Breakdown of the Fermi liquid due to long-range interactions

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Fermions interacting via a long-range repulsive potential are considered in D spatial dimensions, where $1 < D \leq 2$. The standard screening picture is found to break down, i.e., the screened effective interaction cannot be treated as instantaneous. For a bare potential which behaves at long distances as $r^{-(2-D)}$ [as $\ln r$ in two dimensions (2D)] and within the random-phase approximation, the retardation effects induce an infrared catastrophe that changes the Fermi liquid into a Luttinger liquid in which the occupation number in momentum space is continuous across the Fermi surface. In 2D, the quantum liquid which we investigate may be called a "$Z = 0$ Fermi liquid" (where Z is the strength of the quasiparticle pole at the Fermi surface) since the electron propagator has an isolated pole with a constant residue that scales to zero as the size of the system increases to infinity. For $1 < D < 2$, the quantum liquid resembles the 1D Luttinger liquid as the single-particle propagator exhibits a branch cut structure. Moreover, we present a ground-state wave function which reproduces the Luttinger-liquid exponent of the momentum distribution near the Fermi surface.

I. INTRODUCTION

Recently, models of highly correlated fermions have attracted considerable attention. Experiments on high-T_c superconductors suggest the possible breakdown of standard Fermi-liquid theory.[1] In three dimensions, our understanding of the metallic state is based on Landau's Fermi-liquid theory.[2] The latter is primarily aimed at the description of the particle-hole continuum, though some so-called Fermi-liquid effects are due to the collective density modes.[3] Theoretically, finding a metallic non-Fermi-liquid fixed point is difficult in space dimension higher than one. Some discussions on the breakdown of Fermi liquid in dimension $D > 1$ have been given recently[4] and various generalizations of bosonization in two and higher dimensions have been proposed.[5] For one-dimensional (1D) systems, the breakdown of Fermi liquid is well documented.[6] In one space dimension, the interaction always produces a branch cut structure in the electron propagator which indicates the breakdown of Fermi liquid.[6-8]

In the present paper, we study a class of models with long-range interactions beyond one dimension. We focus on the one- and two-body correlations of these models in order to investigate the possible breakdown of Fermi liquid. Beyond one space dimension, the very concept of "Luttinger liquid" is unclear despite its appearance in the recent literature. By Luttinger liquid we mean a metallic state at zero temperature with well-defined Fermi-surface singularities, yet with a vanishing quasiparticle spectral weight at the Fermi surface. As a consequence, the occupation number in momentum space n_k is continuous across k_F. Nevertheless, the Fermi surface exists and is marked by the singularities in the first or higher derivatives of n_k with respect to k. In our search for a Luttinger liquid beyond one dimension, we were led to consider fermion interacting via *long-range* potentials in space dimension D, i.e.,

$$U(k) = \frac{g_D}{k^\eta} \quad . \tag{1}$$

For simplicity, we discuss here spinless fermions. A generalization of our results to fermions with internal degrees of freedom is presented in Sec. IV. The questions we want to address in this paper are the following: (i) How long ranged should the interaction (1) be for the momentum distribution to be continuous at the free Fermi momentum? (ii) Does Fermi liquid really break down for long-range interactions of the form (1) beyond 1D? (iii) What liquid do the variational wave functions of the Jastrow type $\Psi_0(\{x\}) = \prod_{i<j} f(x_i - x_j)\Psi_{\rm FS}(\{x\})$ describe? (Here $\Psi_{\rm FS}$ denotes the wave function of the free Fermi sea.)

For $\eta \geq 2$, the interaction (1) causes the density fluctuations to open an energy gap in the small momentum regime. It is generally believed that long-range forces have no dramatic effect on the quasiparticle properties of a Fermi liquid. This belief finds its rationale in the known fact that the static (frequency-independent) screened interaction is necessarily short ranged. We argue below that the standard screening picture does not apply when the particles interact via extremely long-range forces. More precisely, within the random-phase approximation (RPA) and taking into account the frequency dependence of the effective interaction, we show that (1) produces an infrared catastrophe (IRC) provided $\eta \geq 2D - 2$. The IRC effectively destroys the Fermi-liquid fixed point. In the marginal case $\eta = 2D - 2$, it is found that the IRC changes the Fermi liquid into a Luttinger liquid. In particular, the strength of the quasiparticle pole at the Fermi surface Z scales to zero in the limit of an infinite system as

$$Z \propto L^{-\varsigma_D} \quad , \tag{2}$$

where L represents the linear size of the system. The exponent ς_D in (2) is given by

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$$\varsigma_D = \frac{\gamma}{2^D \pi^{D/2} \Gamma\left(\frac{D}{2}\right)} \ , \tag{3}$$

where $\gamma = [\frac{m g_D}{\rho_0}]^{1/2}$ denotes a dimensionless coupling constant. The scaling result for Z is obtained by four different methods, namely (i) a hydrodynamical approach (Sec. II), (ii) a poor man's renormalization analysis (Sec. III), (iii) a random-phase approximation for the electron propagator (Sec. III); and (iv) a wave-function calculation (Sec. IV).

Specifically, our main results are for $1 < D < 2$, we find a Luttinger liquid in the sense that the single-particle propagator has a branch cut structure, i.e., there are no true asymptotic quasiparticle states. These results are not applicable to a physical situation due to the peculiar spacial dimension. Nonetheless, carrying such a calculation is intructive because (i) it gives information on how the spectral function evolves as we increase the spacial dimension from one to two and (ii) it explicitly demonstrates how the coupling between the electron and the collective mode leads to the breakdown of Fermi liquid beyond one dimension. For $D = 2$, the quasiparticles form a Fermi liquid yet their overlap with the bare fermion vanishes in the thermodynamic limit. The transport properties of this quantum liquid are expected to be similar to those of the usual Fermi liquids. The momentum distribution is continuous (weak singularity) at the Fermi momentum. We call such a liquid a "$Z = 0$ Fermi liquid." It is widely believed that a continuous momentum distribution implies a Luttinger-liquid behavior. The Z=0 Fermi liquid provides a counterexample. We establish the connection between a class of Hamiltonians containing long-range forces of the type (1) and the Jastrow-type wave functions which have been considered as good variationnal wave functions for the Hubbard and related models. We develop a technique to calculate one- and two-body correlation functions of the Jastrow-type wave functions and show analytically that the momentum distribution is continuous at the free Fermi momentum.

The plan of the paper is as follows: In Sec. II, we deal with an hydrodynamical description of the IRC. In Sec. III, we evaluate the single-particle propagator near the Fermi surface within RPA and confirm thereby the simple hydrodynamical calculations of the previous section. A renormalization-group (RG) analysis is also given which yields results consistent with those of Sec. II. In Sec. IV, we propose and investigate a ground-state wave function which describes the long-wavelength density fluctuations in the system. We show that this wave function reproduces the Fermi-edge singularity (2). Section V is devoted to a generalization of the previous results to fermions interacting via long-range spin-dependent forces. Section VI contains the concluding remarks.

II. INFRARED CATASTROPHE: HYDRODYNAMICAL CALCULATION

In order to clarify the physical content of the IRC,[1] we shall first take the hydrodynamical point of view. At wavelengths which are large compared to the mean particle spacing, the density fluctuations of the interacting system can be described by a quantized theory of longitudinal sound waves. The classical dynamics of small density fluctuations is governed by the Lagrangian[9]

$$\mathcal{L} = \int d^D x \left[\frac{1}{2}\frac{m}{\rho_0}\mathbf{j}^2(x) - \frac{1}{2}V_0\rho^2(x)\right] - \int d^D x \int d^D y \frac{1}{2}\rho(x)U(x-y)\rho(y) \ , \tag{4}$$

where ρ_0 denotes the average number density, $\mathbf{j}$ is the number vector current, and V_0 is proportional to the inverse of the compressibility of the free fermion gas. We assume a two-body potential $U(x) = g|x|^{\eta - D}$ with Fourier transform

$$U(k) = \frac{g_D}{k^\eta} \ , \tag{5}$$

where $k = |\mathbf{k}|$ and g_D is proportional to the coupling constant g in real space of dimension D. At this stage, the exponent η in (5) denotes an arbitrary real number. Below, we shall determine the condition that η must fulfill in order for (5) to induce an IRC.

The continuity equation, $\dot\rho(x) + \nabla \cdot \mathbf{j} = 0$, which expresses the conservation of matter, reads in momentum space

$$\mathbf{j}_k = i\frac{\widehat{\mathbf{k}}}{k}\dot\rho_k \ , \tag{6}$$

where $\widehat{\mathbf{k}}$ is a unit vector pointing in the direction of $\mathbf{k}$. On substitution of (6) into Eq. (4), we have

$$\mathcal{L} = \frac{1}{2V}\sum_k \frac{m}{\rho_0}\ \frac{1}{k^2}|\dot\rho_k|^2 - \frac{1}{2V}\sum_k |\rho_k|^2\left[V_0 + U(k)\right] \ , \tag{7}$$

where V is the volume of the system. The Lagrangian (7) describes a collection of harmonic oscillators with normal coordinates ρ_k. In the quantized theory, the wave function associated with the zero-point density fluctuations is given by

$$\phi_0\left(\{\rho_k\}\right) = \prod_k \frac{A_k^{1/2}}{\pi^{1/4}} e^{-(1/4)A_k|\rho_k|^2} \ , \tag{8}$$

where

$$A_k = \frac{1}{V}\left[\frac{4m\left[V_0 + U(k)\right]}{\rho_0 k^2}\right]^{1/2} \ . \tag{9}$$

From (7), we can evaluate the density-density correlation function $\Pi_{\text{eff}}(x,t) = \langle T\left\{\rho(x,t)\rho(0,0)\right\}\rangle$ in Fourier space,

$$\Pi_{\text{eff}}(k,\omega) = \frac{1}{\frac{m}{\rho_0}\frac{\omega^2}{k^2} - \left[U(k) + V_0\right]} \ . \tag{10}$$

The equal-time density-density correlation follows readily by contour integration,

$$\langle \rho_k \rho_{-k}\rangle = \frac{1}{2}\left[\frac{\rho_0}{m}\right]\frac{k}{\left[U(k) + V_0\right]^{1/2}} \ . \tag{11}$$

To discuss the IRC, we inject a single electron into the interacting system. Due to screening effects, the electron will be dressed by the charge-density fluctuations. The interaction between the additional particle and the collective density oscillations leads to a term $\Delta\mathcal{L}$ in the Lagrangian (4),

$$\Delta\mathcal{L} = -\int d^D x \ \rho(x) U(x) \ . \qquad (12)$$

The wave function for the density fluctuations is modified accordingly, i.e.,

$$\phi'(\{\rho_k\}) = \phi_0(\{\rho_k - \delta\rho_k\}) \ , \qquad (13)$$

where $\phi'(\{\rho_k\})$ corresponds to displaced harmonic oscillators. In (13), $\delta\rho_k$ represents the displacement of the k mode and is obtained by minimizing the energy with respect to the density fluctuations, i.e.,

$$\delta\rho_k = -\frac{U(k)}{U(k)+V_0} \ . \qquad (14)$$

The wave-function overlap between the states $|\phi'(\{\rho_k\})\rangle$ and $|\phi_0(\{\rho_k\})\rangle$ is easily calculated,

$$\langle\phi_0(\{\rho_k\})|\phi'(\{\rho_k\})\rangle \propto \exp\left(-\frac{1}{4V}\sum_k \left[\frac{m[U(k)+V_0]}{\rho_0 k^2}\right]^{1/2}\left[1+\frac{V_0^2}{U^2(k)}\right]^{-1}\right) \ . \qquad (15)$$

The quasiparticle weight Z at the Fermi surface in the one-electron propagator is $|\langle\phi_0(\{\rho_k\})|\phi'(\{\rho_k\})\rangle|^2$, and on subtitution of (5) into (15), we find near $k=0$

$$Z \propto \exp\left(-\frac{1}{2}\left[\frac{mg_D}{\rho_0}\right]^{1/2}\int_{q_c}^{\Lambda}\frac{dk^D}{(2\pi)^D}\frac{1}{k^{1+\eta/2}}\right) \ . \qquad (16)$$

The ultraviolet cutoff $\Lambda < \rho_0^{1/3}$ in (16) specifies the range of momentum values where the hydrodynamical description should be valid and the infrared cutoff $q_c \sim L^{-1}$, where L represents the linear size of the system. From (16) we conclude that $Z=0$ for $\eta \geq 2D-2$. Clearly, the vanishing of the coherent piece in the propagator is a direct consequence of the IRC. Specializing to the case $\eta = 2D-2$, we infer

$$Z \propto (L\Lambda)^{-\varsigma_D} \ , \qquad (17)$$

where $\varsigma_D = \gamma/2^D\pi^{D/2}\Gamma\left(\frac{D}{2}\right)$ and $\gamma = [\frac{mg_D}{\rho_0}]^{1/2}$ is a dimensionless constant. The scaling behavior of Z in (17) is reminiscent of that occurring in the 1D Luttinger liquids.[1] A crucial observation is that the scaling exponent ς_D is proportional to the square root of the coupling constant g_D, i.e., is *not analytic* in g_D, in contrast to the one-dimensional case where ς_D depends linearly on the square of g_D in the weak-coupling limit. In one dimension, the breakdown of Fermi-liquid theory can be demonstrated by conventional perturbation techniques while in higher dimensions it is generically a *nonperturbative* effect.

We remark that the IRC discussed here should not be confused with the orthogonality catastrophe occurring in the x-ray spectra of metals (see, for example, Mahan's book in Ref. 10 and references therein). In the latter problem, the absorption edge singularity for x-ray transitions stems from the particle-hole continuum of the conducting electrons. However, to observe the power-law singularity at the absorption edge, one requires a vanishing bandwidth for the core electrons and the effect exists even for short-range interactions. In contrast, the IRC that occurs in our model arises from the collective modes provided the forces between the particles are long ranged. Within RPA, we argue below that the particle-hole continuum does not cause an IRC as a consequence of the finiteness of the electron mass.

III. RANDOM-PHASE APPROXIMATION IN DIMENSIONS $1 < D \leq 2$

In Sec. II, we have been concerned with an hydrodynamical description of the IRC. In the present section, we wish to perform a quantum-mechanical calculation which corroborates and completes the previous discussion. The hydrodynamical analysis has provided us with the precise long-wavelength behavior of the potential expected to produce an IRC in dimension D. In one spacial dimension, we find that a short-range interaction is sufficient while in dimension $D=2$ a long-range logarithmic interaction is at least necessary for the IRC to occur. In three dimensions, the required interaction is so long ranged that the potential becomes confining, i.e., increases linearly with distance at long wavelengths. In the present section, we assume a potential of the form

$$U(k) = \frac{g_D}{k^{2D-2}} \ . \qquad (18)$$

We find it useful to vary continuously the dimension of space D from one to two as we can examine in detail how the electron propagator evolves with increasing spatial dimension.

In order to investigate the nature of the time-ordered electron propagator in dimension $1 < D \leq 2$, we shall evaluate the self-energy in the RPA (see Fig. 1),[10] i.e.,

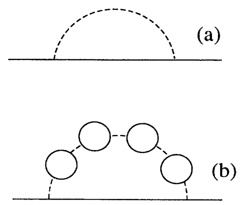

FIG. 1. (a) Hartree-Fock contribution to the self-energy; (b) pure RPA-bubble contribution to the self-energy.

$$\Sigma^{\mathrm{RPA}}(k,\omega) = i\int \frac{d^D q}{(2\pi)^D}\int \frac{d\nu}{2\pi} U(q) G_0(k-q,\omega-\nu) \times \left(1 + \frac{U(q)\Pi_0(q,\nu)}{1-U(q)\Pi_0(q,\nu)}\right), \tag{19}$$

where $G_0(k,\omega)$ denotes the usual free-electron propagator

$$G_0(k,\omega) = \frac{1}{\omega - \epsilon_k + i\delta_k}, \tag{20}$$

with $\epsilon_k = \frac{k^2}{2m} - \frac{k_F^2}{2m}$ and $\delta_k = \delta\,\mathrm{sgn}(k-k_F)$. The polarization $\Pi_0(k,\omega)$ has the familiar form

$$\Pi_0(k,\omega) = -i\int \frac{d^D q}{(2\pi)^D}\int \frac{d\nu}{2\pi} G_0(q+k,\nu+\omega) G_0(q,\nu). \tag{21}$$

The first term in the large parentheses on the right-hand side (rhs) of Eq. (19) [Fig. 1(a)] represents the Hartree-Fock contribution to the self-energy whereas the second term [Fig. 1(b)] is of purely RPA origin.

In two dimensions, a straightforward evaluation of the polarization (21) for $\omega \geq 0$ [with the property $\Pi_0(k,\omega) = \Pi_0(k,-\omega)$] yields

$$\mathrm{Re}\Pi^0(\widetilde{q},\widetilde{\nu},) = \begin{cases} \frac{m}{2\pi\widetilde{q}^2}[\sqrt{(\widetilde{\nu}+\frac{\widetilde{q}^2}{2})^2 - \widetilde{q}^2 M_+^2} + \sqrt{(\widetilde{\nu}-\frac{\widetilde{q}^2}{2})^2 - \widetilde{q}^2 M_-^2} - \widetilde{q}^2], & \widetilde{\nu} \leq \frac{\widetilde{q}^2}{2} \\ \frac{m}{2\pi\widetilde{q}^2}[\sqrt{(\widetilde{\nu}+\frac{\widetilde{q}^2}{2})^2 - \widetilde{q}^2 M_+^2} - \sqrt{(\widetilde{\nu}-\frac{\widetilde{q}^2}{2})^2 - \widetilde{q}^2 M_-^2} - \widetilde{q}^2], & \widetilde{\nu} \geq \frac{\widetilde{q}^2}{2}, \end{cases} \tag{22}$$

$$\mathrm{Im}\Pi^0(\widetilde{q},\widetilde{\nu},) = \begin{cases} \frac{m}{2\pi\widetilde{q}^2}[\sqrt{\widetilde{q}^2 - (\widetilde{\nu}+\frac{\widetilde{q}^2}{2})^2} - \sqrt{\widetilde{q}^2 - (\widetilde{\nu}-\frac{\widetilde{q}^2}{2})^2}], & 0 \leq \widetilde{\nu} \leq \widetilde{q} - \frac{\widetilde{q}^2}{2} \\ -\frac{m}{2\pi\widetilde{q}^2}[\sqrt{\widetilde{q}^2 - (\widetilde{\nu}-\frac{\widetilde{q}^2}{2})^2}], & \left|\frac{\widetilde{q}^2}{2} - \widetilde{q}\right| \leq \widetilde{\nu} \leq \frac{\widetilde{q}^2}{2} + \widetilde{q}, \end{cases} \tag{23}$$

where dimensionless variables $\widetilde{q} = \frac{q}{k_F}$ and $\widetilde{\nu} = \frac{\omega m}{k_F^2}$ have been used and $M_\pm = \min\{|\frac{\widetilde{\nu}}{\widetilde{q}} \pm \frac{\widetilde{q}}{2}; 1\}$. In the limit of long wavelength and finite frequency, the polarization becomes purely real,

$$\Pi_0(q,\omega) \approx \frac{k_F^2 q^2}{4\pi m\omega^2}. \tag{24}$$

The RPA density correlation function is given by

$$\Pi_{\mathrm{eff}}(k,\omega) = \frac{\Pi_0(q)}{1 - U(q)\Pi_0(q)}. \tag{25}$$

From the pole in (25), we infer the plasma frequency

$$\omega_p^2 = \frac{g_D \rho_0}{m}, \tag{26}$$

where $\rho_0 = \frac{k_F^2}{4\pi}$ denotes the 2D fermion density. At this point, it is convenient to introduce the dimensionless coupling constant γ as

$$\gamma = \frac{g_D}{\omega_p} = \left[\frac{m g_D}{\rho_0}\right]^{1/2}. \tag{27}$$

In dimension D, $1 < D < 2$, the imaginary part of the polarization is easily evaluated,

$$\mathrm{Im}\Pi_0(\widetilde{q},\widetilde{\nu}) = -\frac{m}{2^{D-1}\pi^{(D-1)/2}(D-1)k_F^{2-D}\Gamma(\frac{D-1}{2})} \times \frac{1}{\widetilde{q}}[(k_\perp^>)^{D-1} - (k_\perp^<)^{D-1}], \tag{28}$$

where

$$k_\perp^> = \left(\max\left\{0; \left[1 - \left(\frac{\widetilde{\nu}}{\widetilde{q}} - \frac{\widetilde{q}}{2}\right)^2\right]\right\}\right)^{1/2}, \tag{29}$$

$$k_\perp^< = \left(\max\left\{0; \left[1 - \left(\frac{\widetilde{\nu}}{\widetilde{q}} + \frac{\widetilde{q}}{2}\right)^2\right]\right\}\right)^{1/2}. \tag{30}$$

Notice that (28) also applies to one dimension. The fermion density in dimension D is given by

$$\rho_0 = \frac{k_F^D}{2^{D-1}\pi^{D/2} D\Gamma(\frac{D}{2})}. \tag{31}$$

We can calculate $\mathrm{Re}\Pi_0(q,\omega)$ from the definition by expanding (21) for small q to order q^2. The calculation is straightforward, and, at finite frequency, we have

$$\mathrm{Re}\Pi_0(q,\omega) \approx \frac{V_{\mathrm{FS}}}{(2\pi)^D}\frac{q^2}{m\omega^2} = \frac{\rho_0}{m}\frac{q^2}{\omega^2}, \tag{32}$$

where

$$V_{\rm FS} = \frac{2\pi^{D/2}}{\Gamma(\frac{D}{2})}\frac{k_F^D}{D}$$

denotes the volume of the Fermi sphere in dimension D. Equations (32) and (25) imply that the RPA polarization has a pole at $\omega = \omega_p(q) = g_D\gamma^{-1}q^{2-D}$. Therefore, the density mode is gapless for $1 < D < 2$. For $\omega < v_F q$, $\Pi_{\rm eff}(q,\omega)$ has a finite imaginary part coming from the particle-hole continuum. We have checked that, near the Fermi surface, the particle-hole continuum contributes terms that are less singular than those due to the collective density mode. Since we focus in this work primarily on the dominant singularities near the Fermi surface, we shall omit these terms. The effective polarization $\Pi_{\rm eff}(k,\omega)$ in dimension D is then

$$\Pi_{\rm eff}(k,\omega) \approx \frac{1}{\frac{m}{\rho_0}\frac{\omega^2}{q^2} - U(q) + i\delta}. \tag{33}$$

The result (33) agrees with the semiclassical calculation presented in Sec. II. The imaginary part in (33) has been chosen so as to reproduce the correct analytic properties of $\Pi_{\rm eff}(k,\omega)$. The frequency integral in Eq. (19) is easily performed. The Hartree-Fock (HF) contribution [Fig. 1(a)] to the self energy is real, i.e.,

$$\Sigma^{\rm HF}(k,\omega) = -\int \frac{d^Dq}{(2\pi)^D}U(q)n_{k-q}\,, \tag{34}$$

whereas the pure RPA term [Fig. 1(b)] yields

$$\Sigma_0^{\rm RPA}(k,\omega) = \frac{1}{2}\left[\frac{\rho_0}{m}\right]^{1/2}\int\frac{d^Dq}{(2\pi)^D}\frac{q\,[U(q)]^{3/2}}{\omega-\epsilon_{k+q}-{\rm sgn}(\epsilon_{k+q})\{q[\frac{\rho_0}{m}U(q)]^{1/2}-i\delta\}}\,. \tag{35}$$

Before evaluating (35), it is instructive to analyze the propagator within the RG approach. We apply the "poor man's renormalization" method in momentum space. We introduce a ultraviolet cutoff Λ and reduce progressively, together with Λ, the number of degrees of freedom in the system. In this process, the coupling constant g_D does not renormalize. At scale Λ, the response of the real part of the self-energy to a change of scale $-\delta\Lambda$ is found to be

$$\delta\,{\rm Re}\Sigma^{\rm RPA}(k,\omega;\lambda) \approx -\frac{g_D}{2\pi}\frac{\delta\Lambda}{\Lambda^{D-1}}F_D(D-1) - \left\{\frac{\gamma}{2\pi}\frac{\delta\Lambda}{\Lambda}DF_D\left(\frac{D}{2}+1\right)v_F(\Lambda)\right\}\delta k - \left\{\frac{\gamma}{2\pi}\frac{\delta\Lambda}{\Lambda}F_D\left(\frac{D}{2}\right)\right\}\omega\,, \tag{36}$$

where

$$F_D(\alpha) = \frac{1}{2^{D-1}\pi^{(D-1)/2}}\frac{\Gamma\left(\alpha-\frac{D-1}{2}\right)}{\Gamma(\alpha)}\,. \tag{37}$$

In deriving (36), we have linearized the spectrum near the Fermi surface, neglected the curvature of the Fermi surface, and made the assumption that the main contributions to the integral in (35) come from the forward-scattering processes. The first term on the rhs of (36) corresponds to a rescaling of the chemical potential and can be absorbed in the definition of the latter while the two other terms generate a renormalization of the quasiparticle weight $Z(\Lambda)$ and of the Fermi velocity $v_F(\Lambda)$, respectively. Writing the electron propagator at scale Λ as

$$G(k,\omega;\Lambda) = \frac{Z(\Lambda)}{\omega - v_F(\Lambda)\delta k}\,, \tag{38}$$

and comparing with its expression at scale $\Lambda' = \Lambda - \delta\Lambda$, the RG Lie equations for the quasiparticle residue and Fermi velocity follow, respectively,

$$\frac{\partial \ln Z(\Lambda)}{\partial\Lambda} = \frac{\gamma}{2\pi}\frac{F_D(\frac{D}{2})}{\Lambda}\,, \tag{39}$$

$$\frac{\partial \ln v_F(\Lambda)}{\partial\Lambda} = \frac{\gamma}{2\pi}\frac{F_D(\frac{D}{2}) - DF_D(\frac{D}{2}+1)}{\Lambda}\,. \tag{40}$$

Noting that $F_D(\frac{D}{2}) = DF_D(\frac{D}{2}+1)$ and integrating with respect to Λ leads to

$$Z(\Lambda) \approx \Lambda^{(\gamma/2\pi)F_D(D/2)}, \tag{41}$$

$$v_F(\Lambda) \approx {\rm const}\,. \tag{42}$$

From (41) and (42) we conclude that the quasiparticle weight Z renormalizes to zero as $\Lambda \to 0$ with an exponent $\varsigma_D = \frac{\gamma}{2\pi}F_D(\frac{D}{2})$ while the Fermi velocity renormalizes to a constant. Equation (41) is consistent with the hydrodynamical calculations of Sec. II.

We now consider the full RPA self-energy (19). In two space dimensions, the direct evaluation of (35) near the Fermi surface leads to $\Sigma^{\rm RPA}(k,\omega) \approx \frac{\gamma}{4\pi}\Delta\omega_k \ln\left(\frac{q_c}{\Lambda}\right)$, where an infrared cutoff $q_c \sim \frac{1}{L}$ has been used and L denotes the linear size of the system. The occurrence of the logarithmic singularity in the self-energy signals the IRC. The hydrodynamical calculations of Sec. II as well as the RG arguments above suggest that this type of singularity arises from the expansion of an algebraic power in the electron propagator. Promoting the logarithm to a power law, the electron propagator takes the form $G(k,\omega) \approx (\omega-\epsilon_k)^{-1}\left(\frac{q_c}{\Lambda}\right)^{\gamma/4\pi}$. The latter has therefore an isolated pole with a constant residue (independent of frequency and momentum) that scales to zero as $L^{-\gamma/4\pi}$ with increasing size of the system. The quasiparticle is well defined, though its overlap with the bare electron vanishes. Despite the single-particle properties in this model are very different from those encountered in the standard Fermi-liquid theory, we expect the response functions and the transport in the system to be similar to those of the usual Fermi liquids. Therefore, the quantum liquid we find may be called more appropriately a $Z=0$ Fermi liquid.

We now discuss the general case $1 < D \leq 2$. It is convenient to first evaluate the imaginary part of $\Sigma^{\rm RPA}(k,\omega)$, i.e.,

$$\mathrm{Im}\Sigma^{\rm RPA}(k,\omega)=\frac{\pi g_D^2}{2\gamma}\int\frac{d^Dq}{(2\pi)^D}\frac{1}{q^{3D-4}}\{n_{k+q}\delta(\epsilon_{k+q}-\omega-\omega_q)-(1-n_{k+q})\delta(\epsilon_{k+q}-\omega+\omega_q)\}\;, \tag{43}$$

where $\omega_q=\omega_p(q)$ denotes the frequency of the density mode. It is possible to give a rigorous discussion of the region in the (k,ω) plane where the imaginary part of the self-energy is finite. For our purposes, however, it is sufficient to calculate $\mathrm{Im}\Sigma^{\rm RPA}(k,\omega)$ by neglecting the curvature of the Fermi surface as k approaches k_F. Taking into account the most singular contributions in the integral (43) as $\delta k = k-k_F\to 0$ and $\Delta\omega_k=\omega-\epsilon_k\to 0$, we have

$$\begin{aligned}\mathrm{Im}\Sigma^{\rm RPA}(k,\omega)\approx-\frac{\gamma}{2^{D+1}\pi^{D/2-1}(2-D)\Gamma\left(\frac{D}{2}\right)}\Delta\omega_k\{\,&2\Theta[\mathrm{sgn}\delta k\;\Delta\omega_k-\omega_p(q_c)]\\&-\Theta[\mathrm{sgn}\delta k\;\Delta\omega_k-\omega_p(\Lambda)]-\Theta[\mathrm{sgn}\delta k\;\Delta\omega_k-\omega_p(|\delta k|)]\\&-\Theta[-\mathrm{sgn}\delta k\;\Delta\omega_k-\omega_p(\Lambda)]+\Theta[-\mathrm{sgn}\delta k\;\Delta\omega_k-\omega_p(|\delta k|)]\}\;,\end{aligned}\tag{44}$$

where $\omega_p(q)=g_D\gamma^{-1}q^{2-D}$ is the dispersion of the density mode and $\Theta(x)=1$ if $x>0$ and vanishes otherwise. In (44), $\Delta\omega_k=\omega-\epsilon_k$ represents the excess energy with respect to the free fermion of momentum k. When the energy $\Delta\omega_k$ exceeds the lower edge of the density mode frequency $\omega_p(q)|_{q_c\sim L^{-1}}$, the imaginary part of the self-energy is finite and the electron can decay by absorbing (emitting) a collective density excitation of energy $\Delta\omega_k$. In two dimensions, the electrons near the Fermi surface cannot decay since the plasma gap is finite (remember that we neglected the particle-hole continuum).

From (44) and the spectral relation for the retarded self-energy, we can derive $\mathrm{Re}\Sigma^{\rm RPA}(k,\omega)$. Combining real and imaginary parts together yields

$$\begin{aligned}\Sigma^{\rm RPA}(k,\omega)\approx\frac{\gamma}{2^{D+1}\pi^{D/2}(2-D)\Gamma(\frac{D}{2})}\Delta\omega_k\Bigg\{&\ln\left[\frac{\omega_p(q_c)-\mathrm{sgn}\delta k\;\Delta\omega_k}{\omega_p(\Lambda)-\mathrm{sgn}\delta k\;\Delta\omega_k}\right]\\&+\ln\left[\frac{\omega_p(q_c)-\mathrm{sgn}\delta k\;\Delta\omega_k}{\omega_p(\Lambda)+\mathrm{sgn}\delta k\;\Delta\omega_k}\right]+\ln\left[\frac{\omega_p(|\delta k|)+\mathrm{sgn}\delta k\;\Delta\omega_k}{\omega_p(|\delta k|)-\mathrm{sgn}\delta k\;\Delta\omega_k}\right]\Bigg\}\;.\end{aligned}\tag{45}$$

We note that a logarithmic singularity similar to that of (45) (which arises only for $1<D<2$) occurs in the 2D marginal Fermi-liquid theory proposed by Varma *et al.* in the context of high-T_C superconductivity.[11]

When $|\Delta\omega_k|<\omega_p(q_c)=g_D\gamma^{-1}q_c^{2-D}$, $\Sigma^{\rm RPA}(k,\omega)$ simplifies to

$$\Sigma^{\rm RPA}(k,\omega)\approx\varsigma_D\Delta\omega_k\ln\left(\frac{q_c}{\Lambda}\right)\;, \tag{46}$$

where

$$\varsigma_D=\frac{\gamma}{2^D\pi^{D/2}\Gamma\left(\frac{D}{2}\right)}\;. \tag{47}$$

The electron propagator is then

$$G(k,\omega)\approx\frac{1}{\Delta\omega_k\left[1-\varsigma_D\ln\left(\frac{q_c}{\Lambda}\right)\right]}\;. \tag{48}$$

The hydrodynamical considerations of Sec. II as well as the RG arguments in the above discussion indicate that the logarithmic term in (48) may be promoted to an algebraic power, i.e.,

$$G(k,\omega)\propto\frac{\left[\frac{q_c}{\Lambda}\right]^{\varsigma_D}}{\omega-\epsilon_k}. \tag{49}$$

Though not rigorous, we believe here such a procedure to be legitimate. The logarithmic singularity together with its coefficient in (46) are consistent with the results (41) and (42).

When $|\Delta\omega_k|>\omega_p(q_c)=g_D\gamma^{-1}q_c^{2-D}$, the self-energy reads

$$\Sigma^{\rm RPA}(k,\omega)=\frac{\varsigma_D}{2-D}\Delta\omega_k\ln\left(\frac{-\mathrm{sgn}\delta k\;\Delta\omega_k}{g_D\gamma^{-1}\Lambda^{2-D}}\right)\;. \tag{50}$$

In this regime, $\Sigma^{\rm RPA}(k,\omega)$ has a finite imaginary part. The origin of the branch cut singularity in (50) is unclear. For convenience, we write this term in a compact form as an algebraic power and the electron propagator becomes

$$G(k,\omega) \propto \frac{1}{\omega-\epsilon_k}\left[\frac{-\mathrm{sgn}\delta k\ \Delta\omega_k}{g_D\gamma^{-1}\Lambda^{2-D}}\right]^{\varsigma_D/(2-D)}\left[\frac{g_D\gamma^{-1}|\delta k|^{2-D}+\mathrm{sgn}\delta k\ \Delta\omega_k}{g_D\gamma^{-1}|\delta k|^{2-D}-\mathrm{sgn}\delta k\ \Delta\omega_k}\right]^{\varsigma_D/2(2-D)} . \tag{51}$$

We emphasize that when discussing this result, one should keep in mind that (51) is confirmed only up to the first order in ς_D. Clearly, as a function of ω, the electron propagator has three regimes and Eq. (45) describes the crossover between them. We may summarize the results for the propagator near the Fermi surface as follows:

$$G(k,\omega) \propto \begin{cases} \frac{1}{(\omega-\epsilon_k)}\left[\frac{q_c}{\Lambda}\right]^{\varsigma_D}, & |\omega-\epsilon_k| < \omega_p(q_c) \\ \frac{\exp\left(-i\frac{\varsigma_D}{2-D}\pi\Theta[(\omega-\epsilon_k)\eta_k]\right)}{(\omega-\epsilon_k)}\left[\frac{|\omega-\epsilon_k|}{g_D\gamma^{-1}\Lambda^{2-D}}\right]^{\varsigma_D/(2-D)}, & \omega_p(q_c) < |\omega-\epsilon_k| \ll \omega_p(\delta k) \\ \frac{\exp\left(-i\frac{\varsigma_D}{2(2-D)}\pi\right)}{(\omega-\epsilon_k)}\left[\frac{|\omega-\epsilon_k|}{g_D\gamma^{-1}\Lambda^{2-D}}\right]^{\varsigma_D/(2-D)}, & \omega_p(\delta k) \ll |\omega-\epsilon_k| , \end{cases} \tag{52}$$

where $\eta_k = \mathrm{sgn}\delta k$. In dimension $D = 2$, $\omega_c(q_c)$ is finite and, at low energies, the electron propagator (52) has a pole with a residue that scales to zero in the limit of a large system. In marked constrast, for $1 < D < 2$, $G(k,\omega)$ exhibits three different regimes. For $|\Delta\omega_k| \gg \omega_p(q_c)$, the single-particle propagator has a branch cut as in the 1D Luttinger liquids.[1,6,7] It is worth observing that (52) is consistent, in the first regime $[|\omega-\epsilon_k| < \omega_p(q_c)]$, with the scaling arguments of the previous discussion. (See Fig. 2.)

In the same spirit, we evaluate the momentum distribution $n_k = n_k^0 + \delta n_k$ (n_k^0 denotes the free-Fermi-gas momentum distribution) for momenta close to k_F in RPA,

$$\delta n_k = -i\int \frac{d\omega}{2\pi}[G_0(k,\omega)]^2\Sigma^{\mathrm{RPA}}(k,\omega) . \tag{53}$$

Performing the frequency integral, we find

$$\delta n_k = -\frac{[g_D]^2}{2\gamma}n_k^0\int\frac{d^Dq}{(2\pi)^D}\frac{1}{q^{3D-4}}\frac{(1-n_{k+q}^0)}{(\epsilon_{k+q}-\epsilon_k+\omega_q)^2} + \frac{[g_D]^2}{2\gamma}(1-n_k^0)\int\frac{d^Dq}{(2\pi)^D}\frac{1}{q^{3D-4}}\frac{n_{k+q}^0}{(\epsilon_{k+q}-\epsilon_k+\omega_q)^2} . \tag{54}$$

The most singular contributions to the momentum distribution near k_F are due to the forward-scattering processes and, again, can be obtained by assuming a flat Fermi surface, i.e.,

$$\delta n_k = \frac{\gamma}{2^{D+1}\pi^{D/2}\Gamma(\frac{D}{2})}\left[n_k^0\ln\left|\frac{\delta k}{\Lambda}\right| - (1-n_k^0)\ln\left|\frac{\delta k}{\Lambda}\right|\right] , \tag{55}$$

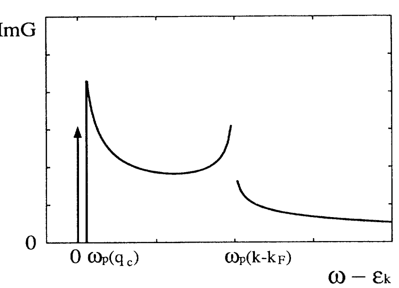

FIG. 2. The spectral function of the electron propagator exhibits three different regimes for $1 < D < 2$ [see Eqs. (51) and (52)]. The first regime $[|\omega-\epsilon_k| < \omega_p(q_c)]$ is a finite-size effect. As the size of the system increases to infinity, the second and third regimes $[|\omega-\epsilon_k| > \omega_p(q_c)]$ survive. At $|\omega-\epsilon_k| = 0$, the spectral function is a δ-function peak with a vanishing weight proportional $L^{-\varsigma_D}$. The peak at $|\omega-\epsilon_k| = \omega_p(k-k_F)$ is due to the resonance with the collective mode. The broad distribution in the spectral function illustrates the incoherence of the electron propagation.

where Λ denotes the ultraviolet cutoff in momentum space and $\delta k = k - k_F$. It is remarkable that, whatever the dimension D, the infrared cutoff q_c cancels in the calculation of δn_k. If we interpret the rhs of (55) as the first term in an expansion in powers of ς_D, we can write

$$n_k = \frac{1}{2} + \frac{1}{2}\left[n_k^0|k-k_F|^{\varsigma_D} - (1-n_k^0)|k-k_F|^{\varsigma_D}\right] . \tag{56}$$

The result (56) shows that the momentum distribution, in the RPA, is a continuous function of k across k_F despite the fact that n_k is not analytic at k_F. This strongly supports the idea that a gas of spinless fermions interacting via logarithmic potential in two dimensions is a Luttinger liquid, though the precise structure of the electron propagator at the Fermi surface differs substantially from that of the 1D Luttinger liquid.[1,6] Again, the power of the algebraic singularity at the Fermi surface matches exactly that of our previous calulations.

IV. ANSATZ FOR THE ASYMPTOTIC FORM OF THE WAVE FUNCTION

Correlated basis functions of the Bijil-Digle-Jastrow type have been studied extensively during the past three decades.[9,10,12,13] In particular, Feenberg has generalized the Jastrow-type trial wave function to deal with fermion systems. This author[9] proposes to write the wave function of a strongly interacting fermion system as a product of single pairwise correlations and a Slater determinant of single-particle states $\Psi_{\mathrm{FS}}(\{x\})$, i.e.,

$$\Psi_0(\{x\}) = \prod_{i<j} f(x_i - x_j)\Psi_{\rm FS}(\{x\}) \ . \tag{57}$$

The Jastrow factor $f(x)$ depends typically on the distance $|x|$ between the particles and is chosen so as to incorporate the correlation effects for the problem under consideration [e.g., in the case of repulsive interactions, one may require $f(|x| = 0) = 0$]. Recently, it was pointed out in Ref. 14 that the Feenberg-Jastrow-type wave function might exhibit some Luttinger-liquid behavior (see also Ref. 15). In this section, we present a Jastrow-type wave function that describes the long-wavelength behavior of the ground state in the Luttinger liquids introduced above.

Consider the wave function (8) associated with the long-wavelength zero-point density fluctuations. We see that its form is dictated by the long-range part of the interaction. Specializing (57) to the potential (18) and using the definition (9), it follows that $A_k \approx \frac{2\gamma}{k^D}$ as $k \to 0$. Therefore, we have

$$\phi_0(\{\rho_k\}) \propto \exp\left(-\frac{\gamma}{2}\sum_k \frac{1}{k^D}|\rho_k|^2\right) \ , \tag{58}$$

where ρ_k represents a c number density fluctuation at wave vector k. For fermions interacting via the long-range potential (18), the long-wavelength density fluctuations in the system are controlled by the Jastrow factor $f(x)$. Therefore, we can determine the large distance form of $f(x)$ from the potential (18). The second quantized version of (57) is

$$\begin{aligned}|\Psi\rangle &= \exp\left(\frac{1}{2}\int d^Dx\, d^Dy\ \ln f(x-y) : \rho_x\rho_y :\right)|\Psi_{\rm FS}\rangle \\ &= \exp\left(\frac{1}{2}\sum_k h_k : \rho_k^\dagger \rho_k :\right)|\Psi_{\rm FS}\rangle \ ,\end{aligned} \tag{59}$$

where h_k is the Fourier transform of $\ln f(x)$ and :: denotes normal order with respect to the Fermi sea. As usual, $\rho_x = c_x^\dagger c_x$ denotes the density operator and ρ_k its Fourier transform.

In the long-wavelength limit, comparison of (58) with (59) suggests that $f(x) = |x|^\lambda$ and so we have

$$h_k = \frac{a_D\lambda}{k^D} \ , \tag{60}$$

where $a_D = 2^{D-1}\pi^{D/2}\Gamma\left(\frac{D}{2}\right)$. The ansatz (59) can be rewritten as

$$|\Psi\rangle = \exp\left(-\frac{\lambda}{2}\sum_k \frac{a_D}{k^D} : \rho_k^\dagger\rho_k :\right)|\Psi_{\rm FS}\rangle \ . \tag{61}$$

Equations (58) and (59) agree provided we identify

$$\gamma = a_D\lambda \ . \tag{62}$$

We now present a more rigorous derivation of (62). Following Feenberg,[9] we evaluate the static form factor $S(k) = \rho_0^{-1}\langle\rho_k\rho_{-k}\rangle$ for the trial wave function (61) as

$$S(k) = \frac{S_0(k)}{1 + C_k S_0(k)} \ , \tag{63}$$

where $S_0(k)$ represents the static form factor for the free Fermi gas and C_k is the Fourier coefficient in the expansion of

$$\lambda \ln|x| = -\frac{1}{2V}\sum_k \frac{C_k}{\rho_0} e^{i\mathbf{k}\cdot\mathbf{x}} \ , \tag{64}$$

i.e.,

$$C_k = 2\lambda\rho_0\frac{a_D}{k^D} \ . \tag{65}$$

At long wavelengths, we have

$$S(k) \approx \frac{1}{C_k} = \frac{k^D}{2\lambda\rho_0 a_D} \ . \tag{66}$$

Alternatively, we can infer $S(k)$ from the results of Sec. II. In the hydrodynamical approach, the long-wavelength behavior of the static form factor follows from (11),

$$S(k) \approx \frac{k^D}{2\gamma\rho_0} \ . \tag{67}$$

Comparison of (66) and (67) immediately leads to (62). The above discussion demonstrates that the wave function (61) correctly reproduces the long-wavelength density fluctuations in the ground state for the interacting system with potential (18).

We were motivated by the above to inquire about the exact analytical structure of the singularities in the momentum distribution n_k near k_F for a wave function of the Feenberg-Jastrow type. The above arguments, an exact solution in 1D (Ref. 16) as well as recent variational Monte Carlo calculations in 2D,[17] suggest that if we choose the two-body Jastrow function $f(x)$ as algebraic function of $|x|$ as in (61), n_k should be continuous at the free Fermi momentum k_F. Nonetheless, we expect the Fermi surface to exist in the sense that n_k has a weak singularity (weaker than a step function) at k_F.

In a remarkable series of papers, Sutherland[16] has solved and discussed the problem of 1D spinless fermions (or bosons) interacting via a r^{-2} potential (more precisely, a periodic version of this potential; a r^{-2} potential with an harmonic restoring force has been considered by Calogero[18]). The ground-state wave function for this problem is given in (57) with $f(x_i - x_j) = |e^{i(2\pi/L)x_i} - e^{i(2\pi/L)x_j}|^\lambda$, where L denotes the size of the system and $\lambda = 0$ for noninteracting fermions (this definition of λ differs by a constant unity from that given by Sutherland[16]). The ground-state wave function (57) is of purely Bijil-Dingle-Jastrow form.

The asymptotic form of the equal-time electron propagator in one dimension can be evaluated by a number of different methods.[6–8] A calculational strategy which does not require knowledge of conformal field theory, Bethe ansatz, or bosonization techniques is the use of the plasma analogy.[19] The electron propagator can be written explicitly as the partition function of a one-component plasma on a one-dimensional ring interacting with external charges. Noting that the one-component plasma behaves like a metal and assuming a continuous distribution of charges on the ring, we can evaluate the energy required to add an external charge to the system by the method of images. Exploiting the holomorphic properties of the norm of the wave function, the large distance form of the propagator follows as

$$G(x,0^-) \propto \frac{\sin(k_F x)}{|x|^{1+2\alpha}} , \tag{68}$$

where $\alpha = \frac{\lambda^2}{4(1+\lambda)}$. The result (68) implies an algebraic singularity in the momentum distribution at k_F as seen by simple Fourier transform, i.e.,

$$n_k \approx \frac{1}{2} + \text{const}|k-k_F|^{2\alpha} . \tag{69}$$

To conclude the discussion of the one-dimensional case, we note that the Calogero-Sutherland wave function reproduces the correct long-wavelength behavior of the electron propagator for the Luttinger liquids in one dimension, including models solved by Bethe ansatz. The Bethe ansatz wave function is notorious for its complexity. The simple Calogero-Sutherland wave function can be viewed as a universal wave function for all 1D spinless Luttinger liquids.[20] For Luttinger liquids with spin, a new wave function (which is not of, but related to, the Jastrow form) is proposed in Sec. V.

In order to extend the analysis to higher dimensions, where no such methods as Bethe ansatz or bozonisation are available, we start with the wave function (61) in second quantization. Let us first take a naive point of view and evaluate the equal-time electron propagator n_k,

$$n_k = \frac{\langle \Psi_{\rm FS}| e^{\lambda A} c_k^\dagger c_k e^{\lambda A}|\Psi_{\rm FS}\rangle}{\langle \Psi_{\rm FS}|e^{2\lambda A}|\Psi_{\rm FS}\rangle} , \tag{70}$$

in a small λ expansion. In (70), A is given by

$$\begin{aligned} A &= \frac{1}{2}\int d^Dx\, d^Dy\, \ln|x-y| : \rho_x\rho_y : \\ &= \frac{1}{2}\sum_k \frac{a_D}{k^D} : \rho_k^\dagger \rho_k : \quad . \end{aligned} \tag{71}$$

Applying Wick's theorem, a straightforward yet tedious calculation to order λ^2 yields[10]

$$\begin{aligned} n_k = n_k^0 &+ \frac{\lambda^2}{V^2}\sum_{k_1,k_2} \hat{f}^2(k_1) n^0_{k+k_1} n^0_{k_2-k_1}(1-n^0_{k_2}) \\ &+ \frac{\lambda^2}{V^2}\sum_{k_1,k_2} \hat f(k_1)\hat f(k_2)[n^0_{k+k_1+k_2}(1-n^0_{k+k_1})(1-n^0_{k+k_2}) - (1-n^0_{k+k_1+k_2})n^0_{k+k_1}n^0_{k+k_2}(1-n^0_k)] + O(\lambda^3) , \end{aligned} \tag{72}$$

where $\hat f(k) = \int d^Dx\, e^{ikx}\ln|x|$ and V denotes the volume of the system. In deriving (72), we have used $f(x) = f(-x) = f^*(x)$. Notice that in (72) there is no term linear in λ, i.e., the first corrections to n_k are of order λ^2. Specializing to one dimension, Eq. (72) reduces near k_F to (with $\delta k = k - k_F$)

$$\begin{aligned} n_k = \frac{1}{2}\left[1+\lambda^2 H(k)\right] &+ \frac{n_k^0}{2}\left[1+\frac{\lambda^2}{2}\ln|\delta k|\right] \\ &- \frac{(1-n_k^0)}{2}\left[1+\frac{\lambda^2}{2}\ln|\delta k|\right] , \end{aligned} \tag{73}$$

where only the most singular terms have been kept and $H(k)$ is finite near k_F. Equation (73) is consistent with the result (68) obtained from the exact solution. Despite the fact that the perturbation theory works well in one dimension, we find that it fails beyond $D=1$.

In higher dimensions, we need to perform a nonperturbative calculation. Our strategy is quite general and applies to arbitrary dimension, so we implicitly assume that the dimension of space is $D>1$. The crucial point is the obsevation that the equal-time propagator $iG(x,0^-)$, i.e.,

$$iG(x-y,0^-) = -\frac{\langle \Psi_{\rm FS}| e^{\lambda A} c_y^\dagger c_x e^{\lambda A}|\Psi_{\rm FS}\rangle}{\langle \Psi_{\rm FS}|e^{2\lambda A}|\Psi_{\rm FS}\rangle} , \tag{74}$$

can be written in terms of a τ-ordered Green's function as

$$iG(x-y,t=0^-) = i\mathcal{G}(x-y,\tau=0^-) . \tag{75}$$

By definition, the τ-ordered Green's function is
$i\mathcal{G}(x-y,\tau)$

$$= \frac{\left\langle \Psi_{\rm FS} \middle| T_\tau\left\{ c_x(\tau)c_y^\dagger(0)\exp\left(\int_{-\lambda}^{\lambda} d\tau A(\tau)\right)\right\}\middle| \Psi_{\rm FS}\right\rangle}{\left\langle \Psi_{\rm FS}\middle| T_\tau\left\{\exp\left(\int_{-\lambda}^{\lambda} d\tau A(\tau)\right)\right\}\middle|\Psi_{\rm FS}\right\rangle} , \tag{76}$$

where T_τ denotes a τ ordering operator. In this formulation, all the operators in the "interaction picture" are τ independent since the "free Hamiltonian" H_0 vanishes identically. This property guaranties dramatic simplifications in the calculations. The "interaction" part of the Hamiltonian is also τ independent, i.e.,

$$\mathcal{H}_1(\tau) = iA = \frac{1}{2}\int d^Dx\, d^Dx'\, \mathcal{U}(x,x') : \rho_x\rho_{x'} : , \tag{77}$$

where $\mathcal{U}(x,x') = i\ln|x-x'|$. In Fourier space, the "potential" is

$$\mathcal{U}(q) = -i\frac{a_D}{q^D} , \tag{78}$$

where $a_D = 2^{D-1}\pi^{D/2}\Gamma\left(\frac{D}{2}\right)$. Since we are interested in the long-wavelength regime, we shall sum over the leading divergences at small momenta. This procedure is equivalent to a RPA calculation and we must evaluate the diagrams of Figs. 1(a) and 1(b).

The τ-ordered propagator for the "noninteracting" system is defined by

$$i\mathcal{G}_0(x-y,\tau) = \langle \Psi_{\rm FS}| T_\tau \left\{ c_x(\tau) c_y^\dagger(0) \right\} |\Psi_{\rm FS}\rangle \, . \tag{79}$$

With $c_x(\tau) = c_x(0)$, we have

$$i\mathcal{G}_0(x-y,\tau) = \begin{cases} i\mathcal{G}_0(x-y,0^+), & \tau > 0 \\ i\mathcal{G}_0(x-y,0^-), & \tau < 0 \, . \end{cases} \tag{80}$$

Equation (80) implies that the zero-order "polarization"

$$\mathcal{P}_0(x,\tau;x',0) = -i\mathcal{G}_0(x,0^+)\mathcal{G}_0(x',0^-) \tag{81}$$

is τ independent. It is therefore convenient to work in the mixed representation (q,τ) where only the space component has been Fourier transformed. Notice that in this representation

$$-i\mathcal{G}_0(k,\tau) = \begin{cases} -(1-n_k^0), & \tau > 0 \\ n_k^0, & \tau < 0. \end{cases} \tag{82}$$

The lowest order polarization bubble is

$$\mathcal{P}_0(q) = -i \int \frac{d^D k}{(2\pi)^D} \mathcal{G}_0(k,0^+)\mathcal{G}_0(k+q,0^-) \, . \tag{83}$$

The τ integrals in the bubble diagrams are trivial, for all integrands are τ independent. The effective polarization is then a geometric series, i.e.,

$$\mathcal{P}_{\rm eff}(q) = \frac{\mathcal{P}_0(q)}{1-2\lambda\mathcal{U}(q)\mathcal{P}_0(q)} \, . \tag{84}$$

The Hartree-Fock contribution [Fig. 1(a)] to the τ-ordered propagator is easily seen to vanish whereas the pure RPA contribution [Fig. 1(b)] leads to

$$i\delta\mathcal{G}(k,\tau) = -\int \frac{dq^D}{(2\pi)^D} \mathcal{U}^2(q)\mathcal{P}_{\rm eff}(q) \int_{-\lambda}^{\lambda} d\tau_1 \int_{-\lambda}^{\lambda} d\tau_2 \mathcal{G}_0(k,\tau-\tau_2)\mathcal{G}_0(k-q,\tau_2-\tau_1)\mathcal{G}_0(k,\tau_1) \, . \tag{85}$$

Because of the form of the free propagator (82), the τ integrals are elementary and we find

$$i\delta\mathcal{G}(k,0^-) = -i\lambda^2 \int \frac{dq^D}{(2\pi)^D} \mathcal{U}^2(q)\mathcal{P}_{\rm eff}(q) \left[n_k^0(1-n_{k-q}^0) - n_{k-q}^0(1-n_k^0) \right] \, . \tag{86}$$

For our purposes, it is sufficient to evaluate (86) with the approximation

$$\mathcal{P}_{\rm eff}(q) \approx -\frac{1}{2\lambda\mathcal{U}(q)} \, , \tag{87}$$

which is valid for $q \to 0$. Since we are interested in the most singular terms only, the integral in (86) can be performed by assuming a flat Fermi surface and expanding around k_F. In dimension D, the dominant singularity of the momentum distribution near k_F is then

$$\delta n_k = -i\mathcal{G}(k,0^-) \approx \frac{a_D\lambda}{2^{D+1}\pi^{D/2}\Gamma(\frac{D}{2})} \times \left[n_k^0 \ln|\delta k| - (1-n_k^0)\ln|\delta k| \right] \, . \tag{88}$$

The result (88) is interesting in two respects: (i) it is nonperturbative in λ; in the naive perturbation approach discussed prior to this analysis, we saw that the first corrections to n_k are of order λ^2, a result which in view of (88) is clearly false. (ii) Since we can identify the dimensionless constant introduced in Sec. III as $\gamma = a_D\lambda$ [compare with (62)], Eq. (88) agrees with (55), i.e., the trial wave function (61) reproduces the Luttinger-liquid exponent of the momentum distribution at k_F for a gas of spinless fermions interacting via the long-range potential (18).

The wave function (61) (with appropriate changes required for $D = 3$) was in fact proposed as early as 1954 in a famous paper on the RPA by Bohm and Pines. In Ref. 21, the authors show that the plasma mode of the three-dimensional electron gas with long-range Coulomb forces leads to the wave function (61) in the high-density limit. Note that the wave function (61) describes a Wigner crystal in the limit of large interaction strength $\lambda \gg 1$. Yet remembering (62), this corresponds to the regime of low densities while our analysis focuses on the high-density limit, i.e. $\rho_0 \gg mg_D$.

V. FERMIONS WITH SPIN

This section is devoted to a generalization of the results of the previous discussions to fermions with internal degrees of freedom (spin, isospin, and so forth). Since the algebraic manipulations offer no difficulty other than those encountered so far, we content ourselves with a brief account of the calculations. In the following, we investigate a model with charge and spin-dependent interaction

$$H_1 = \frac{1}{2} \int d^D x \, d^D x' \, \psi_\lambda^\dagger(x)\psi_\mu^\dagger(x') U(x,x')_{\lambda\lambda',\mu\mu'} \times \psi_{\mu'}(x')\psi_{\lambda'}(x) \, , \tag{89}$$

where the two-body potential $U(x,x')$ is SU(2) symmetric,

$$U(x,x')_{\lambda\lambda',\mu\mu'} = U_c(x-x')\delta_{\lambda\lambda'}\delta_{\mu\mu'} + U_s(x-x')\sigma_{\lambda\lambda'}\cdot\sigma_{\mu\mu'} . \qquad (90)$$

We assume $U_c(q)$ and $U_s(q)$ to be long range, i.e.,

$$U_c(q) = \frac{g_{Dc}}{q^{2D-2}}, \quad U_s(q) = \frac{g_{Ds}}{q^{2D-2}} . \qquad (91)$$

For simplicity, we focus here on spin $s = \frac{1}{2}$ fermions. The extension to fermions with arbitrary spin s interacting via a long-range SU($2s+1$) symmetric potential is presented at the end of this section. We employ the standard notation for the polarization tensor[10]

$$\Pi(x_1,t_1;x_2,t_2)_{\alpha\beta,\gamma\delta} = -i\langle T\left\{\psi^\dagger_\alpha(x_1,t_1)\psi_\beta(x_1,t_1)\psi^\dagger_\gamma(x_2,t_2)\psi_\delta(x_2,t_2)\right\}\rangle . \qquad (92)$$

The polarization in RPA follows from solving

$$\Pi_{\rm eff}(q,\omega)_{\alpha\beta,\gamma\delta} = \Pi_0(q,\omega)_{\alpha\beta,\gamma\delta} + \Pi_0(q,\omega)_{\alpha\beta,\mu\nu}U_{\rm eff}(q,\omega)_{\mu\nu,\rho\eta}\Pi_0(q,\omega)_{\rho\eta,\gamma\delta} , \qquad (93)$$

where the effective interaction is readily evaluated as

$$U_{\rm eff}(q,\omega)_{\alpha\beta,\rho\tau} = \frac{U_c(q)}{1-2\Pi_0(q,\omega)U_c(q)}\delta_{\alpha\beta}\delta_{\rho\tau} + \frac{U_s(q)}{1-2\Pi_0(q,\omega)U_s(q)}\sigma_{\alpha\beta}\cdot\sigma_{\rho\tau} . \qquad (94)$$

The zero-order polarization is given by

$$\Pi_0(q,\omega)_{\alpha\beta,\gamma\delta} = \Pi_0(q,\omega)\delta_{\alpha\delta}\delta_{\beta\gamma} , \qquad (95)$$

with the definition

$$\Pi_0(k,\omega) = -i\int\frac{d^Dq}{(2\pi)^D}\int\frac{d\epsilon}{2\pi}G_0(q+k,\epsilon+\omega)G_0(q,\epsilon) . \qquad (96)$$

A straightforward calculation for $\Pi_{\rm eff}(q,\omega)$ yields

$$\Pi_{\rm eff}(q,\omega)_{\alpha\beta,\gamma\delta} = \frac{\Pi_0(q\omega)}{1-2\Pi_0(q,\omega)U_c(q)}\delta_{\alpha\delta}\delta_{\beta\gamma} + \frac{\Pi_0(q,\omega)}{1-2\Pi_0(q,\omega)U_s(q)}\sigma_{\alpha\beta}\sigma_{\gamma\delta} . \qquad (97)$$

By appropriate contractions of the tensor indices, we obtain the density correlation functions as

$$v\Pi^c_{\rm eff}(q,\omega) = \Pi_{\rm eff}(q,\omega)_{\alpha\alpha,\gamma\gamma} = \frac{2\Pi_0(q,\omega)}{1-2\Pi_0(q,\omega)U_c(q)} \qquad (98)$$

for charge and

$$\Pi^s_{\rm eff}(q,\omega) = \frac{1}{4}\sigma^i_{\alpha\beta}\,\Pi_{\rm eff}(q,\omega)_{\alpha\beta,\gamma\delta}\,\sigma^i_{\gamma\delta} = \frac{3}{4}\frac{2\Pi_0(q,\omega)}{1-2\Pi_0(q,\omega)U_s(q)} \qquad (99)$$

for spin. Equation (98) together with (99) implies that, at small q, the charge and the spin density fluctuations, respectively, have the following dispersion:

$$\omega_c(q) = g_{Dc}\gamma_c^{-1}q^{2-D} , \qquad (100)$$

$$\omega_s(q) = g_{Ds}\gamma_s^{-1}q^{2-D} , \qquad (101)$$

where $\gamma_j = [\frac{mg_{Dj}}{2\rho_0}]^{1/2}$ for $j=c,s$. Note that below two dimensions (100) and (101) lead to different dispersion laws for charge and spin density modes while at $D=2$ the density fluctuations acquire a finite gap, both in the charge and spin channels. The model Hamiltonian with interaction (90) has a certain "spin-charge separation" in the sense that the collective density modes propagate with different dispersions. However, we emphasize that the situation here is very different from that in one dimension.[1]

Integrating (98) and (99) over frequency, we infer the static charge-charge and spin-spin correlation functions, respectively,

$$\langle\rho_k\rho_{-k}\rangle = i\int\frac{d\omega}{2\pi}\Pi^c_{\rm eff}(k,\omega) = \frac{k^D}{2\gamma_c} , \qquad (102)$$

$$\langle S_kS_{-k}\rangle = i\int\frac{d\omega}{2\pi}\Pi^s_{\rm eff}(k,\omega) = \frac{3k^D}{8\gamma_s} . \qquad (103)$$

Equations (102) and (103) are accurate at long wavelengths only. The Hartree-Fock contribution to the self-energy is frequency independent and diagonal in spin space, i.e.,

$$\Sigma^{\rm HF}(k,\omega)_{\alpha\beta} = -\delta_{\alpha\beta}\int\frac{d^Dq}{(2\pi)^D}\left[U_c(q)+3U_s(q)\right]n_{k-q} . \qquad (104)$$

The pure RPA contribution is

$$\Sigma^{\rm RPA}_0(k,\omega)_{\alpha\beta} = i\int\frac{d^Dq}{(2\pi)^D}\int\frac{d\epsilon}{2\pi}U(q)_{\sigma\sigma',\alpha\nu}\Pi_{\rm eff}(q,\omega)_{\rho'\rho,\sigma\sigma'}U(q)_{\mu\beta',\rho'\rho}G_0(k-q,\omega-\epsilon)_{\nu\mu} , \qquad (105)$$

where $G_0(k,\omega)_{\alpha\beta} = \delta_{\alpha\beta}G_0(k,\omega)$. On substitution of (97) into (105) it is readily seen that the full self-energy is diagonal in spin space. Performing the frequency integral in (105), we have

$$\Sigma_0^{\rm RPA}(k,\omega)_{\alpha\beta} = \delta_{\alpha\beta}1/2\left[\frac{2\rho_0}{m}\right]^{1/2}\int\frac{d^Dq}{(2\pi)^D}\left[\frac{q\,[U_c(q)]^{3/2}\,(q)}{\omega-\epsilon_{k+q}-\mathrm{sgn}(\epsilon_{k+q})\left(q\left[\frac{2\rho_0}{m}U_c(q)\right]^{1/2}-i\delta\right)} + \frac{3\,q\,[U_s(q)]^{3/2}}{\omega-\epsilon_{k+q}-\mathrm{sgn}(\epsilon_{k+q})\left(q\left[\frac{2\rho_0}{m}U_s(q)\right]^{1/2}-i\delta\right)}\right]. \quad (106)$$

Applying the same scheme of approximations as in Sec. III and keeping only the most singular terms, we arrive at

$$\Sigma^{\rm RPA}(k,\omega)_{\alpha\beta} \approx \delta_{\alpha\beta}\frac{\Delta\omega_k}{2^{D+1}\pi^{D/2}(2-D)\Gamma(\frac{D}{2})} \times\sum_{a=c,s}\gamma_a\left\{\ln\left[\frac{\omega_{pa}(q_c)-\mathrm{sgn}\delta k\;\Delta\omega_k}{\omega_{pa}(\Lambda)-\mathrm{sgn}\delta k\;\Delta\omega_k}\right]+\ln\left[\frac{\omega_{pa}(q_c)-\mathrm{sgn}\delta k\;\Delta\omega_k}{\omega_{pa}(\Lambda)+\mathrm{sgn}\delta k\;\Delta\omega_k}\right]+\ln\left[\frac{\omega_{pa}(|\delta k|)+\mathrm{sgn}\delta k\;\Delta\omega_k}{\omega_{pa}(|\delta k|)-\mathrm{sgn}\delta k\;\Delta\omega_k}\right]\right\}, \quad (107)$$

where $\omega_{pa}(q) = g_{Da}\gamma_a^{-1}q^{2-D}$ for $a = c,s$. When $|\Delta\omega_k| < \omega_{pc}(q_c)$ and $|\Delta\omega_k| < \omega_{ps}(q_c)$, the self-energy can be approximated by

$$\Sigma^{\rm RPA}(k,\omega) \approx \varsigma_D^{cs}\Delta\omega_k\ln\left(\frac{q_c}{\Lambda}\right), \quad (108)$$

where

$$\varsigma_D^{cs} = \frac{\gamma_c+\gamma_s}{2^D\pi^{D/2}\Gamma(\frac{D}{2})}. \quad (109)$$

The electron propagator in this regime takes the form

$$G(k,\omega)_{\alpha\beta} \approx \frac{1}{\Delta\omega_k\left[1-\varsigma_D^{cs}\ln\left(\frac{q_c}{\Lambda}\right)\right]}. \quad (110)$$

The result (110) is similar to that of Sec. III for the spinless fermions [compare with (48)]. In two dimensions, the electron propagator exhibits an isolated pole with a spectral weight that scales to zero as the size of the system increases. For $1 < D < 2$, there are various regimes as in the spinless case and there is a region where the propagator shows no quasiparticle structure but a branch cut reminiscent of that occuring in the 1D Luttinger liquids.[1,6] The spectral function has three singularities at $\omega = v_F(k-k_F)$, $v_F(k-k_F)+\omega_{pc}(|k-k_F|)$, and $v_F(k-k_F)+\omega_{ps}(|k-k_F|)$, while in one dimension there are two singularities located at $\omega = v_c(k-k_F)$, $v_s(k-k_F)$ [$v_{c,s}$ denote the velocities of the charge (c) and spin (s) density fluctuations].

The momentum distribution $n_k = n_k^0 + \delta n_k$ is determined from the trace of the self-energy

$$\delta n_k = -i\int\frac{d\omega}{2\pi}\left[G_0(k,\omega)\right]^2\Sigma_{\alpha\alpha}^{\rm RPA}(k,\omega). \quad (111)$$

Subsituting (106) into (111), we conclude

$$\delta n_k \approx \frac{2\left[\gamma_c+3\gamma_s\right]}{2^{D+1}\pi^{D/2}\Gamma\left(\frac{D}{2}\right)}\left[n_k^0\ln\left|\frac{\delta k}{\Lambda}\right|-(1-n_k^0)\ln\left|\frac{\delta k}{\Lambda}\right|\right]. \quad (112)$$

Following the ideas of Sec. IV, we now consider a wave function related to, but not of, the Jastrow type to describe the long-wavelength charge and spin density fluctuations, i.e.,

$$|\Psi\rangle = \exp\left(\frac{\lambda}{2}\int d^Dx\,d^Dy\,\ln|x-y|:\rho_x\rho_y+\frac{\Delta}{4}S_xS_y:\right)\times|\Psi_{\rm FS}\rangle, \quad (113)$$

where S_x denotes the usual spin operator $S_x = \frac{1}{2}\psi_\alpha^\dagger(x)\sigma_{\alpha\beta}\psi_\beta(x)$. The ansatz (113) is clearly a spin singlet ($S^z = S = 0$). The strategy of Sec. IV applies without essential modifications. Again, it is convenient to introduce a τ ordered Green's function,

$$i\mathcal{G}(x-y,\tau)_{\alpha\beta} = \frac{\left\langle\Psi_{\rm FS}\left|T_\tau\left\{\psi_\alpha(x,\tau)\psi_\beta^\dagger(y,0)\exp\left(\int_{-\lambda}^{\lambda}d\tau A(\tau)\right)\right\}\right|\Psi_{\rm FS}\right\rangle}{\left\langle\Psi_{\rm FS}\left|T_\tau\left\{\exp\left(\int_{-\lambda}^{\lambda}d\tau A(\tau)\right)\right\}\right|\Psi_{\rm FS}\right\rangle}, \quad (114)$$

where

$$A = \frac{1}{2}\int d^Dx\,d^Dx'\,\ln|x-x'|:\rho_x\rho_x'+\frac{\Delta}{4}S_xS_{x'}:. \quad (115)$$

We evaluate (114) within a RPA-like calculation. The λ-dependent polarization $\mathcal{P}_{\rm eff}(q,\lambda)_{\alpha\beta,\gamma\delta}$ has a structure analogous to that of (97) and leads to the charge

$$\mathcal{P}_{\rm eff}^c(q,\lambda) = \mathcal{P}_{\rm eff}(q,\lambda)_{\alpha\alpha,\gamma\gamma} = \frac{2\mathcal{P}_0(q)}{1-4\lambda\mathcal{P}_0(q)\mathcal{U}_c(q)} \quad (116)$$

and spin

$$\mathcal{P}^s_{\text{eff}}(q,\lambda) = \frac{1}{4}\,\sigma^i_{\alpha\beta}\,\mathcal{P}_{\text{eff}}(q,\lambda)_{\alpha\beta,\gamma\delta}\,\sigma^i_{\gamma\delta} = \frac{3}{4}\,\frac{2\mathcal{P}_0(q)}{1-4\lambda\mathcal{P}_0(q)\mathcal{U}_s(q)}\ , \tag{117}$$

density correlation functions at finite interaction strength λ. In (116) and (117), we have

$$\mathcal{U}_c(q) = -i\frac{a_D}{q^D}\ , \tag{118}$$

$$\mathcal{U}_s(q) = -i\frac{a_D\Delta}{q^D}\ , \tag{119}$$

and a_D is given in Sec. IV. The long-wavelength behavior of the static charge and spin correlation functions follows from (116) and (117) as

$$\langle\rho_k\rho_{-k}\rangle = i\mathcal{P}^c_{\text{eff}}(k,\lambda) = \frac{k^D}{2\lambda a_D}\ , \tag{120}$$

$$\langle S_k S_{-k}\rangle = i\mathcal{P}^s_{\text{eff}}(k,\lambda) = \frac{3k^D}{8\lambda a_D\Delta}\ . \tag{121}$$

On comparison of (102) and (103) with (120) and (121), we infer the relation between the interaction characterized by γ_c and γ_s and the ansatz wave function parametrized by λ and $\Delta\lambda$,

$$\gamma_c = a_D\lambda\ , \tag{122}$$

$$\gamma_s = a_D\Delta\lambda\ . \tag{123}$$

Equations (122) and (123) imply that

$$g_{Ds} = \Delta^2 g_{Dc}\ . \tag{124}$$

The mean occupation number distribution in momentum space is determined from

$$\delta n_k = -i\,\mathrm{Tr}\left[\delta\mathcal{G}(k,0^-)\right]\ . \tag{125}$$

Substituting the results of our calculations into (125), we find

$$\delta n_k \approx \frac{2a_D\lambda\,[1+3\Delta]}{2^{D+1}\pi^{D/2}\Gamma(\frac{D}{2})}\left[n^0_k\ln|\delta k| - (1-n^0_k)\ln|\delta k|\right]. \tag{126}$$

Equation (126) is seen to agree with the RPA result (112) on account of (122) and (123). We note that, for $D=2$, the introduction of internal degrees of freedom in the problem does not modify in an essential way the picture obtained in the spinless case.

We now extend these results to fermions of spin s interacting via long-range spin-dependent forces. Consider (89) with the SU$(2s+1)$ symmetric potential

$$U(q)_{\alpha\beta,\gamma\delta} = U_c(q)\delta_{\alpha\beta}\delta_{\gamma\delta} + (2s+1)U_s(q)\sigma^{ab}_{\alpha\beta}\sigma^{ba}_{\gamma\delta}\ , \tag{127}$$

where the convention of summing over repeated indices is adopted. In (127), $\sigma^{ab}_{\alpha\beta}$, with $\alpha,\beta = 1,\dots,2s+1$ and $a,b=1,\dots,2s+1$, represents a set of traceless tensors (in both the upper and lower indices)

$$\sigma^{ab}_{\alpha\beta} = \delta_{a\alpha}\delta_{\beta b} - \frac{1}{2s+1}\delta_{ab}\delta_{\alpha\beta}\ . \tag{128}$$

The greek indices refer to the "matrix" indices while the latin ones parametrize the "basis" set. Notice that there are $4s(s+1)$ linearly independent traceless tensors and so the matrices σ^{ab} are not all linearly independent. To verify that (127) is invariant under the special unitary group, we note that

$$\sigma^{ab}_{\alpha\beta}\sigma^{ba}_{\delta\gamma} = \delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{1}{2s+1}\delta_{\alpha\beta}\delta_{\gamma\delta}\ , \tag{129}$$

where $P_{\alpha\beta,\gamma\delta} = \delta_{\alpha\delta}\delta_{\beta\gamma}$ is the permutation operator acting on the tensor-product space of two interacting particles. But $P_{\alpha\beta,\gamma\delta}$ is clearly connected to the fundamental representations of SU$(2s+1)$ and can be expressed as a polynomial of degree $2s$ in the scalar product $\mathbf{S}(x)\cdot\mathbf{S}(y)$, where $\mathbf{S}(x)$ denotes the usual spin vector operator.

Within RPA, the long-wavelength behavior of the charge and spin tensor correlation functions are readily found,

$$\langle\rho_k\rho_{-k}\rangle \approx \frac{k^D}{2\gamma_c}\ , \tag{130}$$

$$\langle S^{ab}_k S^{ba}_{-k}\rangle \approx \frac{4s(s+1)}{2s+1}\,\frac{k^D}{2\gamma_s}\ , \tag{131}$$

where $\gamma_j = [\frac{m g_{Dj}}{(2s+1)\rho_0}]^{1/2}$ for $j=c,s$. In (131), S^{ab}_k represents the Fourier transform of the tensor spin operators $S^{ab}_x = \psi^\dagger_\alpha(x)\sigma^{ab}_{\alpha\beta}\psi_\beta(x)$.

For the momentum distribution deviation δn_k, we find

$$\delta n_k = \frac{(2s+1)\,[\gamma_c + 4s(s+1)\gamma_s]}{2^{D+1}\pi^{D/2}\Gamma\left(\frac{D}{2}\right)} \times\left[n^0_k\ln\left|\frac{\delta k}{\Lambda}\right| - (1-n^0_k)\ln\left|\frac{\delta k}{\Lambda}\right|\right]\ . \tag{132}$$

The full propagator near the Fermi surface can be evaluated. For brevity, we omit the corresponding discussion here, for no new elements appear in the analysis. We proceed with the ansatz for the wave function associated with the density fluctuations,

$$|\Psi\rangle = \exp\left(\frac{\lambda}{2}\int d^Dx\,d^Dy\,\ln|x-y| : \rho_x\rho_y + \Delta S^{ab}_x S^{ba}_y :\right) \times|\Psi_{\text{FS}}\rangle\ . \tag{133}$$

The state (133) transforms as the singlet representation of SU$(2s+1)$. Following the strategy developed earlier, we calculate the small momentum behavior of the charge

$$\langle\rho_k\rho_{-k}\rangle \approx \frac{k^D}{2\lambda a_D} \tag{134}$$

and the spin tensor

$$\langle S^{ab}_k S^{ba}_{-k}\rangle \approx \frac{4s(s+1)}{2s+1}\,\frac{k^D}{2\lambda a_D\Delta} \tag{135}$$

correlation functions. Comparing (130) and (131) with (134) and (135) leads to

$$\gamma_c = \lambda a_D \ , \tag{136}$$

$$\gamma_s = \lambda a_D \Delta \ , \tag{137}$$

as in the spin $s = \frac{1}{2}$ case. This in turn requires (124) for the ansatz (133) to be applicable to the model Hamiltonian with interaction (127). The momentum occupation number distribution function is evaluated similarly,

$$\delta n_k \approx \frac{(2s+1)a_D\lambda\left[1+4s(s+1)\Delta\right]}{2^{D+1}\pi^{D/2}\Gamma(\frac{D}{2})} \times \left[n_k^0 \ln|\delta k| - (1-n_k^0)\ln|\delta k|\right] . \tag{138}$$

Taking into account (136) and (137), Eqs. (132) and (138) agree. Our results provide strong support to the idea that, in two dimensions, a system of fermions with spin s interacting via SU$(2s+1)$-symmetric long-range forces is indeed a liquid of Luttinger type.

VI. CONCLUSIONS

In this work, we have demonstratively given evidence for the existence of Luttinger liquids in space dimension higher than one. We have invoked the infrared catastrophe as an intuitive guide to the breakdown of the Fermi-liquid fixed point.[1] In two space dimensions, fermions (with and without spin) interacting via long-range logarithmic forces form a liquid of Luttinger type (more precisely a $Z = 0$ Fermi liquid). The single-particle propagator has an isolated pole with a residue that scales to zero as the size of the system increases. For $1 < D < 2$, an interaction of the form $gr^{-(2-D)}$ changes the Fermi liquid into a Luttinger liquid. In this case, the electron propagator is fully incoherent and exhibits a branch cut structure reminiscent of that occurring in the 1D Luttinger liquids.[1,6] The spectral function has singularities at $\omega = v_F(k-k_F)$ and $\omega = v_F(k-k_F)+\omega_p(k-k_F)$ in the spinless case and at $\omega = v_F(k-k_F)$, $v_F(k-k_F)+\omega_{pc}(k-k_F)$, and $v_F(k-k_F)+\omega_{ps}(k-k_F)$ for spin $s=\frac{1}{2}$ electrons. The singularities at the charge $\epsilon_k+\omega_{pc}$ and spin $\epsilon_k+\omega_{ps}$ fluctuation energies also occur in the spectral function of the 1D Luttinger liquid. However, the typical Fermi-liquid dispersion singularity at $\omega = v_F(k-k_F)$ is absent in one space dimension. The Luttinger-liquid exponent ς_D that controls the electron propagator and the momentum distribution singularity near the Fermi surface has been obtained by four different methods for $1 < D \leq 2$.

As a matter of fact, our results are limited by the approximations we employ and terms may arise which have escaped our scrutinity. Nonetheless, our results provide a mechanism for the breakdown of Fermi liquid beyond one space dimension. The models we have been investigating all involve long-range forces and may be considered, at first sight, as irrelevant to real physical systems. We share the conviction that this not so, however. For example, in two spatial dimensions, $U(k) = \frac{g_2}{k^2}$ represents a logarithmic potential which corresponds to the longitudinal part of a two-dimensional gauge interaction. The IRC discussed here might have some consequences for the two-dimensional gauge theories of high-T_c superconductors as well as for the quantum Hall state at filling fraction $\nu = \frac{1}{2}$.[22,23]

In addition to addressing the question of the existence of Luttinger liquids in $D > 1$, we propose an ansatz wave function that correctly reproduces the Luttinger-liquid exponent of the momentum distribution singularity at the Fermi surface as well as the long-wavelength behavior of charge and spin correlation functions in the ground state of the interacting system. Starting from this wave function, we have deviced a strategy to evaluate static properties. The method applies to both fermions with and without spin. In the case of fermions interacting via long-range spin-dependent forces, the ansatz we adopt is not of, but related to, the Jastrow-type wave function. Our approach should be useful for dealing with three-dimensional strongly correlated sytems such as nuclear matter and liquid helium.

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