

## Nonlinear $U(j)$ law from magnetic relaxation in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ single crystals: Flux motion through double-kink formation of the pancake vortices

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We have studied magnetic relaxation in a single crystal of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  and have developed the  $U$ - $j$  relationship by an approach we devised earlier. We explain this nonlinear  $U$ - $j$  relationship based on the essential modification of the local pinning potential profile associated with flux motion through double-kink formation of the pancake vortices. The important microscopic parameters such as condensation energy, screening length, and vortex core energy have been extracted from the experimental data. An implication of the  $U(j)$  law on the voltage-current  $E$ - $j$  characteristics is discussed as well.

Magnetic relaxation in type-II superconductors has been reported earlier by various groups.<sup>1-9</sup> This phenomenon was explained by Anderson and Kim using a thermally activated flux-creep model.<sup>10</sup> According to them the flux motion is thermally activated, and the rate with which flux lines (or bundles) jump over the pinning barriers can be described by an Arrhenius-type expression

$$v = v_0 \exp\{-U(j)/kT\}, \quad (1)$$

where  $v_0$  is an attempt frequency and  $U(j)$  is the effective activation energy.

Although Beasley, Labush, and Webb<sup>1</sup> assumed a linear  $U$ - $j$  relationship leading to the characteristic logarithmic decay of magnetization, they realized that, in general, there exists a nonlinear  $U$ - $j$  relationship leading to a nonlogarithmic decay of magnetization. Nonlogarithmic decay of magnetization was later observed experimentally in type-II superconductors by various groups.<sup>5,6,11-14</sup> A nonlogarithmic decay of magnetization occurs because of three factors: (i) avalanche breakdown, (ii) reverse hopping, and (iii) shape and distribution of the local pinning potential. When a field larger than the full penetration field is applied, the system first forms a supercritical state. It then organizes itself into a critical state by avalanche breakdown governed by avalanche dynamics self-organized criticality.<sup>15</sup> This type of process may be important only near the critical-state regime. As time progresses and the magnetization decays from the critical state, the contribution from avalanche breakdown decreases. In a typical magnetic relaxation experiment we measure magnetization quite far away from the critical state. Thus, nonlogarithmic magnetization observed experimentally is generally due to the other two factors. At the low driving forces, the hopping rate is controlled

by the height of the pinning barrier, regardless of the detailed shape of the potential and hence determined by the reverse hopping. In the presence of high driving force, however, the amount of deformation of the potential well, and hence the resulting barrier height, are determined by the shape of the potential well. Our experiments are conducted at large driving force regime and therefore the contribution from reverse hopping can be safely neglected in this paper.

We shall consider the effect of the double-kink energy,  $U_v(x)$ ,<sup>16-18</sup> of two-dimensional "pancake" vortices on the form of the  $U$  vs  $j$  relationship extracted from the magnetic relaxation data for  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  single crystals. As will be shown, this correction modifies a local potential profile resulting in a nonlinear (logarithmic)  $U(j)$  law. We find, in particular, that the farther the system is from the critical state, the more important is the two-dimensional (2D) double-kink energy contribution of the vortex line. By applying the extracted  $U(j)$  law to the voltage-current characteristics, we show that the negative curvature of the  $E$ - $j$  curve as well as the true zero resistivity is consistent with the predicted  $U$  vs  $j$  relationship.

Magnetic relaxation experiments were performed on a  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  single crystal by using a Quantum Design Superconducting quantum interference device magnetometer. Magnetization as a function of time was measured for both increasing and decreasing magnetic field. For both the cases, the sample was first zero-field cooled to a desired temperature below  $T_c$ . For increasing field, a field of 0.5 T parallel to the  $c$  axis of the single crystal was applied, whereas for decreasing fields a field of 3 T was first applied parallel to the  $c$  axis and then reduced to 0.5 T. For both cases, the initial magnetization was recorded 120 s after stabilization of the field. The travel length of

the sample in each scan was 3 cm to avoid any field inhomogeneity. The current density was estimated by using a standard Bean model.<sup>19</sup>

The effective activation energy,  $U(j)$ , at a given temperature was extracted from the magnetic relaxation experiment performed at various temperatures by a procedure developed earlier.<sup>20–22</sup> The typical behavior of the  $U(j)$  curve for  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  single crystals at  $T=5.5$  K with a field of 0.5 T applied parallel to the  $c$  axis is shown in Fig. 1. We obtain a best fit using the function

$$U_{\text{exp}}(j) = a - b(1 - j/j_0) + c \ln(j_0/j), \quad (2)$$

where  $a = 0.0002$  eV,  $b = 0.005$  eV, and  $c = 2b = 0.01$  eV.

In a hard type-II superconductor, the pinning potential is randomly distributed.<sup>23</sup> Instead of considering the distribution, one can consider an equivalent local pinning potential,  $U_p(x)$ . One can expect to obtain  $U_p(x)$  from the actual pinning potentials by averaging them over all ranges of pinning energies. Since the distribution of the pinning potentials is unknown, the actual shape of  $U_p(x)$  is also unknown. However,  $U_p(x)$  may be obtained by different trial functions based on the  $U$ - $j$  relationship developed. Various shapes have been proposed to explain the nonlinear decay of magnetization.<sup>5,24–26</sup> It is well established that because of the highly anisotropic nature of layered  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$ , the flux lines have a two-dimensional characteristic, often described as pancake vortices.<sup>17</sup> As reported earlier that it is energetically favorable for vortices to move through the formation of double kinks.<sup>16,18</sup> To give a possible explanation of the above  $U$ - $j$  law [Eq. (2)], we assume that the effective local pinning potential,  $U_p(x)$ , is given by

$$U_p(x) = U_0 + U_v(x), \quad (3)$$

where  $U_0$  is the barrier height of pinning potential encountered by a single vortex line at the defect location. The potential energy required for a pancake vortex to be displaced by a distance  $x$  from its origin through double-kink formation has the form<sup>17,18</sup>

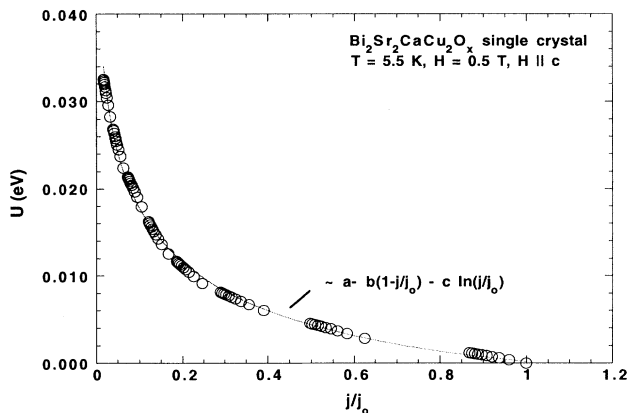


FIG. 1.  $U$ - $j/j_0$  curve for a single crystal of  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  at a field of 0.5 T applied parallel to the  $c$  axis at  $T=5.5$  K. The curved line is fit to Eq. (2), with  $a=0.0002$  eV,  $b=0.005$  eV,  $c=2b=0.01$  eV.

$$U_v(x) = 2U_1 \ln\{(x + x_0)/2x_0\}. \quad (4)$$

Here  $U_1 = \Phi_0^2/4\pi\mu_0\Lambda$  is the condensation energy of a single vortex line,  $\Lambda = 2\lambda_{ab}^2/d_s$  is the effective two-dimensional screening length in the copper-oxide plane of the thickness  $d_s$ ,<sup>18</sup> and  $x_0$  is the vortex core size [ $x_0 = \xi(T)$ ], where  $\xi(T)$  is the coherence length. Figure 2 shows  $U_p(x)/U_0$  as a function of  $x/x_0$ .

In the presence of a driving force, the resultant barrier becomes

$$U(x) = U_p(x) - jVBx, \quad (5)$$

where  $j$  is the current density,  $B$  is the magnetic induction, and  $V$  is the bundle volume. According to Eqs. (3)–(5), the local effective barrier,  $U(x)$ , has a metastable extremum at  $x=x_0$ , which gives rise to the linear (Anderson-Kim) law, namely

$$U_l(j) = U(x_0) = U_0 - jBVx_0 = U_0(1 - j/j_c), \quad (6)$$

with the critical current density  $j_c = U_0/BVx_0$ .

At the same time,  $U(x)$  has a true (stable) extremum at  $x=x_c$ , where  $x_c$ , a stable solution of the equation  $dU(x)/dx = 0$ , has the form

$$x_c(j) = -x_0 + 2U_1/jBV. \quad (7)$$

Thus using Eqs. (4), (5), and (7), the  $U(j)$  law due to the vortex line 2D double-kink energy contribution reads

$$U(j) = U(x_c) = (U_0 - U_1) - U_1(1 - j/j_0) + 2U_1 \ln(j_0/j), \quad (8)$$

with an “effective” critical current density  $j_0 = U_1/BVx_0$ .

According to Eq. (8), the fitting parameters in Eq. (2) are physically meaningful. The parameters  $a$  and  $b$  in Eq. (2) are related to the pinning barrier energies,  $U_0$  and  $U_1$ , as follows:  $a = U_0 - U_1$ , and  $b = U_1$ . Thus, the coefficient  $a$  defines the change of the barrier height,  $U_0$ , as a result of the double-kink formation of the vortex line with energy  $U_1$ . In fact,  $U_0 - U_1 \cong U_{\text{core}}$ , where  $U_{\text{core}} = \epsilon U_1$  is the vortex core energy ( $\epsilon = 0.08$  in normal three-dimensional

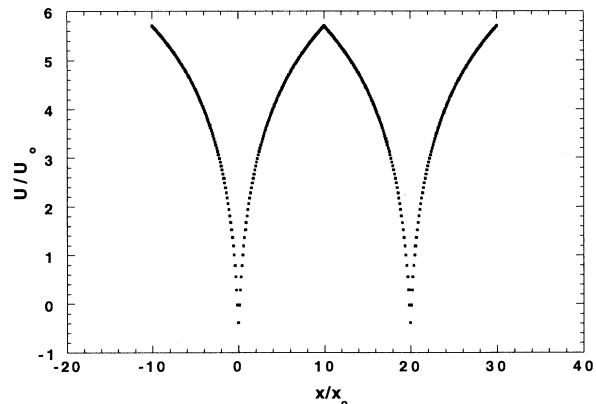


FIG. 2. Effective local pinning potential  $U_p(x)/U_0$  as a function of the displacement  $x/x_0$ .

Abrikosov vortices<sup>27</sup>). Using absolute values of the fitting parameters  $a$  and  $b$  [Eq. (2)], we get the two-dimensional (pancake) vortex core energy  $U_{\text{core}}=0.0002$  eV, which gives  $\varepsilon=0.04$ . In turn, if we take into account the definition of  $U_1$  [Eq. (4)], the above estimates predict a screening length  $\Lambda=2\lambda_{ab}^2/d_s=250$   $\mu\text{m}$ , in good agreement with what is reported for  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  (Ref. 25) with values of  $\lambda_{ab}=270$  nm and  $d_s=6$   $\text{\AA}$ .

It should be noted that by the definitions given in Eqs. (6) and (8), the critical current densities  $j_c$  and  $j_0$  are related to each other as  $j_c/j_0=U_0/U_1$ . Since  $U_0 \geq U_1$ , we get that  $j_c \geq j_0$ . Furthermore, near the critical state (where  $j \leq j_0$  and the vortex energy contribution can be neglected, i.e.,  $U_0 \cong U_1$  and  $j_0 \cong j_c$ ),  $x_c \cong x_0$ . Thus the nonlinear  $U(j)$  law given by Eq. (8) reduces to the more recognized Anderson-Kim linear relationship [Eq. (6)]. Finally, assuming that<sup>26</sup>  $V=\pi\xi_c(T)d^2$ , where  $d=(\Phi_0/B)^{1/2}$  is the distance between the vortices,  $x_0=\xi_c(T)$ , and using the experimental value of  $\xi_c(T=5.5\text{K}) \cong 10$   $\text{\AA}$  and the value of  $U_1=0.005$  eV, we estimate the "effective" critical current density of our sample [Eq. (8)]  $j_0=U_1/BVx_0 \cong 10^6$  A/cm<sup>2</sup>, which is comparable with the typical values reported for  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  single crystals.<sup>18</sup>

Let us now consider the implication of this type of  $U-j$  relationship on the electric field ( $E$ ) as a function of  $j$ . In the flux-creep regime

$$E \sim \exp\{-CU(j)\}, \quad (9)$$

where  $C$  is a constant. In Fig. 3 we have plotted  $\ln E$  as a function of  $j/j_c$  for various  $U-j$  relationships. As can be seen in the figure, the linear  $U-j$  relationship predicts a finite resistance at all current densities while a zero-resistance state has been predicted by the vortex-glass<sup>28</sup> and collective-creep<sup>29</sup> models with a power-law form of the  $U(j)$  function. Interestingly, similar to the above-mentioned models, our  $U-j$  relationship [Eq. (8)] also predicts a negative curvature to the  $E-j$  curve and a zero resistance at finite  $j$  in the vortex state. A similar  $E-j$  behavior using the shape of  $U_p(x)$  has been earlier observed by Zeldov *et al.*<sup>24</sup>

In summary, we have conducted magnetic relaxation

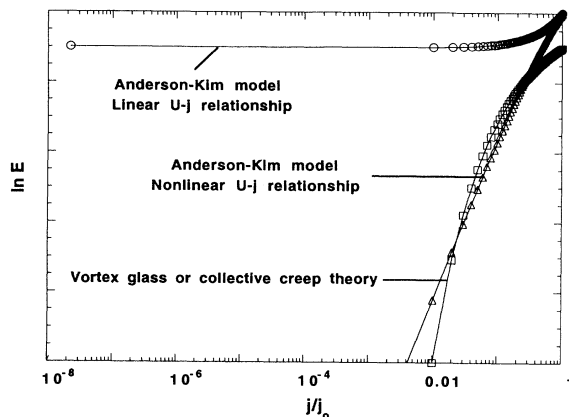


FIG. 3.  $E-j$  curves developed using the Anderson-Kim model with a linear  $U-j$  relationship [ $U=U_0(1-j/j_0)$ ], collective-creep or vortex-glass theory [ $U=U_0(j_0/j)^\mu$  with  $\mu=0.2$ ], and the Anderson-Kim model with nonlinear  $U-j$  relationship [ $U=a-b(1-j/j_0)+c \ln(j_0/j)$ ] Eq. (2). Note:  $U_0$  value is 0.005 eV for all models shown in the figure.

experiments on a  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  single crystal and developed a nonlinear  $U-j$  relationship. We have proposed a possible explanation for this nonlinear relationship based on the modification of the local pinning potential profile by double-kink formation of the pancake vortices. Based on this development, we have calculated the important superconducting parameters of the  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x$  system. We have also shown that the zero-resistivity state and the negative curvature of  $E-j$  curves as predicted by vortex-glass and collective-creep theory can also be explained by the observed  $U-j$  relationship.

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<sup>1</sup>M. R. Beasley, R. Labusch, and W. W. Webb, *Phys. Rev.* **181**, 682 (1969).

<sup>2</sup>K. A. Muller, M. Takashige, and J. G. Bednorz, *Phys. Rev. Lett.* **58**, 1143 (1987).

<sup>3</sup>Y. Yeshurun and A. P. Malozemoff, *Phys. Rev. Lett.* **60**, 2202 (1988).

<sup>4</sup>H. S. Lessure, S. Simizu, and S. G. Sankar, *Phys. Rev. B* **40**, 5165 (1989).

<sup>5</sup>Y. Xu, M. Suenaga, A. R. Modenbaugh, and D. O. Welch, *Phys. Rev. B* **40**, 10 882 (1989).

<sup>6</sup>D. Shi and M. Xu, *Phys. Rev. B* **44**, 4548 (1991); D. Shi, M. Xu, A. Umezawa, and R. F. Fox, *ibid.* **42**, 2062 (1990); D. Shi

and S. Salem-Sugui, Jr., *ibid.* **44**, 7647 (1991); S. Salem-Sugui, Jr. and Donglu Shi, *ibid.* **46**, 6618 (1992).

<sup>7</sup>S. Sengupta, P. McGinn, N. Zhu, and W. Chen, *Physica C* **171**, 174 (1990).

<sup>8</sup>M. Turchinskaya, L. H. Bennett, L. J. Swartzendruber, A. Roitburd, C. K. Chiang, M. Hill, J. E. Blendell, and K. Sawano, in *High Temperature Superconductors: Fundamental Properties and Novel Materials Processing*, edited by D. Cristen, J. Narayan, and L. Schneemeyer, MRS Symposia Proceedings No. **169** (Materials Research Society, Pittsburgh, 1990), p. 931; U. Atzmony, R. D. Shull, C. K. Chiang, L. J. Swartzendruber, L. H. Bennett, and R. E. Watson, *J. Appl. Phys.* **63**, 4179 (1988).

<sup>9</sup>M. D. Lan, J. Z. Liu, and R. N. Shelton, *Phys. Rev. B* **44**, 2751 (1991).

<sup>10</sup>P. W. Anderson and Y. B. Kim, *Rev. Mod. Phys.* **36**, 39

- (1964).
- <sup>11</sup>L. Burlachkov, Phys. Rev. B **47**, 8056 (1993).
- <sup>12</sup>Y. Ren and Peter A. J. de Groot, Physica C **196**, 111 (1992).
- <sup>13</sup>M. P. Maley, J. O. Willis, H. Lessure, and M. E. McHenry, Phys. Rev. B **42**, 2639 (1990).
- <sup>14</sup>M. E. McHenry, S. Simizu, H. Lessure, M. P. Maley, Y. C. Coulter, I. Tanaka, and H. Kojima, Phys. Rev. B **44**, 7614 (1991).
- <sup>15</sup>X. Ling, D. Shi, and J. I. Budnick, Physica C **185**, 2181 (1991).
- <sup>16</sup>V. Geshkenbein, A. Larkin, M. V. Feigel'man, and V. M. Vinokur, Physica C **162-164**, 239 (1989).
- <sup>17</sup>J. R. Clem, Phys. Rev. B **43**, 7837 (1991).
- <sup>18</sup>J. T. Kucera, T. P. Orlando, G. Virshup, and J. N. Eckstein, Phys. Rev. B **46**, 11 004 (1992).
- <sup>19</sup>C. P. Bean, Phys. Rev. Lett. **8**, 250 (1962).
- <sup>20</sup>B. M. Lairson, J. Z. Sun, T. H. Geballe, M. R. Beasley, and J. C. Bravman, Phys. Rev. B **43**, 10 405 (1991).
- <sup>21</sup>S. Sengupta, Donglu Shi, S. Salem-Sugui, Jr., Zuning Wang, P. J. McGinn, and K. DeMoranville, J. Appl. Phys. **72**, 592 (1992).
- <sup>22</sup>S. Sengupta, Donglu Shi, Zuning Wang, M. Smith, and P. J. McGinn, Phys. Rev. B **47**, 5165 (1993).
- <sup>23</sup>C. W. Hagen and R. Grieseen, Phys. Rev. Lett. **62**, 2857 (1989).
- <sup>24</sup>E. Zeldov, N. M. Amer, G. Koren, A. Gupta, and M. W. McElfresh, Appl. Phys. Lett. **56**, 680 (1990); E. Zeldov, N. M. Amer, G. Koren, and A. Gupta, *ibid.* **56**, 1700 (1990).
- <sup>25</sup>P. Manuel, C. Aguilon, and S. Senoussi, Physica C **177**, 281 (1991).
- <sup>26</sup>D. O. Welch, IEEE Trans. Magn. **27**, 1133 (1991); D. O. Welch, M. Suenaga, Y. Xu, and A. Ghosh, in *Advances in Superconductivity II*, edited by T. Ishiguro and K. Kajimura (Springer-Verlag, Tokyo, 1990).
- <sup>27</sup>M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).
- <sup>28</sup>M. P. A. Fisher, Phys. Rev. Lett. **62**, 1415 (1989).
- <sup>29</sup>M. V. Feigel'man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Phys. Rev. Lett. **63**, 2303 (1989).