Experimental evidence for finite-width edge channels in integer and fractional quantum Hall effects

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We report our transport study of a narrow and low-density two-dimensional electron system (2DES) defined by a pair of split gates, in the integer (IQHE) and fractional quantum Hall effect (FQHE) regimes. The gate-bias-dependent breakdown on the high and low magnetic field side of the IQHE and FQHE diagonal resistance minima is studied. In the IQHE case, edge states of width comparable to the magnetic length, in the single-electron Landau-level picture, cannot explain the observed breakdown data and much wider edge channels (wide band of edge states) are needed. These wide edge channels are interpreted as the compressible regions when the 2DES near the sample boundary phase separates into compressible and incompressible regions. The same breakdown behavior is observed in the fractional quantum Hall state and is interpreted as a similar phase separation in the FQHE regime.

I. INTRODUCTION

The theoretical concept of edge states[1,2] is widely used in describing many interesting properties of the quantum Hall effect in ultrasmall structures fabricated on two-dimensional electron systems (2DES). Two most interesting examples are the conductance oscillations in the zero-dimensional quantum dot structure[3] and the resistance fluctuations in narrow Hall bars.[4] The former come from the resonant tunneling between two edge states through the zero-dimensional states in the dot, and the latter come from the resonant backscattering of the edge states through the bound states around a potential hill in the channel.[5]

There have also been experiments demonstrating the existence of edge states. The first was the quantization of the diagonal resistance (R_{xx}) measured across two regions of different filling factors in series by Haug *et al.* and Washburn *et al.*[6] This quantization in R_{xx} is derived from the Buttiker-Landauer formula[2] by counting the number of edge states penetrating the gated region. When the number of edge states in the ungated region is ν and the number of edge states in the gated region is ν', R_{xx} is quantized to $(h/e^2)[1/\nu'-1/\nu]$. Soon after, van Wees *et al.*[7] made controllable current and voltage probes, using quantum point contacts,[8] which can selectively populate and detect edge states. Using this technique, they demonstrated that the value of the Hall conductance G_H measured across a bulk 2DES was determined by the number of edge states reaching the voltage and current probes, rather than by the filling factor in the bulk of the sample.

The position of the edge state is determined by the intersection between the Landau-level energy and the chemical potential (μ). Its width is simply the spatial extent of the wave function at the Landau level, and is order of the magnetic length $l_0=(\hbar c/eB)^{1/2}$. To explain most of the experimental observations mentioned above, this simple physical picture of an edge state is enough. However, the picture requires an abrupt density change within l_0 wherever μ interests a Landau level. In an actual sample, the density decreases gradually near the boundary at $B=0$. There is a cost of electrostatic energy to deform the gradual density profile at $B=0$ into a steplike one at $B\neq0$. On the other hand, a partially filled Landau level at μ can accommodate extra charge without changing its energy (perfect screening[9]). The system can therefore avoid this cost of electrostatic energy by having a spatially wider region of the Landau level at μ. As a result, each edge state widens into a band of edge states, or an edge channel with a finite width, and the density profile becomes gradual rather than the steplike one. The edge channel, where μ aligns with the partially filled Landau level, is compressible in that an extra electron can be added without changing the size of the region or costing additional energy. The region where μ resides in between two Landau levels is at a constant density (or integer filling factor) and is incompressible. To add an extra electron in this region costs finite energy because it has to go to the next higher Landau level. As a result, there is a series of alternating compressible and incompressible regions as the sample boundary is approached.

Using the experimental technique used by van Wees *et al.*[7] described above, Kouwenhoven *et al.*[10] showed that G_H could be quantized at $\frac{2}{3}(e^2/h)$ when the filling factor of the bulk was at 1. Chang and Cunningham[11] measured the quantization in R_{xx} when the gated region in the middle of the Hall bar was at the fractional state of $\frac{1}{3}$ or $\frac{2}{3}$ and the ungated region was at $\frac{2}{3}$ or 1. In both experiments G_H or R_{xx} can be calculated using a generalized Buttiker-Landauer formula when the fractional edge states are included. MacDonald[12] showed that there could be many branches of fractional edge states for a single partially filled Landau level. At the same time with MacDonald, Beenakker[13] and Chang[14] have proposed a physical model in which the system phase separates into compressible and incompressible regions near the sample boundary. Their ideas are equivalent to what was described in the previous paragraph for the integer quantum Hall effect (IQHE), except for the fact that the incompressible regions occur at the special fractional filling factors where the energy gap exists. Recently, Chklovskii, Shklovskii, and Glazman (CSG) (Ref. 15) quantitatively studied the phase separation near the boundary of 2DES and analytically calculated the width of the compressible and incompressible regions.

In this paper, we wish to report our study of the R_{xx} across a narrow and low-density 2DES defined by a pair of split gates, in the IQHE and the fractional quantum Hall effect (FQHE) regimes. We systematically measured and analyzed the breakdown on both the high magnetic-field (B) side and the low B side of the IQHE and FQHE R_{xx} minima as a function of the gate bias (V_g). The breakdown on the high B side of the minima is a very sensitive function of V_g, while the breakdown on the low B side of the minima is independent of V_g. In the IQHE regime, we find that the backscattering between two opposite simple edge states with widths $\sim l_0$ cannot explain our data, and that bands of edge states much wider than l_0, or edge channels of finite widths are needed. To account for these finite-width edge channels, phase separation into compressible and incompressible regions alternating with each other is needed, and these finite-width edge channels correspond to the compressible regions. The gate-bias-dependent breakdown on the high B side of the R_{xx} minimum happens when the two compressible edge channels, one on each side of the sample, are brought into contact. In the FQHE regime we observe the same breakdown behavior as in the IQHE regime, suggesting a similar phase separation of the 2DES and the presence of compressible edge channels with finite widths.

II. SAMPLE AND MEASUREMENT

The device is designed to realize a narrow 2DES whose width is controllable by negatively biasing a pair of split gates. The material is a low-density ($n_{2D}=6.46\times10^{10}$ cm^{-2}) and high mobility ($\mu=10^6$ cm^2/V sec) single interface GaAs/AlGaAs heterojunction. The heterointerface is 3100 Å, and the donor layer is 1900 Å from the surface. We wet etched $\sim$3000-Å deep before evaporating Cr/Au as the metal split gates. The wet etching depletes the 2D electrons underneath the split-gate region, and as a result, a narrow 2DES is already defined at zero bias and biasing the split gate negatively immediately squeezes the narrow 2DES. More detailed processing steps are given in Ref. 16. Due to a lateral etching of $\sim$3000 Å, the actual width of the device is ~ 2.4 μm. The estimated conducting width of the device is <1 μm in the bias range where we are interested,[17] as a result of the very wide depletion layers expected in a low-density sample.

Our sample was placed in the mixing chamber of a He^3-He^4 dilution refrigerator and the magnetotransport measurement was made using a AC lock-in technique with a frequency 13.77 Hz. To prevent heating, excitation currents less than 1 nA were used. The inset of Fig. 1(a) shows a schematic of the sample geometry. Several Ohmic contacts were made on both sides of the split gates to characterize the wide part of the sample simultaneously with the narrow 2DES, defined by the split gates. We used contacts 1 and 4 for current and 6 and 5 for voltage to measure the R_{xx} across the narrow 2DES (R_n). The R_{xx} of the wide 2DES was obtained by measuring the voltage across contacts 2 and 3 (R_w).

III. DATA

Figure 1 shows the magnetotransport data of both the wide and the narrow 2DES at 20 mK. R_w is shown in Fig. 1(a), and R_n's taken at three different gate biases, $V_g=0$, -0.25, and -0.5 V, are shown in Fig. 1(b). At the integer filling factors indicated by the arrows in Fig. 1(a), R_w shows vanishing resistances in wide ranges of B. For the $\nu=1$ minimum, we draw dashed vertical lines indicating breakdown of the dissipationless transport occurring on the high B side of the minimum at $B_H=2.93$ T and on the low B side at $B_L=2.26$ T. It is clear from the dashed lines extended into Fig. 1(b) that

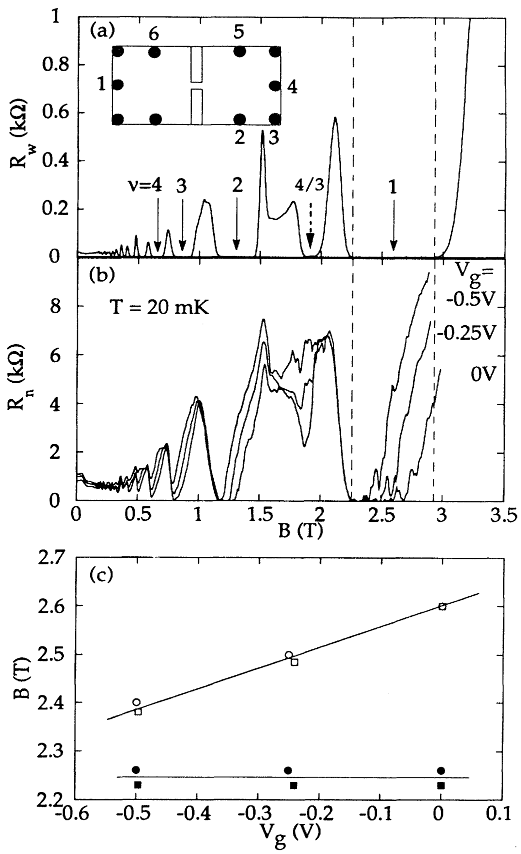

FIG. 1. (a) The diagonal resistance across the wide 2DES, R_w. It shows vanishing resistances at the filling factors indicated by the vertical arrows. The two dashed lines indicate the breakdown on the high and low B sides of the minimum at $\nu=1$. (b) The diagonal resistance across the narrow 2DES, R_n, at $V_g=0$, 0.25, and -0.5 V, respectively. As can be clearly seen from the dashed lines extended from (a) the breakdown on the high B side shifts to lower B at a larger $-V_g$ while the breakdown on the low B side is independent of V_g. (c) V_g dependence of the breakdown B fields; B_H on the high B side (open symbols) and B_L on the low B side (filled symbols), for two different cooldowns.

the most outstanding effect of the confinement is the shifting of the breakdown on the high B side to smaller B fields: $B_H=2.60$, 2.50, and 2.40 T in the R_n at $V_g=0$, -0.25, and -0.5, V, respectively. On the other hand, the breakdown on the low B side of the minimum occurs at the same B as that of R_w, and is independent of V_g. In Fig. 1(c) we plot the B_H and B_L of the $\nu=1$ minimum as a function of V_g from two different cooldowns of the sample. Filled symbols are B_L's and the empty symbols are B_H's. It is clear that B_H decreases as $-V_g$ increases, but B_L is constant, independent of V_g.

This shift of the breakdown field on the high B side as a function of V_g is also apparent in the other IQHE minima. At $\nu=2$ the rise of R_n on the low B side of the minimum of all the traces in Fig. 1(b) remains the same, but on the high B side it moves to lower B, as $-V_g$ increases from 0 to 0.5 V. The same behavior persists for $\nu=3$ and 4, even though the minima do not go close to zero. More importantly, this breakdown behavior does not change on different cooldowns, as is shown in Fig. 1(c). Thermal cycling of the sample changes only the small size fluctuations, but not the overall line shape of R_n, especially the breakdown B field positions. Furthermore, the same breakdown behavior is observed in a narrow sample of the same geometry, fabricated on a high mobility p-type wafer with a 2D hole density 7.3×10^{10} cm^{-2}. This gate-bias dependence of the breakdown is generic to narrow samples of low disorder, low-density 2D systems; it does not depend on the details of the microscopic potential fluctuations in the sample.

IV. ANALYSIS

Current understanding of the IQHE in a narrow 2DES is in terms of the simple edge states of width $\sim l_0$. Figure 2(a) is an illustration of the edge-state picture, showing, on the left-hand side three Landau levels and the simple edge states (closed circles), defined by their intersections with μ. On the right-hand side, a sketch of R_n is given to show the relation between R_n and the position of μ relative to the Landau levels. When μ lies between the bottoms of two neighboring Landau levels (the middle dashed line) the current carrying states are the left and right edge states. As long as there is negligible tunneling between the left and right edge states, R_n vanishes at the IQHE minimum. At the moment when electrons start populating the next higher Landau level and the bottom of the level aligns with μ (the top dashed line), the IQHE breaks down, and this breakdown corresponds to the onset of R_n observed at B_L. On the other hand, the breakdown at B_H occurs when μ is sufficiently close to the bottom of the lower Landau level that tunneling between the left and right edge states is large enough to give rise to a measurable R_n (the bottom dashed line).

Qualitatively, the observed gate-bias dependence of B_H can be explained within the model described above. In Fig. 2(b) we sketch the Landau levels in a gated sample. Here, the dashed parabolas are the Landau levels at $V_g=0$ V and the solid parabolas are the Landau levels at $V_g=-0.5$ V. The fact that B_L is observed to be independent of V_g suggests that the bottom of the Landau level remains at the same energy in our V_g range. The application of a negative V_g only increases the curvature of the parabolas and thus decreases the spatial separation between the left and right edge states at a fixed μ. Since breakdown at B_H is determined by the tunneling distance between the left and right edge states, it follows that with a negative V_g the same tunneling distance can be realized for μ at an energy higher above the bottom of the lower Landau level. In a sweeping B experiment, the breakdown is expected to be observed at a lower B_H, and the B_H is expected to decrease with increasing $-V_g$.

Quantitatively, however, the estimated tunneling probability P between two simple edge states with width $\sim l_0$ [as pictured in the lower left panel of Fig. 2(c)] is much too small to give a finite R_n at B_H. From Ref. 18, $P\sim\exp(-W^2/l_0^2)$, where W is the tunneling distance between the left and right edge states. W can be estimated

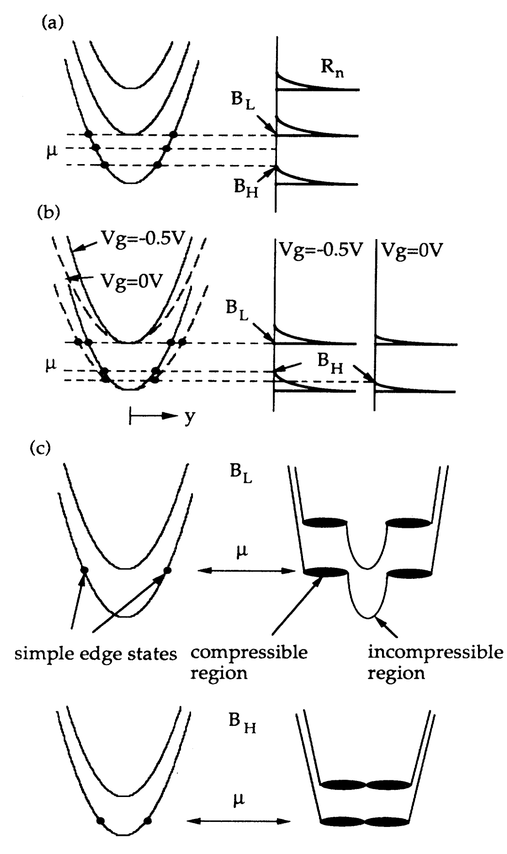

FIG. 2. (a) An illustration of the simple edge states of width $\sim l_0$. On the right-hand side, a sketch of R_n is given to show the relation between R_n and the position of μ relative to the Landau levels. (b) A sketch of Landau levels in a gated sample. The dashed parabolas are the Landau levels at $V_g=0$ V and the solid parabolas are the Landau levels at $V_g=-0.5$ V. On the right-hand side a sketch of R_n is given for $V_g=0$ and -0.5 V. (c) A sketch of single-electron Landau levels and simple edge states at $B=B_L$ and $B=B_H$. On the right-hand side corresponding phase-separated Landau levels with compressible and incompressible regions are drawn.

from experiment by assuming a parabolic potential confining the electron motion in the y direction.[19,20] When B is applied perpendicular to the narrow 2DES with parabolic confining potential, the Landau-level energy is given by

$$E=\frac{1}{2}m^*\frac{\omega_0^2\Omega^2}{\omega_C^2}y^2+\hbar\Omega(n+\tfrac{1}{2})\pm g\mu_B B\ , \qquad (1)$$

where g is the effective g factor of the electron, μ_B is the Bohr magneton, $\omega_C=eB/(m^*c)$ is the cyclotron frequency, ω_0 is the curvature of the confining parabola, and $\Omega=(\omega_0^2+\omega_C^2)^{1/2}$. W is the distance between the two intersects of the parabolic Landau level with μ. Therefore, W is given by $2y'$, where

$$y'=\frac{\omega_C}{\Omega\omega_0}\sqrt{2E'/m^*} \qquad (2)$$

and E' is the energy difference between μ and the bottom of the lower Landau parabola. We estimate W at $B_H=2.40$ T and $V_g=-0.5$ V by recognizing that at $B=B_L=2.26T$, $g\mu_B B_L+(\frac{1}{2})\hbar\Omega=\mu$. Assuming that g does not change with V_g and using the fact that $E'=\mu-\frac{1}{2}\hbar\Omega+g\mu_B B_H=0.61$ meV and ω_0, estimated from our $B=0$ data, is $\sim 2.7\times 10^{11}$ sec^{-1} at $V_g=-0.5$ V (Ref. 17), we obtain $y'\sim 2100$ Å. Since $W=2y'\sim 4200$ Å and $l_0=168$ Å at $B=B_H$, P is infinitesimally small and tunneling between the simple edge states cannot explain the observed R_n at B_H.

We believe that the fact that the tunneling distance is too large to give the observed R_n at $B=B_H$ is evidence that the edge channels must have width much wider than l_0. A naive picture is that the left and right edge channels, located at $\pm y'$, each has a width of $\sim W$. The breakdown on the high B side occurs when the innermost channels from the opposite edges are in contact, as illustrated on the right-hand side sketches of Fig. 2(c). The R_n observed at B_H results from the dissipation in the combined conducting stripe of the two edge channels. This picture is also consistent with our observation that the R_n follows an approximately linear dependence on $1/W$ (see Fig. 3), and it does not have the exponential dependence on W^2 expected from tunneling.

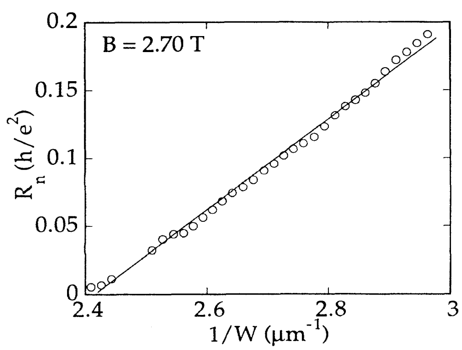

FIG. 3. R_n vs $1/W$ at $B=2.7$ T ($\nu=1$). A linear dependence is clear.

The wide edge channels are a result of the phase separation into the compressible and incompressible regions already proposed by Beenakker,[13] Chang,[14] and by CSG (Ref. 15), not from level broadening. In our sample, $\mu=10^6$ cm^2/V sec, the level broadening calculated from self-consistent Born approximation[21] is $\Delta E\sim 0.264$ meV. The broadening in y' estimated from this ΔE is $\Delta y'\sim 420$ Å, much smaller than W, and cannot account for the wide edge channels. According to the calculation by CSG (Ref. 15), the width of the compressible region is comparable to the depletion width. In our case, $W\sim 4000$ Å and the depletion layer width of our sample is ~ 7000 Å.

V. FQHE REGIME

The gate-bias-dependent breakdown of the vanishing R_{xx} also exists in the FQHE regime. While it is barely discernible in Fig. 1 for the $\frac{4}{3}$ FQHE (indicated by the dashed arrow), it is quite clear in Fig. 4, which shows the R_n trace near $\nu=\frac{1}{3}$ for $V_g=0$ and -0.241 V. The V_g dependence of the $\frac{1}{3}$ FQHE minimum shows the same behavior as that shown by the integer minima. On the low B side of the minimum, the breakdown is independent of V_g; on the high B side, it occurs at lower B for larger $-V_g$. This similarity in breakdown behavior to the IQHE suggests that there is a similar phase separation into the compressible and incompressible regions in the FQHE: Two wide compressible edge regions separated by the incompressible region of the $\frac{1}{3}$ FQHE state. The breakdown on the high B side of the minimum occurs when the two compressible regions are in close contact.

Finally, we remark that our experiment is distinctly different from previous FQHE edge-state experiments. In all previous experiments[10,11,22] the phase separation is demonstrated by the existence of a FQHE state of filling factor different from that of the bulk 2DES's by measuring the nonequilibrated transport coefficients. For example, Chang and Cunningham[22] more recently demonstrated the phase separation near the boundary of a sample. They showed the existence of the $\frac{1}{3}$ and $\frac{2}{3}$ states at the edges when the bulk was at $\nu=1$ and also the existence of

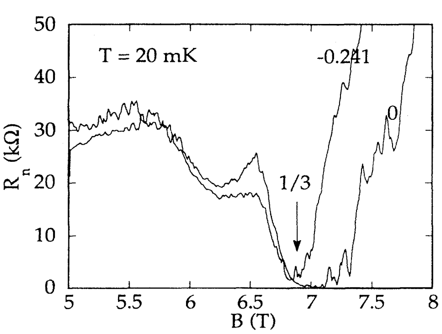

FIG. 4. The breakdown behavior at the $\nu=\frac{1}{3}$ minimum. The diagonal resistance, R_n, at $V_g=0$ and -0.241 V are shown. The same breakdown behavior as that of the IQHE minima is observed.

the $\frac{1}{3}$ state when the bulk was at $\frac{2}{3}$. In our experiment, the existence of the phase separation is shown by analyzing the breakdown of R_n, observed in a simple narrow 2DES at a single filling factor, not from the measurement in specially designed geometries with regional differences in filling factor. From our experiments, we cannot tell whether the net current is carried by the compressible regions or by the incompressible regions. However, the similarity of the breakdown mechanism between IQHE and FQHE suggests that the edge channels in FQHE regime are also the compressible regions.

VI. CONCLUSION

In conclusion, we have studied the gate-bias-dependent breakdown of the quantum Hall effect on the high and low B sides of the R_{xx} minima in a narrow 2DES. In the IQHE case, simple edge states of width l_0 cannot explain the observed breakdown on the high B side of the minima, and much wider edge channels ($\sim$4000 Å) are needed. These wide edge channels are interpreted as the compressible regions when the 2DES near the sample boundaries phase separates into compressible and incompressible regions. The same breakdown behavior is observed in the FQHE states in the R_{xx} minima at $\nu=\frac{1}{3}$ and $\frac{4}{3}$. We interpret this observation as evidence for the existence of a similar phase separation in the FQHE states.

Note added in proof. The phase-separated Landau levels at B_H sketched in Fig. 2(c) were theoretically predicted by D. B. Chklovskii, K. A. Matveev, and B. I. Shklovskii [Phys. Rev. B **47**, 12 605 (1993)] in both IQHE and FQHE regimes.

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[1] B. I. Halperin, Phys. Rev. B **25**, 2185 (1982); A. H. MacDonald and P. Streda, *ibid.* **29**, 1616 (1984); P. Streda, J. Kucera, and A. H. MacDonald, Phys. Rev. Lett. **59**, 1973 (1987); J. K. Jain and S. A. Kivelson, Phys. Rev. B **37**, 4276 (1988).

[2] M. Buttiker, Phys. Rev. Lett. **57**, 1761 (1986); Phys. Rev. B **38**, 9375 (1988).

[3] B. J. van Wees, L. P. Kouwenhoven, C. J. P. M. Harmans, J. G. Williamson, C. E. Timmering, M. E. I. Broekaart, C. T. Foxon, and J. J. Harris, Phys. Rev. Lett. **62**, 2523 (1989).

[4] J. A. Simmons, H. P. Wei, L. W. Engel, D. C. Tsui, and M. Shayegan, Phys. Rev. Lett. **63**, 1731 (1989), and references therein.

[5] J. K. Jain and S. A. Kivelson, Phys. Rev. Lett. **60**, 1542 (1988); J. A. Simmons, S. W. Hwang, D. C. Tsui, H. P. Wei, L. W. Engel, and M. Shayegan, Phys. Rev. B **44**, 12 933 (1991).

[6] R. J. Haug, A. H. MacDonald, P. Streda, and K. von Klitzing, Phys. Rev. Lett. **61**, 2797 (1988); S. Washburn, A. B. Fowler, H. Schmid, and D. Kern, *ibid.* **61**, 2801 (1988).

[7] B. J. van Wees, E. M. M. Willems, C. J. P. M. Harmans, C. W. J. Beenakker, H. van Houten, J. G. Williamson, C. T. Foxon, and J. J. Harris, Phys. Rev. Lett. **62**, 1181 (1989).

[8] See, for example, H. van Houten, C. W. J. Beenakker, and B. J. van Wees, in *Semiconductors and Semimetals,* edited by M. A. Reed (Academic, New York, 1990).

[9] S. Luryi, in *High Magnetic Fields in Semiconductor Physics,* edited by G. Landwehr, Springer Series in Solid State Sciences Vol. 71 (Springer-Verlag, Berlin, 1987), p. 16

[10] L. P. Kouwenhoven, B. J. van Wees, N. C. van der Vaart, C. J. P. M. Harmans, C. E. Timmering, and C. T. Foxon, Phys. Rev. Lett. **64**, 685 (1990).

[11] A. M. Chang and J. E. Cunningham, Solid State Commun. **72**, 651 (1989).

[12] A. H. MacDonald, Phys. Rev. Lett. **64**, 220 (1990).

[13] C. W. J. Beenakker, Phys. Rev. Lett. **64**, 216 (1990).

[14] A. M. Chang, Solid State Commun. **74**, 871 (1990).

[15] D. B. Chklovskii, B. I. Shklovskii, and L. I. Glazman, Phys. Rev. B **46**, 4026 (1992).

[16] S. W. Hwang, J. A. Simmons, D. C. Tsui, and M. Shayegan, Phys. Rev. B **44**, 13 497 (1991).

[17] S. W. Hwang, D. C. Tsui, and M. Shayegan (unpublished). The conducting width of the sample is obtained, assuming a parabolic confining potential, from the conductance quantization data at $B=0$.

[18] J. K. Jain and S. Kivelson, Phys. Rev. A **36**, 3476 (1987); Phys. Rev. B **37**, 4111 (1988).

[19] K. F. Berggren, T. J. Thornton, D. J. Newson, and M. Pepper, Phys. Rev. Lett. **57**, 1769 (1986); D. A. Wharam, U. Ekenberg, M. Pepper, D. G. Hasko, H. Ahmed, J. E. F. Frost, D. A. Ritchie, D. C. Peacock, and G. A. C. Jones, Phys. Rev. B **39**, 6283 (1989); J. F. Weisz and K. F. Berggren, *ibid.* **40**, 1325 (1989).

[20] S. E. Laux, D. J. Frank, and F. Stern, Surf. Sci. **196**, 101 (1988).

[21] T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

[22] A. M. Chang and J. E. Cunningham, Phys. Rev. Lett. **69**, 2114 (1992).