Magnetic excitations of a doped two-dimensional antiferromagnet

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Magnetic excitations of the two-dimensional (2D) t-J model are considered in the presence of a small concentration of holes c. The spin-wave approximation used implies long-range antiferromagnetic ordering from the beginning. Migdal's theorem is shown to be valid for the model considered. The energy spectrum of the magnons is determined with the help of the one-pole approximation for the hole Green's function. If the concentration of mobile holes is larger than a critical value an additional branch of overdamped magnons arises near the Γ and M points of the Brillouin zone. This is connected with the generation of electron-hole pairs (the Stoner excitations) by magnons. The appearance of such excitations means the destruction of the long-range antiferromagnetic order. For parameters presumably realized in cuprate perovskites this happens for several percent of holes per site. The relation between the critical concentration and the hole concentration destroying the 3D long-range ordering in $La_{2-x}Sr_xCuO_4$ is discussed. The arising short-range order is characterized by the instantaneous spin correlation length $\xi \sim c^{-1/2}$, in coincidence with the experimentally observed dependence in this crystal.

I. INTRODUCTION

Magnetic properties of cuprate perovskites have attracted considerable attention due to their presumable connection with high-T_c superconductivity. Some of these properties are rather unusual and still have no satisfactory explanation. One of them is the extreme sensitivity of the long-range antiferromagnetic order to any deviation from the ideal crystal. In $La_{2-x}Sr_xCuO_4$ already at $x \approx 0.02$ the three-dimensional (3D) long-range order is destroyed and the short-range order with an instantaneous spin correlation length $\xi \approx a/\sqrt{x}$ in the CuO_2 planes arises[1,2] (a is the distance between copper sites in the planes). Sometimes the explanation of this experimental fact is based on arguments of Ref. 3 about competing antiferromagnetic and ferromagnetic interactions generated by a hole localized on an oxygen orbital. Such interactions can be described in the framework of the extended Hubbard model[4] in the limit of strong repulsion. However, it is known[5] that in this limit the low-energy dynamics of the model can be described by the t-J model. For the latter model, calculations[6,7] show that for reasonable sets of parameters perturbations produced by a *solitary* hole in the spin subsystem are too small to explain the extremely small dopant concentrations which are sufficient for the destruction of the ordering.

In the present paper an attempt is undertaken to analyse these questions on the basis of the 2D t-J model in the presence of a *finite concentration* of mobile holes. The model is supposed to be a good candidate for the description of transport and magnetic properties of the CuO_2 planes in the cuprate perovskites.[8] To make this problem tractable we use the spin-wave approximation.[9,10,7] This approximation has been shown to give results which are in quantitative agreement with exact calculations on small lattices in the cases of one and two holes.[7,11] For the considered model vertex corrections will be shown to be small (in analogy to Migdal's theorem for the electron-phonon interaction in metals[12]); this makes the system of Dyson's equations self-consistent. Under the supposition that the homogeneous Fermi-liquid description is valid and by making use of the one-pole approximation for the hole Green's function the energy spectrum of the magnetic excitations is determined. Above some critical concentration of holes the usual magnon branch (slightly renormalized by the hole-magnon interaction) is supplemented by a new branch of magnetic excitations existing only in the vicinity of the Γ and M points of the Brillouin zone. The radii of these regions are approximately equal to the Fermi momentum. The imaginary part of the frequency of the new excitations is considerably larger than the real part; i.e., the excitations are overdamped magnons. This fact and the position of the new branch in the Brillouin zone mean that with the appearance of such excitations the long-range antiferromagnetic order, implied from the very beginning in the spin-wave approximation, is destroyed. The correlation length ξ of the arising short-range order is connected with the hole concentration per site c (which equals to the concentration x of Sr for compositions of interest[13]) and the intersite distance a by the relation $\xi \approx a/\sqrt{c}$ cited above. The appearance of these overdamped excitations is attributed to an interaction between magnetic excitations of different nature: the magnons of localized spins and the Stoner excitations[14] of mobile holes. For parameters presumably realized in $La_{2-x}Sr_xCuO_4$ the obtained results allow us to explain the extremely low concentrations of holes destroying the long-range ordering in this material.

II. DYSON'S EQUATIONS

As mentioned above, our investigation is based on the t-J Hamiltonian in the spin-wave approximation.[9,10,7] A 2D square lattice is considered. After the unitary transformation which takes into account transversal spin fluctuations this Hamiltonian can be represented in the following form (for details of the derivation of the Hamiltonian see Ref. 7):

$$\mathcal{H} = -\mu \sum_{\mathbf{k}} h_{\mathbf{k}}^{\dagger} h_{\mathbf{k}} + \sum_{\mathbf{k}} \omega_{\mathbf{k}}^{0} b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} + \sum_{\mathbf{kk'}} (g_{\mathbf{kk'}} h_{\mathbf{k}}^{\dagger} h_{\mathbf{k}-\mathbf{k'}} b_{\mathbf{k'}} + \text{H.c.}), \tag{1}$$

where $h_{\mathbf{k}}^{\dagger}$ is the fermion creation operator of a hole with wave vector $\mathbf{k}$ in the classical Néel state $|\mathcal{N}\rangle$ which is the reference state for the boson spin-wave operator $b_{\mathbf{k}}$, i.e., $b_{\mathbf{k}}|\mathcal{N}\rangle = 0$. With good accuracy these two operators can be considered as independent for the states of interest.[7] In Eq. (1), μ is the chemical potential of holes, $\omega_{\mathbf{k}}^{0} = 2J\sqrt{1-\gamma_{\mathbf{k}}^{2}}$ is the unperturbed magnon frequency with the superexchange constant J and $\gamma_{\mathbf{k}} = [\cos(k_x a) + \cos(k_y a)]/2$. The interaction constant $g_{\mathbf{kk'}} = -4t(\gamma_{\mathbf{k}-\mathbf{k'}} u_{\mathbf{k'}} + \gamma_{\mathbf{k}} v_{\mathbf{k'}})/\sqrt{N}$ comprises the effective hopping constant t and the number of sites N, $u_{\mathbf{k}} = \cosh(\alpha_{\mathbf{k}})$, $v_{\mathbf{k}} = -\sinh(\alpha_{\mathbf{k}})$, $\alpha_{\mathbf{k}} = \ln[(1+\gamma_{\mathbf{k}})/(1-\gamma_{\mathbf{k}})]/4$.

For rather general conditions the effective t-J Hamiltonian can be obtained from the Hamiltonian of the extended Hubbard model[5,15] which is widely accepted to give a realistic description of CuO_2 planes of cuprate perovskites. In this case the t-J Hamiltonian and its spin-wave counterpart contain terms describing the static attraction between holes.[15] These terms can lead to the superconducting transition. However, for the description of normal state properties these terms are unessential, and therefore they are omitted in Eq. (1).

The temperature (in units of energy) is supposed to be much smaller than μ and J. This allows us to use the zero-temperature Green's functions

$$G(\mathbf{k}, t-t') = -i\langle \mathcal{T} h_{\mathbf{k}}(t) h_{\mathbf{k}}^{\dagger}(t')\rangle, \quad D(\mathbf{k}, t-t') = -i\langle \mathcal{T} b_{\mathbf{k}}(t) b_{\mathbf{k}}^{\dagger}(t')\rangle, \tag{2}$$

for the holes and magnons, respectively. Here $\mathcal{T}$ is the time ordering operator. From Eq. (1) it is clear that with some modifications which are due to the more complex interaction Dyson's equations for these Green's functions are the same as those for the electron-phonon system in a metal:[12,16]

$$\begin{aligned} D(\mathbf{k}\omega) &= [\omega - \omega_{\mathbf{k}}^{0} - \Pi(\mathbf{k}\omega)]^{-1}, \\ G(\mathbf{k}\omega) &= [\omega + \mu - \Sigma(\mathbf{k}\omega)]^{-1}, \\ \Pi(\mathbf{k}\omega) &= -i \sum_{\mathbf{k'}} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} g_{\mathbf{k'k}} \Gamma(\mathbf{k'}\omega'; \mathbf{k}\omega) G(\mathbf{k'}\omega') G(\mathbf{k'}-\mathbf{k}, \omega'-\omega), \\ \Sigma(\mathbf{k}\omega) &= i \sum_{\mathbf{k'}} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} [g_{\mathbf{kk'}} \Gamma(\mathbf{k}\omega; \mathbf{k'}\omega') D(\mathbf{k'}\omega') \\ &\quad + g_{\mathbf{k}-\mathbf{k'},-\mathbf{k'}} \Gamma(\mathbf{k}-\mathbf{k'}, \omega-\omega'; -\mathbf{k'}, -\omega') D(-\mathbf{k'}, -\omega')] G(\mathbf{k}-\mathbf{k'}, \omega-\omega'), \end{aligned} \tag{3}$$

where the vertex part $\Gamma(\mathbf{k'}\omega'; \mathbf{k}\omega)$ is the sum of all diagrams which connect one magnon and two hole lines. The bare vertex $g_{\mathbf{k'k}}$ and the diagram in Fig. 1 are the terms of the two lowest orders in t in this sum.

In Sec. IV the vertex correction in Fig. 1 will be shown to be much smaller than the lowest order term $g_{\mathbf{k'k}}$ for wave vectors and frequencies of interest. The same is supposed to be also true with respect to higher order corrections. This allows us to substitute $\Gamma(\mathbf{k'}\omega'; \mathbf{k}\omega)$ by the bare vertex $g_{\mathbf{k'k}}$. In this way Eqs. (3) become self-consistent.

The energy spectrum of the one-hole states has been considered by different methods in a number of papers (see, e.g., Refs. 6, 7, 9–11, and 17). The obtained results can be summarized as follows: The spectrum consists of branches of single-particle excitations characterized by different values of the z component S_z of the total spin (in the spin-wave approximation) and a hole-magnon continuum of scattering states. For parameters presumably realized in cuprate perovskites[18] the lowest single-particle branch corresponds to $S_z = \pm 1/2$ and has four equivalent minima at $\mathbf{k}^0 = (\pm\pi/\sqrt{2}a, 0)$ and $(0, \pm\pi/\sqrt{2}a)$. We use the system of coordinates with axes along the diagonals of the plane; in this system the effective mass tensor of the hole band is diagonal. In the vicinity of the minima the band energy $\varepsilon_{\mathbf{k}}$ can be represented in the form $\varepsilon_{\mathbf{k},+1} = (k_x - k_x^0)^2/2m_1 + k_y^2/2m_2$ for the first two minima and $\varepsilon_{\mathbf{k},-1} = k_x^2/2m_2 + (k_y - k_y^0)^2/2m_1$ for the latter two minima. In these regions the band is strongly anisotropic,[17] $m_2/m_1 \approx 5-7$.

For small hole concentrations the overall character of

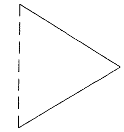

FIG. 1. Lowest order vertex correction. Solid and dashed lines denote hole and magnon Green's functions, respectively.

the hole spectrum is supposed to be the same as for the solitary hole (we shall discuss this supposition in more detail below). We assume additionally that the magnetic properties which we want to consider are mainly determined by the lowest hole band. Consequently, the hole Green's function in Eq. (3) can be approximated by the one-pole expression

$$G(\mathbf{k}\omega) \approx \frac{Z}{\omega - \varepsilon_{\mathbf{k}} + \mu + i\eta\,\mathrm{sgn}(\varepsilon_{\mathbf{k}} - \mu)}, \tag{4}$$

where $\eta \to +0$ and the quasiparticle residue reads

$$Z = \left[1 - \frac{d}{d\omega}\Sigma(\mathbf{k}\omega)\right]^{-1}. \tag{5}$$

III. THE MAGNON SPECTRUM

By making use of the approximations discussed in the previous section, after the integration over ω' the magnon self-energy can be represented in the form

$$\Pi(\mathbf{k}\omega) = Z^2\mathcal{P}\sum_{\mathbf{k}'} g^2_{\mathbf{k}'+\mathbf{k}/2,\mathbf{k}} \frac{n(\mathbf{k}'+\mathbf{k}/2) - n(\mathbf{k}'-\mathbf{k}/2)}{\varepsilon(\mathbf{k}'+\mathbf{k}/2) - \varepsilon(\mathbf{k}'-\mathbf{k}/2) - \omega} + i\pi\,\mathrm{sgn}(\omega)\, Z^2 \sum_{\mathbf{k}'} g^2_{\mathbf{k}'+\mathbf{k}/2,\mathbf{k}}[n(\mathbf{k}'+\mathbf{k}/2) - n(\mathbf{k}'-\mathbf{k}/2)]\delta(\varepsilon(\mathbf{k}'+\mathbf{k}/2) - \varepsilon(\mathbf{k}'-\mathbf{k}/2) - \omega), \tag{6}$$

where $n(\mathbf{k}) = \theta(\mu - \varepsilon_{\mathbf{k}})$ is the occupation number, $\theta(x)$ is the Heavyside step function, and the symbol $\mathcal{P}$ denotes Cauchy's principal value of the integration over $\mathbf{k}'$ into which the summation is transformed. It can be seen from Eqs. (3) and (6) that the self-energy is connected with the generation process of electron-hole pairs corresponding to the bubble diagram.

It will be shown below that dramatic changes of the magnon spectrum are limited to the regions of the Brillouin zone around the Γ $[\mathbf{k}_\Gamma = (0,0)]$ and M $[\mathbf{k}_M = (\sqrt{2}\pi/a, 0)]$ points. The sizes of these regions are shown below to be of the order of the Fermi momentum $k_F \approx \sqrt{2\mu m_1}$. Outside of these regions the spectrum remains practically unchanged. The formulas given below correspond to the region around the Γ point. Respective formulas for the M point can be obtained by substituting $\mathbf{k} - \mathbf{k}_M$ for $\mathbf{k}$.

For small $\mathbf{k}$ vectors near the Γ point the occupation numbers limit the range of integration over $\mathbf{k}'$ in Eq. (6) to vicinities of the minima of the hole band. There the effective mass approximation can be used. For these regions the interaction constant $g_{\mathbf{k}'+\mathbf{k}/2,\mathbf{k}}$ can be approximated by $-(t/\sqrt{N})\sqrt[4]{2/(ka)^2}[\pm(k_x - k_y) \pm (k_x + k_y)]a$ where the signs correspond to the four equivalent minima of the hole band. With these simplifications the imaginary part of the self-energy, which describes the magnon damping, can be transformed to the following form:

$$\mathrm{Im}\,\Pi(\mathbf{k}\omega) = -2\,\mathrm{sgn}(\omega)\,\frac{\pi Z^2 t^2}{B}\sum_{\sigma=\pm 1}\left(ka + \sigma\frac{k_x^2 - k_y^2}{k}a\right)\epsilon_{\mathbf{k}\sigma}^{-1/2}[\theta(\nu_{\mathbf{k}\sigma} + \omega)\sqrt{\nu_{\mathbf{k}\sigma} + \omega} - \theta(\nu_{\mathbf{k}\sigma} - \omega)\sqrt{\nu_{\mathbf{k}\sigma} - \omega}], \tag{7}$$

where $B = (\pi/a)^2/2\sqrt{m_1 m_2}$ is of the order of the hole bandwidth, $\nu_{\mathbf{k}\sigma} = 2\mu - \epsilon_{\mathbf{k}\sigma}/2 - \omega^2/2\epsilon_{\mathbf{k}\sigma}$, $\epsilon_{\mathbf{k},+1} = k_x^2/2m_1 + k_y^2/2m_2$, and $\epsilon_{\mathbf{k},-1} = k_x^2/2m_2 + k_y^2/2m_1$.

Let us analyze the damping as given by Eq. (7). For this purpose one should find the quasiparticle residue (5). Using approximation (4) for small hole concentrations one finds from Eq. (3)

$$\frac{d}{d\omega}\Sigma(\mathbf{k}^0, \varepsilon_{\mathbf{k}^0}) = -Z\sum_{\mathbf{k}}\frac{g^2_{\mathbf{k}^0\mathbf{k}}}{(\varepsilon_{\mathbf{k}^0} - \varepsilon_{\mathbf{k}^0-\mathbf{k}} - \omega^0_{\mathbf{k}})^2}, \tag{8}$$

where $\mathbf{k}^0$ is the wave vector of one of the hole band minima. The usage of the one-pole approximation (4) in Eq. (8) gives an upper bound for the value of Z (see the corresponding discussion in Ref. 10). For parameters[18] of La_2CuO_4 the effective value of J/t can be estimated[7] to fall into the range 0.1–0.3 where the hole bandwidth is $B \approx 3J$.[11,17] This bandwidth is only slightly larger than the limiting magnon frequency. By making use of the effective mass approximation for the integration in the vicinity of the hole band minima in (8), one finds

$$Z^2 \approx w\frac{JB}{\pi t^2}, \tag{9}$$

for these parameters, where $w \approx 0.4$–0.7. With this value we obtain from Eq. (7) the rough estimate for the damping

$$\mathrm{Im}\,\Pi(\mathbf{k}\omega) \approx Jka\sqrt{\frac{\mu}{\epsilon_{\mathbf{k}\sigma}}}. \tag{10}$$

In accordance with Eq. (7), the following conditions have to be fulfilled to have a finite damping:

$$\epsilon_{\mathbf{k}\sigma} - 2\sqrt{\mu\epsilon_{\mathbf{k}\sigma}} < \omega_{\mathbf{k}} < \epsilon_{\mathbf{k}\sigma} + 2\sqrt{\mu\epsilon_{\mathbf{k}\sigma}},$$

where $\omega_{\mathbf{k}}$ is the magnon frequency. These inequalities give conditions for the Landau damping[16] due to the generation of electron-hole pairs by magnons. For magnons with wave vectors $k \lesssim k_F$ these conditions can be reduced to the inequality

$$u \lesssim \sqrt{\frac{2\mu}{m_1}}, \tag{11}$$

where $u = \omega_{\mathbf{k}}/k$ is the velocity of the spin waves. For unperturbed magnons ($\omega^0_{\mathbf{k}} \approx \sqrt{2}Jak$) and small hole concentrations, condition (11) cannot be fulfilled, since in this case the hole bandwidth is only slightly larger than the unperturbed magnon bandwidth as already mentioned. Of course, even if long-wavelength magnons do not decay, there can be a finite damping for short-wavelength high-frequency unperturbed magnons. However, for small hole concentrations this damping is negligibly small in comparison with the real part of their frequency; cf. Eq. (10). It will be shown in the following that the interaction leads to the appearance of an additional branch of low-frequency magnons for which condition (11) can be fulfilled even for small hole concentrations. Moreover, it turns out that inequality (11) coincides with the condition for the existence of this branch.

Near the Γ point the real part of the magnon self-energy (6) can be represented in the form

$$\mathrm{Re}\,\Pi(\mathbf{k}\omega) = \frac{\sqrt{2}\pi Z^2 t^2}{B} \sum_{\sigma=\pm 1} \left(ka + \sigma \frac{k_x^2 - k_y^2}{k} a \right) \mathcal{I}(s_{\mathbf{k}\sigma}, q_{\mathbf{k}\sigma}), \tag{12}$$

with $s_{\mathbf{k}\sigma} = \omega/2\sqrt{\mu\epsilon_{\mathbf{k}\sigma}}$ and $q_{\mathbf{k}\sigma} = \sqrt{\epsilon_{\mathbf{k}\sigma}/4\mu}$. The function

$$\mathcal{I}(s,q) = \begin{cases} -1 + \cos\Phi - \frac{1}{2\pi}\frac{|s|}{q} \int_0^{\Phi} \frac{d\phi}{\cos^2\phi} \ln \left| \frac{(1-q^2)\cos^2\phi + s^2 - 2|s|\cos\phi\sqrt{q^2\cos^2\phi - q^2 + 1}}{(1-q^2)\cos^2\phi + s^2 + 2|s|\cos\phi\sqrt{q^2\cos^2\phi - q^2 + 1}} \right|, & q > 1, \\ -1 - \frac{1}{2\pi}\frac{|s|}{q} \int_0^{\pi/2} \frac{d\phi}{\cos^2\phi} \ln \left| \frac{s^2 - (1 - q^2 + 2|s|q)\cos^2\phi}{s^2 - (1 - q^2 - 2|s|q)\cos^2\phi} \right|, & q < 1, \end{cases} \tag{13}$$

with $\cos\Phi = \sqrt{1 - q^{-2}}$ is shown in Fig. 2. We note that $\mathcal{I}(-s,q) = \mathcal{I}(s,q)$.

It follows from Eq. (3) that the renormalized magnon frequencies are determined by the equation

$$\frac{\omega}{k} = \sqrt{2}Ja + \frac{1}{k}\mathrm{Re}\,\Pi(\mathbf{k}\omega). \tag{14}$$

The difference in the components of the effective mass tensor introduces an unessential dependence of the magnon frequencies on the wave vector direction. To simplify the further discussion we put $m_1 = m_2 = m$. In this case $\epsilon_{\mathbf{k},+1} = \epsilon_{\mathbf{k},-1}$, $s = u/v$, $q = k/2k_F$, and $v = k_F/m$ is the hole velocity at the Fermi level. Thus, the renormalized magnon frequencies are determined by the intersection of the plane $u = vs$ [the left hand side of Eq. (14)] with the surface $I(s,q) = \sqrt{2}Ja + \mathrm{Re}\,\Pi/k$. The latter has the shape shown in Fig. 2 with the respective shift and change of scale for the $\mathcal{I}$ axis. In accordance with estimate (9), the absolute value of I on the plateau in the region $|s| + q < 1$ is approximately one order of magnitude smaller than the unperturbed magnon velocity $\sqrt{2}Ja$. Therefore, an intersection of the plateau by the plane vs will mean an appearance of a slow low-frequency magnon. An example of such an intersection is shown in Fig. 3.

Let us derive a condition for the existence of the low-frequency magnons. Since the plateau is positioned in the region $|s| + q < 1$ and "the height" of the plateau is equal to $u_0 = \sqrt{2}Ja - 2\sqrt{2}\pi a Z^2 t^2/B$, the low-frequency magnons will appear if

$$u_0 < v = \sqrt{\frac{2\mu}{m}}. \tag{15}$$

This coincides with the condition for a nonzero damping, Eq. (11). The magnons exist in the region

$$\frac{k}{2k_F} < 1 - \frac{u_0}{v} \tag{16}$$

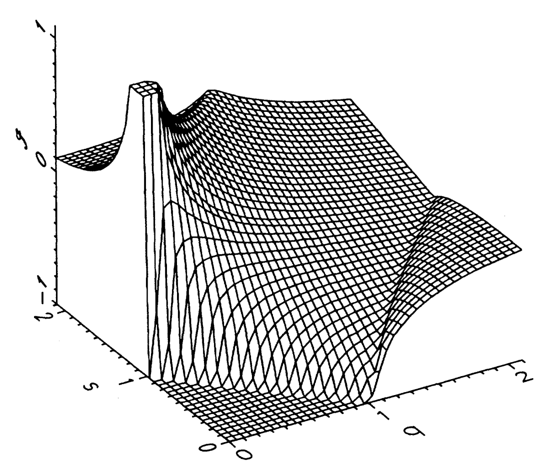

FIG. 2. Function $\mathcal{I}(s,q)$, Eq. (13), which is proportional to the real part of the magnon self-energy. $\mathcal{I}(s,q)$ diverges when $q \to 0$ and $s \to 1$.

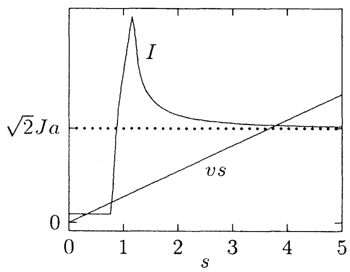

FIG. 3. Graphical solution of Eq. (14) for renormalized magnon frequencies in the case $q = 0.2$. The dotted line is the velocity of unperturbed magnons.

near the Γ point and in the region of an analogous radius near the M point. The damping (10) is much larger than the real part of the magnon frequency $u_0 k$ [see Eqs. (15) and (9)]. Thus, when the low-frequency magnons exist they are overdamped.

The overdamping of the low-frequency branch means that the occupation numbers of the respective magnon modes become indefinite. This contradicts the initial assumption of the spin-wave approximation about small values of these numbers. Thus, the appearance of such excitations means that the spin-wave approximation becomes inapplicable to regions of CuO_2 planes which are larger than the characteristic length

$$\xi \approx \frac{1}{k_F} \approx \frac{a}{\sqrt{x}}, \tag{17}$$

i.e., the inverse of the limiting wave vector of the overdamped branch [see Eq. (16)]. The relaxational excitations of this branch can be interpreted as a movement of the axes of the staggered magnetization in regions of size (17). This statement can be reformulated in the following form: The spin correlation function decays with distance as $\langle s_{\mathbf{l}}^z s_{\mathbf{0}}^z \rangle \sim \exp(-|\mathbf{l}|/\xi)$, where $s_{\mathbf{l}}^z$ denotes the z component of the spin at the position $\mathbf{l}$. Thus, ξ is the instantaneous spin correlation length. As mentioned above, the dependence $\xi(x)$ described by Eq. (17) has been observed[1] in $La_{2-x}Sr_xCuO_4$.

From Eq. (9) and condition (15) it is easy to estimate the critical concentration of holes which corresponds to the appearance of the overdamped magnons and to the destruction of the long-range antiferromagnetic order. For the parameters mentioned above it appears to be equal to several percent. This is close to the hole concentration at which the 3D long-range antiferromagnetic order in $La_{2-x}Sr_xCuO_4$ is destroyed.[1,2] Since we are dealing with a 2D model, this comparison needs some additional comments. In the previous considerations the long-range antiferromagnetic ordering has been implied from the very beginning. However, it is known[19] that such ordering is impossible for any nonzero temperature in the 2D case. In the real crystal the ordering is possible due to weak 3D corrections. Nevertheless we suppose that our consideration is applicable even to the ideal 2D case at finite but low temperatures (the temperature T has to be much smaller than μ/k_B and $\sqrt{2}Jak_F/k_B$ where k_B is the Boltzmann constant). Indeed, the correlation length of the temperature induced spin fluctuations is exponentially large,[20] $\xi' \sim \exp(J/k_BT)$ for $J \approx 0.1$eV (cf. Ref. 18) and low temperatures. This correlation length exceeds considerably the correlation length (17) of the spin fluctuations generated by the holes. Because of this difference in scale our result that an additional branch of overdamped magnons appears at some hole concentration remains unchanged. But the above observation that for this hole concentration the infinite instantaneous spin correlation length decreases to the value ξ given by Eq. (17) has to be changed: Now the exponentially large correlation length ξ' decreases to ξ. The small value of the interplane exchange constant[1] means that large 2D regions of ordered spins play the crucial role in establishing the 3D long-range ordering.[20] Thus, the critical value of the hole concentration, which corresponds to the drastic decrease of the 2D correlation length, can be identified with the hole concentration destroying the 3D long-range antiferromagnetic ordering and can therefore be compared with the respective experimentally determined concentration of Sr in $La_{2-x}Sr_xCuO_4$.

It follows from estimation (9) that the value of u_0 and the real part of the excitation frequency $\omega_{\mathbf{k}}$ may be negative [as mentioned, $\mathcal{I}(s,q) = \mathcal{I}(-s,q)$; the intersection of the surface I and the plane vs occurs at a negative value of s in this case]. However, the imaginary part of the frequency remains much larger than $|\mathrm{Re}\,\omega_{\mathbf{k}}|$ and therefore this case is equivalent to the case $\mathrm{Re}\,\omega_{\mathbf{k}} > 0$. The large damping does not allow one to interpret the negative sign of $\mathrm{Re}\,\omega_{\mathbf{k}}$ as an indication that a superstructure (like the spiral state, etc.) occurs in the spin alignment.

From Figs. 2 and 3 it can be seen that, besides the discussed intersection with the plateau in the region of small s, the plane vs crosses the wall surrounding the plateau and, for the small hole concentrations considered, the nearly horizontal part of the surface in the region of large s. This second plateau of I is positioned at a value of approximately $\sqrt{2}Ja$, i.e., the velocity of unperturbed magnons. The intersection with the wall corresponds to excitations with negligibly small quasiparticle residues [see Eq. (5) with Π substituted for Σ]. Indeed,

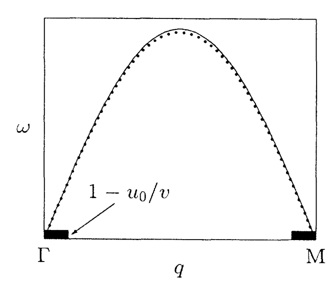

FIG. 4. Magnon branches along the line connecting the Γ and M points. Overdamped magnons are indicated by thick lines. Solid and dotted lines are the renormalized and initial branches of usual magnons, respectively.

the wall has a large slope and for small hole concentrations $d\Pi(\mathbf{k}\omega)/d\omega = 2w(\sqrt{2}Ja/v)d\mathcal{I}/ds \gg 1$. Therefore these excitations can be neglected. The intersection with the second plateau yields the branch of usual magnons which are only slightly perturbed by a small hole concentration. The magnon branches are displayed in Fig. 4. As explained above, a part of the branch of overdamped magnons exists also near the M point.

It follows from Eqs. (1) and (6) that the overdamped magnons appear and the long-range antiferromagnetic order is destroyed, because electron-hole pairs are generated by magnons. In the considered situation the magnons are the excitations of localized spins, while the electron-hole pairs are the Stoner excitations[14] of mobile spins connected with holes. Equations (11) and (15), which give the condition for the appearance of the branch, coincide with the condition that the magnon branch submerges into the continuum of the Stoner excitations where the magnons undergo the Landau damping.[16]

The overdamped magnons contribute to hole self-energy (3). Because of the magnon damping, for small hole concentrations their influence is small in contrast to that of the usual magnon branch. This small contribution is the main change in the equation for the hole energies in comparison with the case of a solitary hole. Therefore the corresponding deviations of the hole spectrum from the spectrum of a solitary hole can be neglected as done above.

IV. MIGDAL'S THEOREM

In the previous sections it was supposed that vertex corrections could be neglected. Now the validity of this assumption will be verified by comparing the bare vertex $g_{\mathbf{k}_1\mathbf{k}}$ with the first correction given by the diagram in Fig. 1.

The diagram corresponds to the following formula:

$$\Gamma^{(1)}(\mathbf{k_1}\omega_1,\mathbf{k}\omega) = i\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\sum_{\mathbf{k}'}[g_{\mathbf{k_1}+\mathbf{k}',\mathbf{k}'}g_{\mathbf{k_1}+\mathbf{k}'-\mathbf{k},\mathbf{k}'}D(\mathbf{k}'\omega') + g_{\mathbf{k_1}-\mathbf{k},-\mathbf{k}'}g_{\mathbf{k_1},-\mathbf{k}'}D(-\mathbf{k}',-\omega')]$$
$$\times\, g_{\mathbf{k_1}+\mathbf{k}',\mathbf{k}}G(\mathbf{k_1}+\mathbf{k}',\omega_1+\omega')\,G(\mathbf{k_1}+\mathbf{k}'-\mathbf{k},\omega_1+\omega'-\omega). \quad (18)$$

In accordance with the previous considerations the magnon momentum $\mathbf{k}$ in Eq. (18) can be considered to be small while the hole momentum $\mathbf{k_1}$ is approximately equal to one of wave vectors $\mathbf{k^0}$ of the hole band minima. By making use of approximation (4) for the hole Green's function and integrating over ω', one can see that the contribution from the poles of the magnon Green's function becomes negligibly small after the integration over $|\mathbf{k}'|$. The main contribution to correction (18) is due to the poles of the hole propagators. The occupation numbers, which arise in $\Gamma^{(1)}$ after the integration over ω', and the small values of $\mathbf{k}$ limit the integration over $\mathbf{k}'$ to the regions where the wave vector $\mathbf{k_1}+\mathbf{k}'$ is near the Fermi surface. Out of the four regions of this kind, two around $\mathbf{k}'=\mathbf{k}_\Gamma$ and $\mathbf{k}'=\mathbf{k}_M$, give the main and equal contributions. We consider the former case for which the hole energies in Eq. (18) can be approximated[16] as $\varepsilon_{\mathbf{k_1}+\mathbf{k}'}-\mu \approx v(|\mathbf{k}'|-k_F)$ and $\varepsilon_{\mathbf{k_1}+\mathbf{k}'-\mathbf{k}}-\mu \approx v(|\mathbf{k}'|-k_F) - v\mathbf{k}\cdot\mathbf{k}'/|\mathbf{k}'|$.

After these simplifications the integration over $|\mathbf{k}'|$ in Eq. (18) can be easily performed. Contributions of the usual and the overdamped magnon branches should be considered separately. For small hole concentrations the velocity u of the former spin waves is much larger than the hole velocity v. This allows us to estimate the contribution of the branch as

$$\Gamma_1^{(1)} \approx g_{\mathbf{k_1}\mathbf{k}}k_F a\frac{v}{u} \ll g_{\mathbf{k_1}\mathbf{k}}. \quad (19)$$

The contribution of the overdamped magnons is nearly purely imaginary due to the large magnon damping. The absolute value of this contribution is approximately given by

$$|\Gamma_2^{(1)}| \approx g_{\mathbf{k_1}\mathbf{k}}k_F a \ll g_{\mathbf{k_1}\mathbf{k}}. \quad (20)$$

Thus, both contributions are small in comparison with the bare vertex. For the electron-phonon system considered by Migdal[12,16] the smallness of the vertex corrections is provided by a small ratio of the limiting frequency of phonons to the Fermi energy. In our case in which the ratio of the analogous energy parameters is the opposite the smallness is connected with the small Fermi energy.

V. CONCLUDING REMARKS

Our consideration was essentially based on the picture of mobile holes which is inherent to the metallic phase. In fact, it has been found experimentally[21] that in the doping regime $0.02 < x \lesssim 0.06$, where the destruction of the long-range ordering takes place, the resistance within the CuO_2 planes is metallic and linear in T over a wide range of temperatures above 80 K. However, below this temperature logarithmic corrections to the resistivity appear, leading to a crossover to semiconducting behavior. Such logarithmic corrections are pertinent to the case of weak localization when the wave functions can still be labeled approximately by wave vectors and the hole-magnon interaction differs from the interaction in the metallic phase only for extremely small wave vectors. We suppose that our considerations are applicable also to this case. In support of this conjecture the following experimental fact can be adduced: The dependence $\xi(x)$ described by Eq. (17) is observed not only in the metallic but also in the semiconducting phase.[1] Besides, there are experimental indications of a smooth variation of the magnetic properties across the metal-semiconductor boundary.[1] Similarly the experimental result[1,2] that the critical concentration of holes for temperatures down to $T \approx 10$ K remains practically the same as for $T \approx 80$ K indicates that the processes which destruct the long-range order are the same in the metallic and semiconducting phases.

Apart from Ref. 3, several investigations were undertaken to explain the extreme sensitivity of the Néel state on the doping with holes. Two papers[22,23] are based on models analogous to the one considered here. The major difference to the present one is the following: In both studies[22,23] expansions in powers of $x \sim k_F^2$ of the magnon self-energy are used. Such an approach does not allow one to describe the self-energy properly in the region $k \lesssim k_F$ where the plateau of I is positioned. We note that only in this region the self-energy differs significantly from zero. As a result, the branch of overdamped magnons, which is in our opinion the central result of the present paper explaining the destruction of the long-range antiferromagnetic ordering, has not been found there. Instead, in Ref. 22 the vanishing spin-wave velocity of long-wavelength excitations has been interpreted as an indication of the destruction of the ordering. As mentioned above, in our opinion the applicability of the used power expansion to the long-wavelength magnons is doubtful. In Ref. 23 a damping of long-wavelength magnons of the slightly renormalized usual branch has been considered as the criterion for the destruction of the long-range ordering. The analogous criterion has also been used in Ref. 24 for explaining the empirically established dependence $\xi \approx a/\sqrt{x}$. We have already pointed out that in $La_{2-x}Sr_xCuO_4$ a damping of these magnons is possible only for hole concentrations much larger than those which destroy the Néel state in this crystal.

In summary, we have shown that for values of parameters presumably realized in $La_{2-x}Sr_xCuO_4$ doping of the 2D t-J model by several percent of holes leads to the appearance of an additional branch of overdamped magnons. This means that at this critical concentration of holes the long-range antiferromagnetic ordering, which is assumed to exist at lower concentrations, is substituted by the short-range ordering with an instantaneous spin correlation length $\xi \approx a/\sqrt{x}$. These results allow one to explain the extremely small dopant concentration destroying the Néel state in $La_{2-x}Sr_xCuO_4$ and the observed values of the correlation length. Besides, we have demonstrated that for the model considered Migdal's theorem is valid.

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