Frequency-to-voltage converter based on Bloch oscillations in a capacitively coupled GaAs-$Ga_xAl_{1-x}As$ quantum well

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A charged capacitor which is placed with a semiconductor in a closed loop will normally discharge in a characteristic time $\tau = 1/RC$. We show, however, that if a semiconductor of high purity is subjected to a rf bias with frequency ω, discharge does not occur if the voltage across the capacitor is proportional to an integer multiple of $\hbar\omega/e$. Thus a rf frequency may be converted to a dc voltage. The ac to dc conversion is made possible by the interplay between Bloch oscillations induced by the electric field of the external rf source and those induced by the field from the charge residing on the capacitor. The process resembles the frequency-to-voltage conversion by a hysteretic Josephson junction, except that the evolution of the Josephson phase is replaced by the evolution of the crystal momentum of conduction electrons in the semiconductor. We discuss the parameters and conditions for which this behavior may be observed.

I. INTRODUCTION

In 1928 Bloch[1] showed that under the influence of a dc field electrons in a perfect solid will oscillate, giving rise to an ac rather than a dc current. Until recently, however, convincing evidence for the existence of such coherent oscillations [alternatively viewed as the resonances of a Wannier-Stark (WS) ladder] has been sparse, due largely to the fact that scattering times in typical solids tend to be shorter than the period of a Bloch oscillation.[2] Nevertheless, observations of negative differential conductivity at high electric fields in Esaki-Tsu superlattices[3] indicate the presence of Bloch oscillations, and the direct observation of the WS ladder via optical absorption in GaAs-$Ga_xAl_{1-x}As$ superlattices has clearly demonstrated the unique features of this phenomenon.[4] We propose here a simple circuit in which the manifestations of Bloch oscillations should be clearly visible. The circuit consists of a two-dimensional GaAs-$Ga_xAl_{1-x}As$ quantum well which is connected in series to a capacitor, as shown in Fig. 1. The semiconductor is to be uniformly irradiated by a Gunn diode oscillator at a frequency ω of 6×10^{11} $\sec^{-1}$. Initially, the capacitor will be charged by a dc source to a voltage V. When the voltage source is disconnected, the capacitor tends to discharge across the semiconductor. However, as we will show below, the capacitor remains charged when the voltage V is approximately equal to $(l/a)\times(\hbar/e)n\omega$, where a and l are the lattice constant and the overall length of the semiconductor, respectively, and n is an integer.

Such a sustained dc voltage is reminiscent of the dc voltage steps observed in hysteretic Josephson junctions which are subjected to a rf field.[5] We recall, for example, that the phase difference ϕ across a Josephson junction evolves according to the same second-order nonlinear equation which describes the rotational phase of a driven damped pendulum,[6]

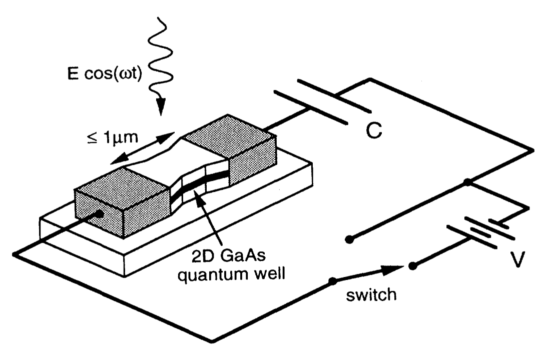

FIG. 1. A schematic diagram of the proposed frequency-to-voltage converter. The GaAs-$Ga_xAl_{1-x}As$ quantum well is uniformly irradiated by a Gun diode oscillator. Initially, the capacitor is charged by a battery to a voltage V. When the battery is disconnected, the capacitor discharges, unless V is proportional to a multiple of $(\hbar/e)\omega$.

$$\ddot{\phi}+\alpha\dot{\phi}+A\sin(\phi)=B\sin\omega t\ . \tag{1.1}$$

In (1.1), the damping coefficient α, the "depth of the potential" A, and the strength of the rf drive B are specified in terms of junction attributes such as capacitance, quasiparticle resistance, critical current, etc. The voltage V across the junction is proportional to the time derivative of the phase via the relation $V=(\hbar/2e)\dot{\phi}$. When the driving is strong enough, the phase in Eq. (1.1) may lock to a state of sustained rotation such that the average of $\dot{\phi}$ is equal to an integral multiple of the driving frequency ω. In such a case, it follows that the average voltage across the Josephson junction is given by $V=(\hbar/2e)n\omega$. For $A \ll \omega^2$ the condition for locking is given approximately by[5]

$$|\alpha n\omega| < |AJ_n(B/\omega^2)|\ , \tag{1.2}$$

where J_n is the ordinary Bessel function of the first kind of order n. We shall now show that the operation of a capacitively coupled semiconductor is analogous in many respects.

II. CIRCUIT DYNAMICS

To calculate the current in the two-dimensional semiconductor quantum well, we consider the evolution of the distribution function $f(k,t)$, which describes the probability that an electron in the lowest conduction miniband has crystal momentum $\hbar k$ at time t. We consider only the evolution of the wave vector k in the direction of the superlattice. For a spatially uniform charge density, the Boltzmann equation takes on the following form in the relaxation-time approximation:[2]

$$\frac{\partial}{\partial t}f(k,t)-\frac{e}{\hbar}E(t)\frac{\partial}{\partial k}f(k,t)=-\alpha[f(k,t)-f_T(k)]\ . \tag{2.1}$$

In (2.1), $E(t)$ is the applied electric field, α is the scattering rate which, for simplicity, we take to be the same for all values of k, and $f_T(k)$ is the equilibrium distribution in the absence of an applied field. The solution of (2.1) is

$$f(k,t)=\frac{1}{a}\int_0^t ds\, e^{-\alpha(t-s)}\dot{\phi}(s)\frac{\partial}{\partial k}f_T\{k-[\phi(t)-\phi(s)]/a\}+f_T(k)\ , \tag{2.2}$$

where we have introduced the phase $\phi(t)$ as the integration over time of the voltage across a unit cell $\phi(t)=(ea/\hbar)\int_0^t ds\, E(s)$. For a cosine band of width W, the contribution to the current from each state of crystal momentum $\hbar k$ is $(N\sigma Wae/2\hbar)\sin(ka)$, where N is the number of carriers per unit volume and σ is the cross-sectional area of the semiconductor. Thus the current $I(t)$ is given by

$$I(t)=(N\sigma Wae/2\hbar)\times\int_0^t ds\, e^{-\alpha(t-s)}\dot{\phi}(s)\left\{\int_0^{2\pi/a}dk/2\pi\sin(ka)\frac{\partial}{\partial k}f_T\{k-[\phi(t)-\phi(s)]/a\}\right\}\ . \tag{2.3}$$

To facilitate the integration of (2.3), we approximate $f_T(k)$ by the Dirac delta function $\delta(k)$ in the low-temperature limit. Equation (2.3) becomes

$$I(t)=(N\sigma Wae/2\hbar)\int_0^t ds\, e^{-\alpha(t-s)}\dot{\phi}(s)\cos[\phi(t)-\phi(s)]\ . \tag{2.4}$$

In the closed loop, the phase $\phi(t)$ is related to the electric field across the semiconductor;

$$\dot{\phi}(t)=\frac{ea}{\hbar}E(t)=\frac{ea}{\hbar}\left[\frac{Q(t)}{Cl}+\epsilon\cos(\omega t)\right]\ , \tag{2.5}$$

where l is the length of the semiconductor, $Q(t)$ is the charge on the capacitor, and ϵ is the amplitude of the rf field. In addition, the current $I(t)$ in the semiconductor is equal to the time rate of change of the charge on the capacitor, $\dot{Q}$. Therefore, from the combination of (2.5) and (2.4) we obtain a second-order nonlinear, temporally nonlocal, equation for the phase $\phi(t)$,

$$\ddot{\phi}(t)+A\int_0^t ds\, e^{-\alpha(t-s)}\dot{\phi}(s)\cos[\phi(t)-\phi(s)]=B\sin(\omega t)\ , \tag{2.6}$$

where $A=N\sigma Wa^2e^2/2Cl\hbar^2$ and $B=-(e/\hbar)a\epsilon\omega$. Equation (2.6) describes the evolution of charge in the circuit of Fig. 1. In the remainder of this paper we analyze Eq. (2.6) and demonstrate that "phase-locked" solutions exist as a consequence of the superposition of Bloch oscillations from the two sources of voltage.

III. NUMERICAL AND ANALYTICAL INVESTIGATIONS

In order to determine the qualitative behavior of the proposed circuit, we first analyze (2.6) in two limiting cases. In the limit of large scattering, the integrand in (2.6) decays quickly, and $\dot{\phi}(s)\sim\dot{\phi}(t)$ and $\cos[\phi(t)-\phi(s)]\sim 1$. If we neglect exponentially decaying transients, (2.6) becomes

$$\ddot{\phi}(t)+(A/\alpha)\dot{\phi}(t)=B\sin(\omega t)\ . \tag{3.1}$$

The capacitor always discharges in this uninteresting limit; $\dot{\phi}(t)$ simply oscillates at long times. On the other hand, in the limit of no scattering, the integration in (2.6) may be performed, giving

$$\ddot{\phi}+A\sin(\phi)=B\sin\omega t\ . \tag{3.2}$$

Thus in the absence of scattering, our system is identical to the Josephson junction without damping [cf. (1.1)]. By analogy, then, one might expect that solutions of (2.6) exist for which the time average of $\dot{\phi}(t)$ is equal to an integral multiple of the rf frequency. We again stress the striking similarity between the two systems; the voltage across a Josephson junction is equal to $(\hbar/2e)\dot{\phi}$; here, the voltage across the semiconductor is equal to $(\hbar/e)\times(l/a)\dot{\phi}$. In both cases, a sustained rotation of the phase is indicative of a persistent dc voltage. With parameters which permit phase-locked solutions of (1.1), that is, A, B, and α, which obey the stability criterion (1.2), we have obtained numerical solutions for (2.6) which show that $\dot{\phi}$ does not decay at long times, and that the average of $\dot{\phi}$ is approximately equal to an integral multiple of the rf frequency

$$\langle \dot{\phi} \rangle \sim n\omega \ . \tag{3.3}$$

Examples of these results for $n=1$ are shown in Fig. 2. If $\dot{\phi}$ were truly locked to an integer multiple of the rf frequency, then the phase plot would show a single track. The multiple tracks observed here indicate that the average rotation rate is slightly slower than the rf frequency, an effect which becomes more pronounced for larger damping. This phenomenon may be understood through the subsequent stability analysis of (2.6). The deviations from the stable rotational solutions of (1.1) may be described by the second-order differential equation for a harmonic oscillator. In contrast, we will find that deviations from the rotational solutions of (2.6) are described by a first-order differential equation.

The stability analysis of (2.6) proceeds as follows. Let us rewrite the phase $\phi(t)$ in (2.6) as $\phi(t)=\phi_{\text{fast}}+\phi_{\text{slow}}-(B/\omega^2)\sin(\omega t)$, so that we explicitly keep track of its fast and slow components. As will soon become apparent, we will assume that ϕ_{slow} (ϕ_{fast}) evolves slowly (quickly) with respect to the scattering rate α. Equation (2.6) becomes

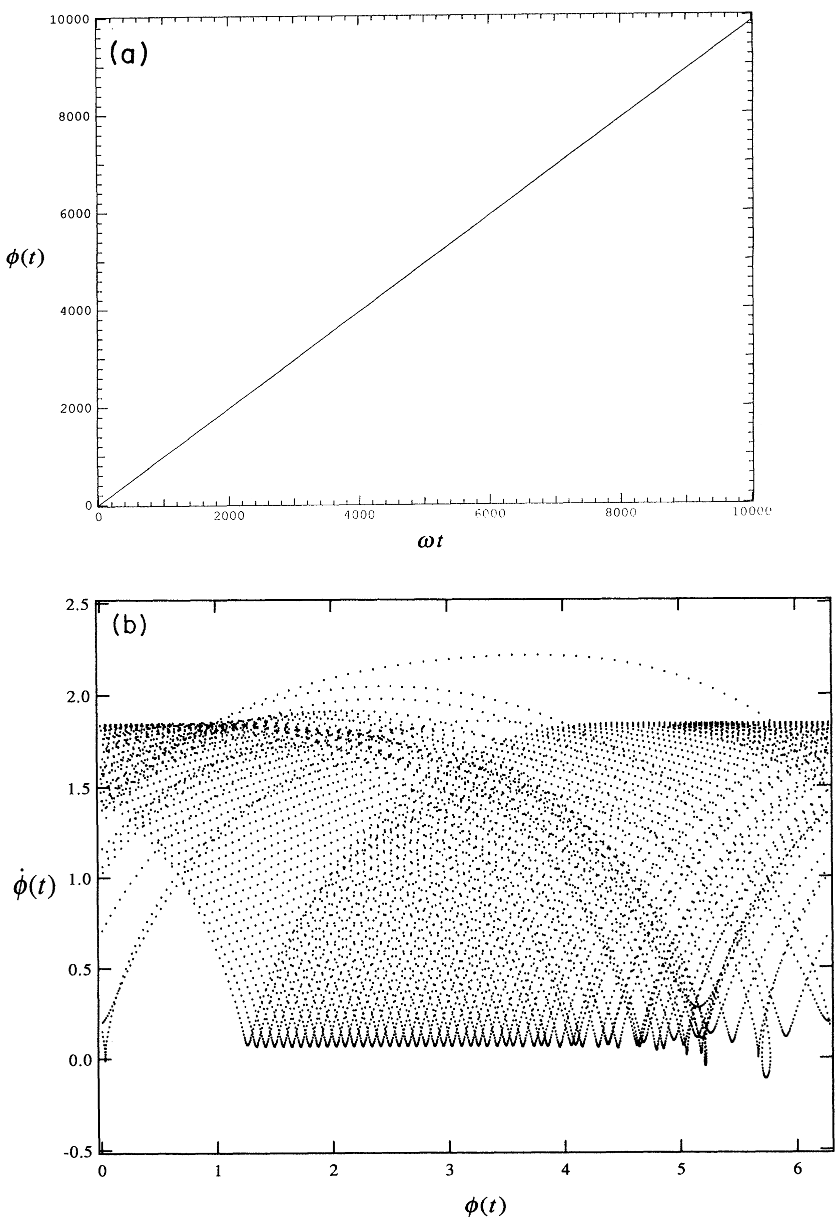

FIG. 2. (a) A graph of the phase ϕ as a function of time. Equation (2.6) was numerically integrated using fourth-order Runge-Kutta with a time step of $\Delta(\omega t)=0.01$. The parameters are $A/\omega^2=0.4$, $B/\omega^2=1.0$, $\alpha/\omega=0.05$, $\dot{\phi}(0)=1.1\omega$, and $\phi(0)=0$. The straight line is indicative of a phase-locked state. (b) The phase plot $\dot{\phi}(t)$ versus $\phi(t)$ for the same parameters. The average of $\dot{\phi}(t)$ is slightly less than the driving frequency ω. This is the reason there are multiple trajectories, rather than a single track.

$$\ddot{\phi}_f+\ddot{\phi}_s+A\int_0^t ds\, e^{-\alpha(t-s)}[\dot{\phi}_f(s)+\dot{\phi}_s(s)-(B/\omega)\cos(\omega s)]$$
$$\times\cos\{\dot{\phi}_s(t)(t-s)+\phi_f(t)-\phi_f(s)-(B/\omega^2)[\sin(\omega t)-\sin(\omega s)]\}=0\ , \quad (3.4)$$

where we have expanded $\phi_s(s)$ in the cosine in (2.6) to first order in $s-t$. Next, we neglect $\phi_f(t)$ in the integration in (3.4), under the assumption that its contribution to the dynamics is small compared to the effect of $(B/\omega)\cos(\omega t)$. The remaining integration can be written as an expansion in Bessel functions. At long times, we discard exponentially decaying terms, and (3.4) becomes

$$\ddot{\phi}_f+A\alpha\sum_{m\neq n}J_m\left[\frac{B}{\omega^2}\right]J_n\left[\frac{B}{\omega^2}\right]\frac{(\dot{\phi}_s-m\omega)\cos[(m-n)\omega t]+\alpha\sin[(m-n)\omega t]}{(\dot{\phi}_s-m\omega)^2+\alpha^2}$$
$$+\ddot{\phi}_s+A\alpha\sum_{m=-\infty}^{m=\infty}J_m^2\left[\frac{B}{\omega^2}\right]\frac{(\dot{\phi}_s-m\omega)}{(\dot{\phi}_s-m\omega)^2+\alpha^2}=0\ . \quad (3.5)$$

Let us set the top and the bottom parts of (3.5) to zero separately, so that ϕ_f and ϕ_s are associated with the ac and the dc parts of the expansion, respectively. Thus it is in this manner that the phases ϕ_f and ϕ_s are explicitly defined. We find that our neglect of ϕ_f in the integration of (3.4) is in fact justified in the limit in which $A \ll B$. We now focus our attention on the bottom part of (3.5), which is a first-order equation in the rotational velocity $\dot{\phi}_s$. The rotational velocity $\dot{\phi}_s$ is proportional to the average charge, $\bar{Q}(t)$, which remains on the capacitor at long times: $\bar{Q}(t)=(C\hbar l/ea)\dot{\phi}_s(t)$. Bearing this in mind, from here on we will substitute the letter q for $\dot{\phi}_s$. As a first-order equation for q, the bottom part of (3.5) becomes

$$\dot{q}+q/\tau(q)=0\ . \quad (3.6a)$$

The effective time constant is a function of velocity,

$$1/\tau(q)=\frac{A\alpha}{q}\sum_{m=-\infty}^{m=\infty}J_m^2\left[\frac{B}{\omega^2}\right]\frac{(q-m\omega)}{(q-m\omega)^2+\alpha^2}\ . \quad (3.6b)$$

Were it not for the q dependence of the time constant $\tau(q)$, (3.6a) would be simply the first-order equation which determines the charge in a RC series circuit.

In the neighborhood of $q=0$, $\tau(q)$ goes to a constant: $\tau(0)=\alpha/AJ_0^2(B/\omega^2)$. This indicates that a small initial charge will dissipate. For $\alpha\ll\omega$, the weighted Lorentzians in the sum in (3.6b) are peaked at distinctly separate values of q. They pass through zero regularly when $q\cong n\omega$ [cf. Fig. 3(a)]. At these points, the effective dc conductivity is zero, so on the average the flow of charge is completely inhibited. Thus a capacitor which is initially charged so that $q\cong n\omega$ will remain charged. Moreover, the stability of these zero crossings is argued as follows. If the charge on the capacitor is increased from a crossing point, the conductivity becomes positive, and discharge occurs. On the other hand, if the charge on the capacitor is decreased from a crossing point, the resistance becomes negative, causing the capacitor to recharge again. Thus the average charge on the capacitor will remain stable in the neighborhood of a zero crossing. It can be argued that any zero crossing for positive q with a positive slope (or for negative q with negative slope) is stable. If the scattering rate is increased, zero crossings occur less often and also at sporadic locations. The tails from neighboring weighted Lorentzians in the sum in (3.6b) will displace the zeros of the function $1/\tau(q)$ from the perfect integer crossings seen in the limit in which $\alpha\to 0$, so that stable points may occur when q is slightly above or below integer multiples of the frequency. A large scattering rate will tend to lift the function $1/\tau(q)$ from the axis, reducing the number of zero crossings, especially for large q. Thus as a general rule, stable points exist when the scattering rate is much smaller than the ac frequency. This is in keeping with the fact that the Bloch oscillations are not likely to be noticed if electrons are scattered in a time which is short compared to the period of oscillation. We point out, however, that through a judicious choice of the rf amplitude, it is possible to find stable locking points in this circuit when the scattering time is of the order of the period of a Bloch oscillation. For example, if the intensity is such that B/ω^2 is a root of a Bessel function, neighboring terms in the sum become more prominent, and are more likely to cause the sum to pass through zero as a function of q. We note in passing that if the intensity is such that B/ω^2 is a root of J_0, the effective conductance goes to zero as a result of dynamic localization,[7–10] and the point at $q=0$ becomes unstable. As a result, charge will be spontaneously driven onto an initially uncharged capacitor by the rf field.

IV. CONCLUDING REMARKS

The key mechanism of this circuit which allows for the stable points discussed above is best described as the relative timing of the Bloch oscillations. These are induced jointly by the external rf field ϵ and the dc field, which is proportional to the average charge on the capacitor. If the Bloch oscillation frequency due to the dc field is slightly greater than ω, then on the average the crystal momentum spends more time traversing the positive half of the Brillouin zone, and the dc conductance is positive. However, if the Bloch oscillation frequency due to the dc field is slightly less than ω, more time is spent traversing the negative half of the Brillouin zone, and the conductance is negative. In the case of negative conductance, the dc current is actually in a direction opposite to that of the dc field. When equal time is spent on both sides of the Brillouin zone, the dc conductance is zero. As we have argued above, sign changes of the conductivity give rise to the observed stable points. Such timing arguments are easier to envision if one considers the example of a rf square wave rather than a sinusoidal field. Related discussions may be found in Ref. 8.

In this paper, we have chosen to describe the circuit dynamics as an evolution in momentum space. However, useful insights into the nature of the phase lock may also be gleaned from a description in terms of an electron hopping from site to site along a WS ladder. As in the analysis of Eq. (2.6), it is helpful to separate the field across the capacitor into its ac and (approximately) dc components, E_{ac} and E_{dc}, respectively. For the moment, let us ignore the ac field; the dc field alone sets up a WS ladder in the semiconductor with an energy spacing $eE_{dc}a$. Since the dc field changes slowly if the system is slightly perturbed from the phase-locked state, we regard the WS ladder as an approximate basis of adiabatic eigenstates. Let us now examine the electron-phonon scattering as a perturbation from this basis. The ability to emit or absorb phonons will allow the electron to hop both "up" and "down" the Stark ladder. At low temperatures, however, while the electron can always emit a phonon, there are few available for absorption. Thus hops will be preferentially in the downward direction, and the capacitor will begin to discharge.

Let us reintroduce the ac field, with a frequency such that $eE_{dc}a = \hbar\omega$. Now hops can be made along the Stark ladder without the help of the phonons, since the appropriate addition or subtraction of the quanta $\hbar\omega$ brings the levels into coincidence. Efficient transport may be achieved in this manner, and there is no preferred upward or downward direction, since phonons are not required for energy conservation. For this reason, this particular photon-assisted transport channel cannot contribute to the discharge of the capacitor. Nevertheless, there still remains the probability that an electron will make a phonon-assisted transition without absorbing or emitting a photon. Therefore, in spite of the ac field, electrons will continue to drift "down" the Stark ladder. How then is the phase lock achieved? The key is that since the electrons which leak down the Stark ladder are discharging

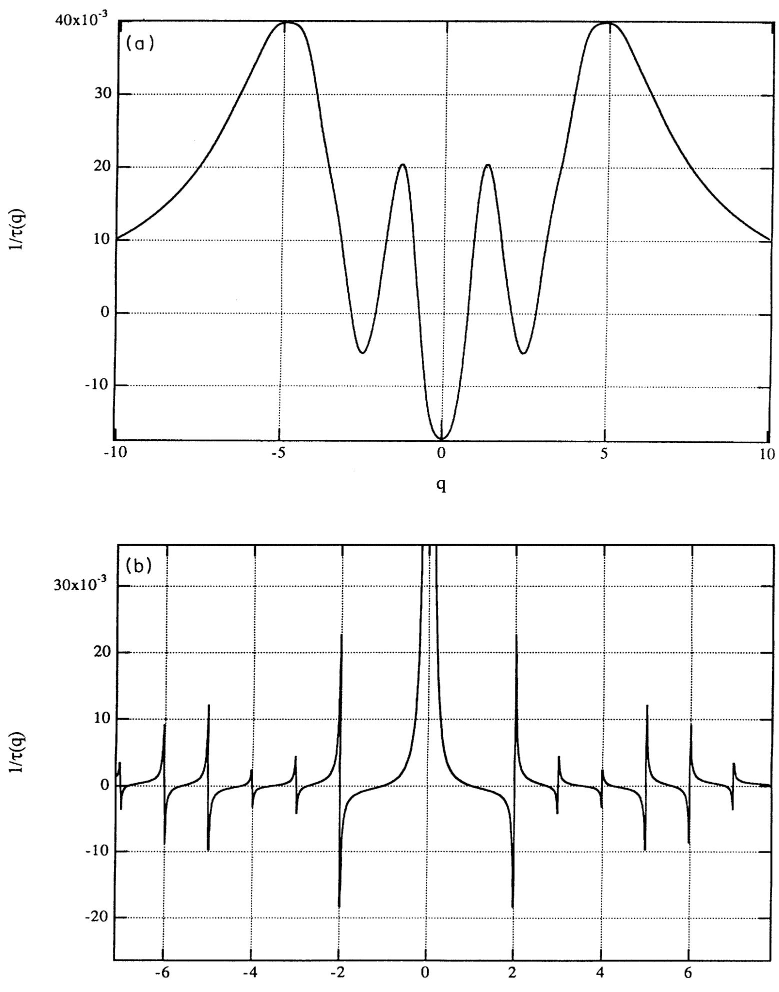

FIG. 3. (a) A graph of the effective RC decay rate, $1/\tau(q)$, as a function of the rotational velocity q, for parameters as follows: $B/\omega^2=5$ and $\alpha/\omega=0.9$. The zero crossings at $q/\omega \cong \pm 1, \pm 3$ are stable points: if the system is prepared at these points initially, the capacitor will not discharge. The zero crossings at $q/\omega \cong \pm 2$ are unstable points. (b) The effective RC decay rate is plotted as a function of q for the following parameters: $B/\omega^2=7$ and $\alpha/\omega=0.01$. For the small scattering rate, the stable zero crossings occur near integer multiples of the frequency ω.

the very capacitor which is causing the dc bias in the first place, the average electric field across the sample is slightly reduced, such that $eE_{dc}a < \hbar\omega$. When this situation occurs, an electron may absorb a quanta $\hbar\omega$ and hop "up" the Stark ladder if it *emits* a phonon, or it may emit a quanta $\hbar\omega$ and hop "down" the Stark ladder only if it *absorbs* a phonon. Again, since phonon emissions are more likely than absorptions, the hops "up" the Stark ladder are preferred. If the contribution to the overall electron transport from this channel is large enough, the capacitor will *charge back up*. It is in this way that a stable situation is created such that the capacitor remains charged for all time, in spite of the interactions with a heat bath.

The most important criterion for the achievement of a phase-locked state in this system is that the scattering rate should be smaller than the ac frequency. For a two-dimensional GaAs-Ga_xAl_{1-x}As quantum well with a carrier density of 1×10^{11} cm^{-2} and a carrier mobility of 1×10^{6} cm^2/V s, the value of the scattering rate $\alpha\sim30$ GHz at 0.3 K. This is smaller than the applied bias frequency ω, which can easily be in the range $70\leq\omega/2\pi\leq100$ GHz for existing Gunn diode oscillators. A stable phase lock at these frequencies for a lattice constant of about 2 Å implies an electric-field strength of 10^6 V/m, which is below breakdown. In order that B/ω^2 be of order 1, so that the Bessel function coefficients in (3.6b) are not too small, we require that the rf amplitude be also of the order of 10^6 V/m. This implies a microwave intensity of 10^{11} W/m^2, which is within the capacity of the Gunn diode source. Finally, for the conditions discussed in this paper, we require that $A/B \ll 1$, i.e.,

$$(NWae/2\chi\hbar\omega\epsilon)\times\frac{d\sigma_{\rm semi}}{l\sigma_{\rm cap}} \ll 1\ , \tag{4.1}$$

where χ is the dielectric constant and d is the width of the capacitor, respectively, and $\sigma_{\rm semi}/\sigma_{\rm cap}$ is the ratio of the surface area of the semiconducting sample to the area of the capacitor. For a 0.1-eV bandwidth, this implies that $d\sigma_{\rm semi}/l\sigma_{\rm cap} \ll 10^{-2}$, which is easily designed.

V. CONCLUSION

We have argued that a semiconductor-based frequency-to-voltage converter is operable at low temperatures. The intrinsic precision of this device cannot compare with that of a phase-locked Josephson junction, for as we have seen, scattering pulls the voltage away from the perfect quantized steps. Nevertheless, the device may be useful as a highly accurate transfer standard. It has the advantage that the voltage output is scaled by the length of the semiconductor in the direction of the field. Thus a nominal 1-V output could be achieved with a 1-μm-length quantum well.

ACKNOWLEDGMENTS

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