are rather hard to interpret unambiguously.

We turn now to a comparison of the remanant magnetization as a function of system size. We have seen in Sec. IV D above, that in the presence of algebraic order, one expects that the magnetization will fall off as $L^{-\eta/2}$. In the $D = 0$, XY limit, log plots of m against L indeed yield this behavior, with η increasing from 0 to $T = 0$ to 0.25 at $T \cong 0.93$, and with values of η consistent with, though very slightly lower than, those derived from the correlation function data. At higher temperatures the observed magnetization is due to statistical fluctuations and thus $m \sim N^{-1/2} \sim L^{-1}$; there is a sharp delineation between the two types of behavior.

At higher values of D, the data become much harder to interpret unambiguously. As an example, in Fig. 6 we show a log plot of m against L for various different temperatures for $D = 0.4$. In the low-temperature regime the plot is clearly no longer linear, and indeed can be fitted to a scaling form $m(L) \sim f_D(T) g_D(L)$. This form seems not to apply after about $D \cong 0.4$, and is replaced by a somewhat bumpy, but apparently linear, form which can be interpreted as giving a $L^{-\eta}$ behavior, and thus is consistent with a Kosterlitz-Thouless phase in this intermediate-temperature regime. However, in contrast to the correlation function results, which give a value of η identical to that of the Kosterlitz-Thouless phase at any given temperature, here the implied value of η seems to be increased from its value in the pure XY model. We also find a high-temperature regime, above $T \cong 1$, in

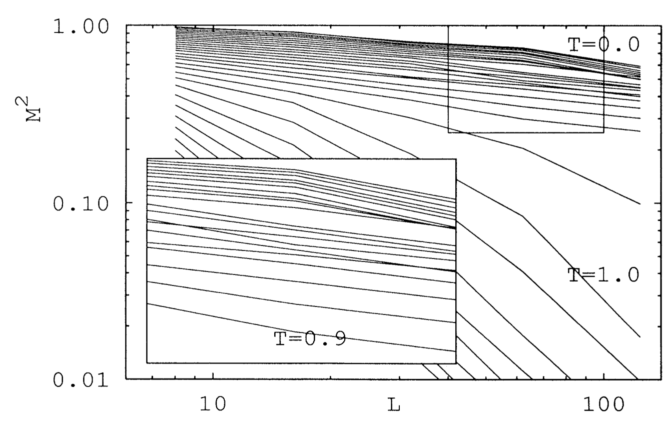

FIG. 6. Log plot of M^2 vs L at $D = 0.4$. Each set of points corresponds to a different value of T; temperatures increase from zero in steps of 0.05 down the graph. Inset shows the nonlinear low-temperature behavior. Sets of points replaced by continuous lines are an aid to the eye.

which $m(L) \sim L^{-1/2}$ for high L; this is clearly the paramagnetic phase. Our interpretation is that if we observe a linear dependence of $\ln m$ on $\ln L$ with an apparent value of $\eta < 0.25$, then we accept that this is indeed an indication of an algebraically ordered phase, but if $\eta > 0.25$, we suppose that at higher L, paramagnetic behavior will dominate.

Although the results of this subsection are broadly consistent with the picture derived from individual simulations, in the sense that low-, intermediate-, and high-temperature regimes are clearly discernible, nevertheless it is important to emphasize that ambiguity still remains. A glance at Fig. 6 shows that the crossover between the low- and intermediate-temperature regimes is somewhat fuzzy. The data in this region are rather bumpy, perhaps due to results being irreproducible simply because the system is random. It is possible to suppose that there are other secrets hidden at higher finite sizes which our scaling has failed to reveal.

G. Phase diagram

The different kinds of data presented in the previous sections permit the construction of a tentative phase diagram, which has been plotted in (T,D) space in Fig. 7. This diagram exhibits a low-temperature glassy phase, an intermediate Kosterlitz-Thouless phase, and a high-temperature paramagnetic phase. Above $D=1$, the Kosterlitz-Thouless phase is suppressed. It has not been possible to say whether in this regime there really is a transition between two separate phases, or whether the paramagnetic phase extends to low temperatures in this regime, and at lower D there is a reentrant transition to paramagnetism.

H. The low-temperature phase

In this section we make some further comments about the nature of the low-temperature phase. In Sec. III A above we have seen that the Imry-Ma-Chudnovsky arguments lead one to expect magnetic coherent clusters of characteristic size $\xi_m \sim D^{-1}$. We have seen as well in Sec. IV C above that in the low-T regime it is not in fact possible to make a good fit of the correlation function to an exponentially decaying form. If, nevertheless, we force $g(r)$ for the annealed simulations into this form, we are able to examine the dependence of ξ_m upon D. This is shown in Fig. 8. It will be seen that the Imry-Ma-Chudnovsky theory is well verified over the range examined. However, it is interesting to note that the numerical value of ξ_m is such that the speromagnetic regime, for which $\xi_m \sim 1$, is not reached in the regime of our simulations; we can extrapolate it to appear at $D \sim 20$, which seems rather high compared to the (albeit rather shallow) specific-heat peak occurring at $D \sim 4$.

We have referred to the low-temperature phase in a rather shorthanded kind of way as a glassy phase. Nevertheless, as we have already made clear, the definition of glassy in this context is by no means unambiguous. The qualitative data presented in Sec. IV A suggest some kind of nonequilibrium behavior at low temperatures, and at the same time it is clear that the low-temperature phase is less magnetically ordered than it would be if the random anisotropy were not present. In the spin-glass literature a true glassy regime would have multiple ground states separated by free-energy barriers whose magnitude grows with system size. Figure 1 suggests the presence of energy barriers, but gives no indication of the size dependence. Given the nature of the Hamiltonian, it is clear that the ground state must be at least twofold degenerate. It would be desirable to examine these questions in more detail, and to carry out some of the tests which are usually carried out in numerical simulations of Ising spin glasses.[35]

In fact, Chakrabarti[16] made measurements of a spin-glass order parameter in a RAM of three-dimensional spins. These results have been criticized by Fisch,[11] who suggests that Chakrabarti had failed to reach equilibrium. We have tried to make measurements of a spin-glass order parameter,

$$q(t)=\frac{1}{N}\sum_i \langle \mathbf{m}_i(\tau)\cdot\mathbf{m}_i(t+\tau)\rangle , \tag{10}$$

where τ is chosen at a time when the system has already

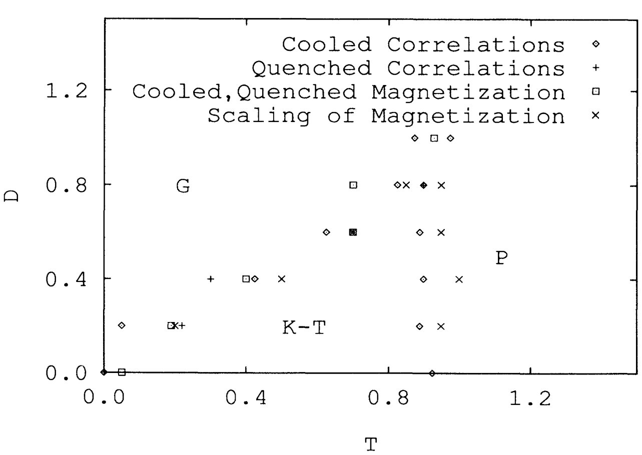

FIG. 7. Phase diagram derived using magnetization and correlation data comparing quenched and cooled systems, showing paramagnetic (P), Kosterlitz-Thouless (KT), and glassy (G) phases.

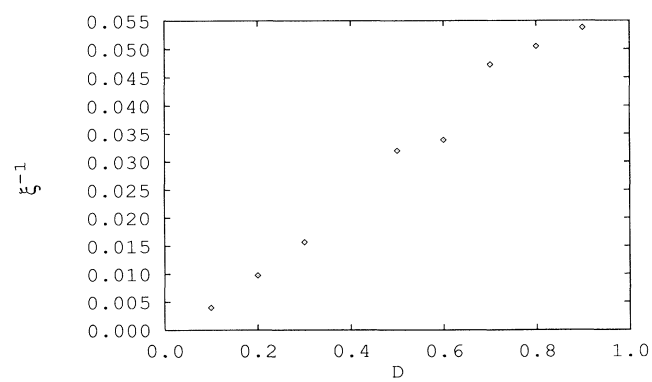

FIG. 8. Dependence of ξ_m^{-1} upon anisotropy D in the very low-T limit, using correlation data of Sec. IV C.

equilibrated, and where the Edwards-Anderson spin-glass order parameter $q = \lim_{t\to\infty} q(t)$.[23]

Unfortunately, however, it proved impossible to fit the long-time behavior of $q(t)$ to the exponential or stretched exponential forms which have been proposed in the literature, and it did not turn out to be possible to correlate the existence of a nonzero q to the other measures of the existence of a glassy low-temperature phase.

V. FINITE FIELD SIMULATIONS

A. Hysteresis curves

A common criterion for the existence of magnetism in a given material is the presence of hysteresis loops in the M versus H characteristic. These loops continue to exist in amorphous rare-earth alloys, presumably as nonequilibrium features, despite the disappearance of macroscopic long-range magnetic order. We have simulated hysteresis loops, varying both the jump in field at each step, and the number of Monte Carlo cycles at each fixed point. In order to maintain manageable data, we have kept our Monte Carlo runs at each field point relatively short, between 3000 and 10 000 cycles.

In Fig. 9, we show a typical hysteresis loop observed in one of our computer experiments. Irreproducible steps in the M vs H characteristic, which bear considerable resemblance to features observed experimentally, can be seen. Similar curves were seen by Dieny and Barbara[29] in their zero-temperature simulations of the same model. These authors identify the steps with the mutual destruction of topological vortices.

In Fig. 9 we can identify two crucial magnetic fields. These are the coercive field H_{co} required to (just) reverse the magnetization direction, and the critical reversible field H_{rev}, which is that field required for the sample to exhibit a history-free magnetization.

It is helpful to collect the hysteresis data from many simulations. The curve $H_{rev}(T)$ for a given D may be thought of as analogous to the de Almeida–Thouless line,[36] which separates the ergodic and nonergodic regions in the spin-glass phase diagram. We find that it is easier computationally to plot the coercive field $H_{co}(T)$. The ratio of the two quantities is more or less constant in any given situation. A series of such plots, for various different values of D, is shown in Fig. 10. It will be seen that all hysteresis ceases above $T_c \cong 1.0$, which is also the Kosterlitz-Thouless ordering transition, although for low values of D, hysteresis appears to cease at low temperatures. There is, however, only a very weak dependence of these curves on the magnetic sweep rate dH/dt, suggesting that we are dealing with quasistatic rather than dynamic phenomena.

We can identify a critical temperature $T^*(D)$, above which, for a given D, hysteresis no longer exists, and a zero-temperature coercive field $H^*_{co}(D)$. The resemblance between the coercive field curves suggests that a universal curve may exist, of the form $H_{co}(D,T) = H^*_{co}(D)\not{f}[T/T^*(D)]$. In fact, however, we have found it impossible to collapse the different curves onto each other by such a transformation. There is, however, a degree of uncertainty about the precise values of $T^*(D)$ which suggests that further work may be useful in this regard.

B. Approaching saturation

We now pass to a discussion of the high-field regime, the theory of which has already been summarized in Sec. III B. In this section we concentrate on the magnetization in the high-field limit, and more particularly on the quantity $\delta m(H) = 1 - M(H)$, the difference between the mean magnetization and its saturated value. A crucial feature of the expected results is the crossover between $\delta m(H) \sim H^{-2}$ and $\delta m(H) \sim H^{-1}$ at $H \sim D^2/T$.

In Fig. 11 we show the results of a series of simulations for different system sizes for $T = 0.01$ and $D = 0.5$. The magnetic field is systematically increased in the same simulation with the run at each H lasting 5000 cycles. Despite this apparently short run, it is clear, from the system size independence of the magnetization at fields $H \gtrsim 0.2$, that equilibrium has been reached. There are two interesting features in this graph. The first is that the system size dependence disappears at $H \sim 0.2$, and this is the same, within experimental error, as the critical reversible field H_{rev} derived from the hysteresis results of the

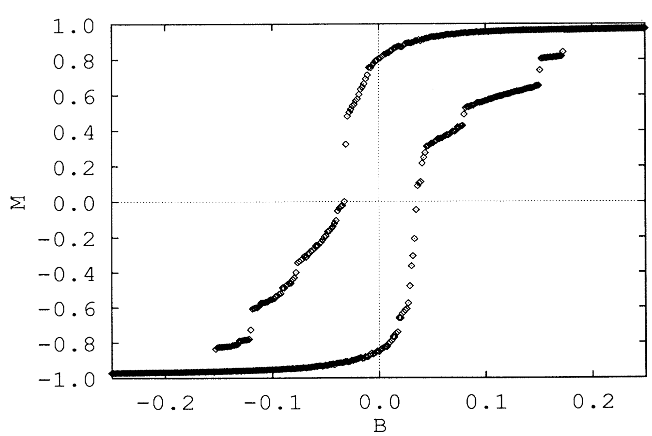

FIG. 9. A typical hysteresis curve at $T/J = 0.1$, $D/J = 0.4$, $\Delta H = 0.002$, showing irreproducible steps.

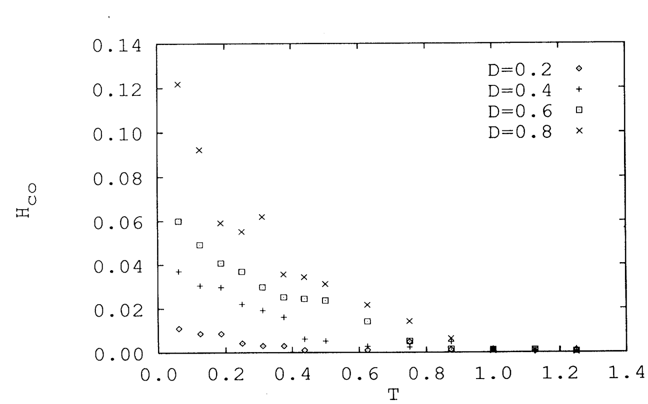

FIG. 10. Coercive field $H_{co}(T)$, for various different values of D.

last subsection. The second is the $\delta m \sim H^{-1}$ behavior, as predicted, for high fields.

It seems that at $T=0.01$, $D=0.5$ is not sufficiently high in anisotropy for the random $\delta m(H) \sim H^{-2}$ behavior to be exhibited. However, also shown in Fig. 11 is the behavior at $D=10$. In the regime of fields over which we have measurements it is immediately clear that in the $D=10$ case, $\delta m \sim H^{-2}$ behavior is observed. A more quantitative examination of the crossover between the two behaviors is shown in Fig. 12, where we examine the form

$$\delta m = a\frac{T}{H} + b\frac{D^2}{H^2} . \tag{11}$$

This form is verified by plotting $\delta m H^2/T$ against H; we find a graph of the form $A(H+H_3)$, with A independent of T and D, and where the intercept $H_3 \sim D^2/T$ is the crossover field between the H^{-2} and H^{-1} behaviors. In Fig. 13, we have plotted H_3 against D^2 at $T=0.01$ (actually using a log plot) and indeed verify the predicted linear relationship.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have carried out an exhaustive study of the two-dimensional random-anisotropy HPZ model over a wide range of temperatures, anisotropies, and magnetic fields. The primary aim of the study was as a pilot for studies of more experimentally realistic three-dimensional systems. In three dimensions, it will be recalled, the Imry-Ma-Chudnovsky theory[17–21] predicts the destruction of ferromagnetism in the presence of anisotropy, and a breakup into domains. In fact the complications of a two-dimensional system with two spin degrees of freedom preclude any direct comparison with real magnetic systems. Nevertheless, because $d=2, n=2$ is a natural limit to consider, there have been a number of direct theoretical studies of this particular system,[6,25–27] and particular interest focuses on the effect of random anisotropy on the low-temperature Kosterlitz-Thouless phase which possesses algebraic long-range order. Of particular interest are the predictions of Cardy and Ostlund[6] that in the system we have considered this phase would disappear, but if, by contrast, there had been three, rather than two, easy directions, then it would have been retained in an intermediate-temperature regime. Other theoreticians have concurred with this general picture.

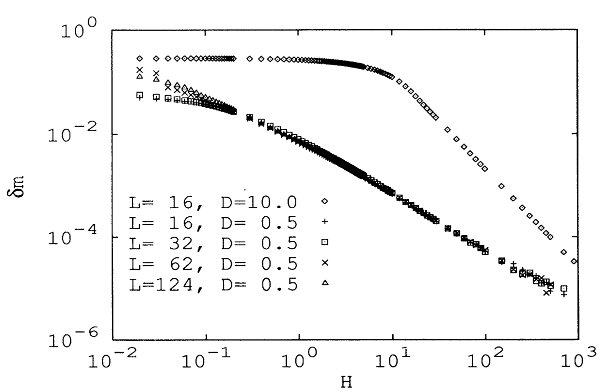

FIG. 11. Simulation results for δm with increasing H, as described in text, $D=0.5$, $T=0.01$ for $L=16,32,62$, and 124, showing $\delta m \sim H$ behavior, and $D=10.0, L=16$ showing $\delta m \sim H^{-2}$ behavior.

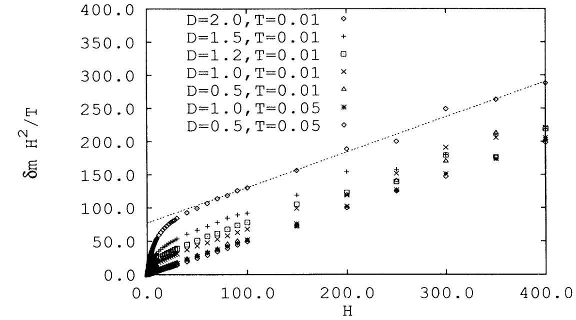

FIG. 12. Plot of $\delta m H^2/T$ vs H for different values of D, as discussed in the text. Note the linear relationship with intercept H_3.

Despite the extremely comprehensive set of simulations that we have carried out, we cannot give a completely convincing answer to this question, though we believe that we have, in the process, illuminated many features of the statistical properties of this model. We have found evidence for three separate regimes which could be identified with a low-temperature glassy phase, an intermediate Kosterlitz-Thouless phase and a high-temperature paramagnetic phase, with the algebraically ordered phase disappearing at higher D.

As the HPZ model was invented in order to provide a model for magnetic glasslike phases in rare-earth alloys, the glassy phase is in one way a heartening feature. On the other hand, there has been much speculation in the literature as to the precise nature of the glassiness under consideration here. A particular point of comparison is with the Ising spin glass.[23,35,6] In that case, the spin-glass phase is essentially defined dynamically. We have had great difficulty in measuring any spin-glass order parameter and correlating it with other apparent features of the phase diagram. There is a certain degree of irreversibility, evidenced, for instance, in the hysteresis curves, and

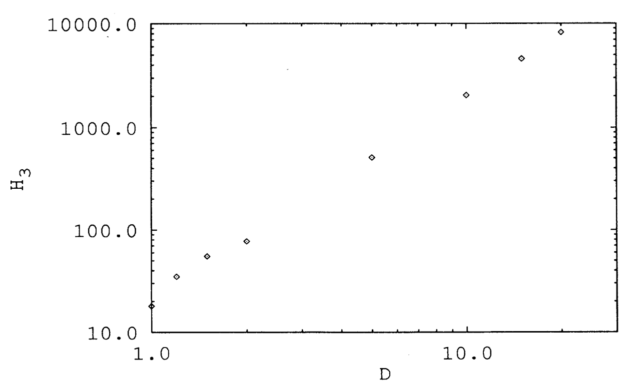

FIG. 13. Log plot of H_3 from Fig. 12 against D^2, showing dependence of crossover field, for $T=0.01$.

we are able in this case to identify the limits of that irreversibility with the de Almeida-Thouless line discussed in the spin-glass literature.

What does seem to be the case is that the low-temperature phase is magnetically inhomogeneous, and that the decay in the magnetic correlations has an exponentially decreasing component, although it cannot be fitted to a pure exponential form. If we insist on the exponential form, then the predictions of the Imry-Ma-Chudnovsky theory, with $d=2$ substituted where necessary, are satisfied. This is rather too good to be true, especially in view of the consensus that topological excitations, not considered in that theory, must be important. If we go to the high-field limit, the Imry-Ma-Chudnovsky picture also works well, even where we have modified it to finite temperatures. Here, of course, the topological considerations no longer play a role.

For $D \lesssim 1$ the low-temperature phase seems to stop at a glass temperature. There is no thermodynamic signature of this whatsoever, and one must therefore be hesitant about a positive identification with a true thermodynamic phase transition. On the other hand, in the low-temperature regime we observe (a) a different form for the magnetic correlation function, (b) a different form for the finite-size dependence of the magnetization, (c) at particular L a magnetization which is noticeably different from that in the $D=0$ case, (d) a much slower dynamics whose final state seems to be capricious and depend on initial conditions, and (e) hysteresis at finite fields. The precise point at which each of these features ends differs, though not dramatically. The phase signature is clearly not completely dynamic. Although changing the Monte Carlo algorithm clearly might have some effect, the signature of the phase does have some static features. In addition, the dependence on D of the phase transition as determined by each of these methods is consistent.

Let us now examine the hypothesized intermediate Kosterlitz-Thouless regime. This is identified by a calculation of the exponent η, which is accessible either directly by examining correlations (at shortish range), or indirectly through finite-size scaling of the magnetization $M(L)$. At $D=0$ these two methods are consistent; the phase ends at about $T=1.0$, below which $\eta<0.25$, and above which the finite-size scaling yields $\eta=1$, and the correlations show a clear exponential component. For $D \lesssim 1$, the specific heat is essentially unaffected by the anisotropy, as is η, calculated from the correlations. One might therefore expect that the Kosterlitz-Thouless temperature was similarly unaffected. However, η calculated from finite-size scaling is affected, and it would not be completely unexpected if the phase transition decreased in this circumstance. The theoretical predictions are equivocal.

If there were no predictions of the disappearance of the Kosterlitz-Thouless phase, there is no doubt that we would interpret our simulations as evidence of its existence in an intermediate regime, as shown in the phase diagram in Fig. 7 though we would be concerned by the detailed lack of consistency of the various pieces of evidence. With those predictions, however, it would be possible to interpret the ambiguity in the data as warning signals that the Kosterlitz-Thouless phase will disappear, albeit at extremely long distances and/or big system sizes. Perhaps the difficulty in analyzing the results is because the system is close to a critical d and p.

Finally, at higher D, where the Kosterlitz-Thouless phase certainly no longer survives, we observe a clear distinction between the glassy and the paramagnetic regimes. Apart from the fact that the finite-size dependence of $M(L)$ appears to be different, there is also a large specific-heat peak, and also a large peak in the magnetic susceptibility χ_m. We do find a dramatic suppression of χ_m at low T, although we have not discussed this property in this paper because of the lack of self-averaging and the need to discuss its dependence on the time scale of the simulation. However, this suppression of χ_m at low T is usually regarded as an experimental signature of a random-anisotropy system. On the other hand, what is not clear is whether there is a true phase transition or a smooth crossover between markedly different regimes.

In conclusion, we have found interesting phenomena in the two-dimensional random-anisotropy HPZ model, which are consistent with some general theoretical ideas on the nature of random magnetism. A more detailed comparison with theory and experiment awaits, on the one hand, a more sophisticated theory which includes dynamic effects, and on the other, experimental manifestations of these extremely interesting systems.

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