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Giant magnetophonon oscillations of the tunneling relaxation rate in double quantum wells

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Electron tunneling relaxation, accompanied by spontaneous emission of optical phonons, is considered in a double quantum well subject to a perpendicular magnetic field. When the energy splitting Δ_T, between the (lowest) levels in the left and right wells, is larger than the optical phonon energy, electrons can emit optical phonons and tunnel from the upper set of Landau levels to the lower one. As a result the relaxation rate exhibits giant magnetophonon oscillations whose amplitude at strong magnetic fields is larger than the zero-field value by one to two orders of magnitude. Moreover, the rate shows oscillations as function of Δ_T. Collision broadening and phonon dispersion are taken into account.

Tunneling relaxation of photoexcited electrons in coupled double quantum wells has been studied actively in past years using time-resolved luminescence techniques.[1-6] When the splitting Δ_T of the (lowest) electron levels of the left and right well is larger than the optical phonon energy $\hbar\omega_0$ spontaneous optical phonon emission is a dominant relaxation mechanism at low temperatures and (electron) concentrations. Both numerical[7] and analytical[8] descriptions of such a process have been reported for zero magnetic field B. However, when B is present and perpendicular to the interfaces, the motion parallel to the latter becomes quantized, the energy spectrum becomes discrete (Landau levels), and the tunneling rate may change considerably.

In this paper we report giant magnetophonon oscillations of the tunneling relaxation rate due to spontaneous emission of optical phonons in double quantum wells when a transverse field B is present. The rate oscillates as function of B and of the splitting Δ_T; the latter can be controlled by the application of a transverse voltage. To our knowledge this effect has not been treated. It happens for $\Delta_T > \hbar\omega_0$ and is the result of electron transitions from the upper set of Landau levels (labeled by n) to the lower one (labeled by n') as shown schematically in Fig. 1. Electron transitions occur whenever the condition

$$\Delta_T - (n' - n)\hbar\omega_c = \hbar\omega_0 \tag{1}$$

is satisfied, ω_c being the cyclotron frequency. This resonance condition is different than that for the usual three- or two-dimensional systems by the term Δ_T. In what follows we evaluate the relaxation rate associated with emission of longitudinal-optical phonons as function of the magnetic field and the splitting Δ_T which depends sensitively on the parameters of the structure. In the calculations we take into account both broadening of the Landau levels and phonon dispersion. For simplicity we consider only bulk phonons and assume that localized modes[9] give qualitatively similar results.

For transitions between the state $|\alpha)$ and the states $|\alpha')$, with corresponding energies E_α and $E_{\alpha'}$, due to spontaneous emission of optical phonons, the relevant rate ν_α is given[8] by

$$\nu_\alpha = \frac{2\pi}{\hbar}\sum_{\alpha'\mathbf{q}} |C(q)|^2 |(\alpha|e^{i\mathbf{q}\cdot\mathbf{r}}|\alpha')|^2\, \delta(E_\alpha - E_{\alpha'} - \hbar\omega_q), \tag{2}$$

where ω_q and $\mathbf{q}$ are the frequency and wave vector of the emitted phonons, and $C(q)$ the electron-phonon interaction matrix element for bulk phonons.

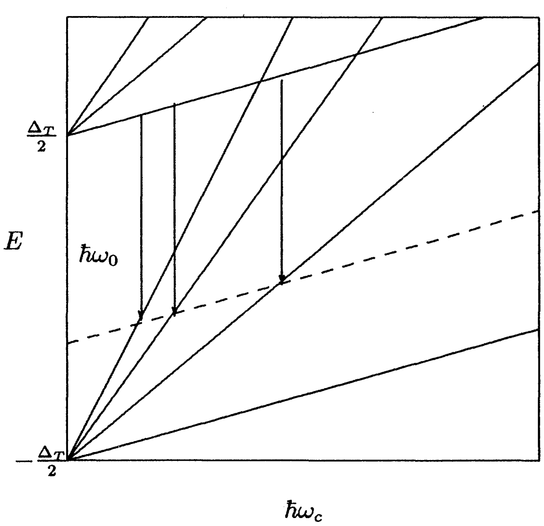

FIG. 1. Scheme of tunneling transitions between two sets of Landau levels under spontaneous emission of optical phonons.

The eigenvalues E_α and eigenfunctions $|\alpha)$ have been obtained earlier[10] assuming that $\hbar\omega_c$ is much smaller than the heterojunction band offsets. Then the magnetic field B, taken along the z direction, does not change the tunneling matrix element T. If only the lowest two levels of the isolated left (l) and right (r) wells are occupied the eigenstates are $|\alpha) \equiv |\pm)|nk_y\rangle$ and the $|\pm)$ states are described by the linear combinations $|+) = N\{|l\rangle + [2T/(\Delta_T + \Delta)]|r\rangle\}$, and $|-) = N\{|r\rangle - [2T/(\Delta_T+\Delta)]|l\rangle\}$, where $N = \sqrt{(\Delta_T + \Delta)/2\Delta_T}$ is the normalization factor and $|nk_y\rangle$ the eigenfunction in the (x, y) plane. Δ_T and Δ are the energy splitting between the $|+)$ and $|-)$ states with and without tunneling, respectively. The energy spectrum is

$$E_{\pm n} = (n + 1/2)\hbar\omega_c \pm \Delta_T/2, \quad \Delta_T = \sqrt{\Delta + 4T^2}. \tag{3}$$

We now consider transitions from the state $|\alpha) = |+)|nk_y\rangle$ to the states $|\alpha') = |-)|n'k_y'\rangle$, cf. Fig. 1. Using the results of Refs. 7, 8, and 10, we obtain

$$|(+nk_y|e^{i\mathbf{q}\cdot\mathbf{r}}| - n'k_y')|^2 = \left(\frac{T}{\Delta}\right)^2 \Phi(q_z) \times|\langle nk_y|e^{iq_x x}|n'k_y'\rangle|^2 \times\delta_{k_y,k_y'+q_y}, \tag{4}$$

where

$$\Phi(q_z) = \chi^2(q_z d_l)+\chi^2(q_z d_r)-2\cos(q_z\Delta z)\chi(q_z d_l)\chi(q_z d_r), \tag{5}$$

and $\chi(a) = (2/a)\sin(a/2)/[1-a^2/4\pi^2]$; d_l and d_r are the widths of the left and right well, respectively, and Δz the distance between the centers of the wells. We assume that the barrier penetration by the $|l\rangle$ and $|r\rangle$ orbitals is small and that intrawell scattering is symmetric in both wells. This ensures that the tunneling superposition of the $|l\rangle$ and $|r\rangle$ states remains valid and is guaranteed when the condition $\Delta_T \simeq \Delta > \hbar\omega_0 \gg T$.

We now substitute Eq. (4) in Eq. (2), integrate out the dependence on k_y' using relative coordinates, and treat collision broadening phenomenologically by replacing the δ function by a Gaussian function $\delta_\Gamma(\ )$ of width Γ assumed to be independent of the indices n and n'. The result is

$$\nu_{+n} = \frac{2\pi}{\hbar} \sum_{n'k_y q_x q_z} |C(q)|^2|\langle n, k_y/2|e^{iq_x x}|n', -k_y/2\rangle|^2 \times\Phi(q_z)\delta_\Gamma(\Delta_T + E_n - E_{n'} - \hbar\omega_q)|_{q_y=k_y}. \tag{6}$$

We have taken into account phonon dispersion using $\omega_q \approx \omega_0(1 - \bar{a}^2q^2)$, where $\bar{a}$ is of the order of the lattice constant. Now the magnetophonon oscillations occur for $\Delta_T > \hbar\omega_0 > \hbar\omega_c$, i.e., when the magnetic length ℓ_c, which determines the characteristic wave vectors q_x and k_y, is larger than $\bar{d} = \sqrt{d_l d_r}$, which determines the scale of the wave vectors along the z direction. As a result $|C(q)|^2$ and the phonon energy $\hbar\omega_q$ depend weakly on q_x and k_y; negleting these dependences, we can carry out the integrations over q_z and q_x, k_y independently. Further, we use the identity[11]

$$\sum_{q_y q_x} |\langle n, k_y|e^{iq_x x}|n', -q_y\rangle|^2 = \frac{L^2}{2\pi\ell_c^2}, \tag{7}$$

where L^2 is the area and introduce the dimensionless variable $g = q_z\bar{d}$. The result for ν_{+n} is

$$\nu_{+n} = \nu_0 \sum_{n'} \mathcal{F}(\Delta_T + E_n - E_n' - \hbar\omega_0), \tag{8}$$

where ν_0 is the relaxation rate[8] in the absence of the magnetic field, and

$$\mathcal{F}(E) = \frac{\hbar\omega_c}{\Gamma} \int_{-\infty}^{\infty} \frac{dg}{g^2} e^{-[E+\hbar\omega_0(\bar{a}/\bar{d})^2 g^2]^2/\Gamma^2} \Phi(g/\bar{d}). \tag{9}$$

The shape of the oscillations is determined by $\mathcal{F}(E)$. A simple expression for $\mathcal{F}(\ )$ is obtained when $\Gamma \gg \Gamma_0 = (\bar{a}/\bar{d})^2\hbar\omega_0$ and collision broadening (CB) is dominant or when $\Gamma \ll \Gamma_0$ and dispersion broadening (DB) is dominant. In the first case (CB) we may neglect the phonon dispersion in the Gaussian function; the result for the integral over g is $A = 4.47$. In the second case, the Gaussian function is replaced by a δ function. These two asymptotic results are

$$\mathcal{F}(E) \approx A(\hbar\omega_0/\Gamma)e^{-E^2/\Gamma^2} \quad \text{(CB)} \tag{10}$$

and

$$\mathcal{F}(E) \approx \theta(-E)\ (2\sqrt{\pi}\hbar\omega_c/E)\ (\bar{a}/\bar{d})\ \Phi\left(\frac{\sqrt{|E|/\hbar\omega_0}}{\bar{a}}\right) \quad \text{(DB)}. \tag{11}$$

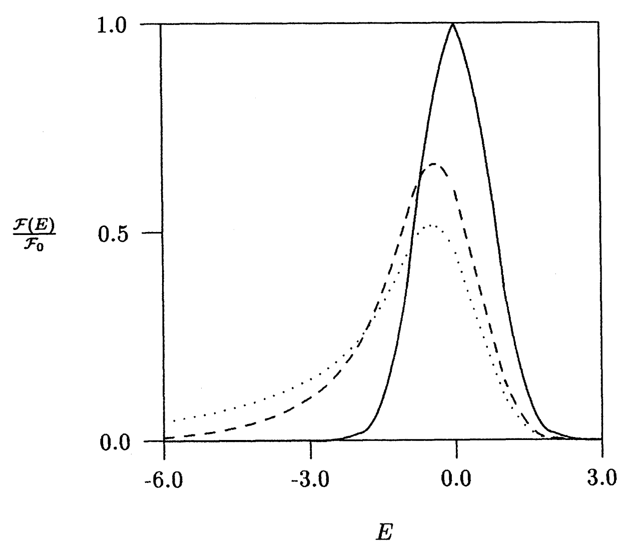

FIG. 2. Shape of the magnetophonon tunneling peak as function of energy for different values of the phonon dispersion parameter $b = (\hbar\omega_0/\Gamma)(\bar{a}/\bar{d})^2$. The solid, dashed, and dotted curves correspond to $b = 0, 1.5$, and 3, respectively.

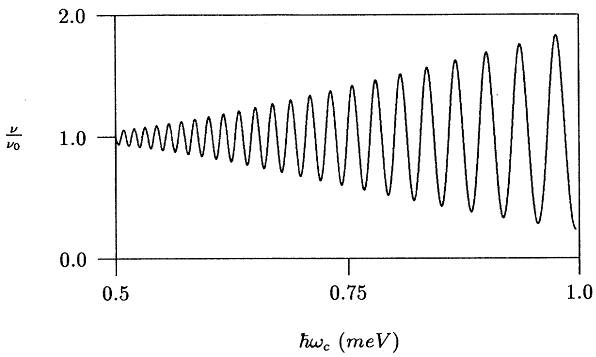

FIG. 3. Magnetic-field dependence of relaxation frequency vs $\hbar\omega_c$ when collision broadening is dominant.

For the intermediate case, $\mathcal{F}(E)$, evaluated numerically, is shown in Fig. 2 as function of E/Γ and of $b = (\bar{a}/\bar{d})^2(\hbar\omega_0/\Gamma)$. The results are normalized to the value $\mathcal{F}_0$ of the integral for $E = b = 0$ and the values of g are given in the caption. As can be seen when the phonon dispersion is significant the magnetophonon peaks are asymmetric and $\mathcal{F}(E)$ decreases[12] very slowly for negative E.

The magnetophonon oscillations are shown in Figs. 3–5. We consider only the CB case with $\Gamma = 0.3$ meV for a GaAs-Al_xGa_{1-x}As heterostructure with $d_l = 65$ Å, $d_r = 90$ Å, and $d = 50$ Å. The oscillations shown describe transitions from the lowest Landau level of the upper set $(+, n = 0)$ to the Landau levels of the lower set and $\nu \equiv \nu_{+0}$. This relaxation rate ν is normalized to ν_0; under experimental conditions,[3–5] ν_0^{-1} is of the order of 100×10^{-12} sec and in agreement with calculations.[7,8] Figure 3 shows the relaxation rate as function of $\hbar\omega_c$ in the range 0.5–1 meV and Fig. 4 in the range 1–5 meV. As can be seen the oscillations are well resolved in both weak and strong magnetic fields B whereas usually[11] they are observed only at strong B. For larger values of $\hbar\omega_c$ and the same parameters, there are four more peaks at $\hbar\omega_c = 5.8, 7.8, 11.7$, and 23.4 meV; the corresponding ν/ν_0 values are 10.2, 14.1, 21.2, 42.3. Comparing these values with those of Figs. 3 and 4 for weak B fields we see that the oscillation amplitude can become one to two orders of magnitude larger at strong fields. Finally, in Fig. 5 we show the relaxation frequency, normalized to ν_0, as function of the splitting $\tilde{\Delta}_T = \Delta_T - \hbar\omega_0$ for fixed magnetic field. Changing Δ_T can be easily achieved by the application of a transverse voltage. As can be seen, when $\hbar\omega_c$ increases the distance between the peaks increases and the first peak occurs at a higher value of $\hbar\omega_c$.

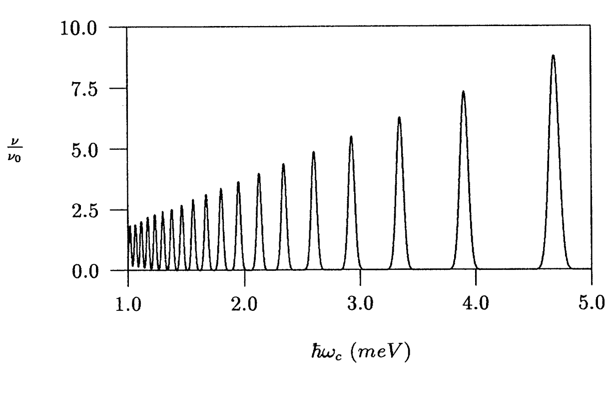

FIG. 4. Same as in Fig. 3 for a different range of $\hbar\omega_c$.

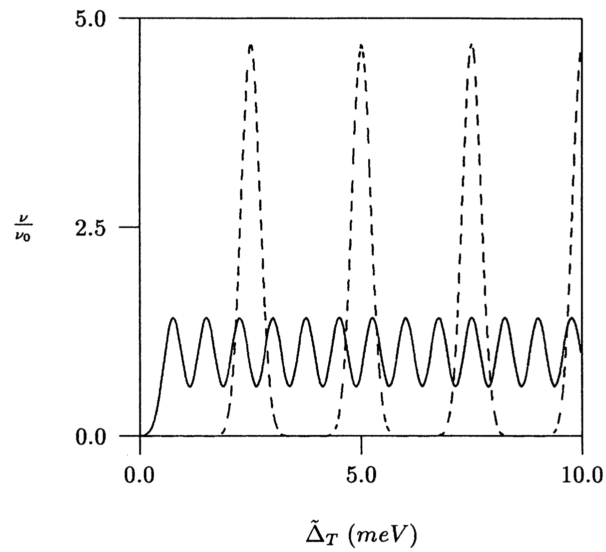

FIG. 5. Relaxation frecuency as function of the splitting $\tilde{\Delta} = \Delta - \hbar\omega_0$ for fixed $\hbar\omega_c$. The solid and dashed curves are for $\hbar\omega_c = 0.75$ meV and $\hbar\omega_c = 2.5$ meV, respectively.

In conclusion, we have shown that the relaxation frequency in double quantum wells, subject to a transverse magnetic field B, shows magnetophonon oscillations as function of B and of the splitting Δ_T due to spontaneous emission of optical phonons by electrons that tunnel from the upper set of Landau levels to the lower one. The resonance condition is given by Eq. (1). A more detailed treatment of this effect, especially with regard to the level broadening, would involve a self-consistent calculation of the energy spectrum. However, for small barrier pentration that we assumed, our results will not change qualitatively. The process considered here will be supressed if condition (1) is not satisfied. In such a case elastic relaxation channels, such as scattering by impurities and/or acoustic phonons, may be important.

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