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Spin waves in quasiperiodic layered ferromagnets

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We consider a layered ferromagnetic superlattice with spins S_a and S_b. The exchange energies are J_1 and J_2 between layers, which are arranged in a quasiperiodic Fibonacci sequence, and J in each layer. When $J_1=J_2=J$ we use a rescaling approach to obtain an exact *decimation transformation for local magnon of layers* $\alpha,\beta,(\gamma)$, and $\delta(\tau)$ in the ferromagnets. Iteration of the transformation provides numerical results for the local density of states (LDOS) and the magnetization. We found that the bandwidths of the LDOS of layers are the functions of $\eta=S_b/S_a$.

The experimental discovery of quasicrystalline order has opened a new field of research to both experimentalists and theoreticians.[1-9] The theoretical investigation of quasicrystals has concentrated on the electronic and phonon spectra, and magnetic properties. Recent studies concerning the dispersion relation of spin waves, energy structures, and magnetic specific heats of quasiperiodic superlattices have pointed out some nontrivial results.[2-9] Other theoretical work is devoted to the study of the low-temperature properties of such superlattices for a variety of artificial structures.[10-12]

Among analytic methods, the real-space renormalization-group (RSRG) scheme developed by Kohomoto, Kadanoff, and Tang (KKT) has been widely used by many authors.[2] Another decimation scheme was proposed by Niu and Nori on the transfer model of a Fibonacci chain.[3] The KKT scheme establishes a recursion relation between the transfer matrices on a Fibonacci chain and provides information on the global energy spectrum and wave functions. An altogether different RSRG scheme than the KKT scheme is a decimation RSRG scheme proposed by Ashraff and Stinchombe to obtain the average electronic density of states.[5] Chakrabarti, Karmakar, and Moitra and Zhong *et al.*[6] developed the method to find the local electronic density of states.

In this paper we present a model of a quasiperiodic layered ferromagnet with exchange interactions J_1,J_2 and spins S_a,S_b, which is a different artificial model from the quasiperiodic superlattice in Ref. 8 and the periodic superlattice with impurity layers in Ref. 11. A RSRG approach had been used to obtain the Green's functions of the system, with the resulting rescaling transformation of dynamic variables; we examined the effect on the local density of states (LDOS) and magnetization of various layers with various $\eta=S_b/S_a$.

We consider a simple cubic ferromagnetic layered superlattice. Each layer with spin $S_a(S_b)$ is a two-dimensional (2D) square lattice with lattice constant a and nearest-neighbor exchange interaction J. Between the layers closest to each other, the exchange interactions can be J_1 or J_2, and are arranged according to a Fibonacci sequence. If we consider the layers with various spins S_a and S_b and exchange couplings J_1 and J_2, each of the layers must be in any one of five different nearest-neighbor surroundings. We shall adopt the following notation for the site energy in layer i, $\varepsilon_i(i=\alpha,\beta,\gamma,\delta,\text{ and }\tau)$. The layer α with spin S_a is located between two J_1s, the layers β and γ with spin S_b are located between J_1 and J_2 and have nearest-neighbor layers with spins S_a and S_b; the layers δ and τ have both nearest-neighbor layers with spin S_b and share common characteristics with sites β and γ. In the xz plane the structure and all kinds of layers are depicted in Fig. 1. There are five kinds of layers in this model which is two more than in the model of Ref. 9, since we take spins S_a and S_b. The Hamiltonian can be written as follows:

$$H=-\sum_{ij}\sum_{\nu\mu}J_{ij}^{\nu\mu}\mathbf{S}_{i\nu}\cdot\mathbf{S}_{i\mu}\ , \tag{1}$$

where i,j are the index numbers of layers, and ν,μ are the index numbers of sites belonging to layer $i(j)$. Nearest-neighbor interactions are considered only. We define the Green's function thus

$$G_{ij}^{\nu\mu}(\omega)=\langle\langle S_{i\nu}^{+};S_{j\mu}^{-}\rangle\rangle_{\omega}\ , \tag{2}$$

and it satisfies the equation of motion

$$\omega G_{ij}^{\nu\mu}(\omega)=\langle(S_{i\nu}^{+},S_{j\mu}^{-})\rangle+\langle\langle(S_{i\nu}^{+},H);S_{j\mu}^{-}\rangle\rangle_{\omega}\ . \tag{3}$$

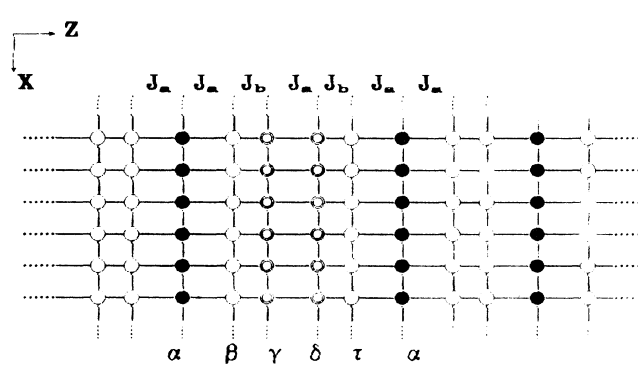

FIG. 1. The quasiperiodic layered ferromagnetic structure in the xz plane. There are five kinds of sites along the z direction, where ● S_a, (○)(⊙) S_b.

Because there is translation invariance in the xy plane, we can perform a Fourier transformation,

$$G_{ij}^{\nu\mu}(\omega)=\frac{2}{N}\sqrt{S_{i\nu}S_{j\mu}}\sum_{\mathbf{K}}F_{ij}(\omega,\mathbf{K})\exp[i\mathbf{K}\cdot(\mathbf{R}_\nu-\mathbf{R}_\mu)]\,, \tag{4}$$

where N is the number of sites in the xy plane, $\mathbf{K}=(K_x,K_y)$ and $\mathbf{R}=(R_x,R_y)$ are the wave vector and coordinate vector in the xy plane, respectively. We use the random-phase approximation, and taking a layer α as a reference layer, from (3) we obtained

$$\begin{aligned}
(\omega-\varepsilon_\alpha)F_{00}&=1+t_1F_{-10}+t_1F_{10}\,,\\
(\omega-\varepsilon_\beta)F_{10}&=t_1F_{00}+t_2F_{20}\,,\\
(\omega-\varepsilon_\delta)F_{20}&=t_2F_{10}+t_1F_{30}\,,\\
(\omega-\varepsilon_\tau)F_{30}&=t_1F_{20}+t_2F_{40}\,,\\
(\omega-\varepsilon_\gamma)F_{40}&=t_2F_{30}+t_1F_{50}\,,
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
&F_{ij}=F_{ij}(\omega,\mathbf{K})\,,\\
&\varepsilon_\alpha=4J_1S_b+4JS_a(2-\cos K_xa-\cos K_ya)\,,\\
&\varepsilon_\beta=\varepsilon_\gamma=2(J_1S_a+J_2S_b)\\
&\qquad+4JS_b(2-\cos K_xa-\cos K_ya)\,,\\
&\varepsilon_\delta=\varepsilon_\tau=2(J_1+J_2)S_b\\
&\qquad+4JS_b(2-\cos K_xa-\cos K_ya)\,,\\
&t_1=-2J_1\sqrt{S_aS_b}\,,\\
&t_2=-2J_2S_b\,.
\end{aligned} \tag{6}$$

The matrix element F_{ij} can be obtained from (5) using a rescaling or decimation transformation.[9-13] The decimation is achieved by removing appropriate sites following rule I: $AB\rightarrow A', A\rightarrow B'$ or rule II: $BA\rightarrow A', A\rightarrow B'$. The transformation decimations associated with a length rescaling factor $b=\tau=(1+\sqrt{5})/2$ according to rules I and II are given by

$$\begin{aligned}
&t_1'=\frac{t_1t_2}{\omega-\varepsilon_\beta},\quad t_2'=t_1\,,\\
&\varepsilon_\alpha'=\varepsilon_\delta+\frac{t_1^2}{\omega-\varepsilon_\tau}+\frac{t_1^2}{\omega-\varepsilon_\beta}\,,\\
&\varepsilon_\beta'=\varepsilon_\gamma+\frac{t_2^2}{\omega-\varepsilon_\tau},\quad \varepsilon_\gamma'=\varepsilon_\alpha+\frac{t_1^2}{\omega-\varepsilon_\beta}\,,\\
&\varepsilon_\alpha'=\varepsilon_\alpha+\frac{t_1^2}{\omega-\varepsilon_\beta},\quad \varepsilon_\tau'=\varepsilon_\gamma+\frac{t_1^2}{\omega-\varepsilon_\beta}\,,
\end{aligned} \tag{7a}$$

and

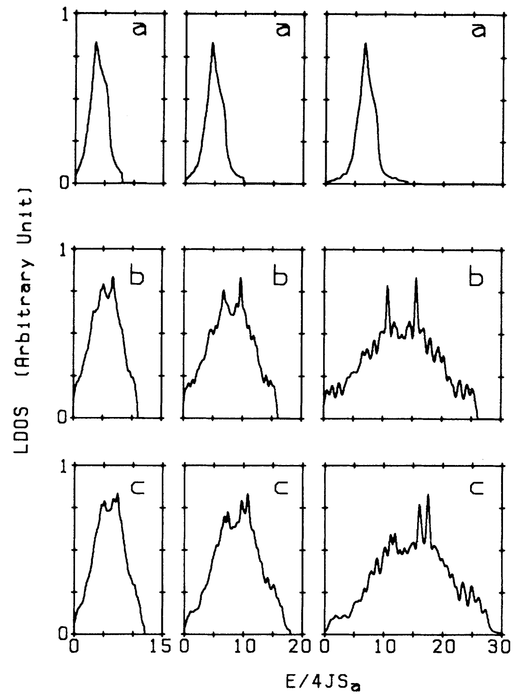

FIG. 2. The local density of states (LDOS). $J_1=J_2=J, \eta=S_b/S_a=2,3,5$, the energy unit is $4JS_a$, where a is for layer α, b for layer $\beta(\gamma)$, and c for layer $\delta(\tau)$.

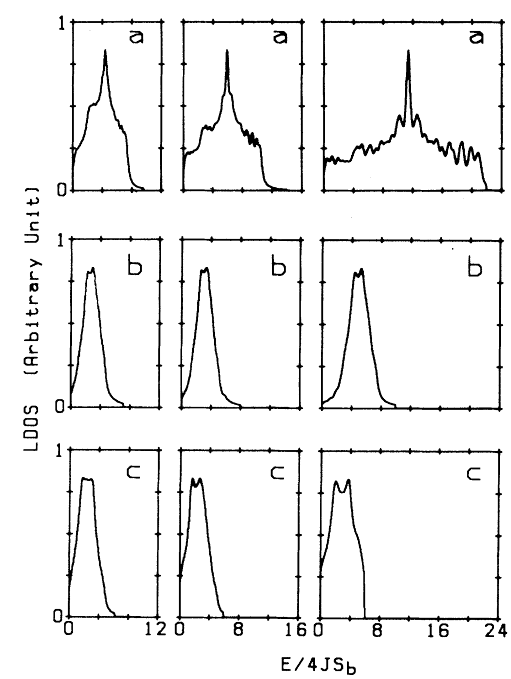

FIG. 3. The local density of states (LDOS). $J_1=J_2=J, \eta=S_b/S_a=1/2,1/3,1/5$, the energy unit is $4JS_b$, where a is for layer α, b for layer $\beta(\gamma)$, and c for layer $\delta(\tau)$.

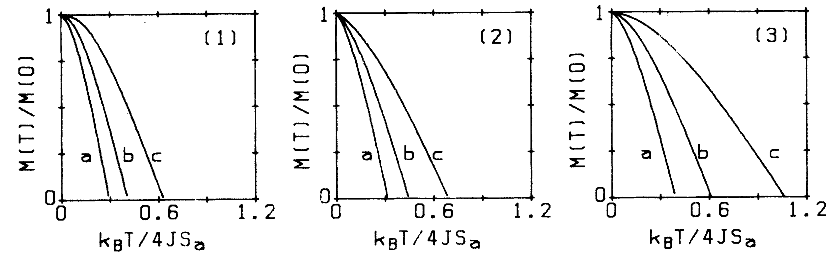

FIG. 4. The magnetization M/M_0-k_BT, (1) in layer α, (2) in layer $\beta(\gamma)$, (3) in layer $\delta(\tau)$, the curves a,b,c correspond to $\eta=2,3,5$, respectively.

$$t_i'=\frac{t_1t_2}{\omega-\varepsilon_\delta},\quad t_2'=t_1\ ,$$
$$\varepsilon_\alpha'=\varepsilon_\tau+\frac{t_1^2}{\omega-\varepsilon_\delta}+\frac{t_2^2}{\omega-\varepsilon_\gamma}\ ,$$
$$\varepsilon_\beta'=\varepsilon_\alpha+\frac{t_2^2}{\omega-\varepsilon_\gamma},\quad \varepsilon_\gamma'=\varepsilon_\beta+\frac{t_2^2}{\omega-\varepsilon_\delta}\ , \tag{7b}$$
$$\varepsilon_\delta'=\varepsilon_\beta+\frac{t_2^2}{\omega-\varepsilon_\gamma},\quad \varepsilon_\tau'=\varepsilon_\gamma+\frac{t_1^2}{\omega-\varepsilon_\gamma}\ ,$$

respectively. The LDOS $\rho_i(\omega)$ is related to the imaginary part of the local Green's function $G(\omega)$. By using Eq. (4),

$$\rho_i(\omega)=-\frac{1}{\pi}\,\mathrm{Im}\sum_{\mathbf{K}\in BZ}F_{ii}(\omega,\mathbf{K})\ , \tag{8}$$

where i ($i=\alpha,\beta,\gamma,\delta$, and τ). The low-temperature local magnetization can be obtained by

$$M_i=M_0\left[1-\int_0^\infty\frac{\rho_i(\omega)}{e^{\beta\omega}-1}d\omega\right]\ , \tag{9}$$

where $\beta=1/k_BT$ and M_0 is the local magnetization when $T=0$ K.

To obtain the LDOS of each layer, we can use the various combinations of rules I and II to calculate the local Green's functions. The transform T_i is for layer i ($i=\alpha,\beta,\gamma,\delta$, and τ), with T_α (I−I−II), T_β(II−I−II), T_γ(I−II−I), T_δ(I−II−II) and T_τ(II−I−I). Because $\varepsilon_\beta=\varepsilon_\gamma,(\varepsilon_\delta=\varepsilon_\tau),T_\beta$ (T_δ) is the same as T_γ (T_τ), so we only need three transforms T_α (for layer α), T_β (for layers β and γ), and T_δ (for layers δ and τ). Obviously when $S_a=S_b$, $\varepsilon_\beta=\varepsilon_\gamma=\varepsilon_\delta=\varepsilon_\tau$, we only need the transforms T_α and T_β.[9]

Our previous work dealt with the effect of different $\lambda=J_2/J_1$, for $S_a=S_b$; now for an understanding of the effect of various S_a and S_b, we take $J_1=J_2=J$, $\eta=S_b/S_a$. This model is different from other artificial structures,[8–12] in fact, our model describes a structure in which the layers of impurity spins S_a are embedded nonuniformly in the layered base-material spins S_b; the nearest exchange interactions between layers are arranged on Fibonacci sequences. We calculated the LDOS of layers α, $\beta(\gamma)$, and $\delta(\tau)$ associated with various η by the above decimation procedure. The results in Figs. 2 and 3 show that the deviations of the LDOS in quasiperiodic systems compared with the 3D periodic system depend on various η values. The bandwidths Δ_i of layers i ($i=\alpha,\beta,\gamma,\delta$, and τ) are functions of η. For various $\eta>1$, the bandwidths are $\Delta_\alpha=8JS_a(2+\eta)$, $\Delta_\beta=\Delta_\gamma=4JS_a(1+5\eta)$, $\Delta_\alpha=\Delta_\tau=24JS_a\eta$. For various $\eta<1$, the bandwidths are $\Delta_\alpha=16JS_b(1/2+1/\eta)$, $\Delta_\beta=\Delta_\gamma=4JS_b(1/\eta+5)$, $\Delta_\delta=\Delta_\tau=24JS_b$. When $\eta>1$, the magnetizations of each layer increase as η increases and the magnetization of layer α(δ or τ) is minimum (maximum) for a fixed η and a given temperature. In spite of the procedure for determining the magnetization from the spin-wave spectrum, it is only accurate near zero temperature, and estimating the tendency of the Curie temperature T_C is possible. From Fig. 4 we can see that the Curie temperature T_C is mainly determined by spin S_a and increases as η increases. When $\eta<1$ the magnetizations of layers α, β, and γ increase as η decreases, the magnetizations of layers $\delta(\tau)$ is almost unaffected by η. The magnetization of layer α(δ or τ) is maximum (minimum) for a fixed η and a given temperature, therefore the Curie temperature T_C of the system mainly depends on S_b (see Fig. 5). For both of the cases $\eta>1$ and $\eta<1$, the LDOS show a single main peak for the layer α and double main peaks for the others, but there is a difference between the two cases, so the magnetizations are different when $\eta>1$ and $\eta<1$.

We studied a layered structure ferromagnet with various spins S_a and S_b, where in each layer the exchange

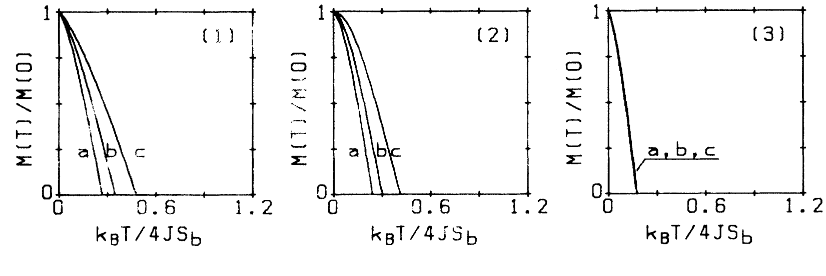

FIG. 5. The magnetization M/M_0-k_BT, (1) in layer α, (2) in layer $\beta(\gamma)$, (3) in layer $\delta(\tau)$, the curves a,b,c correspond to $\eta=1/2$, 1/3,1/5, respectively.

constant is J. The exchange constants between nearest layers J_1 and J_2 are arranged according to a Fibonacci sequence. Within the random-phase approximation, an exact decimation approach for obtaining the local Green's functions has been presented, and the exact results for the LDOS of layers in the quasiperiodic superlattice are obtained. The numerical procedure based on RSRG methods provides a very direct way of obtaining information about the quasiperiodic superlattice spectrum properties. We have calculated the LDOS and the reduced magnetization of layers i ($i=\alpha,\beta,\gamma,\delta$, and τ) for various η when $J_1=J_2=J$. We found that the bandwidths and the magnetizations of the layers are related to η. The LDOS and the local magnetizations of different layers are quite different for different η.

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