Theory of the perpendicular magnetoresistance in magnetic multilayers

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By starting with the Boltzmann equation, we calculate the transport properties of magnetic multilayers for currents perpendicular to the layers. Our model takes into account both volume and interface spin-dependent scattering. We show that the macroscopic equations already used by Johnson *et al.* or van Son *et al.* are justified if the spin-diffusion length is much longer than the mean free path, even for individual layer thicknesses of the order of the mean free path. But, second, we show that Johnson's assumption of additive effects from independent interfaces in multilayers is incorrect and we obtain different results by taking into account the interplay between successive interfaces. The simple expressions derived for individual thicknesses much shorter than the spin-diffusion length are in agreement with the analysis of experimental results already proposed. It turns out that data on the perpendicular magnetoresistance can be used to separate clearly the volume and interface contributions to the spin-dependent scattering.

I. INTRODUCTION

The discovery of giant magnetoresistance (GMR) effects in Fe/Cr multilayers[1,2] has triggered a large number of studies on the transport properties of magnetic multilayers. In most experiments the current flows in the plane of the layers, this is the CIP (current in plane) geometry. However, Pratt *et al.*[3] have extended the MR measurements to the case where the current is perpendicular to the plane, the so-called CPP geometry. For Ag/Co multilayers the CPP-MR is several times larger than the CIP-MR.

For the CIP geometry, classical[4] and quantum[5,6] models, both based on the existence of spin-dependent scattering, have been worked out. Their common physical content is that the electrons average the properties of the multilayers in the perpendicular direction on the length scale of the electron mean free path (MFP) λ. This implies that the GMR vanishes when the period of the multilayer becomes larger than the MFP.[4-6]

For the CPP geometry, the experimental results of Pratt *et al.*[3] have been accounted for in a two-current scheme with volume and interface resistances in series for each spin direction.[7] Alternatively, Johnson[8] proposed to explain the CPP-MR of multilayers by using the concept of "spin-coupled interface resistance." This concept was previously introduced by Johnson and Silsbee[9] and independently by van Son, van Kempen, and Wyder[10] to describe the electron transport through an interface between ferromagnetic and nonmagnetic metals. The point is as follows: if, in the ferromagnet, the current is spin polarized, there will be spin accumulation around the interface with the nonmagnetic metal. This spin accumulation gives rise to an extra potential drop ΔV_I, proportional to the current density J, $\Delta V_I = J r_{SI}$ where r_{SI} is the

"spin-coupled interface resistance" (this effect does not appear in the CIP geometry because there is no net charge or spin transport through the interfaces and therefore no spin accumulation). In his extension to multilayers[8] Johnson assumes that the spin-coupled interface resistances of successive interfaces (calculated for an isolated interface[9,10]) are additive.

We show here that in the limit appropriate for the experiments, i.e., with thicknesses much shorter than the spin-diffusion length (SDL) l_{sf}, the assumption of additive interface resistances is incorrect. The spin accumulations induced by successive interfaces interfere and partly balance each other. This gives a behavior of the CPP-MR definitely different from that obtained by Johnson.[8] The second point is that the macroscopic equations used in Refs. 8–10 are, *a priori*, not valid if the layer thicknesses become of the order of the MFP. Thus, we will first investigate a Boltzmann equation model to assess more precisely the domain of validity of the macroscopic approach. It will be shown that this approach is justified for the limit where the SDL is much longer than the MFP.

Other models of the CPP-MR have been recently proposed by Zhang and Levy,[11] and Bauer.[12] These models, respectively developed in the Kubo and Landauer formalisms of quantum transport, both assume no spin-flip scattering (i.e., independent spin ↑ and spin ↓ channels). The present paper, which does not assume this from the beginning, shows that this is justified only if the layer thicknesses are much smaller than the SDL. This condition was fulfilled in the first experiments of Pratt and co-workers[3,7] (see also Appendix A), but not in the most recent ones where Mn impurities have been intentionally introduced in the silver layers of Co/Ag multilayers in order to shorten the SDL.[13] Here we will get analytical

expressions of the CPP-MR, valid whatever the ratio between the layer thicknesses and the SDL (if we assume MFP $\ll$ SDL). We think this is essential to gain a deeper physical understanding of the phenomenon, and to extract physically significant parameters from the experiments.[13]

Section II will be devoted to our Boltzmann equation model and to the justification of the macroscopic equations for $\lambda \ll l_{sf}$. Then, in Sec. III, we will calculate the CPP resistance of magnetic multilayers in this limit, first in illustrative simple cases, then in the most general situation. In Sec. IV, we will summarize and comment on our theoretical results.

II. BOLTZMANN EQUATION MODEL

In this section we show that a Boltzmann equation model reduces to macroscopic transport equations when the SDL is much longer than the MFP (whatever the ratio of the layer thicknesses to the MFP is).

We consider a structure where single-domain ferromagnetic (F) metal layers alternate with nonferromagnetic (N) metal layers. In all the layers we assume an oversimplified single parabolic conduction band, with the same effective mass m and Fermi velocity v_F which simplifies the calculation. However our calculation can also be extended without essential change to the case with different values of m and v_F in the magnetic and normal layers. A given current density J flows along the z axis perpendicular to the plane of the layers, and we will consider only configurations where the magnetization of a given F layer is "up" or "down" along the x axis taken as spin quantization axis. Because the magnetizations are all colinear, we can introduce the local velocity distribution function of the conduction electrons for the spin direction s, $f_s(z,\mathbf{v})$. Throughout the paper our notation is $\pm$ for the absolute spin direction ($s_x = \pm\frac{1}{2}$) and $\uparrow, \downarrow$ for majority and minority spin directions (respectively, spin antiparallel and parallel to the local magnetization) in a magnetic layer. We limit our analysis to zero temperature, where the electron-magnon spin-flip scattering is frozen out so that the spin-flip scattering events are through spin-orbit interactions on defects or impurities, and also through exchange scattering by paramagnetic moments diluted in the nonmagnetic layers.[14] Thus, we look for a distribution function $f_s(z,\mathbf{v})$ solution of the linearized Boltzmann equation:

$$v_z \frac{\partial f_s}{\partial z}(z,\mathbf{v}) - eE(z)v_z \frac{\partial f^0}{\partial \varepsilon}(v) = \int d^3v' \delta[\varepsilon(v') - \varepsilon(v)]P_s[z,\varepsilon(v)][f_s(z,\mathbf{v}') - f_s(z,\mathbf{v})]$$

$$+ \int d^3v' \delta[\varepsilon(v') - \varepsilon(v)]P_{sf}[z,\varepsilon(v)][f_{-s}(z,\mathbf{v}') - f_s(z,\mathbf{v})] , \tag{1}$$

where $-e$ and $\varepsilon(v) = \frac{1}{2}mv^2$ denote, respectively, the charge and energy of the electrons, and $E(z) = -\partial V(z)/\partial z$ is the local electric field. The $P_s(z,\varepsilon)$ and $P_{sf}(z,\varepsilon)$ are, respectively, spin conserving and spin-flip transition probabilities. They are assumed to be isotropic in velocity space, so that $P_{sf}(z,\varepsilon)$ does not transfer momentum between the two spin channels. $f_s(z,\mathbf{v})$ is written by adding up the Fermi-Dirac distribution $f^0(v)$ and small perturbations:

$$f_s(z,\mathbf{v}) = f^0(v) + \frac{\partial f^0}{\partial \varepsilon}\{[\mu^0 - \mu_s(z)] + g_s(z,\mathbf{v})\} , \tag{2}$$

where μ^0 is the equilibrium chemical potential. In addition to the anisotropic part of the electron distribution perturbation $(\partial f^0/\partial \varepsilon)g_s(z,\mathbf{v})$ existing in CIP geometry (see Ref. 4), we have introduced an isotropic term expressing the local variations of the chemical potential for spin s, $\mu_s(z)$, in order to account for spin accumulation.[10] By introducing Eq. (2) in the Boltzmann Eq. (1), and keeping only linear terms in perturbation, we obtain

$$v_z \frac{\partial g_s}{\partial z}(z,\mathbf{v}) + \left[\frac{1}{\tau_s} + \frac{1}{\tau_{sf}}\right]g_s(z,\mathbf{v})$$

$$= \left[v_z \frac{\partial \bar{\mu}_s}{\partial z}(z) + \frac{\bar{\mu}_s(z) - \bar{\mu}_{-s}(z)}{\tau_{sf}}\right] , \tag{3}$$

where $\bar{\mu}_s(z) = \mu_s(z) - eV(z)$ is the electrochemical potential for spin s. The relaxation times τ_s and τ_{sf} are related to the corresponding transition probabilities in the standard way. The term proportional to τ_{sf}^{-1} in the right-hand side of Eq. (3) expresses the relaxation of the spin polarization (spin accumulation) by spin-flip scattering. As a noticeable difference from CIP geometry, instead of a given homogeneous electric field, the driving term in Eq. (3) is an unknown function of z. However, and this represents a great simplification compared to the CIP case, the problem has a cylindrical symmetry around the z axis, thus $g_s(z,\mathbf{v})$ can be developed in Legendre polynomials (LP) of $\cos\theta$, where θ is the angle between the velocity and the z axis:

$$g_s(z,\mathbf{v}) = \sum_{n=1}^{\infty} g_s^{(n)}(z)P_n(\cos\theta) \tag{4}$$

where the zero-order (isotropic) term is absent since $(\partial f^0/\partial \varepsilon)g_s(z,\mathbf{v})$ was defined by Eq. (2) as the anisotropic part of the electron distribution perturbation. This means that the CPP geometry allows us to separate the dependence in $\mathbf{v}$ and z. Thus substituting Eq. (4) in Eq. (3) and projecting on the complete basis of LP, as detailed in Appendix B, yields an infinite chain of differential equations:

$$\frac{\partial g_s^{(1)}}{\partial z} = \lambda_s \frac{\bar{\mu}_s - \bar{\mu}_{-s}}{l_s^2} \ ,$$

$$\frac{2}{5} \frac{\partial g_s^{(2)}}{\partial z} - \frac{\partial \bar{\mu}_s}{\partial z} = -\frac{g_s^{(1)}}{\lambda_s} \ , \tag{5}$$

$$\frac{n+1}{2n+3} \frac{\partial g_s^{(n+1)}}{\partial z} + \frac{n}{2n-1} \frac{\partial g_s^{(n-1)}}{\partial z} = -\frac{g_s^{(n)}}{\lambda_s} \ , \quad n > 1 \ ,$$

where we have introduced

$$\lambda_s = v_F (1/\tau_s + 1/\tau_{sf})^{-1}$$

and

$$l_s = [\tfrac{1}{3}(v_F \lambda_s)\tau_{sf}]^{1/2} = (D_s \tau_{sf})^{1/2} \ ,$$

respectively the local electron mean free path and spin-diffusion length for spin s; D_s is the diffusion constant. The physical meaning of Eq. (5) appears more clearly when using the identity $J_s = \kappa g_s^{(1)}$ [where $\kappa = \sigma_s/(e\lambda_s)$ is independent of s and of the material in our single band model, and σ_s is the conductivity for spin s], with J_s the current density of spin s (see Appendix B). This leads to the new system of equations

$$\frac{e}{\sigma_s} \frac{\partial J_s}{\partial z} = \frac{\bar{\mu}_s - \bar{\mu}_{-s}}{l_s^2} \ ,$$

$$\frac{\partial \bar{\mu}_s}{\partial z} = \frac{e}{\sigma_s} J_s + \frac{2}{5} \frac{\partial g_s^{(2)}}{\partial z} \ , \tag{6a}$$

$$g_s^{(2)} + \frac{3}{7} \lambda_s \frac{\partial g_s^{(3)}}{\partial z} = -\frac{2}{3\kappa} \lambda_s \frac{\partial J_s}{\partial z} \ ,$$

$$\frac{n}{2n-1} \lambda_s \frac{\partial g_s^{(n-1)}}{\partial z} + g_s^{(n)} + \frac{n+1}{2n+3} \lambda_s \frac{\partial g_s^{(n+1)}}{\partial z} = 0 \ , \tag{6b}$$

$$n > 2 \ .$$

Equations (6a) are just the macroscopic transport equations in 1D including spin relaxation, already introduced in slightly different forms by Johnson and Silsbee in Ref. 9 and van Son, van Kempen, and Wyder in Ref. 10, *except for the supplementary term* $\frac{2}{5} (\partial g_s^{(2)}/\partial z)$ *on the right-hand side*. At this stage, we can introduce a formal solution of Eq. (6b) as

$$g_s^{(n)}(z) = -\frac{2}{3\kappa} \int_{-\infty}^{\infty} d\bar{z}\, G_s^{(n)}(z,\bar{z}) \left[\lambda_s(\bar{z}) \frac{\partial J_s}{\partial \bar{z}} \right] \ , \tag{7}$$

where $G_s^{(n)}(z,\bar{z})$ is the Green function of Eq. (6b), i.e., satisfies:

$$G_s^{(2)}(z,\bar{z}) + \frac{3}{7} \lambda_s(z) \frac{\partial G_s^{(3)}(z,\bar{z})}{\partial z} = \delta(z - \bar{z}) \ ,$$

$$\frac{n}{2n-1} \lambda_s(z) \frac{\partial G_s^{(n-1)}(z,\bar{z})}{\partial z} + G_s^{(n)}(z,\bar{z})$$

$$+ \frac{n+1}{2n+3} \lambda_s(z) \frac{\partial G_s^{(n+1)}(z,\bar{z})}{\partial z} = 0 \ , \quad n > 2 \ . \tag{8}$$

Note the explicit dependence of $G_s^{(n)}(z,\bar{z})$ on both variables, and not only on their difference, because of the lack of translational invariance in the z direction. Substituting Eq. (7) in Eq. (6a), we can write

$$\frac{e}{\sigma_s} \frac{\partial J_s}{\partial z} = \frac{\bar{\mu}_s - \bar{\mu}_{-s}}{l_s^2} \ ,$$

$$\frac{\partial \bar{\mu}_s}{\partial z}(z) = \frac{e}{\sigma_s(z)} \left[J_s(z) - \frac{4}{15} \lambda_s(z) \frac{\partial}{\partial z} \int_{-\infty}^{+\infty} d\bar{z}\, G_s^{(2)}(z,\bar{z}) \left[\lambda_s(\bar{z}) \frac{\partial J_s}{\partial \bar{z}}(\bar{z}) \right] \right] \ . \tag{9}$$

Equation (9) shows explicitly that the "Boltzmann correction" beyond the macroscopic transport equations breaks the locality of the linear response relation between the electrochemical potential gradient and the current. $\partial \bar{\mu}_s/\partial z$ at a given point no longer depends only on the current at the same point; but also on the current divergence integrated over a domain centered at this point, and extending upon the decay length of $G_s^{(2)}(z,\bar{z})$, i.e., upon a length of the order of the MFP because this is the unique scaling length of Eq. (8). However, in the present case, it is possible to obtain a little more from Eq. (9) without explicit calculation of the Green function. As a matter of fact, the appearance of a current divergence for spin s occurs [despite the conservation of the total current verified by Eq. (6a)] because of the spin relaxation mechanisms which take place on the length scale of the SDL. This means that quite generally

$$\lambda_s \frac{\partial J_s}{\partial z} \approx \frac{\lambda_s}{l_{sf}} J \ ,$$

with

$$(1/l_{sf})^2 = (1/l_\uparrow)^2 + (1/l_\downarrow)^2 \ ,$$

see Ref. 10. For a multilayer with layer thicknesses $t \ll l_{sf}$, it turns out, in fact, that

$$\lambda_s \frac{\partial J_s}{\partial z} \approx \frac{t}{l_{sf}} \frac{\lambda_s}{l_{sf}} J$$

as we will show in the following; and thus the whole "Boltzmann correction" in Eq. (9) is proportional to λ_s/l_{sf}.

Thus we can make the statement that, regardless of the layer thicknesses, the macroscopic transport equations

$$\frac{e}{\sigma_s}\frac{\partial J_s}{\partial z}=\frac{\bar{\mu}_s-\bar{\mu}_{-s}}{l_s^2}\ ,\tag{10}$$

$$J_s=\frac{\sigma_s}{e}\frac{\partial \bar{\mu}_s}{\partial z}\tag{11}$$

are recovered in the limit $\lambda_s/l_{sf}\ll 1$. Equation (11) is just *the Ohm's law,* and Eq. (10) expresses that, in steady state, *the spin accumulation related to the spin current divergences is balanced by the spin-flip processes.* And λ_s/l_{sf}, which is of the same order of magnitude as $(\tau_s/\tau_{sf})^{1/2}$, is quite small in the systems of interest: $(\tau_s/\tau_{sf})^{1/2}\leq 10^{-1}$ for noble and $3d$ metals at liquid helium temperature (see Appendix A). As a reasonably good approximation for the systems of interest, we will thus assume that Eqs. (10) and (11) are valid from here on.

III. MACROSCOPIC MODEL

In Sec. II, we have shown that, in the limit where the SDL is much longer than the MFP, the Boltzmann equation model reduces to a macroscopic model, i.e., a model in which the current densities are related to the electrochemical potentials by Eqs. (10) and (11) involving macroscopic transport coefficients, σ_s (conductivity) and l_s (SDL). As shown in Appendix A, this condition is reasonably fulfiled, at least at low temperature and for metals with moderate spin-orbit coupling. The macroscopic approach is that previously adopted by Johnson and co-worker[8,9] and van Son, van Kempen, and Wyder.[10] Our calculation differs from these previous ones in that we consider the specific case of a multilayer and we treat the interferences between spin accumulations at successive interfaces.

First of all we transform Eqs. (10) and (11) in a more directly usable way. We write the spin-dependent electrochemical potential as $\bar{\mu}_\pm=\bar{\mu}\pm\Delta\mu$, where $\Delta\mu$ is the term related to the spin accumulation. In the free-electron model we consider here, $\Delta\mu$ is related to the out-of-equilibrium magnetization ΔM by

$$|\Delta\mu|=2\mu_0|\Delta M|/(3n\mu_B)\ ,$$

where n is the electron density and μ_B the Bohr magneton. The gradient of the spin-independent part of $\bar{\mu}_\pm$ divided by e:

$$F(z)=\frac{1}{e}\frac{\partial\bar{\mu}}{\partial z}\tag{12}$$

is equivalent to an electric field. Thus Eqs. (10) and (11) transform into

$$\frac{e}{\sigma_\pm}\frac{\partial J_\pm}{\partial z}=\pm 2\frac{\Delta\mu}{l_s^2}\ ,$$

$$J_\pm(z)=\sigma_\pm\left[F(z)\pm\frac{1}{e}\frac{\partial\Delta\mu}{\partial z}\right]\ .\tag{13}$$

This leads to a spin-diffusion type equation for $\Delta\mu$ (Ref. 10):

$$\frac{\partial^2\Delta\mu}{\partial z^2}=\frac{\Delta\mu}{l_{sf}^2}\tag{14}$$

where

$$(1/l_{sf})^2=(1/l_\uparrow)^2+(1/l_\downarrow)^2\ ,$$

is the appropriate "average" spin-diffusion length, as already mentioned by van Son, van Kempen, and Wyder[10]; it also leads to

$$\frac{\partial^2}{\partial z^2}(\sigma_+\bar{\mu}_++\sigma_-\bar{\mu}_-)=0\ .\tag{15}$$

These two second-order differential equations have the following general solution in an homogeneous medium:

$$\Delta\mu=A\,\exp(z/l_{sf})+B\,\exp(-z/l_{sf})\ ,\tag{16}$$

$$(\sigma_+\bar{\mu}_++\sigma_-\bar{\mu}_-)=Cz+D\ .\tag{17}$$

At this stage, if we introduce a bulk spin asymmetry coefficient β in the F layers,

$$\rho_{\uparrow(\downarrow)}=1/\sigma_{\uparrow(\downarrow)}=2\rho_F^*[1-(+)\beta]\tag{18}$$

and write in the N layers $\rho_{\uparrow(\downarrow)}=2\rho_N^*$, we obtain the general expressions of $\bar{\mu}_\pm(z)$, $F(z)$, and $J_\pm(z)$ given in Appendix C.

Obviously, they still depend on various integration constants, which have to be determined in each layer by taking into account the proper boundary conditions at each interface. At an interface located at $z=z_0$, J_+ and J_- must be continuous (we neglect the spin relaxation at the interfaces[15]):

$$J_s(z=z_0^+)-J_s(z=z_0^-)=0\tag{19}$$

while $\bar{\mu}_+$ and $\bar{\mu}_-$ are continuous only if there is no interface scattering. In the presence of interface scattering (i.e., *significant scattering localized in an interfacial zone that is supposed to be infinitesimally thin*), the potential conditions are

$$\bar{\mu}_s(z=z_0^+)-\bar{\mu}_s(z=z_0^-)=r_s[J_s(z=z_0)/e]\ ,\tag{20}$$

where r_s is the spin-dependent boundary resistance for a unit surface of the F/N interfaces. In the same way as in the bulk, we introduce an interfacial spin asymmetry coefficient γ according to

$$r_{\uparrow(\downarrow)}=2r_b^*[1-(+)\gamma]\ .\tag{21}$$

Using the general solutions Eqs. (C1)–(C6) given in Appendix C, and taking into account the boundary conditions Eqs. (19) and (20) at each interfaces, all the quantities of interest in any multilayer structure can be calculated.

To allow the reader to distinguish clearly our calculation for multilayers from that by Johnson,[8] we will present calculations for isolated and interfering interfaces successively. As the calculation in the most general case, i.e., with both bulk and interface spin-dependent scattering, is needlessly intricate, we differ the general case until the end of the Sec. III C; and we begin by illustrating the main physical ideas with calculations in simpler cases:

(A) isolated interface with bulk spin-dependent scattering only; (B) multilayer with bulk spin-dependent scattering only.

A. Isolated interface

We illustrate this case with the example of an interface located at $z = 0$, assumed to have zero resistance, separat-
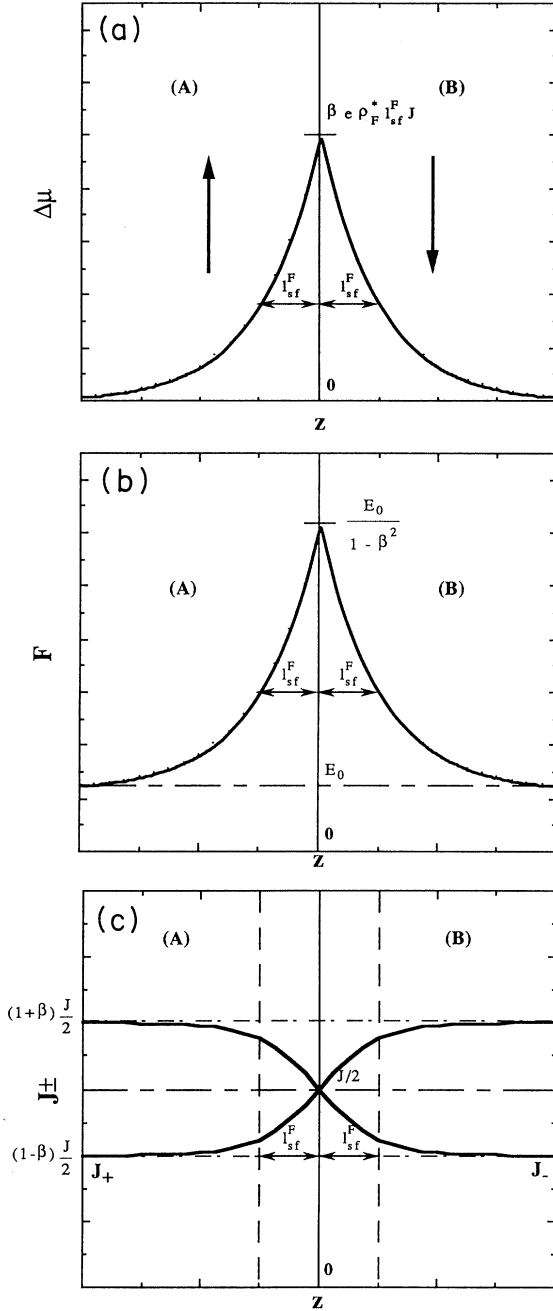

FIG. 1. (a) Chemical potential $\Delta\mu$, (b) electric field F, and (c) current densities J_+ and J_- vs z in the case of an isolated interface separating two semi-infinite domains with opposite magnetizations. The plotted variations are derived from Eqs. (22)–(24). The arrows represent the magnetization of the layers.

ing two semi-infinite ferromagnetic metals having opposite magnetizations, see Fig. 1(a). According to Eq. (18), we have on the left (A), $\sigma_\pm = [2\rho_F^*(1\pm\beta)]^{-1}$; and on the right (B), $\sigma_\pm = [2\rho_F^*(1\mp\beta)]^{-1}$. The figure is drawn for $\beta > 0$ (i.e., $\sigma_\uparrow > \sigma_\downarrow$) and $J > 0$ (electrons flowing from the right to the left). In such a case of semi-infinite domains, one has to retain in the expressions (C1)–(C3) given in Appendix C, only the exponential terms which decay when going away from the interface. Furthermore, because we assume $r_b^* = 0$ in Eq. (21), we have continuity of both electrochemical potentials and currents at the interface. This leads to the following solution in (A):

$$\Delta\mu(z) = \frac{\beta}{1-\beta^2} eE_0 l_{sf}^F \exp\left[\frac{z}{l_{sf}^F}\right], \tag{22}$$

$$F(z) = E_0\left[1 + \frac{\beta^2}{1-\beta^2}\exp\left[\frac{z}{l_{sf}^F}\right]\right], \tag{23}$$

$$J_+(z) = (1-\beta)\frac{J}{2}\left[1 + \frac{\beta}{1-\beta}\exp\left[\frac{z}{l_{sf}^F}\right]\right],$$

$$J_-(z) = (1+\beta)\frac{J}{2}\left[1 - \frac{\beta}{1+\beta}\exp\left[\frac{z}{l_{sf}^F}\right]\right], \tag{24}$$

where E_0 is the unperturbed electric field (i.e., far from the interface): $E_0 = (1-\beta^2)\rho_F^* J$. The corresponding expressions in (B) are simply obtained by changing $\exp(z/l_{sf}^F)$ into $\exp(-z/l_{sf}^F)$ and positive into negative signs in Eqs. (22)–(24).

The overall variations of $\Delta\mu$, F, and $J_\pm$ with z are represented in Figs. 1(a)–1(c). In comparison with the case where the direction of magnetization is the same from $z = -\infty$ to $z = +\infty$, the second term of Eqs. (23) introduces an additional voltage drop

$$\Delta V_I = \int_{-\infty}^{+\infty} [F(z) - E_0]dz = 2\beta^2\rho_F^* l_{sf}^F J .$$

This means that the reversal of the magnetization introduces an interface resistance per unit area:

$$r_{SI} = 2\beta^2\rho_F^* l_{sf}^F . \tag{25}$$

r_{SI} is the spin-coupled interface resistance of Johnson and Silsbee[9] or van Son, van Kempen, and Wyder,[10] calculated here for a pedagogically simple case. Its physical meaning is that, for $\beta\neq 0$, *the spin accumulation significantly reduces the current asymmetry and increases the electric field over a length l_{sf} on both sides of the interface. This adds an interface resistance of the order of magnitude of $\rho_F^* l_{sf}^F$.*

B. Multilayer, bulk spin-dependent scattering

Now, we consider a multilayer composed of ferromagnetic layers (ρ_F^*, $\beta\neq 0$, thickness t_F) alternating with nonmagnetic layers (ρ_N^*, thickness t_N). We still neglect any true interfacial resistance r_b; and, for simplicity, we assume the same value of the spin-diffusion length l_{sf} in the magnetic and nonmagnetic layers.

We consider first the case where the magnetizations of adjacent magnetic layers are antiparallel (AP). In a mag-

netic layer with "up" magnetization [layer A in Fig. 2(a)], Eqs. (C1)–(C6) of Appendix C leads to

$$\Delta\mu(z) = 2\mu_F^{AP} \sinh\left[\frac{z - z_A}{l_{sf}}\right], \tag{26}$$

$$F(z) = E_F + \frac{2\beta\mu_F^{AP}}{el_{sf}} \cosh\left[\frac{z - z_A}{l_{sf}}\right], \tag{27}$$

$$J_\pm(z) = (1 \mp \beta)\frac{J}{2} \pm \frac{\mu_F^{AP}}{e\rho_F^* l_{sf}} \cosh\left[\frac{z - z_A}{l_{sf}}\right], \tag{28}$$

where E_F would be the electric field within an infinitely thick layer of the magnetic material in the presence of the current J, i.e., $E_F = (1 - \beta^2)\rho_F^* J$, and z_A is the center position of the layer A. In the magnetic layers with reverse magnetization [layer C in Fig. 2(a)], $F(z)$ is unchanged, the sign of $\Delta\mu$ is reversed, J_+ exchanges it expression with J_-; and obviously z_A becomes z_C. In a nonmagnetic layer [layer B in Fig. 2(a)] we have, very similarly,

$$\Delta\mu(z) = 2\mu_N^{AP} \cosh\left[\frac{z - z_B}{l_{sf}}\right], \tag{29}$$

$$F(z) = E_N = \rho_N^* J, \tag{30}$$

$$J_\pm(z) = \frac{J}{2} \pm \frac{\mu_N^{AP}}{e\rho_N^* l_{sf}} \sinh\left[\frac{z - z_B}{l_{sf}}\right]. \tag{31}$$

The coefficients μ_F^{AP} and μ_N^{AP} are determined by the continuity requirements on $\mu_\pm$ and $J_\pm$ at each interface. Their general and intricate expressions are not required here. The profile of the periodic variation of $\Delta\mu(z)$, $F(z)$, and $J_\pm(z)$ is shown in Figs. 2(a)–2(c).

The case of a multilayer with parallel (P) arrangement of magnetizations can be treated in the same way. The only qualitative difference is that the z dependence of $\Delta\mu$ in the nonmagnetic layers becomes like $\sinh[(z - z_B)/l_{sf}]$ instead of $\cosh[(z - z_B)/l_{sf}]$, and thus inversely the z dependence of $J_\pm$ becomes like $\cosh[(z - z_B)/l_{sf}]$ instead of $\sinh[(z - z_B)/l_{sf}]$, leading to the new schemes of periodic variations of $\Delta\mu(z)$, $F(z)$, and $J_\pm(z)$ represented in Figs. 2(d)–(f).

In both cases of P and AP configurations, the potential drop over long distance has no contribution from the oscillatory part $\Delta\mu$ of the electrochemical potentials. Thus, the equivalent areal resistance of one period can be calculated by integrating $F(z)$ over z and dividing the results by J. In the spirit of Johnson's paper,[8] we write the resistance per unit area of one bilayer of the multilayer:

$$r^{(P,AP)} = r_0 + 2r_{SI}^{(P,AP)}$$

$$\text{with } r_0 = (1 - \beta^2)\rho_F^* t_F + \rho_N^* t_N, \tag{32}$$

where r_0 is just the resistance of a nonmagnetic layer in series with a magnetic one, when both of them are supposed to be infinitely thick [i.e., if $(t_F, t_N) \gg l_{sf}$], and we sort a factor of 2 in front of the spin-coupled interfacial part $r_{SI}^{(P,AP)}$ because of the two interfaces belonging with one bilayer. In the present case, from Eqs. (27) and (30) and the equivalent ones in the P configuration, we obtain

straightforwardly

$$r_{SI}^{(P)} = 2\beta^2 \frac{\rho_N^* \rho_F^*}{\rho_F^* \coth(t_N/2l_{sf}) + \rho_N^* \coth(t_F/2l_{sf})} l_{sf},$$

$$r_{SI}^{(AP)} = 2\beta^2 \frac{\rho_N^* \rho_F^*}{\rho_F^* \tanh(t_N/2l_{sf}) + \rho_N^* \coth(t_F/2l_{sf})} l_{sf}. \tag{33}$$

Contrary to what Johnson assumed in Ref. 8, it is clear from Eq. (33) that the difference $r_{SI}^{(AP)} - r_{SI}^{(P)}$ (which is the absolute CPP-MR) is not simply related in general to the spin-coupled interfacial resistance r_{SI}^0 of an isolated F/N interface, which is just a generalization of Eq. (25) (see Refs. 9 and 10):

$$r_{SI}^0 = 2\beta^2 \frac{\rho_N^* \rho_F^*}{\rho_F^* + \rho_N^*} l_{sf}. \tag{34}$$

It is only in the limit $(t_F, t_N) \gg l_{sf}$, where obviously one recovers independent interfaces, that both $r_{SI}^{(P)}$ and $r_{SI}^{(AP)}$ admit r_{SI}^0 as an asymptotic value, as this can be easily verified from Eq. (33).

In the opposite limit, $(t_F, t_N) \ll l_{sf}$ which is certainly the most meaningful to discuss existing experimental results (see Appendix A), the whole concept of spin-coupled interfacial resistance becomes irrelevant. As a matter of fact, if we retain only the terms of first order in t_F/l_{sf} and t_N/l_{sf}, we obtain from Eqs. (26)–(31) in the AP configuration $J_\pm = J/2$ in both A and B with:

$$\Delta\mu(z) = \beta e\rho_F^* J(z - z_A), \quad F = \rho_F^* J \tag{35}$$

in A, and

$$\Delta\mu(z) = \beta e\rho_N^* Jt_F, \quad F = \rho_N^* J \tag{36}$$

in B. In comparison with the case of isolated interfaces, Eqs. (22)–(24), the interference between opposite spin accumulations at successive interfaces which are apart by less than l_{sf} reduces the amplitude of $\Delta\mu$ by a factor t/l_{sf}. The oscillations of J_+ and J_- are also reduced and, to the first order in $(t_F, t_N)/l_{sf}$, J_+ and J_- are equal. It is straightforward to show that the amplitude of the oscillations of J_+ and J_- is only of the order $(t_F t_N)/l_{sf}^2$. The picture that emerges for the antiparallel case in the limit $(t_F, t_N) \ll l_{sf}$ is thus very simple: the electrons, in each spin channel, go through a series of layers having effective resistances $\rho_F^* = t_F(\rho_\uparrow + \rho_\downarrow)/2$ and $2\rho_N^* t_N$. The resistance scheme that corresponds to this is represented in Fig. 2(g). The case of a multilayer with ferromagnetic arrangement can be treated in the same way. The final result corresponds to the resistance scheme of Fig. 2(h): there are again two independent spin channels with a succession of resistances $2(1 - \beta)\rho_F^* t_F$ and $2\rho_N^* t_N$ in one spin channel and $2(1 + \beta)\rho_F^* t_F$ and $2\rho_N^* t_N$ in the other. Thus, in order to summarize, if

$$(t_F, t_N) \ll l_{sf},$$

$$r^{(P,AP)} = \frac{1}{1/r_+^{(P,AP)} + 1/r_-^{(P,AP)}}$$

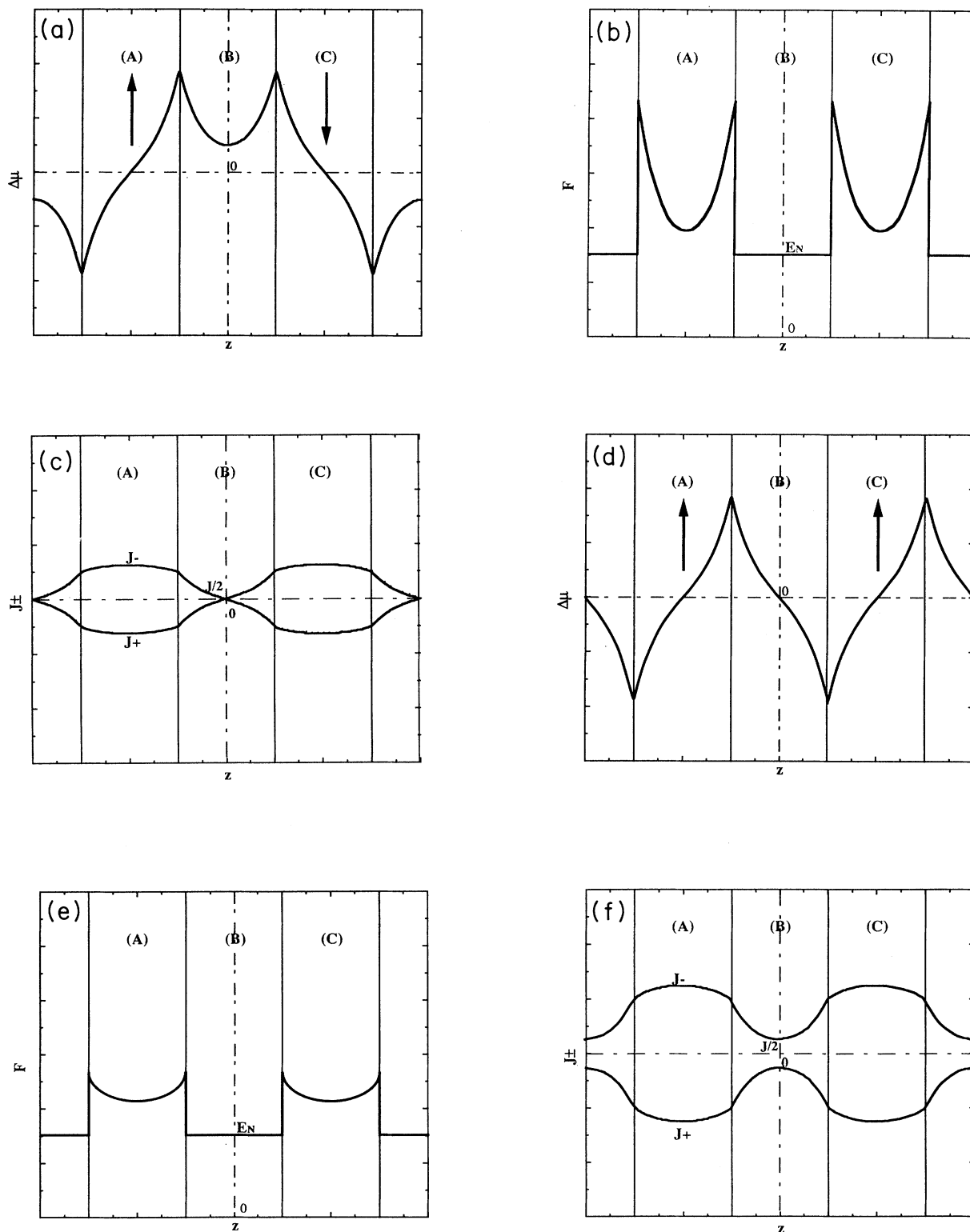$$\text{with } r_{+(-)}^{(AP)} = (r_+^{(P)} + r_-^{(P)})/2 \tag{37}$$

FIG. 2. Chemical potential terms $\Delta\mu$, electric field F, and current densities $J\pm$ for a multilayer with bulk spin-dependent scattering only in the respective cases of (a),(b),(c) AP configuration and (d),(e),(f) P configuration; the plotted variations are derived from Eqs. (26)–(31). The schemes (g) and (h) represent respectively the equivalent resistance array giving the potential drop coming from the pseudoelectric field F ($\Delta\mu$ is periodic and does not contribute to the potential drop over long distances) in the limit $(t_N, t_F) \ll l_{sf}$ for AP and P alignment.
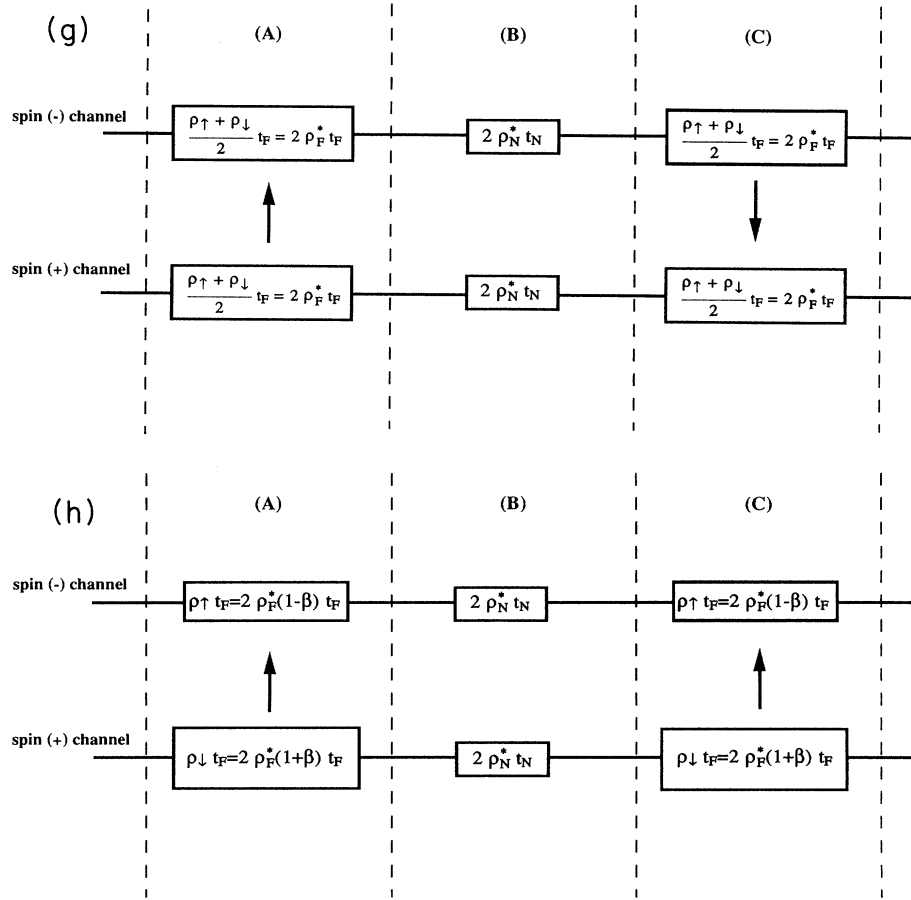
(g)

(A)　(B)　(C)

spin (-) channel

$$\frac{\rho_\uparrow + \rho_\downarrow}{2}\, t_F = 2\,\rho_F^*\, t_F \qquad 2\,\rho_N^*\, t_N \qquad \frac{\rho_\uparrow + \rho_\downarrow}{2}\, t_F = 2\,\rho_F^*\, t_F$$

spin (+) channel

$$\frac{\rho_\uparrow + \rho_\downarrow}{2}\, t_F = 2\,\rho_F^*\, t_F \qquad 2\,\rho_N^*\, t_N \qquad \frac{\rho_\uparrow + \rho_\downarrow}{2}\, t_F = 2\,\rho_F^*\, t_F$$

(h)

(A)　(B)　(C)

spin (-) channel

$$\rho_\uparrow\, t_F = 2\,\rho_F^*(1-\beta)\, t_F \qquad 2\,\rho_N^*\, t_N \qquad \rho_\uparrow\, t_F = 2\,\rho_F^*(1-\beta)\, t_F$$

spin (+) channel

$$\rho_\downarrow\, t_F = 2\,\rho_F^*(1+\beta)\, t_F \qquad 2\,\rho_N^*\, t_N \qquad \rho_\downarrow\, t_F = 2\,\rho_F^*(1+\beta)\, t_F$$

FIG. 2. *(Continued)*.

and

$$r_{+(-)}^{(P)} = 2\rho_F^*[1 + (-)\beta]t_F + 2\rho_N^* t_N \ . \tag{38}$$

One can easily verify that Eqs. (37)–(38) can be obtained directly from Eqs. (32)–(33), by an appropriate expansion in t_N/l_{sf} and t_F/l_{sf}. Finally, in order to emphasize that this is completely different from Johnson's assumption,[8] let us write the absolute resistance change from Eqs. (37) and (38) as

$$r^{(AP)} - r^{(P)} = \beta^2 \frac{(\rho_F^* t_F)^2}{\rho_N^* t_N + \rho_F^* t_F} \ , \tag{39}$$

which has nothing to do with Eq. (34). *In both cases of P and AP configurations, we simply recover the two current model in the limit without spin flip, which appears as the relevant physical picture if* $(t_N, t_F) \ll l_{sf}$.

C. General case, multilayer with both interface and bulk spin-dependent scattering

The calculations are still based on Eqs. (C1)–(C6) and similar to those described for simpler cases in the preceding paragraphs; but now there are true discontinuities of the potentials at the interfaces according to Eqs. (20) and (21) with $r_b^* \neq 0$. We give first the results for the most general situation, i.e., without any assumption for the relative values of the thicknesses and l_{sf}. We obtain now for the total areal resistance $R^{(i)}$ for a multilayer composed of M bilayers in the magnetic configuration (i)

$$R^{(P,AP)} = M(r_0 + 2r_{SI}^{(P,AP)})$$

$$\text{with } r_0 = (1 - \beta^2)\rho_F^* t_F + \rho_N^* t_N + 2(1 - \gamma^2)r_b^* \ , \tag{40}$$

where the spin-coupled interface parts are given by

$$r_{SI}^{(P)} = \cfrac{\dfrac{(\beta - \gamma)^2}{\rho_N^* l_{sf}^{(N)}} \coth\left[\dfrac{t_N}{2l_{sf}^{(N)}}\right] + \dfrac{\gamma^2}{\rho_F^* l_{sf}^{(F)}} \coth\left[\dfrac{t_F}{2l_{sf}^{(F)}}\right] + \dfrac{\beta^2}{r_b^*}}{\dfrac{1}{\rho_N^* l_{sf}^{(N)}} \coth\left[\dfrac{t_N}{2l_{sf}^{(N)}}\right]\dfrac{1}{\rho_F^* l_{sf}^{(F)}} \coth\left[\dfrac{t_F}{2l_{sf}^{(F)}}\right] + \dfrac{1}{r_b^*}\left[\dfrac{1}{\rho_N^* l_{sf}^{(N)}} \coth\left[\dfrac{t_N}{2l_{sf}^{(N)}}\right] + \dfrac{1}{\rho_F^* l_{sf}^{(F)}} \coth\left[\dfrac{t_F}{2l_{sf}^{(F)}}\right]\right]} \ , \tag{41}$$

(a)

    (A)   (B)   (C)

spin (-) channel
$$\frac{\rho_\uparrow+\rho_\downarrow}{2}\, t_F = 2\,\rho_F^* \, t_F \quad \frac{r_\uparrow+r_\downarrow}{2}=2\,r_b^* \quad 2\rho_N^* t_N \quad \frac{r_\uparrow+r_\downarrow}{2}=2\,r_b^* \quad \frac{\rho_\uparrow+\rho_\downarrow}{2}\, t_F = 2\,\rho_F^* \, t_F$$

spin (+) channel
$$\frac{\rho_\uparrow+\rho_\downarrow}{2}\, t_F = 2\,\rho_F^* \, t_F \quad \frac{r_\uparrow+r_\downarrow}{2}=2\,r_b^* \quad 2\rho_N^* t_N \quad \frac{r_\uparrow+r_\downarrow}{2}=2\,r_b^* \quad \frac{\rho_\uparrow+\rho_\downarrow}{2}\, t_F = 2\,\rho_F^* \, t_F$$

(b)

    (A)   (B)   (C)

spin (-) channel
$$\rho_\uparrow\, t_F=2\,\rho_F^*(1-\beta)\, t_F \quad r_\uparrow=2r_b^*(1-\gamma) \quad 2\rho_N^* t_N \quad r_\uparrow=2r_b^*(1-\gamma) \quad \rho_\uparrow\, t_F=2\,\rho_F^*(1-\beta)\, t_F$$

spin (+) channel
$$\rho_\downarrow\, t_F=2\,\rho_F^*(1+\beta)\, t_F \quad r_\downarrow=2r_b^*(1+\gamma) \quad 2\rho_N^* t_N \quad r_\downarrow=2r_b^*(1+\gamma) \quad \rho_\downarrow\, t_F=2\,\rho_F^*(1+\beta)\, t_F$$
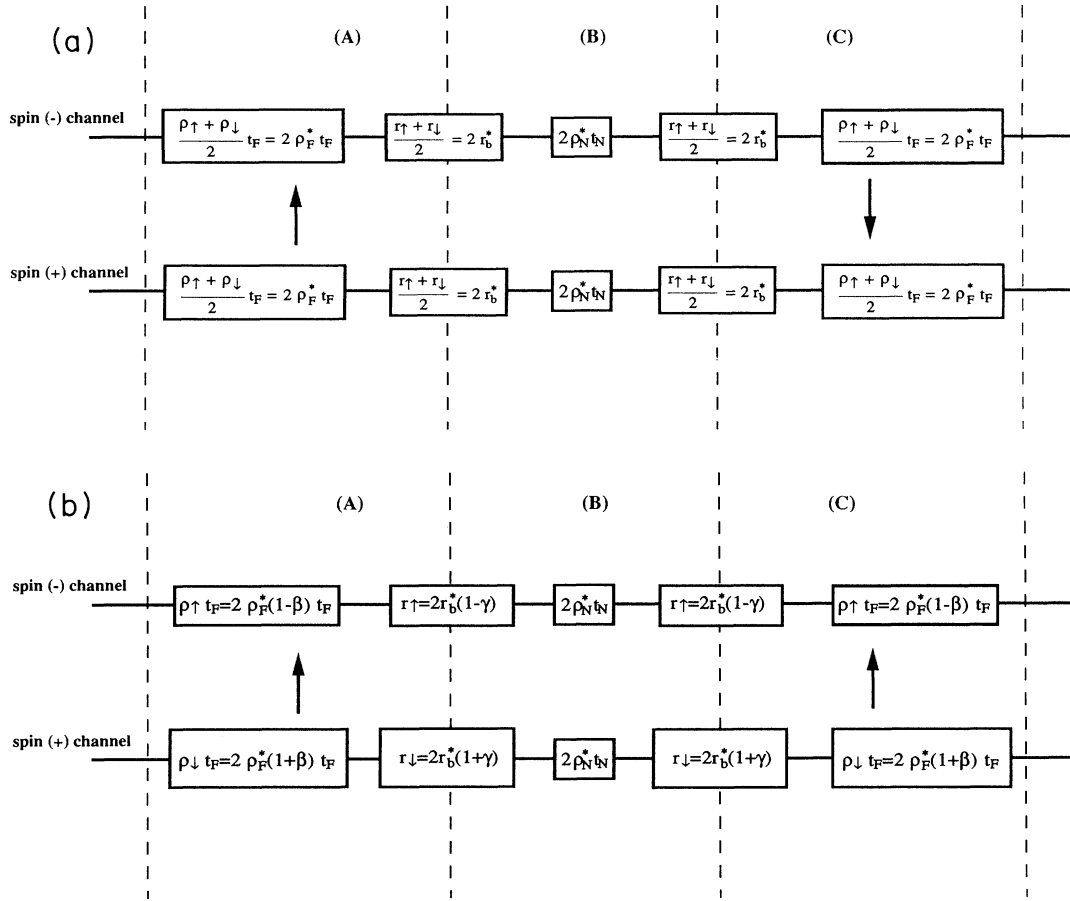
FIG. 3. Equivalent resistance array giving the potential drop coming from the pseudoelectric field F ($\Delta\mu$ is periodic and does not contribute to the potential drop over long distances) in the limit $(t_N, t_F) \ll l_{sf}$, for the general case with both bulk and interface spin-dependent scattering. (a) is for an antiferromagnetic arrangement and (b) is for a ferromagnetic one. $\rho_\uparrow$ and $\rho_\downarrow$ are the resistivities induced by spin-dependent bulk scattering. $r_\uparrow$ and $r_\downarrow$ are the resistance induced by the spin-dependent interface scattering.

$$r_{SI}^{(AP)} = \frac{\dfrac{(\beta-\gamma)^2}{\rho_N^* l_{sf}^{(N)}}\tanh\left[\dfrac{t_N}{2l_{sf}^{(N)}}\right] + \dfrac{\gamma^2}{\rho_F^* l_{sf}^{(F)}}\coth\left[\dfrac{t_F}{2l_{sf}^{(F)}}\right] + \dfrac{\beta^2}{r_b^*}}{\dfrac{1}{\rho_N^* l_{sf}^{(N)}}\tanh\left[\dfrac{t_N}{2l_{sf}^{(N)}}\right]\dfrac{1}{\rho_F^* l_{sf}^{(F)}}\coth\left[\dfrac{t_F}{2l_{sf}^{(F)}}\right] + \dfrac{1}{r_b^*}\left[\dfrac{1}{\rho_N^* l_{sf}^{(N)}}\tanh\left[\dfrac{t_N}{2l_{sf}^{(N)}}\right] + \dfrac{1}{\rho_F^* l_{sf}^{(F)}}\coth\left[\dfrac{t_F}{2l_{sf}^{(F)}}\right]\right]} . \tag{42}$$

In the limit where the thicknesses t_F and t_N are much smaller than $l_{sf}^{(F)}$ and $l_{sf}^{(N)}$, respectively, Eq. (37) is unchanged but the spin-dependent interface resistance adds up in Eq. (38) to give

$$r_{+(-)}^{(P)} = 2\rho_F^*[1+(-)\beta]t_F + 2\rho_N^* t_N + 4r_b^*[1+(-)\gamma] .$$
$$\tag{43}$$

This corresponds to the resistor schemes of Fig. 3(a) for the AP configuration, and of Fig. 3(b) for the P configuration.

Another interesting case is that of randomly distributed up and down magnetizations with a zero total magnetization. As the inhomogeneities are averaged over l_{sf},

this will lead to the same resistance as in the AP configuration if the magnetization averages to zero within l_{sf} (see also Ref. 11).

IV. SUMMARY AND DISCUSSION

In Sec. II we have shown that, in the limit $\lambda \ll l_{sf}$, a Boltzmann equation model reduces to a macroscopic model. Then in Sec. III, we have worked out a macroscopic model of the CPP conduction in magnetic multilayers. The same type of macroscopic model had already been applied by Johnson and Silsbee[9] and van Son, van Kempen, and Wyder[10] to calculate the effective resistance of interfaces between metals having different spin-

dependent conduction properties. Johnson[8] has also extended his calculations to multilayers by simply adding up the resistances calculated for isolated interfaces. We have shown that essentially different results are obtained when one treats the interplay of the successive interfaces in a multilayer.

Our results are particularly simple in the limit where the layer thicknesses are much smaller than the spin diffusion length l_{sf}. In this limit, the resistance of a multilayer can be calculated according to the resistor schemes of Fig. 3. This gives, using Eqs. (37) and (43), the type of expressions already used[7] for the interpretation of experimental results on Ag/Co and Cu/Co multilayers with M bilayers:

$$R^{(AP)} = M(\rho_F^* t_F + \rho_N^* t_N + 2r_b^*) , \tag{44}$$

$$\frac{1}{R^{(P)}} = \frac{1}{M}\left[\frac{1}{2\rho_F^*(1-\beta)t_F + 2\rho_N^* t_N + 4r_b^*(1-\gamma)} \right. $$
$$\left. + \frac{1}{2\rho_F^*(1+\beta)t_F + 2\rho_N^* t_N + 4r_b^*(1+\gamma)} \right] , \tag{45}$$

which leads to

$$R^{(P)} = R^{(AP)} - \frac{\{\beta\rho_F^*[t_F/(t_F+t_N)]L + 2\gamma r_b^* M\}^2}{R^{(AP)}} \tag{46}$$

if we define $L = M(t_F + t_N)$ the total thickness of the multilayer.

In Fig. 4 we show the expected variation of $R^{(AP)}$ and $R^{(P)}$ as a function of M, in one of the experimental conditions of Pratt and co-workers[3,7]: i.e., for a fixed total thickness L and the same thickness for the ferromagnetic and normal layers, $t_F = t_N = t = L/2M$. Figures 4(a) and 4(b) are for $\beta = 0$ (spin dependence from interface scattering only) and $\gamma = 0$ (spin dependence from bulk scattering only), respectively. These figures are meaningful in the range of validity of our expressions, i.e., $t \ll l_{sf}$ or $M \gg L/l_{sf}$ with expected values of l_{sf} at a low temperature above 10^3 Å (see Appendix A).

For interface spin-dependent scattering, Fig. 4(a), $(R^{(AP)} - R^{(P)})$ is an increasing function of M, in agreement with the experimental results on Ag/Co and Co/Cu.[3,7] The resistance $R^{(AP)}$ is a linear function of M. The departure of $R^{(P)}$ from $R^{(AP)}$ starts as M^2 and then, for

$$M \geq M_c = L(\rho_F^* + \rho_N^*)/(2r_b^*) ,$$

$R^{(P)}$ also becomes linear in M with a smaller slope than for $R^{(AP)}$. By increasing ρ_N^* (addition of impurities in Ag), Lee et al.[7] have been able to shift M_c and make the crossover from quadratic to linear appear clearly in the experimental range of M for Ag/Co multilayers.

For bulk spin-dependent scattering only, Fig. 4(b), $(R^{(AP)} - R^{(P)})$ decreases as M increases. This should be observed for systems with predominant spin-dependent scattering within the magnetic layers. As it has been suggested that this occurs for Permalloy layers,[16] CPP-MR measurements on, for example, Permalloy/Cu multilay-

ers should be of interest.

Some other measurements by Pratt and co-workers[3,7] have been performed on samples with a fixed value for t_F while t_N is varying with M. The predicted variation of $R^{(AP)}$ and $R^{(P)}$ could also be plotted for these conditions. It would appear that, in this case, $(R^{(AP)} - R^{(P)})$ is always an increasing function of M.

On an other hand, it is possible to obtain from Eqs. (46) the relation

$$\sqrt{(R^{(AP)} - R^{(P)})R^{(AP)}} = \beta \frac{t_F}{t_F+t_N}\rho_F^* L + 2\gamma r_b^* M \tag{47}$$

with the valuable property of a right-hand side made of additive contributions from bulk and interface spin-dependent scattering. The plot of $\sqrt{(R^{(AP)} - R^{(P)})R^{(AP)}}$
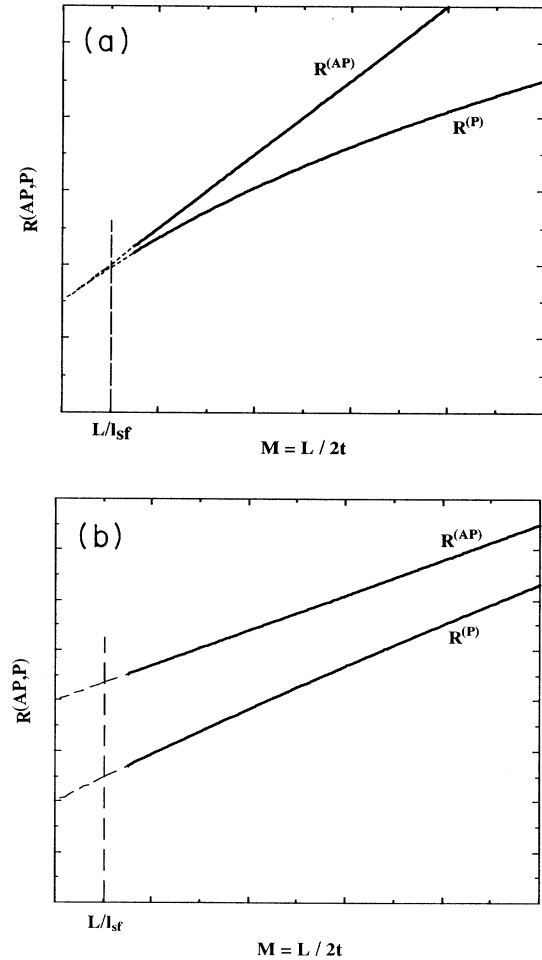

FIG. 4. Predicted variation of the CPP resistance of a multilayer as a function of the number of bilayers M for a fixed total thickness L and the same individual thicknesses for the ferromagnetic and normal layers, $t_F = t_N = t = L/(2M)$. (a) is for a multilayer with surface spin-dependent scattering only ($\beta = 0$) and (b) with bulk spin-dependent scattering only ($\gamma = 0$). The plotted variations are derived from Eqs. (44)–(46). The plots are meaningful in the validity range of these expressions, i.e., $t \ll l_{sf}$ or $M \gg L/l_{sf}$.

vs M for L fixed is thus of great interest to identify where the spin scattering occurs, as this has been done[7] and is also shown schematically in Fig. 5 for the case $t_F = t_N = t$. After independent measurements of ρ_F^* and r_b^*, the intercept with the vertical axis gives β, while the slope is characteristic of γ. Lee et al.[7] have analyzed extensive series of experimental results on Co/Ag and Co/Cu by using equations of the type of Eqs. (44), (46), or (47) [for both cases: $t_F = t_N = L/(2M)$ and $t_F = Cte$, $t_N = (L - Mt_F)/M$]. These analyses have led to the following parameters[7]: $\rho_N^* = (10 \pm 1)$ n$\Omega\,m$, $\rho_F^* = (107 \pm 10)$ n$\Omega\,m$, $\beta = 0.48 \pm 0.05$ (i.e., $\alpha_F = \rho_\downarrow / \rho_\uparrow \approx 2.9$), $r_b^* = (0.56 \pm 0.03)10^{-15}\ \Omega\,m^2$, $\gamma = 0.85 \pm 0.03$ (i.e., $\alpha_b = r_\downarrow / r_\uparrow \approx 12$) for Co/Ag and $\rho_N^* = (7 \pm 2)$ n$\Omega\,m$, $\rho_F^* = (86 \pm 4.5)$ n$\Omega\,m$, $\beta = 0.5 \pm 0.1$, (i.e., $\alpha_F = \rho_\downarrow / \rho_\uparrow \approx 3$), $r_b^* = (0.5 \pm 0.02)10^{-15}\ \Omega\,m^2$, $\gamma = 0.76 \pm 0.05$ (i.e., $\alpha_b = r_\downarrow / r_\uparrow \approx 7.3$) for Co/Cu. It turns out that the contribution to the CPP-MR from interface spin-dependent scattering is predominant at usual thicknesses in both Ag/Co and Cu/Co systems. We, however, point out that the relative magnitude of the interface and bulk contributions in Eq. (46) depends on the layer thicknesses (via M) and, for thicknesses of a few hundred of angstroms, the bulk contribution is expected to exceed the interface one. The values of the resistivities and spin asymmetry coefficients α_F are similar to those found from magnetoresistance data in the conventional CIP geometry.[17] The new parameters, not involved in the models of CIP-MR, are the interface resistances $r_{\uparrow(\downarrow)} = 2r_b^*[1 - (+)\gamma]$. Both the specular reflections of the electrons at the interface potential steps and the diffuse scattering by disordered interfaces are expected to contribute to these interface resistances. The contribution from potential steps has

been recently calculated.[18,19] It turns out that the experimental values of $r_\uparrow$ and $r_\downarrow$ can be accounted for with reasonable values of the potential steps. However, this does not rule out that the contribution from roughness can also be significant.

In the range $M < L/l_{sf}$ where the condition $t \ll l_{sf}$ is not fulfilled, the MR is predicted from our general expressions, Eqs. (40)–(42), to be smaller than expected from Eq. (46) or Eq. (47). For example, in the limit $M \ll L/l_{sf}$ (i.e., $t \gg l_{sf}$) of the plot of Fig. 5,

$$\sqrt{(R^{(AP)} - R^{(P)})R^{(AP)}}$$

is predicted to decrease as

$$\exp\left[-\frac{t}{2l_{sf}}\right] \sim \exp\left[-\frac{L}{4Ml_{sf}}\right].$$

This deviation from a linear variation is represented by a thin solid line in Fig. 5. The linear variation observed by Lee et al.[7] up to layer thicknesses of 300 Å suggests that l_{sf} amounts to a few thousand Å, as expected from the electron spin resonance (ESR) data recalled in Appendix A. However, l_{sf} can be shortened by adding impurities with strong spin-orbit interaction or, in the nonmagnetic layers, paramagnetic impurities inducing spin-flip exchange scattering. This has been done recently by Yang et al.[13] who have introduced Mn impurities in the Ag layers of Ag/Co and found a strong reduction of the CPP-MR with definite deviations from the linear variations predicted by Eq. (47). They could account for the reduction of the MR and for the nonlinear variation by proceeding to our general expressions, Eqs. (40)–(42), with values of $l_{sf}^{(N)}$ as short as 70 Å for the higher concentration of Mn. Finally, we point out that experiments are beginning to be reported which extend the study of CPP-MR up to room temperature by using pillar shaped microstructure and conventional (not using superconductive devices) resistance measurements.[20] At such high temperatures, electron-magnon collisions are becoming important (see Appendix A).

V. CONCLUSION

The main difference between the CPP and CIP problems is due to the existence of spin accumulation effects in the CPP case and to the appearance of the spin-diffusion length l_{sf} as unique damping length of the current inhomogeneities. As discussed in Appendix A, the spin-diffusion length is fixed by the spin-orbit and exchange scattering in the low-temperature limit and is expected to be relatively long—above 10^3 Å—in multilayers alternating $3d$ and pure noble metals. We have demonstrated two consequences of the existence of this very long scaling length.

(a) In Sec. II we have demonstrated that, in the usual conditions where the mean free path is much shorter than the spin-diffusion length, a Boltzmann equation treat-
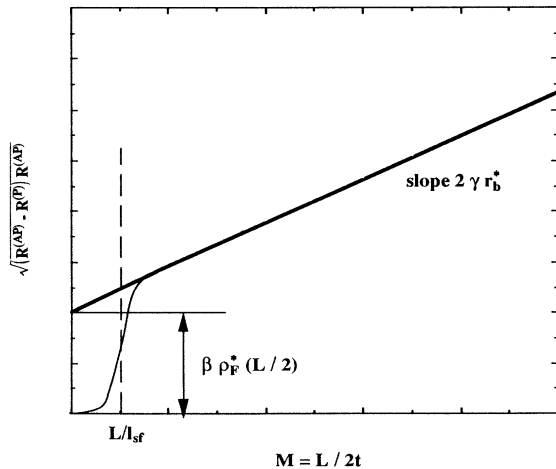

FIG. 5. $\sqrt{(r_{AF} - r_{FM})r_{AF}}$ is plotted as a function of the number of bilayers M for a fixed total thickness L and the same individual thickness for the ferromagnetic and normal layer, $t_F = t_N = t = L/(2M)$. The solid line is the linear variation expected from Eq. (47) for the limit $t \ll l_{sf}$ or $M \gg L/l_{sf}$. For $M < L/l_{sf}$, Eq. (47) is no longer valid and $\sqrt{(r_{AF} - r_{FM})r_{AF}}$ is expected to drop as $\exp(-t/4l_{sf})$, see thin solid line.

ment reduces to the equations of the macroscopic approach introduced by Johnson and Silsbee[9] and van Son, van Kempen, and Wyder.[10]

(b) In Sec. III we have adapted the macroscopic approach of Johnson and Sisbee[9] and van Son, van Kempen, and Wyder[10] to the case of multilayers and treated the interplay between spin accumulations at successive interfaces. Our results for the CPP-MR are definitely different from those obtained by Johnson by adding up "spin-coupled interface resistances" calculated for isolated interfaces. We have shown that, for layer thicknesses much shorter than the spin-diffusion length, the concept of "spin-coupled interface resistance" is no longer meaningful and that the zero spin mixing limit of the two-current model is the relevant picture (Fig. 3). The expressions of the CPP-MR in the limit $(t_F, t_N) \ll l_{sf}$ are particularly simple (see Secs. III and IV) and we have justified the analysis of the experimental results already proposed by Lee et al.[7] The typical behavior expected for a multilayer of fixed total thickness as a function of the number of bilayers is shown in Fig. 4. We emphasize that the appearance of additive contributions from bulk and interface spin-dependent scattering in Eq. (47) is of great interest to separate these contributions (Fig. 5).

Although the simplest results have been obtained for the limit where the spin-diffusion length is much longer than the layer thicknesses, we have also derived general expressions of the CPP-MR. In particular, these general expressions predict a collapse of the CPP-MR as $\exp(-t_N/2l_{sf})$ in the limit of thicknesses larger than the spin-diffusion length (with still $\lambda \ll l_{sf}$). This was also observed recently by Yang et al.[13] by reducing l_{sf} in the silver layers of Co/Ag multilayers with Mn impurities. Finally the regime $\lambda \approx l_{sf}$ is more complex to describe but, in principle, could be treated starting from Eq. (9) or even directly from Eq. (6).

ACKNOWLEDGMENTS

APPENDIX A: GENERALITIES ABOUT SPIN-FLIP SCATTERING

For ferromagnetic metals, the spin-flip scattering due to electron-magnon collisions is frozen in the low-temperature limit and the residual spin-flip scattering is due to spin orbit. This spin-flip scattering by spin-orbit interactions has been extensively studied by ESR.[21,22] Its theory has been developed by Yafet.[23] From Fig. 3 in Ref. 22 it turns out that the $3d$ impurities are generally stronger spin-orbit scatterers than s or p impurities (even Au has a smaller spin-flip cross section than Co or Fe). For $3d$ elements the ratio between the spin-flip and non-spin-flip cross sections is around 10^{-2}. From the expressions of l_{sf} given in Sec. II, we are expecting

$$l_{sf}/\lambda \approx [\lambda_{sf}/(3\lambda)]^{1/2}$$

and therefore $l_{sf} \approx 10\lambda$ for structures composed of $3d$ elements with Cu or Ag. This shows that the condition $\lambda \ll l_{sf}$ is fulfilled in all the investigated samples, and this justifies the use of the macroscopic approach.

Furthermore, with the mean free path estimated to a few hundred angstrom in Cu and Ag layers,[8,9] this means that the spin-diffusion length should amount to a few thousand angstrom. In the Co layers, even if the mean free path is somewhat shorter than in Ag or Cu (i.e., around 10^2 Å)[3] we are still expecting spin-diffusion lengths above 10^3 Å. In these conditions for all the samples with pure Cu or Ag spacers investigated by Pratt and co-workers,[3,7] the layer thickness t should be definitely shorter than the spin-diffusion length and the simple expressions we have obtained for this limit can be applied. This certainly explains the good agreement obtained by Lee et al.[7] by using similar simple expressions.

Deviations from the expressions obtained in the limit $t \ll l_{sf}$ are expected either in samples with thick layers (with smaller MR) or if impurities are introduced to enhance the spin-orbit effects and reduce l_{sf}. The best candidates should be $5d$ impurities, Pt for example, or sp impurities in the same line of the periodic table (Pb or Bi, for example). Another possibility to shorten l_{sf} is to introduce exchange scattering on paramagnetic impurities in the nonmagnetic layers, which will open an indirect channel for spin relaxation: transfer of magnetization from the conduction electron to the impurities by exchange scattering, then localized spin to lattice relaxation through spin orbit. Demonstrative results in this latter direction have already been obtained by Yang et al.[13]

Finally, at relatively high temperatures, the onset of electron-magnon collisions should bring a third channel for the spin relaxation: spin transfer from light conduction electrons to heavy d electrons and then spin-lattice relaxation via spin-orbit interactions. Furthermore, it will induce exchange between the two spin channels, leading to a reduced effective value of the resistivity asymmetry between the two spin channels. The resulting effect on the spin accumulation process is difficult to predict accurately but this should affect significantly the temperature dependence of the MR. The new experimental approach of Gijs, Lenczowski, and Giesbers,[20] which allows CPP-MR measurements up to room temperature, is certainly of great interest to address this interesting problem.

APPENDIX B: DECOMPOSITION OF BOLTZMANN EQUATION IN LEGENDRE HARMONICS

In this appendix we will detail the mathematical procedure which has allowed us to derive Eq. (5). In order to do so we will use several properties of Legendre polynomials. The reader interested in this matter can refer to standard handbooks of mathematical functions.[24]

Substituting the development (4) in the Boltzmann

equation (3), when taking into account

$$\frac{\partial f_0}{\partial \varepsilon} = \frac{1}{mv}\frac{\partial f_0}{\partial v} = \frac{-\delta(v-v_F)}{mv_F}$$

due to the degeneration of the electron gas in metals, leads to

$$\cos\theta \sum_{n=1}^{\infty} \frac{\partial g_s^{(n)}}{\partial z} P_n(\cos\theta) + \frac{1}{\lambda_s}\sum_{n=1}^{\infty} g_s^{(n)} P_n(\cos\theta)$$

$$= \cos\theta \frac{\partial \bar{\mu}_s}{\partial z} + \frac{\bar{\mu}_s - \bar{\mu}_s}{v_F \tau_{sf}} \quad \text{(B1)}$$

with $\lambda_s = v_F(1/\tau_s + 1/\tau_{sf})^{-1}$ the MFP for spin s. If we project now the equation (B1) on the Legendre polynomials of order $\bar{n}$, taking into account that $\cos\theta = P_1(\cos\theta)$, we get

$$\sum_{n=1}^{\infty} \frac{\partial g_s^{(n)}}{\partial z} \int_{-1}^{1} du\, P_1(u)P_{\bar{n}}(u)P_n(u) + \frac{1}{\lambda_s}\sum_{n=1}^{\infty} g_s^{(n)} \int_{-1}^{1} du\, P_{\bar{n}}(u)P_n(u)$$

$$= \frac{\partial \bar{\mu}_s}{\partial z} \int_{-1}^{1} du\, P_1(u)P_{\bar{n}}(u) + \frac{\bar{\mu}_s - \bar{\mu}_{-s}}{v_F \tau_{sf}} \int_{-1}^{1} du\, P_{\bar{n}}(u) \; . \quad \text{(B2)}$$

The only integral appearing in Eq. (B2) not directly determined by the orthogonality relation between Legendre polynomials, that is

$$\int_{-1}^{1} du\, P_{\bar{n}}(u)P_n(u) = \frac{2}{2n+1}\delta_{n,\bar{n}} \; , \quad \text{(B3)}$$

where $\delta_{n,\bar{n}}$ is the usual Kronecker symbol, is

$$\int_{-1}^{1} du\, P_1(u)P_{\bar{n}}(u)P_n(u)$$

In order to compute its value, we recall a well-known recurrence formula:

$$(n+1)P_{n+1} - (2n+1)P_1 P_n + nP_{n-1} = 0 \; , \quad \text{(B4)}$$

which leads to

$$P_1 P_n = \frac{(n+1)}{(2n+1)}P_{n+1} + \frac{n}{(2n+1)}P_{n-1} \; . \quad \text{(B5)}$$

From Eq. (B5), it is thus obvious that

$$\int_{-1}^{1} du\, P_1(u)P_{\bar{n}}(u)P_n(u)$$

$$= \frac{(n+1)}{(2n+1)} \int_{-1}^{1} du\, P_{\bar{n}}(u)P_{n+1}(u)$$

$$+ \frac{n}{(2n+1)} \int_{-1}^{1} du\, P_{\bar{n}}(u)P_{n-1}(u) \quad \text{(B6)}$$

and using the orthogonality relation (B3) we finally obtain

$$\int_{-1}^{1} du\, P_1(u)P_{\bar{n}}(u)P_n(u)$$

$$= \frac{2(n+1)}{(2n+1)(2n+3)}\delta_{\bar{n},n+1}$$

$$+ \frac{2n}{(2n+1)(2n-1)}\delta_{\bar{n},n-1} \; . \quad \text{(B7)}$$

Thus, taking into account Eqs. (B3) and (B7), Eq. (B2) becomes

$$\sum_{n=1}^{\infty} \frac{\partial g_s^{(n)}}{\partial z}\left[\frac{2(n+1)}{(2n+1)(2n+3)}\delta_{\bar{n},n+1} + \frac{2n}{(2n+1)(2n-1)}\delta_{\bar{n},n-1}\right] + \frac{1}{\lambda_s}\sum_{n=1}^{\infty} g_s^{(n)}\frac{2}{(2n+1)}\delta_{\bar{n},n}$$

$$= \frac{2}{3}\frac{\partial \bar{\mu}_s}{\partial z}\delta_{\bar{n},1} + 2\frac{\bar{\mu}_2 - \bar{\mu}_{-s}}{v_F \tau_{sf}}\delta_{\bar{n},0} \; . \quad \text{(B8)}$$

Now, if we particularize the general expression (B8) for given values of $\bar{n}$, we obtain

$$\frac{\partial g_s^{(1)}}{\partial z} = \lambda_s \frac{\bar{\mu}_s - \bar{\mu}_{-s}}{l_s^2}, \quad \text{for } \bar{n}=0 \; ,$$

$$\frac{2}{5}\frac{\partial g_s^{(2)}}{\partial z} - \frac{\partial \bar{\mu}_s}{\partial z} = -\frac{g_s^{(1)}}{\lambda_s}, \quad \text{for } \bar{n}=1 \; , \quad \text{(B9)}$$

$$\frac{\bar{n}+1}{2\bar{n}+3}\frac{\partial g_s^{(\bar{n}+1)}}{\partial z} + \frac{\bar{n}}{2\bar{n}-1}\frac{\partial g_s^{(\bar{n}-1)}}{\partial z} = -\frac{g_s^{(\bar{n})}}{\lambda_s}, \quad \text{for } \bar{n}>1 \; ,$$

where we have defined $l_s = [\frac{1}{3}(v_F\lambda_s)\tau_{sf}]^{1/2}$ the spin-diffusion length for spin s. Equation (B9) is just Eq. (5) as referred to in the main part of this paper, which is thus fully justified.

In order to apprehend the physical significance of Eq. (B9) it is worthwhile to establish the exact relation between J_s, the current of spin s, and g_s. By definition of g_s, given by the Eq. (2), we have

$$J_s = -e\left[\frac{m}{h}\right]^3 \int d^3v \, \frac{\partial f^0}{\partial \varepsilon} g_s(\mathbf{v})v_z \; , \quad \text{(B10)}$$

where h is the Planck constant. Using again the degeneration of the electron gas

$$\frac{\partial f_0}{\partial \varepsilon} = \frac{1}{mv}\frac{\partial f_0}{\partial v} = \frac{-\delta(v - v_F)}{mv_F} ,$$

and the cylindrical symmetry of the problem around the z axis, we deduce from Eq. (B10)

$$J_s = \frac{3}{mv_F}2\pi\left[\frac{mv_F}{h}\right]^3\int_0^\pi d\theta\,\sin\theta\,\cos\theta\,g_s(\cos\theta)$$

$$= \frac{e}{mv_F}2\pi\left[\frac{mv_F}{h}\right]^3\int_{-1}^1 du\,P_1(u)g_s(u) . \tag{B11}$$

And from the development (4) in Legendre harmonics of g_s, using the orthogonality relation (B3), we establish immediately

$$\int_{-1}^1 du\,P_1(u)g_s(u) = \frac{2}{3}g_s^{(1)} .$$

Thus

$$J_s = \frac{1}{e}\frac{1}{v_F(1/\tau_s + 1/\tau_{sf})^{-1}}\frac{[\frac{4}{3}\pi(mv_F/h)^3]e^2(1/\tau_s + 1/\tau_{sf})^{-1}}{m}g_s^{(1)}$$

$$= \frac{1}{e}\frac{1}{\lambda_s}\frac{n_s e^2(1/\tau_s + 1/\tau_{sf})^{-1}}{m}g_s^{(1)} = \frac{\sigma_s}{e\lambda_s}g_s^{(1)} \tag{B12}$$

with

$$n_s = \frac{4}{3}\pi(mv_F/h)^3$$

the number of electrons with spin s. We have thus established the identity $J_s = (\sigma_s/e\lambda_s)g_s^{(1)}$ that we have used throughout the paper.

APPENDIX C: GENERAL SOLUTIONS IN AN HOMOGENEOUS LAYER

From Eqs. (14)–(16) it is straightforward to obtain the general expression of $\bar\mu_\pm(z)$, $\Delta\mu(z)$, $F(z)$, and $J_\pm(z)$ in the layer (n), with only few constants of integration $K_i^{(n)}$ which have to be determined by taking into account the boundary conditions. Thus, *in the F layers with "up" magnetization*

$$\bar\mu_+(z) = (1-\beta^2)e\rho_F^* Jz + K_1^{(n)} + (1+\beta)\left[K_2^{(n)}\exp\left[\frac{z}{l_{sf}^F}\right] + K_3^{(n)}\exp\left[-\frac{z}{l_{sf}^F}\right]\right] ,$$

$$\bar\mu_-(z) = (1-\beta^2)e\rho_F^* Jz + K_1^{(n)} - (1-\beta)\left[K_2^{(n)}\exp\left[\frac{z}{l_{sf}^F}\right] + K_3^{(n)}\exp\left[-\frac{z}{l_{sf}^F}\right]\right] , \tag{C1}$$

$$\Delta\mu(z) = K_2^{(n)}\exp\left[\frac{z}{l_{sf}^F}\right] + K_3^{(n)}\exp\left[-\frac{z}{l_{sf}^F}\right] , \tag{C2}$$

$$F(z) = (1-\beta^2)\rho_F^* J + \frac{\beta}{el_{sf}^F}\left[K_2^{(n)}\exp\left[\frac{z}{l_{sf}^F}\right] - K_3^{(n)}\exp\left[-\frac{z}{l_{sf}^F}\right]\right] ,$$

$$J_+(z) = (1-\beta)\frac{J}{2} + \frac{1}{2e\rho_F^* l_{sf}^F}\left[K_2^{(n)}\exp\left[\frac{z}{l_{sf}^F}\right] - K_3^{(n)}\exp\left[-\frac{z}{l_{sf}^F}\right]\right] , \tag{C3}$$

$$J_-(z) = (1+\beta)\frac{J}{2} - \frac{1}{2e\rho_F^* l_{sp}^F}\left[K_2^{(n)}\exp\left[\frac{z}{l_{sf}^F}\right] - K_3^{(n)}\exp\left[-\frac{z}{l_{sf}^F}\right]\right] .$$

Obviously, *in the F layers with "down" magnetization*, one has simply to interchange the positive and negative indices to get the new expressions. This leads to a change in sign for $\Delta\mu$ regarding (C2). In the same way, *for the N layers*,

$$\bar\mu_\pm(z) = e\rho_N^* Jz + K_1^{(n)} \pm \left[K_2^{(n)}\exp\left[\frac{z}{l_{sf}^N}\right] + K_3^{(n)}\exp\left[-\frac{z}{l_{sf}^N}\right]\right] , \tag{C4}$$

$$\Delta\mu(z) = K_2^{(n)} \exp\left[\frac{z}{l_{sf}^N}\right] + K_3^{(n)} \exp\left[-\frac{z}{l_{sf}^N}\right] , \tag{C5}$$

$$F(z) = \rho_N^* J ,$$

$$J_{\pm}(z) = \frac{1}{2}J + \frac{1}{2e(\rho_N^* l_{sf}^N)}\left[K_2^{(n)} \exp\left[\frac{z}{l_{sf}^N}\right] - K_3^{(n)} \exp\left[-\frac{z}{l_{sf}^N}\right]\right] . \tag{C6}$$

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