Tilted and curved vortices in anisotropic superconducting films

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The shapes of the Abrikosov vortices and of the magnetic-field lines in c-oriented superconducting slabs or films in a tilted magnetic field B_a are calculated from anisotropic London theory. In thick slabs for not too small $B_a \gg B_{c1}$ the vortices and field lines have the same tilt as B_a apart from small deviations near the surfaces. However, in thin films the vortex lines are nearly straight and perpendicular to the surface even when the field lines are strongly tilted or curved by a high current density. Each field line can thus cross many vortex lines.

The discovery of high-T_c superconductors (HTSC) with pronounced anisotropy caused by their layered structure and with interesting equilibrium and pinning properties of the Abrikosov flux-line lattice (FLL) has revived numerous experimental techniques. At the surface of HTSC, the FLL is observed by decoration with "magnetic smoke"[1-3] and the penetration of magnetic flux by the Faraday effect in a magneto-optic layer on the flat surface.[4-6] Bulk experiments measure the magnetization and its change with time (flux creep) or the nonlinear current-voltage curves revealing features of flux flow and of pinning and depinning of flux lines which in HTSC may occur by thermal activation. Other experiments measure the complex ac susceptibility, or the dissipation and frequency change of HTSC performing tilt vibration in a dc magnetic field,[7,8] or the torque exerted by the anisotropic magnetization.[8,9] In many of these experiments the HTSC is a flat monocrystalline platelet or a thin film with the crystalline c axis perpendicular to the flat surface.

This paper presents the slope and curvature of the flux lines (vortex lines) and of the magnetic-field lines in such flat HTSC. The shape of the magnetic-field lines inside the specimen in general does not coincide with the vortex shape and in principle can be measured by small-angle neutron scattering. The curvature of the flux lines influences pinning by the misalignment between flux-line core and linear or flat pins.

The theory of tilted vortices in thin and thick films near the upper critical field B_{c2},[10] where the linearized Ginzburg-Landau theory applies, is discussed in an excellent review by Thompson.[11] Here I shall use anisotropic London theory, which for HTSC applies to the more realistic range $B \ll B_{c2}$. This means that effects on the length scale of the coherence length (or vortex core radius) $\xi = \xi_{ab}$ will not be considered. Since we are interested mainly in the *tilt* of the vortex cores and field lines, the periodic spatial variation of the internal field and of the external stray field (caused by the magnetic monopoles of the flux-line ends and depending on the effective penetration depth $2\lambda^2/d$) will be disregarded; this is allowed for vortex spacing $a < \pi \max(\lambda, 2\lambda^2/d)$ and uniform tilt. For larger a the stray field modifies the elasticity of the FLL near the surface[12] and the xy dependence of the internal and external B becomes important (x and y along the surface, z along the c axis).

For the present purpose it suffices to consider the linear response of the FLL with respect to the parallel component B_s of the applied $B_a = (B_s, 0, B_z)$. Nonlinear results may be obtained from the general solution $\mathbf{B}(\mathbf{r})$ of anisotropic London theory given in Ref. 13. The linear elastic response of the FLL in anisotropic superconductors with B along c has a large range of validity[14] if distortion wavelengths smaller than the penetration depth $\lambda = \lambda_{ab}$ are considered, namely, $|\tan\Theta| \ll \Gamma$, where Θ is the angle between the tilted flux line and the c axis and $\Gamma = \lambda_c/\lambda = \xi/\xi_c$ is the anisotropy ratio ($\Gamma \approx 5$ for Y-Ba-Cu-O, $\Gamma > 60$ for Bi-Sr-Cu-Cu-O). This large range of validity follows also from the explicit treatment of layered HTSC (Ref. 15) and from a scaling concept.[16]

To find the slopes of vortices and field lines, I calculate the vortex displacement $u(z)$ and the smooth internal transverse field $B_x(z)$ inside a flat infinite superconductor ($-d/2 \le z \le d/2$). Due to demagnetizing effects B_z penetrates completely, and the local slope of the field lines is $B_x(z)/B_z$. Thus the shape of one field line is $v(z) = B_z^{-1} \int B_x(z) dz$. Two problems may be distinguished: (a) a superconductor in tilted applied field $B_a = (B_s, 0, B_z)$ yields *odd* displacements $u(-z) = -u(z)$ and *even* field $B_x(-z) = B_x(z)$ with $B_x(\pm d/2) = B_s$; (b) a superconductor carrying a transport current with density $\mathbf{J} = [0, J(z), 0]$ in perpendicular field $B_a = (0,0,B_z)$ has *even* $u(-z) = u(z)$ and *odd* $B_x(-z) = -B_x(z)$ with derivative $B_x'(z) = \mu_0 J(z) = v''(z) B_z$; at the surfaces one has $B_s = B_x(d/2) = -B_x(-d/2) = \mu_0 I/2$, where $I = \int_{-d/2}^{d/2} J(z) dz$ is the sheet current.[17]

Here the equilibrium case (a) will be considered. In the current-carrying case (b) the Lorentz force density $B_z J(z)$ has to be balanced, e.g., by a constant frictional force (drag) $-\eta v = B_z J(z)$ (v = drift velocity, η = volume viscosity of the FLL). This balance implies that in the stationary flux-flow state, the current density and field curvature is spatially constant, $J(z) = J = \text{const}$, $B_x(z) = \mu_0 J z$. If pinning of the FLL is considered, the resulting $u(z)$, $B_x(z)$, and $J(z)$ depend on the assumed distribution of the pinning-force density $B_z J_c(z)$ and on the magnetic history.

I consider first a thick ($d \gg \lambda$) isotropic flat infinite superconductor in tilted field $(B_s 0, B_z)$ and disregard image

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vortices and elastic nonlocality of the FLL. The parallel field B_s generates a Meissner surface field $B_M(z)$ along x and a shielding current $J_M(z)$ along y,

$$B_M(z)=B_s\cosh(z/\lambda)/\cosh(d/2\lambda)\ , \qquad (1)$$

$$J_M(z)=(B_s/\mu_0\lambda)\sinh(z/\lambda)/\cosh(d/2\lambda)\ . \qquad (2)$$

The surface current exerts a force per unit area B_zB_s/μ_0 on the flux-line ends. This shearing force along x is balanced by the elastic restoring force per unit area of the tilted FLL, $c_{44}t$ where $t=\tan\Theta=du/dz$ is the flux-line tilt and $c_{44}=B_zH(B_z)$ the modulus for uniform tilt of the FLL,[18] $H(B_z)$ is the thermodynamic magnetic field which would be in equilibrium with the induction B_z. Thus, the flux lines and field lines are straight with constant tilt and slope (Fig. 1),

$$t=B_zB_s/\mu_0c_{44}=B_s/\mu_0H(B_z)\ . \qquad (3)$$

For large fields $B_z \gg B_{c1}(B_{c1}=\mu_0H_{c1}=$ lower critical field) one has $\mu_0H(B_z)\approx B_z$, and thus $t=B_s/B_z\equiv t_s$ is

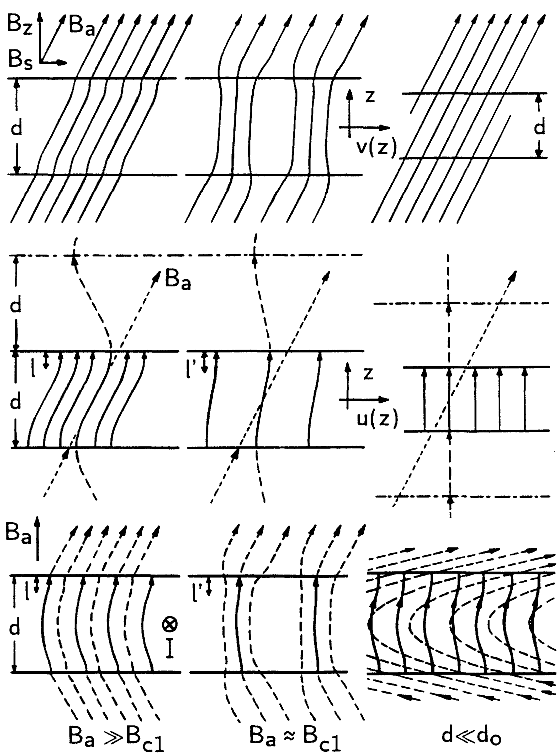

FIG. 1. Magnetic-field lines and vortex lines in a thick slab in magnetic field $B_a \gg B_{c1}$ (left column), $B_a \approx B_{c1}$ (middle column), and in a thin slab with thickness $d \ll d_0=\min(\lambda,a/\pi)/\Gamma$ (right column). Top: Field lines through a slab in the inclined field. Middle: Vortex lines (solid) and their images (dashed) for the same cases as above. Some of the image surfaces are indicated as dash-dotted lines. Bottom: Vortex lines (solid) and field lines (dashed) in slabs carrying a transport current I in perpendicular field B_a. Here a spatially constant pinning force or drag force is assumed. For clarity the figures at the right ($d \ll d_0$) are stretched along z.

just the slope of the internal and external field. For small fields $B_z < B_{c1}$ one has $H(B_z)\approx H_{c1}$, and thus $t=B_s/B_{c1}<t_s=B_s/B_z$. Thus, for B_z/B_{c1} the slopes of the vortex lines and of the average field lines inside the superconductor are smaller than the slope of the applied field, but for $B_z \gg B_{c1}$ the tilted field lines penetrate the superconductor without change of slope; the superconductor is "transparent" to fields $B \gg B_{c1}$.

I show now that consideration of the correct boundary conditions and finite range λ of the vortex-vortex interaction [the nonlocality or dispersion of $c_{44}(k)$] modifies the simple result (3) near the surface and in thin films. For uniform tilt one has in general,

$$c_{44}(B,\Theta)=BH(B,\Theta)+\partial^2F(B,\Theta)/\partial\Theta^2\ , \qquad (4)$$

where F is the free energy of the FLL and Θ is the angle between B and the c axis. In our example, $B=B_z$ and $\Theta=0$ (i.e., $B\|c\|z$). For $B \gg B_{c1}$ one has $F(B,\Theta)\approx B^2/\mu_0$, thus the second term in (4) is much smaller (proportional to the small magnetization) than the first term, and the isotropic result (3) still applies. For $B \ll B_{c1}$ one has $F(B,\Theta)\approx BH_{c1}$, thus $c_{44}=BH_{c1}(\Theta)+BH''_{c1}(\Theta)$, which for $\Gamma \gg 1$ and $B\|c$ is much smaller than the isotropic value $c_{44}=BH_{c1}$ since H''_{c1} is negative because the self-energy of the flux lines $\Phi_0H_{c1}(\Theta)$ is maximum for $\Theta=0$ and minimum for $\Theta=\pi/2$. Explicit results for $t(B_z)$ are given below.

The exact treatment has to account for the boundary conditions. Since the stray field is unimportant when $a<\pi\max(\lambda,2\lambda^2/d)$ and the tilt is uniform, it suffices to add the magnetic field of image vortices, which provides that no current flows through the surface. As a result, the flux-line displacement $u(z)$ and the perturbation $B_1(z)$ and formally also the Meissner field (1) and shielding current (2), become Fourier series which have physical meaning only for $|z|\le d/2$ (Fig. 1),

$$u(z)=\sum u_K\sin(Kz)\ , \qquad (5)$$

$$B_1(z)=\sum B_K\cos(Kz)\ , \qquad (6)$$

$$J_M(z)=\sum J_K\sin(Kz),\quad J_K=\frac{4B_s}{\mu_0 d}\frac{(-1)^n}{1+K^2\lambda^2}\ . \qquad (7)$$

In (5)–(7) the sums are over all $K=K_n=(\pi/d)(2n+1)>0, n=0,1,2,3,\ldots$. In (7) J_K was derived from (2). From (5) we see that the flux lines are perpendicular to the surface, $u'(z=\pm d/2)=0$, and the field change (6) caused by the flux-line tilt vanishes at the surface, $B_1(z=\pm d/2)=0$. The total transverse internal field

$$B_x(z)=B_M(z)+B_1(z) \qquad (8)$$

equals B_s at the two surfaces. Linearizing the anisotropic London field $\mathbf{B}(\mathbf{r})$ (Ref. 13) one obtains for our geometry,

$$B_K=u_KB_zK/(1+K^2\lambda^2)\ , \qquad (9)$$

which is independent of the anisotropy Γ. For the field lines $v(z)$ one obtains a Fourier series like (5) with

$$v_K=u_K/(1+K^2\lambda^2)+4t_s(-1)^n/d(\lambda^{-2}+K^2)\ . \qquad (10)$$

The vortex displacement $u(z)$ is the linear response to the Lorentz force density $B_z J_M$ exerted by the shielding current $J_M(z)$. Minimizing the elastic energy $\sum[\frac{1}{2}c_{44}(K)K^2u_K^2 - B_z J_K u_K]$ one obtains

$$u_K = J_K B_z / c_{44}(K)K^2 \ . \tag{11}$$

Here $c_{44}(K) = c_{44}(0,0,K)$ is the dispersive tilt modulus of the FLL with a tilt which is constant in the xy plane and periodic along z. In the London limit $\lambda \gg \xi$, $B \ll B_{c2}$ one has for $B \| c \| z$ (Refs. 18–20) (as above, $\lambda = \lambda_{ab}$),

$$c_{44}(K) = (B^2/\mu_0)\left[\frac{1}{1+K^2\lambda^2} + \frac{f(K)}{k_{\rm BZ}^2\lambda^2}\right] , \tag{12}$$

$$f(K) = \frac{1}{2\Gamma^2}\ln\frac{\xi_c^{-2}}{\lambda^{-2}+K^2+\Gamma^2k_0^2} + \frac{\ln[1+K^2/(\lambda^{-2}+k_0^2)]}{2K^2\lambda^2} ,$$

with $k_{\rm BZ} = (4\pi B/\Phi_0)^{1/2} = (8\pi/\sqrt{3}a^2)^{1/2} = 3.81/a$ the Brillouin-zone radius and $k_0 \approx k_{\rm BZ}$ a cutoff radius, $\xi_c = \xi/\Gamma, \Phi_0 = h/2e$ the flux quantum, and a the flux-line spacing. The first term in (12) originates from the interaction between flux lines, and the second term mainly from the tension of isolated flux lines. Due to the last term in $f(K)$ the tilt modulus (12) is slightly more general than in Ref 19 and yields the correct limit $B \to 0$, $\Gamma \to \infty$, $k \to 0$, which gives the finite line tension P of a stack of pancake vortices along c for zero Josephson coupling between the CuO layers.[21] One has $P = c_{44}\Phi_0/B = \Phi_0(H_{c1} + d^2H_{c1}/d\Theta^2)$,[19]

$$P \approx (\Phi_0^2/4\pi\mu_0\lambda^2)(\Gamma^{-2}\ln\kappa + 1/2) \ . \tag{13}$$

In the limit $\Gamma \to \infty$ the line tension (13) of a straight flux line stays finite.[20] From (6), (9), and (11) follows

$$u_K = \frac{4t_s(-1)^n}{dK^2[1+(\lambda^{-2}+K^2)k_{\rm BZ}^{-2}f(K)]} , \tag{14}$$

with $t_s = B_s/B_z$ the slope of the applied field. The Fourier coefficients $u_K(14)$, $B_K'(9)$, and $v_K(10)$ are the central result of this paper. I will now discuss special cases.

For *not too small* fields $B_z > 2B_{c1}$ one has flux-line spacing $a \ll \pi\lambda$ and $k_{\rm BZ} \gg \lambda^{-1}$ and (14) gives

$$u_K = 4t_s(-1)^n/d(K^2+K^4l^2) , \tag{15}$$

$$u(z) = t_s z - t_s l \sinh(z/l)/\cosh(d/2l) , \tag{16}$$

for $|z| \le d/2$; (16) is obtained by evaluating the Fourier series (5). The new length l is given by

$$l \approx \frac{a}{3.8\Gamma}\left[\ln\frac{a}{3.8\xi}\right]^{1/2} \approx \frac{a}{\pi\Gamma} \ . \tag{17}$$

The results (15)–(17) apply to all thicknesses $d \gg \xi$. For not too thin films or slabs with $d \gg l \approx a/\pi\Gamma$, (16) means the flux lines are straight lines with the same slope t_s as the applied field, but at a distance $l \ll d$ from the surfaces they curve in order to hit the surface at a right angle, $u(z) \approx t_s(z - le^{-x})$ with $x = (d/2 - z)/l$ (for $z > 0$). For very thin films with $d < l \ll a \ll \pi\lambda$, (15) gives $u_K \propto K^{-4}$, thus,

$$u(z) \approx u_0 \sin(\pi z/d) , \tag{18}$$

$$u_0 = 4t_s d^3\Gamma^2/a^2\pi^2\ln(d/\pi\xi_c) , \tag{19}$$

or $u_0 \approx 4t_s d^3/\pi^4 l^2$. The maximum vortex slope is now $t = u'(0) \approx (4d^2/\pi^3 l^2)t_s \ll t_s$.

For the transverse field $B_x(z)$ one obtains from (6), (8), (9), and (15),

$$B_K = 4B_s(-1)^n/dK(1+K^2\lambda^2)(1+K^2l^2) , \tag{20}$$

$$B_x(z) = B_s + \frac{B_s l^2}{\lambda^2 - l^2}\left[\frac{\cosh(z/l)}{\cosh(d/2l)} - \frac{\cosh(z/\lambda)}{\cosh(d/2\lambda)}\right] , \tag{21}$$

$$v(z) = t_s z + \frac{t_s l^2}{\lambda^2 - l^2}\left[\frac{l\sinh(z/l)}{\cosh(d/2l)} - \frac{\lambda\sinh(z/\lambda)}{\cosh(d/2\lambda)}\right] . \tag{22}$$

Since $l \ll \lambda$, (21) means $B_x(z) \approx B_s = \text{const}$. The magnetic-field lines $v(z) \approx t_s z$ (22) go straight through the specimen, but near the surface they exhibit a small dip of width λ and relative depth $l^2/d\lambda \ll 1$, which originates from the curvature of the flux lines.

For *small* fields $B_z < B_{c1}(a > \pi\lambda)$ the first term in $c_{44}(12)$ (originating from the vortex interaction) and the unity in $u_K(14)$ may be disregarded and one obtains

$$u_K = \frac{4t_s(-1)^n k_{\rm BZ}^2}{dK^2(\lambda^{-2}+K^2)f(K)} \ . \tag{23}$$

Though the derivation of (23) assumes $\pi\lambda < a \ll \pi\lambda^2/2d$ and thus $d \ll \lambda$, it yields the correct u and average B in the bulk even for thick films with $d \gg \lambda$; (23) then gives $u_K(15)$ and $u(z)(16)$ with t_s replaced by t and l by l',

$$t \approx t_s(\pi\lambda/a)^2/[\Gamma^{-2}\ln(\Gamma\kappa) + \tfrac{1}{2}] < t_s , \tag{24}$$

$$l' = \lambda/(1+\Gamma^2\ln\Gamma/\ln\kappa)^{1/2} \approx \lambda/\Gamma \ . \tag{25}$$

This means that for $a > \pi\lambda, d \gg \pi\lambda$ only a small fraction of the transverse field $B_x(0)/B_s \approx t/t_s \ll 1$ penetrates, i.e., in the bulk both flux lines and average field lines have smaller slope t (24) than the applied field. The ratio of these slopes coincides with the above estimate [below Eq. (4)] $t/t_s = B_z/[B_{c1}(\Theta) + B_{c1}''(\Theta)]_{\Theta=0}$. Note that in this case the periodic variation of $B_z(x,y,z)$ can be large. In fact it is the "bundling" of field lines along the vortex lines which for $a > \pi\lambda$ enhances the line tension (=tilt modulus per flux line) and thus decreases the tilt inside the specimen, cf. the middle column of Fig. 1. Our approximation has thus a large range of applicability. The stray field is negligible since in thick specimens the bulk is far from the surfaces, and in thin films the field modulation is small due to the large effective penetration depth $2\lambda^2/d$.

If the film is thin ($d \ll l' \approx \lambda/\Gamma$ and $a > \pi\lambda$) one has the same solution $u(z) \approx u_0 \sin(\pi z/d)$ Eqs. (18) and (19), as found above for $d \ll a/\pi\Gamma$ and $a \ll \pi\lambda$. Thus in the general thin-film limit $d \ll d_0 = \min(\lambda, a/\pi)/\Gamma$ the flux lines are sinusoidal with very small maximum slope $t = u'(0) = u_0\pi/d = [4d^2\Gamma^2/a^2\pi \ln(d/\pi\xi_c)]t_s \ll t_s$.

In conclusion, for *thick* flat superconductors with $d \gg \pi\lambda$ in not too small perpendicular field, $B_z > 2B_{c1}$ an applied transverse field component $B_s = t_s B_z$ tilts the flux lines such that their slope coincides with that of the applied field. The resulting internal field lines have the same slope. Near the two surfaces the flux lines curve over a short length $l \approx a/\Gamma\pi \ll d$ in order to hit the surface at a right angle, cf. $u(z)$, Eq. (16). This vortex curvature causes a weak dip in the transverse field component $B_x(z)$ (21) and in the field lines $v(z)$ (22). In small fields the slope t of the flux lines and field lines falls below the slope t_s of the external field, and the length of the curved section near the surface is $l' = \lambda/\Gamma$. Thus for arbitrary B_z this length $d_0 = \min(\lambda, a/\pi)/\Gamma$ is small for large anisotropy Γ. The slope t follows also from the modulus for uniform tilt $c_{44} = BH + \partial^2 F/\partial\Theta^2$.

For *thin* films with $d \ll d_0$ one has the interesting situation that the flux lines are practically perpendicular to the film even when the field lines are strongly tilted and just traverse the film. Applied to the case of a thin film with transport current I (bottom of Fig. 1) this finding means that the current $J(z) = v''(z)B_z/\mu_0$ is carried by the curvature of the field lines, not the flux lines, which are practically straight and perpendicular to the surface even when the field lines curve strongly. At large current densities each curved and tilted field line can thus cross several vortex lines.

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