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Perimeter–maximum-diameter method for measuring the fractal dimension of a fractured surface

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A perimeter–maximum-diameter method is proposed for measuring the fractal dimension of a fractured surface. The measured values of the fractal dimension by this method seem independent of the length of the yardstick. However, the fractal dimensions D_m measured by the perimeter-area method decrease as the yardstick length increases. When the length of the yardstick is small enough, the value of D_m measured by the latter method approaches the value measured by the present method.

I. INTRODUCTION

Since Mandelbrot, Passoja, and Paullay[1] used the perimeter-area relation, which is sometimes called slit-island analysis, to determine the fractal dimensions of fractured surfaces of 300-grade maraging steel, this method has been used in many cases in the studies of materials processes, such as the fracture toughness,[2] toughness of materials,[3] fatigue,[4] pitting corrosion,[5] recrystallization,[6] etc. The theoretical base of the perimeter-area method is that the ratio

$$\alpha_D(\epsilon)=[L(\epsilon)]^{1/D}/[A(\epsilon)]^{1/2} \tag{1}$$

should be size independent. Where L and A are the perimeter and area of the Koch bright zone, respectively, ϵ is the normalized yardstick length with respect to L_0, the length of initiator ($\epsilon=\eta/L_0$), and η is the absolute length of measuring yardstick.

However, based on experiments, Lung and Mu,[7] have pointed out that the fractal dimension determined by the perimeter-area method is independent on the yardstick length chosen, and the measured value does not seem to be the real fractal dimension of fractured surfaces. In practical measurements, η is usually kept constant, but it is not easy to keep ($\epsilon=\eta/L_0$) constant due to different sizes of the Koch bright zones, L_0.

For the perimeter of the Koch bright zone,

$$L(\eta)=L_0^D\eta^{1-D}, \tag{2}$$

the ratio is as follows:

$$\alpha_D(\eta)=[L(\eta)]^{1/D}/[A(\eta)]^{1/2} =L_0\eta^{(1-D)/D}[A(\eta)]^{-1/2}. \tag{3}$$

Therefore, $\alpha_D(\eta)$ is dependent on the size of the Koch bright zone, L_0 and $\alpha_D(\eta)$ would not be a constant even with a constant yardstick length η.

Furthermore, $\epsilon_i=\eta/L_{0i}$. For larger bright zones, ϵ is smaller and the smaller normalized yardstick yields more generations. In addition, $\alpha_D(\epsilon)$ is yardstick dependent, $\alpha_D(\eta_i)\neq\alpha_D(\eta_j)$. The measured values of D_m must be yardstick dependent. Our experimental data verified the above conclusion.[7] This might, then, be one of the reasons why the negative correlations between measured D_m and the toughness of some materials have been observed. Indeed, we changed the yardstick length, and a positive correlation between D_m and $\log K_{IC}$ was obtained. Then we may draw the conclusion that the perimeter-area method is one of the causes for the negative correlation between D_m and the toughness of materials. In this paper, we propose another method for determining the fractal dimension of a fractured surface, i.e., the perimeter–maximum-diameter method.

II. THE PERIMETER–MAXIMUM-DIAMETER METHOD FOR DETERMINING FRACTAL DIMENSION

It is well known that, for bright zones with fractal coastlines, the length $L(\epsilon)$ of the coastline depends on the yardstick ϵ used to measure its length, that is,

$$L(\epsilon)=\epsilon^{1-D}, \tag{4}$$

where D is the fractal dimension of the Koch perimeter.

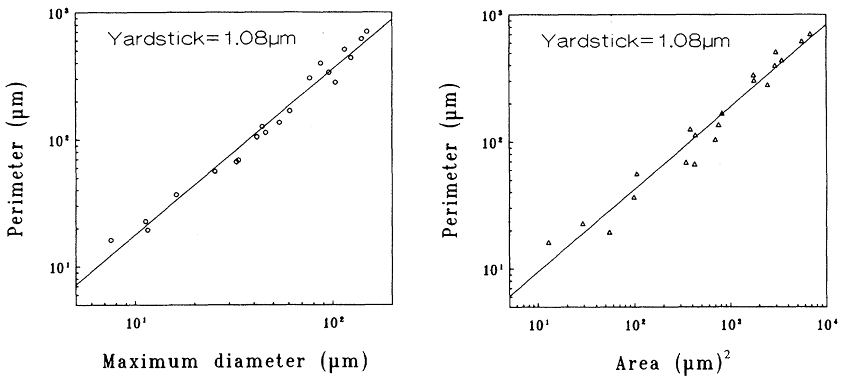

FIG. 1. Fractal perimeter–maximum-diameter relationship and perimeter-area relationship for slit bright zones. Yardstick = 1.08 μm.

Substituting $\epsilon = \eta / L_0$ into (4) gives

$$L(\eta) = L_0^D \eta^{1-D} , \tag{5}$$

where η is an absolute length of the yardstick and L_0 is a length of an initiator. It is difficult to determine the perimeter of the initiator of a self-similar Koch bright zone of a fractured surface. Although the value D can be determined from a slope of $\log L(\eta)$-$\log\eta$ plot by means of changing the measuring yardstick η, sometimes it is more difficult to change the measuring yardstick η in experiments. So a method was proposed to determine the fractal dimension of the fractured surfaces. We assume that the perimeters of initiator L_{0i} are directly proportional to their maximum diameters $(d_{\max})_i$ for various sizes of self-similar bright zones, that is,

$$L_{0i} = m (d_{\max})_i , \tag{6}$$

where m is a constant. Inserting Eq. (6) into Eq. (5) and taking logarithm values of both sides leads to

$$\log L_i(\eta) = D \log m + (1 - D) \log\eta + D \log (d_{\max})_i . \tag{7}$$

Letting the measuring yardstick η be constant, Eq. (7) becomes

$$\log L_i(\eta) = \text{const} + D \log (d_{\max})_i . \tag{8}$$

Then the value of fractal dimension D can be obtained from the slope of the straight line of $\log L_i(\eta)$-$\log(d_{\max})_i$ plots.

III. EXPERIMENTAL PROCEDURE AND RESULTS

The materials used for this investigation are 30CrMnSiNi$_2$$A$ steels. The heat treatments are as follows: austenitize at 900 °C for 30 min, transfer to a salt bath at 260 °C for 60 min, temper at 270 °C for 120 min, and then air cool. Three-point bend (TPB) tests were performed on the specimens used at -60 °C for this fractal dimension investigation. The fractured surfaces of the specimens were plated with electroless nickel by vacuum impregnation in order to ensure edge retention. The specimens were then polished parallel to the fractured surfaces with 0.5 μm synthetic diamond abrasive paste. In the microscope field of vision, the polished zones appear bright, and the unpolished zones appear dark. The perimeters and maximum diameters were measured for 20 bright zones with a Kontron IBAS KAT 386 image analyzer. The areas of the bright zones were also measured so as to compare with the perimeter-area method.

In order to investigate the effects of the fractal dimensions measured with both methods, the square grid method was used to change the measuring yardsticks. The method is to cover the bright zone with a square grid. Let the square cells of grid be $\eta \times \eta$. The number $N(\eta)$ of such squares needed to cover the coastline of the bright zone is roughly equal to the number of steps used when walking a divider with an opening η along the coastline. Decreasing η again gives a large increase in the number of cells needed to cover the bright

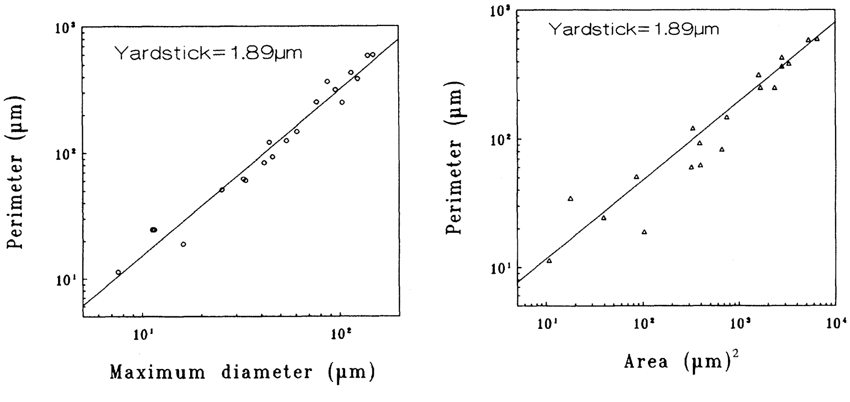

FIG. 2. Fractal perimeter–maximum-diameter relationship and perimeter-area relationship for slit bright zones. Yardstick = 1.89 μm.

zone. Using the side length of the square η as a measuring yardstick, and letting the number of such squares needed to cover the bright zone be $M(\eta)$, then the perimeter of the bright zone, $L(\eta)=N(\eta)\eta$, and the area, $A(\eta)=M(\eta)\eta^2$. The size of the square varied from small to large and the side length of the square η was calibrated. This measurement and calculation were progressively repeated. The perimeters and areas of the bright zone were obtained with different measuring yardsticks η. The maximum diameter d_{max} of the bright zone was measured from the maximum distance between two points at the coastline of the bright zone. Twenty bright zones with different sizes from small to large were measured using different yardsticks η. $\mathrm{Log}L(\eta)$-$\log d_{max}$ and $\log L(\eta)$-$\log A(\eta)$ plots were drawn under different yardsticks η. The slope of a straight line fitted to these data points by a least-squares program was used to calculate the fractal dimension D for both methods.

Figures 1 and 2 show $\log L_i(\eta)$-$\log(d_{max})_i$ and $\log L_i(\eta)$-$\log A_i(\eta)$ plots, respectively, under two different measuring yardsticks. Figure 3 shows the fractal dimensions versus the measuring yardstick for both methods. From Figs. 1 and 2 it can be seen that when the measuring yardstick is smaller, both plots $\log L_i(\eta)$-$\log(d_{max})_i$ and $\log L_i(\eta)$-$\log A_i(\eta)$ have a good linear relation and the fractal dimensions measured by both methods are close to the same value. When the measuring yardsticks become larger, $\log L_i(\eta)$-$\log(d_{max})_i$ plots still have a good linear relation, but the data points of $\log L_i(\eta)$-$\log A_i(\eta)$ plots depart from a straight line. From Fig. 3 it can be seen that the fractal dimensions measured by the perimeter–maximum-diameter method are nearly constant and do not vary with an increase of the measuring yardsticks. However, those measured by the perimeter-area method decrease with increasing measuring yardsticks. In Fig. 3 the fractal dimensions measured are separated into two parts with a perpendicular line. In the range of smaller yardsticks, the fractal dimensions measured by both methods are lower than those in the range of larger yardsticks. The reason may probably be that the fractured surfaces possess two different fractal structures in the different ranges of measuring yardsticks, which correspond to two different fractal dimensions.

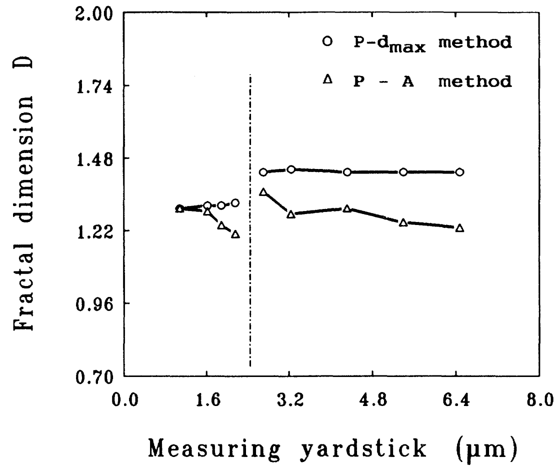

FIG. 3. Measured fractal dimensions D as a function of the measuring yardstick for both the perimeter–maximum-diameter and the perimeter-area methods.

IV. CONCLUSIONS

The fractal dimensions measured by the perimeter–maximum-diameter method are independent of the measuring yardsticks, but those measured by the perimeter-area method depend on the yardstick lengths.

The fractal dimensions measured by both methods are close to the same value when the measuring yardsticks are small enough. But the values measured by the perimeter-area method decrease with an increase in the length of the measuring yardstick.

Fractured surfaces possess different fractal structures with different fractal dimensions in different ranges of measuring yardstick lengths.

ACKNOWLEDGMENT

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