Collective-pinning theory for magnetically coupled layered superconductors

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We calculate the tilt modulus C_{44} and make collective-pinning estimates for a vortex lattice in magnetically coupled superconducting layers. Complete analysis of possible pinning regimes is made. The tilt modulus has unusual nonlocal structure which leads to peculiar pinning behavior. Namely, under certain conditions for the pinning strength and the magnetic coupling, a transition from a regime of independently pinned two-dimensional (2D) vortex lattices in the layers to a state of three-dimensional (3D) collective pinning occurs as a first-order phase transition. The critical current in 3D collective-pinning regime has characteristic nonmonotonic field dependence.

I. INTRODUCTION

Flux pinning in high-temperature superconductors (HTS) depends sensitively on the anisotropy of these materials, i.e., the strength of the coupling between the superconducting layers.[1-3] The anisotropy is expressed by the parameter $\gamma=\lambda_c/\lambda_{ab}$ where λ_{ab} and λ_c are the penetration depths of the screening currents in the *ab* planes and in the *c* direction. Quite similar behavior is anticipated for superconductor-normal (SN) and superconductor-insulator (SI) multilayers. For these artificial structures γ can be changed at will by a proper choice of the material and thickness of the nonsuperconducting layer. With increasing γ the vortex lattice becomes increasingly softer. When the magnetic field is perpendicular to the layers ($H\|c$), especially the nonlocal tilt modulus C_{44} is reduced by a factor γ^2.[4,5] In the concept of collective pinning[6] the size of the correlated regions (Larkin domains) is determined by the balance between deformation energy and pin energy. Consequently, these domains decrease in size when the anisotropy is enhanced, and therefore the low-temperature critical current j_c increases, while the energy barrier for flux creep decreases.[1] Hence, there is a direct interplay between the practical properties of the HTS and the anisotropy.

Depending on the value of γ the tilt modulus follows from the anisotropic Ginzburg-Landau description[4] or from the Lawrence-Doniach model.[5] The former applies when $\xi_c\equiv\xi_{ab}/\gamma>s$, where ξ_{ab} is the coherence length and s the periodicity of the multilayer or HTS. For $\xi_c\ll s$ the Lawrence-Doniach formulation has to be used, but one may still discriminate between two distinct regimes depending on whether the coupling between the superconducting layers is predominantly Josephson-like or mainly magnetic. The vortex lattice properties in ideal magnetically coupled multilayers (i.e., without pinning centers) where studied in several theoretical papers.[7-10] Collective pinning and creep in the Josephson regime has been theoretically investigated by Feigel'man, Geshkenbein, and Larkin[2] and Vinokur, Kes, and Koshelev.[3] In this paper we study the opposite case of predominant magnetic coupling which occurs for $\gamma>\gamma_{cr}\equiv\lambda_{ab}/s$. Experimentally, this situation has been realized, e.g., in Pb/Ge superlattices (Ref. 11), in *a*-MoGe/Ge (Ref. 12) multilayers, and $YBa_2Cu_3O_x/PrBa_2Cu_3O_y$ (Ref. 13) multilayers with sufficiently large thickness of the insulating layer. Investigations of the field dependence of the critical current j_c were performed only in the former case. This dependence demonstrates an unexpected minimum which position depends upon the multilayers parameters and temperature.

Another interesting situation may occur when the Josephson coupling is suppressed either by thermal fluctuations or by a magnetic-field component parallel to the layers, i.e., $\gamma=\gamma(T,H_{\|})>\gamma_{cr}$. This may be the case for $Bi_2Sr_2CaCu_2O_x$ single crystals in which a very large γ value has been observed in magnetic torque measurements above 77.4 K:[14-16] $\gamma>150$, while $\gamma_{cr}\sim 100-200$.

This paper is organized as follows. In Sec. II we outline the derivation of the tilt modulus C_{44} for magnetically coupled layers and present the final results for small and large fields. The derivation is given in detail in Appendix A. The tilt modulus has an unusual nonlocal structure, namely, it does not depend on the transverse wave vector $k_\perp$ but only on the longitudinal wave vector k_z. Another feature of C_{44} is that it decreases sharply when the vortex lattice parameter a becomes smaller than the multilayer periodicity s. In Sec. III the consequences of this peculiar C_{44} for the collective-pinning behavior are investigated. Under specific conditions for the pinning strength and the magnetic coupling a first-order phase transition is predicted from a regime of independently pinned two-dimensional (2D) vortex lattices in the layers to a state of three-dimensional (3D) collective pinning. Expressions for the size of the Larkin domains (correlated regions[6]), the pinning energy, and the critical current density in the different regimes are derived. The implications for several "multilayer system," e.g., BiSSCO, *a*-Nb_3Ge/Ge, Pb/Ge, and Nb/Ge, are analyzed in Sec. IV. The experimental input parameters are used to investigate in which system the predicted phase transition may occur.

Finally, in Appendix B, we discuss the relation between the different parameters characterizing the strength of the disorder and the link with the experimental critical current.

II. TILT MODULUS FOR SUPERCONDUCTING MULTILAYERS WITHOUT JOSEPHSON COUPLING

We consider material composed of superconducting layers of thickness d_s separated by insulating layers of thickness d_i, (see inset of Fig. 1). The period of the multilayer is given by $s=d_i+d_s$. The Josephson coupling between the layers is considered to be negligibly weak. The superconducting properties of this system are determined by the London penetration depth λ_s and the coherence length ξ_s of the superconducting material. The average London penetration depth of the multilayer λ is connected with λ_s by the relation $\lambda=\lambda_s\sqrt{s/d_s}$. In the following we restrict ourselves to the case $\xi_s \ll \lambda$. The vortex lattice in such a system is composed of 2D triangular lattices of pancake vortices, located in the layers. The interaction between pancakes in different layers is mediated by the magnetic field and is much weaker than the interaction between pancake vortices in the same layer. The properties of such a system for the case $d_s \ll d_i$ have been considered in a number of theoretical papers.[7-10] However, a precise calculation of the tilt modulus is absent. Below we obtain the tilt modulus for such a system.

In the limit $H \ll H_{c2}$ the free energy F of the system is given by

$$F=\int d^3\mathbf{r}\Theta_m(z)\frac{(\mathbf{\Phi}_n-\mathbf{A})^2}{8\pi\lambda_s^2}+\int d^3\mathbf{r}\frac{B^2}{8\pi}, \tag{1}$$

the dimensionless function $\Theta_m(z)$ is defined by the relation

$$\Theta_m(z)=\begin{Bmatrix}1, \text{ at } |z-ns|<d_s/2\\ 0, \text{ at } |z-(n+1/2)s|<d_i/2\end{Bmatrix}$$

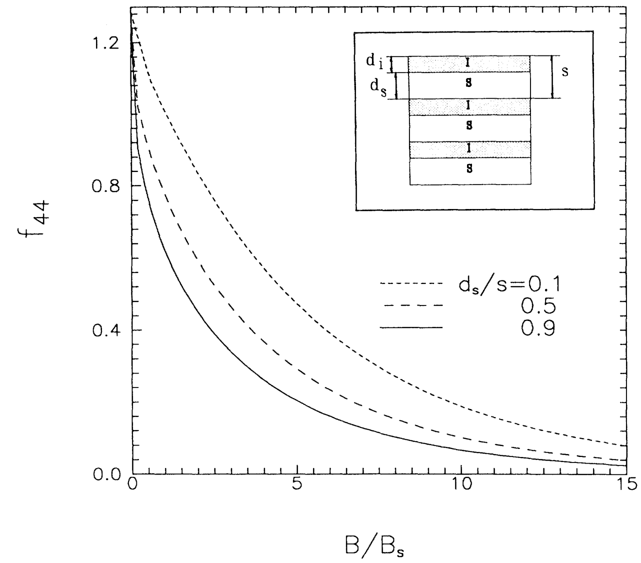

FIG. 1. The function f_{44} describing the field dependence of the tilt modulus for long-wavelength deformations in the small field regime ($B \ll H_{c2}$) for three values of the parameter d_s/s. The inset shows the configuration of the multilayer.

and $\mathbf{\Phi}_n(\mathbf{r})=\Phi_0/2\pi\nabla\phi_n(\mathbf{r})$ is the London vector for the nth layer,

$$\mathbf{\Phi}_n(\mathbf{r})=\frac{\Phi_0}{2\pi}\sum_m\frac{(\mathbf{r}-\mathbf{R}_{nm})\times\mathbf{n}_z}{|\mathbf{r}-\mathbf{R}_{nm}|^2}.$$

$\mathbf{R}_{nm}$ are positions of the vortices in the nth layer. The derivation of the tilt modulus C_{44} from the energy (1) for arbitrary d_s, d_i, and λ_s in the field range $H \ll H_{c2}$ is given in Appendix A. In the practically interesting case of sufficiently large fields, $B>\Phi_0/(4\pi\lambda_s)^2$, the tilt modulus is given by

$$C_{44}(k_z)=\frac{B^2s^3}{32\pi\lambda^4 d_s}\sum_{\mathbf{Q}}\frac{1-[1-\exp(-Qd_s)]/(Qd_s)}{Q^2\tanh(Qs/2)\{[\sinh(Qs/2)]^2+(\tilde{k}_z s/2)^2\}}, \tag{2}$$

where the sum is performed over the reciprocal-lattice vectors $\mathbf{Q}$ of the triangular vortex lattice and $\tilde{k}_z=(2/s)\sin(sk_z/2)$. The expression for small k_z, $C_{44}(0)$ can be written in a scaling form

$$C_{44}(0)=\frac{9\Phi_0^2}{\pi(4\pi\lambda)^4}f_{44}(B/B_s,d_s/s) \tag{3}$$

with

$$B_s=\frac{\sqrt{3}\Phi_0}{2\pi^2 s^2}\approx\frac{1.75[T]}{(s/10\text{ nm})^2}. \tag{4}$$

Plots of the function f_{44} vs B/B_s at different values of the ratio d_s/s are presented in Fig. 1.

For layered superconducting compounds (e.g., $Bi_2Sr_2CaCu_2O_x$) $s\approx 1.5$–2.0 nm which means that the inequality $Qs \ll 1$ ($B \ll B_s$) is valid in an accessible field range. In this limiting case the complex expression (2) simplifies significantly. For not too small fields ($K_0\lambda \gg 1$, $K_0=\sqrt{4\pi B/\Phi_0}$) it takes the form:

$$C_{44}(k_z)=\frac{B^2}{8\pi\lambda^4}\sum_{Q\neq 0}\frac{1}{Q^2(Q^2+\tilde{k}_z^2)}. \tag{5}$$

It is seen that the long-range nature of the interaction leads to nonlocal behavior of the tilt stiffness at wave vectors $k_z>K_0$. In contrast to 3D superconductors[17,5] the tilt modulus does not depend on the transverse wave vector $k_\perp$. As we will show below this kind of nonlocality leads to a very peculiar behavior of the pinning properties. The limiting value of $C_{44}(k_z)$ at $k_z \ll K_0$ is

$$C_{44}(0)=3.68\frac{\Phi_0^2}{(4\pi\lambda)^4} . \quad (6)$$

A reasonable interpolation approximation for $C_{44}(k_z)$ can be obtained by changing the summation in Eq. (5) to an integration and choosing the lower limit of integration such that the asymptotic expression Eq. (6) is reproduced. This gives

$$C_{44}(k_z)=\frac{B\Phi_0}{32\pi^2\lambda^4k_z^2}\ln\left[1+\alpha\frac{k_z^2}{K_0^2}\right] , \quad (7)$$

with $a \approx 0.585$.

The above equations are valid for small fields, $B \ll H_{c2}$. In Appendix A we also obtain the asymptotic expression of C_{44} for $B \to H_{c2}$ valid in the case $H_{c2} < B_s$, namely

$$C_{44}(k_z)=0.74\cdot\frac{(1-b)^2\Phi_0^2}{(4\pi\lambda)^4}\frac{1}{1+\tilde{k}_z^2/Q_0^2} \quad (8)$$

with $b = B/H_{c2}$. To match the regions $B \ll H_{c2}$ [Eq. (6)] and $B \sim H_{c2}$ [Eq. (8)] we use the following interpolation formula for $C_{44}(0)$:

$$C_{44}(0)=3.68\cdot(1-b)^2(1-0.8b)\frac{\Phi_0^2}{(4\pi\lambda)^4} . \quad (9)$$

In the following this expression should be supplemented by the known result for the shear modulus[17]

$$C_{66}=(1-b)^2(1-0.58b+0.29b^2)\frac{B\Phi_0}{(8\pi\lambda)^2} . \quad (10)$$

Note that $C_{44}(k_z) \ll C_{66}$ in most practical cases.

III. COLLECTIVE-PINNING ESTIMATES

A. General

A vortex lattice in layered superconductors has three relevant energy scales, namely the typical tilt energy $U_{\rm tilt}=C_{44}(\pi/s)[(\pi r_p)^2a^2/2s]$, the typical shear energy $U_{\rm shear}=\frac{1}{2}C_{66}s(\pi r_p)^2$, and the pinning energy U_p with r_p being the range of the pinning force (see Appendix B) and a being the intervortex spacing. Depending on the relative magnitude of these energies one can distinguish four possible regimes of pinning: (1) Independently pinned vortex lines ($U_{\rm tilt} > U_p > U_{\rm shear}$), (2) Independently pinned vortex pancakes ($U_p > U_{\rm tilt}, U_{\rm shear}$), (3) 2D collectively pinned state (2DCP) in which the 2D vortex lattices in the layers are pinned independently from each other ($U_{\rm shear} > U_p > U_{\rm tilt}$), and (4) 3D collectively pinned state (3DCP) ($U_{\rm tilt}, U_{\rm shear} > U_p$).

Regimes 1 and 2 occur at low magnetic fields or very close to H_{c2}. Which regime prevails depends on the relation between the strength of disorder and the interlayer coupling energy. We will characterize the disorder by the mean-square amplitude of the random force γ_p, $\gamma_p=(U_p/r_p)^2$ (see Appendix B). The crossover between regimes 1 and 2 takes place when γ_p exceeds the typical value γ_{p0}, which can be estimated from the condition $U_p \sim U_{\rm tilt}$,

$$\gamma_{p0}\approx\left[\frac{\Phi_0^2r_ps}{(4\pi)^2\lambda^4}\right]^2 , \quad (11)$$

(see Appendix B). The important parameter γ_p/γ_{p0} characterizes the strength of disorder in the multilayers in comparison to the tilt stiffness. It can be connected with experimentally accessible parameters by the relation

$$\gamma_p/\gamma_{p0}=\left[0.76\frac{(j_{c0}/[10^9\ {\rm A/m^2}])(\lambda/[100\ {\rm nm}])^4}{r_p/[{\rm nm}]}\right]^2 , \quad (12)$$

with j_{c0} the maximum critical current at small fields. When the parameter γ_p/γ_{p0} is too large only regimes 2 and 3 occur, i.e., the system behaves as a collection of independent thin films. The exact criterion for this case

$$\gamma_p/\gamma_{p0} > \min[\lambda^2/s^2,\lambda^2/(4\xi_s)^2]$$

will be obtained below. In the following we restrict ourselves to the regime of intermediate pinning

$$1 < \gamma_p/\gamma_{p0} < \min[\lambda^2/s^2,\lambda^2/(4\xi_s)^2] .$$

Realization of the other regimes depends on the value of the magnetic field. The typical fields separating different regimes are determined by the length parameters of the system (d_s,d_i,λ,ξ_s) and the average square amplitude of the random force γ_p. Depending on the relation between these parameters different types of behavior are possible which will be considered below.

B. Pinning estimates at small fields ($B \ll H_{c2},B_s$). Jumplike transition between different pinning configurations

To distinguish between the pinning regimes we use the standard collective-pinning scheme[6] for the estimation of the pinning correlation lengths R_c and L_c which determine the geometrical size of the Larkin domain in the direction perpendicular and parallel to the field, respectively.[18] They follow from the balance between the elastic energy and the interaction with the random potential. The random potential is characterized by the mean-square amplitude of the random force W, $W=B\gamma_p/\Phi_0 s$, and the typical interaction range r_p. The energy per unit volume is estimated as

$$E_p(R,L)\approx\frac{C_{66}}{2}\frac{(\pi r_p)^2}{R^2}+\frac{C_{44}(\pi/L)}{2}\frac{(\pi r_p)^2}{L^2}-\frac{W^{1/2}r_p}{(LR^2)^{1/2}} , \quad (13)$$

where the k_z dependence of $C_{44}(k_z)$ is given by Eq. (7). The parameters R_c and L_c are given by the values of R and L which minimize the function $E_p(R,L)$. Minimization with respect to R gives

$$E_p(L)\approx-\frac{W}{2\pi^2C_{66}L}+\frac{C_{44}(0)r_p^2K_0^2}{2\alpha}\ln\left[1+\alpha\frac{\pi^2}{L^2K_0^2}\right] , \quad (14)$$

with $C_{44}(0)$ given by Eq. (6) and

$$R_c(L)=\frac{\pi^2 C_{66} r_p L^{1/2}}{W^{1/2}}=2\frac{B}{B_{2D}}\left[\frac{L}{s}\right]^{1/2}. \tag{15}$$

At small fields the vortex pancakes are pinned independently (regime 2). As the field increases, a crossover from regime 2 to the 2D collectively pinned state takes place at the typical field B_{2D}, which can be estimated from Eq. (15) by putting $R_c(L)=a$ and $L=s$,

$$B_{2D}\approx\left[\frac{\gamma_p}{\gamma_{p0}}\right]^{1/2}\frac{8\Phi_0}{(\pi\lambda)^2}. \tag{16}$$

The pinning energy per unit volume for this state, $E_p^{(2D)}$, can be estimated from Eq. (14) by putting $L=s$,

$$E_p^{(2D)}\approx-\frac{W}{2\pi^2 s C_{66}}+\frac{C_{44}(0)r_p^2K_0^2}{2\alpha}\ln\left[\frac{\Phi_0}{s^2B}\right]. \tag{17}$$

The second term arises from the loss in magnetic coupling energy between the layers. In the case of 2DCP this term is smaller than the first one.

To estimate the field at which the formation of the 3DCP state becomes more favorable we analyze Eq. (14). We start with the simplest case when the minimum of the function $E_p(L)$ occurs at $L=L_c>a$ so that nonlocality of C_{44} is not important. In this case the pinning energy and the correlation lengths are given by

$$E_p^{(3D)}=-\frac{W^2}{(2\pi^2)^3C_{66}^2C_{44}(0)r_p^2}, \tag{18}$$

$$R_c=\frac{\sqrt{2}\pi^4[C_{44}(0)]^{1/2}C_{66}^{3/2}r_p^2}{W}, \tag{19}$$

$$L_c=\frac{2\pi^4C_{44}(0)C_{66}r_p^2}{W}. \tag{20}$$

The condition $L_c>a$ is fulfilled when the parameter W is sufficiently small, i.e., $W<W_{3D}^{(1)}$ with

$$W_{3D}^{(1)}=2\pi^3C_{66}C_{44}(0)r_p^2K_0\propto B^{3/2}. \tag{21}$$

The parameter W is smaller than $W_{3D}^{(1)}$ for magnetic fields $B>B_{3D}^{(1)}$ with

$$B_{3D}^{(1)}\approx\left[\frac{\gamma_p}{\gamma_{p0}}\right]^2\frac{\phi_0 s^2}{\lambda^4}. \tag{22}$$

In the opposite limiting case $W>W_{3D}^{(1)}$ ($B<B_{3D}^{(1)}$) the only minimum of the function $E_p(L)$ occurs at $L=s$, which corresponds to the 2DCP state (curve 1 in Fig. 2). In some field region above $B_{3D}^{(1)}$ the 3DCP state is metastable because it does not give the global minimum to the energy of the system (curve 2 in Fig. 2). This can be easily understood from an analysis of Eq. (14). Due to the nonlocality the tilt contribution to the energy weakly depends on L in the region $L<a$, namely as $\ln(a/L)$. On the other hand the energy of interaction with the random potential still decreases with decreasing L as $-L^{-1}$. This leads to a maximum in $E_p(L)$ at $L\approx L_{\max}$, $L_{\max}\approx aW/W_{3D}^{(1)}$. In the region $L<L_{\max}$ the energy decreases with decreasing L and reaches a second minimum at $L=s$ which corresponds to the 2DCP state. Comparing Eqs. (17) and (18) one can conclude that 3DCP becomes more favorable when the parameter W becomes smaller than $W_{2D\to3D}$,

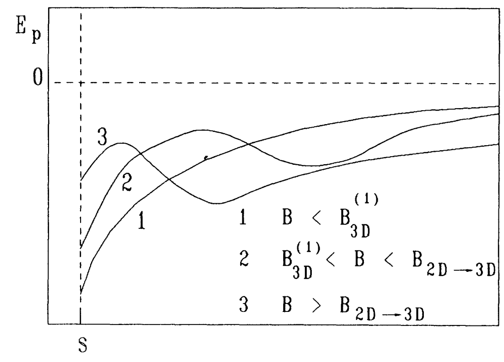

FIG. 2. Schematic dependence of the pinning energy on the size of the Larkin domain in the vortex lattice in a magnetically coupled multilayer for three different field regimes: (1) the only minimum of E_p is at $L=s$ (2DCP); (2) a second minimum with larger E_p appears at $L>a$, the pinning is still 2D; (3) the minimum at $L>a$ belongs to the most favorable configuration (3DCP).

$$W_{2D\to3D}=\frac{sC_{66}B\Phi_0r_p^2}{32\lambda^4}\ln\left[\frac{\Phi_0}{s^2B}\right], \tag{23}$$

which corresponds to the magnetic field $B_{2D\to3D}$,

$$B_{2D\to3D}=\frac{\gamma_p}{\gamma_{p0}}\frac{4\Phi_0}{\pi^2\lambda^2\ln[(\gamma_{p0}/\gamma_p)^{1/2}(\lambda/s)]}. \tag{24}$$

When the field reaches $B_{2D\to3D}$ the equilibrium configuration of the system should jump from almost independently pinned 2D vortex lattices to a collectively pinned 3D lattice. This means that in layered superconductors without Josephson coupling *a transition from 2DCP to 3DCP occurs as a first-order phase transition.* This scenario is realized when the field $B_{2D\to3D}$ is smaller than both B_s and $0.2H_{c2}$.[19] Comparison gives two new conditions for the strength of disorder

$$\frac{\gamma_p}{\gamma_{p0}}\frac{s^2}{\lambda^2}<1, \tag{25}$$

$$\frac{\gamma_p}{\gamma_{p0}}\frac{(4\xi_s)^2}{\lambda^2}<1. \tag{26}$$

If at least one of these conditions is violated then 3DCP is not realized at all. Note that Eqs. (25) and (26) also give conditions of the existence of the so-called "Giaever transformer" effect[20–22] in the multilayer system.

The critical current j_c is connected with the pinning energy by the simple relation

$$(B/c)j_c\approx|E_p|/r_p. \tag{27}$$

In the quasi-2D region [see Eq. (17)] the critical current differs from the value for a single layer by a small negative correction induced by the magnetic coupling between the layers,

$$j_c = j_c^{(2D)}(1 - B/B_{2D\to 3D}) \; . \tag{28}$$

Here $j_c^{(2D)} = j_{c0} B_{2D}/B$ is the critical current for a single layer. Note that the coupling-induced correction is weakly field dependent in the region $B \ll H_{c2}$.

At the field $B \approx B_{2D\to 3D}$ a first-order phase transition to the 3DCP takes place. As the pinning energies of the 2DCP and 3DCP states are the same in the transition point, no strong singularity in the critical current is expected if the lattice is always in the equilibrium state. However, due to the existence of an energy barrier between the 2DCP and 3DCP states "overcooling" is possible, which means that the system remains in the 2DCP nonequilibrium state in some region above $B_{2D\to 3D}$ and then jumps to the 3DCP state. In the latter case a jump in j_c is expected.

In the 3DCP state the critical current can be estimated as $(B \ll H_{c2})$

$$j_c = 0.1 \frac{B_{2D}}{B} \frac{\gamma_p}{\gamma_{p0}} \frac{s^2}{\lambda^2} j_{c0} \; . \tag{29}$$

Note that the critical current in the 3DCP state has the same field dependence as in the 2DCP state ($j_c \propto B^{-1}$) but with a smaller coefficient.

The behavior of the critical current at large fields depends on the relation between ξ_s and s, i.e., between the fields B_s and H_{c2}. Below we separately consider two cases corresponding to different relations between these parameters.

C. Large fields. Case $\xi_s > s$ $(B \le H_{c2} < B_s)$

For multilayers with a coherence length ξ_s larger than the periodicity s the 3DCP survives up to some field close to H_{c2}. Using interpolation formulas for C_{44} (9), C_{66} (10), and W (Appendix B) it is possible to obtain an estimate for the field dependence of j_c in the 3DCP state which is valid in the whole field range $B \le H_{c2}$,

$$j_c = \frac{j_{c1}}{b(1-b)^2(1-0.8b)(1-0.58b+0.29b^2)^2} \; , \tag{30}$$

with $j_{c1} = 0.1(B_{2D}/H_{c2})(\gamma_p/\gamma_{p0})(s^2/\lambda^2) j_{c0}$. It follows from this equation that the field dependence of j_c has a minimum $J_{c\min} \approx 11.5 j_{c1}$ at a field $B_{\min}$, $B_{\min} \approx 0.23 H_{c2}$.

To estimate the field value $B_{3D\to 2D}$ at which the system returns from the 3DCP to the 2DCP state near H_{c2}, we once more compare the energies of these states given by Eqs. (17) and (18) using the asymptotic expressions for C_{44} and C_{66} given by Eqs. (8) and (10). This gives

$$(1 - b_{3D\to 2D})^2 = \frac{\gamma_p}{\gamma_{p0}} \frac{(4\xi_s)^2}{\lambda^2} \; , \tag{31}$$

with $b_{3D\to 2D} = B_{3D\to 2D}/H_{c2}$. Note that due to condition (26) the right-hand side of Eq. (31) is much smaller than unity. In the case of not a too small parameter $(\gamma_p/\gamma_{p0})[(4\xi_s)^2]/\lambda^2$ Eq. (31) should be replaced by a more precise condition which takes into account the field dependencies of the elastic moduli in the field range $B \le H_{c2}$.

$$7.1(1-0.58b+0.29b^2)(1-0.8b)b(1-b)^2 = \frac{\gamma_p}{\gamma_{p0}} \frac{(4\xi_s)^2}{\lambda^2}, \quad \text{at } b = b_{3D\to 2D} \; . \tag{32}$$

Analogous to the transition at weak fields the transition between 3DCP and 2DCP is jumplike. Particularly, the domain length L_c jumps from $L_c \approx \xi_s$ to $L_c = s$.

At the transition field a maximum critical current occurs which can be estimated to be

$$j_{c\max} \approx 0.53 \frac{\gamma_{p0}}{\gamma_p} \frac{\lambda_s^2 \ln(\xi_s/s)}{\xi_s^2} j_{c1} \; . \tag{33}$$

Above the field $B_{3D\to 2D}$ the vortex lattices in the layers are pinned independently from each other and the behavior of the critical current should be the same as for thin films.[23]

D. Large fields. Case $\xi_s < s$ $(B_s < B \le H_{c2})$

For artificially created multilayers with $\xi_s \ll s$ the distance between vortices a becomes smaller than the period of the multilayers s when the magnetic field becomes larger than the typical field B_s given by Eq. (4). The tilt modulus starts to decrease rapidly at $B > B_s$, because the interaction between the layers becomes exponentially weak [see Eq. (2) and Fig. 1]. This decrease leads to nonmonotonic behavior of the critical current with a minimum at $B \sim B_s$. This specific behavior is studied in detail in this section.

In the 3D collective-pinning region the nonlocality in the tilt stiffness can be neglected and we can use Eqs. (2), (18) and (27) to obtain the field dependence of the critical current. Using Eq. (3) the result can be written in the following scaling form:

$$j_c(B_s/B, d_s/s) = j_s \frac{B_s/B}{f_{44}(B/B_s, d_s/s)} \; , \tag{34}$$

with

$$j_s \approx 0.0083 \frac{c\gamma_p^2 (4\pi\lambda)^8}{\Phi_0^7 r_p^3} \approx 1.3 \left[\frac{\gamma_p}{\gamma_{p0}} \right]^{3/2} \left[\frac{s}{\lambda} \right]^4 j_{c0} \; . \tag{35}$$

The plots of j_c/j_s vs B/B_s for different d_s/s are given in Fig. 3. The rapid increase of j_c at $B > B_s$ is accompanied by the rapid decrease of L_c,

$$\frac{L_c}{s} \approx 0.88 \frac{\gamma_{p0}}{\gamma_p} \frac{\lambda^2}{s^2} f_{44} \; , \tag{36}$$

with $f_{44} \approx (B_s/B)\exp(-2\sqrt{B/B_s})$ at $B \gg B_s$ and $d_s/s \sim 1$. The increase of j_c continues as long as the system remains in the 3DCP state, i.e., $L_c > s$. This corresponds to magnetic fields $B < B_{3D\to 2D}$,

$$B_{3D\to 2D} = \frac{B_s}{4} \left[\ln \left[\frac{\gamma_{p0}}{\gamma_p} \frac{\lambda^2}{s^2} \frac{B_s}{B_{3D\to 2D}} \right] \right]^2 \; . \tag{37}$$

At fields $B \sim B_{3D\to 2D}$ a smooth crossover to the 2DCP state takes place, which is characterized by a maximum

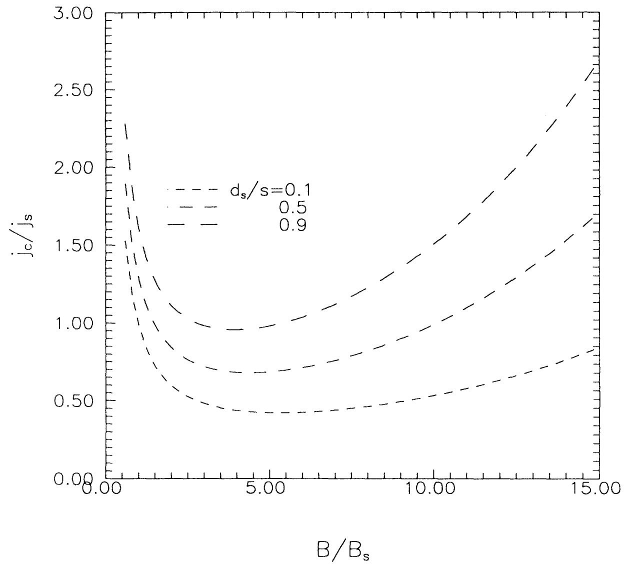

FIG. 3. The behavior of $j_c(B)$ in 3DCP for the case $B_s \ll H_{c2}$ for three values of the parameter d_s/s. The fast decrease of C_{44} causes the minimum and successive increase of j_c.

in the field dependence of the critical current,

$$j_{c\max}=\frac{\gamma_{p0}}{\gamma_p}\frac{\lambda^2}{s^2}\left[\frac{2}{\ln[(\gamma_{p0}/\gamma_p)(\lambda^2/s^2)]}\right]^2 j_s\ . \tag{38}$$

IV. APPLICATION AND DISCUSSION

In this section we explore in which multilayer system the first-order phase transition may occur within realistic experimental conditions. The phase transitions takes place at $B_{2D\to 3D}$ and $B_{3D\to 2D}$ provided that both these fields are smaller than both B_s and $0.2H_{c2}$. From Eqs. (24) to (26) it follows that the value of $B_{2D\to 3D}$ and the condition that 3DCP can actually be realized are determined by the ratios γ_p/γ_{p0}, λ/s, and λ/ξ_s. The general trend is that pronounced 3DCP behavior should be observed in systems with small pinning strength and small λ. The low-temperature values of parameters for five systems are determined from experimental data. The results are summarized in Table I and the consequences are subsequently discussed below. Note that due to the uncertainty in the numerical constants the values presented in this table should not be taken too literally but rather serve to compare different systems.

(a) Multilayers of amorphous Nb_3Ge/Ge. These multilayers are expected to behave similar to a-MoGe/Ge multilayers.[12] In addition, the pinning is collective and weak.[23] For a single a-Nb_3Ge we have $\xi_s=6.5$ nm, $\lambda_s=700$ nm, and $W_{0s}=7.5\times10^{-6}$ N^2m^{-3}. For a multilayer with $s/d_s=2$ and $s=10$ nm, d_i is large enough to guarantee solely magnetic coupling,[12] r_p is taken to be equal to ξ_s. The parameter values we obtain[24] are $\lambda=990$ nm, $j_{c0}=1.4\times10^8$ A/m^2, $\gamma_p/\gamma_{p0}=2.9\times10^4$, which yields the values for B_{2D}, $B_{2D\to 3D}$, $(\gamma_p/\gamma_{p0})(s/\lambda)^2$, and $(\gamma_p/\gamma_{p0})(4\xi_s/\lambda)^2$ in Table I. Although the pinning in this system is weak, the value of the penetration depth is very large, so that $\gamma_p/\gamma_{p0}\propto\lambda^8$ becomes very large too. Because $B_{c2}\approx5.6$ T, this multilayer system will behave as a collection of independent 2D layers in the entire field range. This situation probably will not change with increasing temperature, since the parameter γ_p/γ_{p0} depends weakly on temperature.

(b) Pb/Ge multilayers.[11] From the $j_c(B)$ curve in Ref. 11 at 4.2 K for a single Pb film with a thickness 14 nm it follows that the vortices are pinned almost independently in the whole field range, i.e., $R_c\approx a_0$. W_0 at 4.2 K turns out to be 1.3×10^{-4} N^2/m^3. From this we estimate for a 20 nm/20 nm multilayer that $j_{c0}\approx2\times10^9$ A/m^2. In this multilayer at $T=5$ K the $j_c(B)$ curve shows a pronounced minimum at low fields followed by a maximum at a field about twice as large as the field at the minimum. Above the maximum $j_c(B)$ behaves like for a single film. The other parameters we need are $\xi_s(5K)$ and $\lambda_s(5K)$. ξ_s given in Table I we estimate directly from H_{c2}. From this value we estimate λ_s using the relationships between parameters in the "clean" and "dirty" limits combined with $\lambda_L(0)=37$ nm and $\xi_0=83$ nm for pure Pb. The resulting value of γ_p/γ_{p0} is smaller than one, so that $L_c>s$ at small fields and $B_{2D\to 3D}$ has no meaning. The maximum in $j_c(B)$ would then denote the 3D to 2D transition at the field $B_{3D\to 2D}$ given by Eq. (32). Substitution of the parameters gives $B_{3D\to 2D}\approx0.6H_{c2}$, which exceeds the experimental value $0.2H_{c2}$ considerably. However we note that the estimated parameter values very sensitively depend on the value of λ, see Eqs. (11) and (12). Therefore, we conclude that according to the parameter values this system should exhibit a weak, first-order phase transition

TABLE I. Parameter values for several multilayer system with magnetic coupling between the superconducting layers. For all systems, except Pb/Ge, the values are taken at $T\sim0$.

	Input parameters				Estimated parameters				
Multilayers	ξ_s [nm]	s [nm]	λ [nm]	$\frac{j_{c0}}{[10^9\ \mathrm{A/m^2}]}$	γ_p/γ_{p0}	$\frac{\gamma_p}{\gamma_{p0}}\left[\frac{s}{\lambda}\right]^2$	$\frac{\gamma_p}{\gamma_{p0}}\left[\frac{4\xi_s}{\lambda}\right]^2$	B_{2D}[T]	$B_{2D\to 3D}$[T]
a-Nb_3Ge/Ge	6.5	10	990	0.14	2.9×10^4	3.0	20	0.07	5.4
Pb/Ge (T=5 K)	41	40	201	2	0.37	0.015	0.24	0.025	
Nb/Ge	12	10	70	50	0.64	0.013	0.3	0.135	
BiSCCO	2	1.54	140	40	3.7×10^3	0.45	12	5	$>H_{c2}$
	∼	∼	∼	8.5	170	0.02	0.55	1.1	18
Y/$PrBa_2Cu_3O_7$	1.7	4.8	200	50	1.3×10^5	74	148	15	$>H_{c2}$

at $B \sim B_{3D \to 2D}$. The absence of sharp features expected at such a phase transition can be attributed to smearing by spatial inhomogeneities.

(c) Nb/Ge multilayers. Starting form $\xi_s = 12$ nm, $\lambda_s = 50$ nm, and $j_{cs0} = 10^{11}$ A/m^2 we obtain for a multilayer with $s = 10$ nm and $s/d_s = 2$ the parameter values in Table I. Note that $B_s > B_{c2}(0)$ and that some fine tuning of λ may easily make $\gamma_p/\gamma_{p0} > 1$. We conclude that this system is a good candidate to observe the first-order phase transition, although the large value of j_{c0} may be an experimental concern regarding contact heating.

(d) BiSCCO. To assume predominant magnetic coupling is merely hypothetical, because the zero resistivity in the c direction indicates superconducting coupling between the CuO_2 planes. On the other hand recent torque measurements[14–16] show that the anisotropy of this compound is very high ($\gamma > 150$) and the magnetic coupling between the layers is not weaker than the Josephson coupling. For j_{c0} we took two values obtained by Van der Beek *et al.*[25] As follows from Table I an accessible value for $B_{2D \to 3D}$ is obtained for sufficiently low current densities only.

(c) YBCO/PBCO multilayers.[13] The large value of γ_p/γ_{p0} confines this system to 2D behavior solely, i.e., $L_c = s$. Going to higher temperature may drastically decrease j_{c0} and γ_p/γ_{p0}. However, since this decrease is related to thermal fluctuations[2,3] which are not taken into account here, our predictions do not apply in the thermal depinning region.

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APPENDIX A: DERIVATION OF THE TILT MODULUS

1. Small fields ($B \ll H_{c2}$)

The vector potential $\mathbf{A}(\mathbf{k}_\perp, z)$ obeys the London equations, which follows from Eq. (1):

$$\begin{aligned} &\frac{d^2\mathbf{A}}{dz^2} - k_s^2\mathbf{A} = -\lambda_s^{-2}\mathbf{\Phi}_n, \quad \text{at } |z - ns| < d_s/2 , \\ &\frac{d^2\mathbf{A}}{dz^2} - k_\perp^2\mathbf{A} = 0, \quad \text{at } |z - (n+1/2)s| < d_i/2 . \end{aligned} \tag{A1}$$

Here $k_\perp$ is the wave vector along the layers, $k_s^2 \equiv k_\perp^2 + \lambda_s^{-2}$. Using these expressions the energy of Eq. (1) can be reduced by:[8]

$$E = \frac{s}{8\pi\lambda^2} \sum_n \int d^2\mathbf{r}_\perp (\mathbf{\Phi}_n - \overline{A}_n)\mathbf{\Phi}_n , \tag{A2}$$

with

$$\overline{\mathbf{A}}_n = \frac{1}{d_s} \int_{ns - d_s/2}^{ns + d_s/2} dz \; \mathbf{A}(\mathbf{r}_\perp, z) .$$

Performing the Fourier transform along n and $\mathbf{r}_\perp$ the latter expression can be rewritten as

$$E = \frac{1}{8\pi\lambda^2} \int \frac{d^3\mathbf{k}}{(2\pi)^3} [\,|\Phi(\mathbf{k})|^2 - \overline{\mathbf{A}}(-\mathbf{k})\Phi(\mathbf{k})] . \tag{A3}$$

Due to the linearity of the Eqs. (A1) it is sufficient to solve the vector potential induced by the London vector in the layer $n=0$, i.e., to obtain the solution of (A1) for $\mathbf{\Phi}_n = \mathbf{\Phi}\delta_n$. The solution for the superconducting layers with $n \neq 0$ and for the insulating layers are given by

$$\begin{aligned} &\mathbf{A} = \mathbf{a}_n \exp[-k_s(z - ns)] + \mathbf{b}_n \exp[k_s(z - ns)], \quad \text{at } |z - ns| < d_s/2 , \\ &\mathbf{A} = \mathbf{c}_n \exp\{-k_\perp[z - (n+1/2)s]\} + \mathbf{d}_n \exp\{k_\perp[z - (n+1/2)s]\}, \quad \text{at } |z - (n+1/2)s] < d_i/2 . \end{aligned} \tag{A4}$$

These solutions should be matched at the boundaries between superconducting and normal layers using the continuity of $\mathbf{A}(\mathbf{k}_\perp, z)$ and $d\mathbf{A}(k_\perp, z)/dz$, which gives the following relation

$$\begin{bmatrix} \mathbf{a}_{n+1} \\ \mathbf{b}_{n+1} \end{bmatrix} = \hat{S} \begin{bmatrix} \mathbf{a}_n \\ \mathbf{b}_n \end{bmatrix} . \tag{A5}$$

The $2 \otimes 2$ matrix $\hat{S}$ is given by

$$\hat{S} = \begin{bmatrix} (c_i - c_k s_i) e_s^{-2} & s_k s_i \\ -s_k s_i & (c_i + c_k s_i) e_s^2 \end{bmatrix} ,$$

where the following notations were used: ${c_k \atop s_k} = \frac{1}{2}[(k_s/k_\perp) \pm (k_\perp/k_s)]$, ${c_i \atop s_i} = {\cosh \atop \sinh}(k_\perp d_i)$, ${c_s \atop s_s} = {\cosh \atop \sinh}(k_s d_s)$, $e_s = \exp(k_s d_s/2)$. The eigenvalues of the matrix $\hat{S}$, r_+, and r_-, can be written as follows:

$$r_\pm = \exp(\pm\chi s) \tag{A6}$$

with

$$\cosh\chi s \equiv c_i c_s + c_k s_i s_s .$$

The eigenvector $\binom{\mathbf{a}_-}{\mathbf{b}_-}$ corresponding to the solution which exponentially decreases for $n \to \infty$ is given by

$$\begin{bmatrix} \mathbf{a}_- \\ \mathbf{b}_- \end{bmatrix} = \mathbf{s} \begin{bmatrix} \cosh(\alpha) + 1 \\ \sinh(\alpha) \end{bmatrix} . \tag{A7}$$

Here

$$\sinh(\alpha) \equiv \frac{s_k s_i}{\sinh(\chi s)}$$

and s is an arbitrary vector. The vector potential in the superconducting layers with $n > 0$ can now be represented by (A4) with

$$\begin{bmatrix} \mathbf{a}_n \\ \mathbf{b}_n \end{bmatrix} = \begin{bmatrix} \mathbf{a}_- \\ \mathbf{b}_- \end{bmatrix} \exp(-n\chi s) . \tag{A8}$$

The vector potential for the superconducting layer with $n=0$ can be written as

$$\mathbf{A}=(k_s\lambda_s)^{-2}\mathbf{\Phi}+\mathbf{a}_0\cosh(k_s z) \; . \tag{A9}$$

To obtain the constant vector $\mathbf{a}_0$ we should match this expression with the exponentially decreasing solution for $n\to\infty$ which gives

$$\mathbf{a}_0=-\left[\cosh\frac{k_s d_s}{2}-\exp(\alpha)\sinh\frac{k_s d_s}{2}\right]\frac{\mathbf{\Phi}}{(k_s\lambda_s)^2} \; . \tag{A10}$$

Using the Eqs. (A4) and (A8)–(A10) we obtain the average vector potential for all superconducting layers:

$$\overline{\mathbf{A}}_n=\overline{\mathbf{A}}_0\exp(-|n|\chi s) \; , \tag{A11}$$

with

$$\overline{\mathbf{A}}_0=\frac{\mathbf{\Phi}}{(k_s\lambda_s)^2}\left[1-\frac{\sinh(k_s d_s)-\exp(\alpha)(\cosh(k_s d_s)-1)}{k_s d_s}\right] \; .$$

The latter expression can be used to obtain the vector potential induced by an arbitrary London vector $\mathbf{\Phi}_n$:

$$\overline{\mathbf{A}}_n(\mathbf{k}_\perp)=\sum_{n'}G(\mathbf{k}_\perp,n-n')\mathbf{\Phi}_{n'}(\mathbf{k}_\perp) \; , \tag{A12}$$

with

$$G(\mathbf{k}_\perp,n)=\frac{\exp(-|n|\chi s)}{(k_s\lambda_s)^2}\left[1-\frac{\sinh(k_s d_s)-\exp(\alpha)[\cosh(k_s d_s)-1]}{k_s d_s}\right] \; .$$

Performing the discrete Fourier transform over n, Eq. (A12) can be rewritten as

$$\overline{\mathbf{A}}(\mathbf{k})=\mathbf{\Phi}(\mathbf{k})G(\mathbf{k}) \; , \tag{A13}$$

with

$$G(\mathbf{k})=\frac{1}{1+\lambda_s^2k_\perp^2}\,\frac{\sinh(\chi s)}{\cosh(\chi s)-\cos(k_z s)}\left[1-\frac{\sinh(k_s d_s)-\exp(\alpha)[\cosh(k_s d_s)-1]}{k_s d_s}\right] \; . \tag{A14}$$

From Eqs. (A3) and (A13) it follows that the energy of an arbitrary vortex configuration is given by

$$E=\frac{1}{8\pi\lambda^2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}|\mathbf{\Phi}(\mathbf{k})|^2[G(\mathbf{k})-1] \; . \tag{A15}$$

The elastic energy of the vortex lattice is determined by the interaction energy between the pancakes $E_{\mathrm{int}}(\mathbf{r}_\perp,n)$. Substituting into Eq. (A15) the London vector induced by two pancake vortices, namely

$$\mathbf{\Phi}(\mathbf{k})=i\Phi_0 s\frac{[\mathbf{k}_\perp,\mathbf{n}_z]}{k_\perp^2}(1+\exp i\mathbf{k}\mathbf{r}) \; ,$$

we obtain

$$E_{\mathrm{int}}(\mathbf{r}_\perp,n)=\int\frac{d^3\mathbf{k}}{(2\pi)^3}E_{\mathrm{int}}(\mathbf{k})\exp i\mathbf{k}\mathbf{r} \; , \tag{A16}$$

with

$$E_{\mathrm{int}}(\mathbf{k})=\frac{s^2\Phi_0^2}{4\pi\lambda^2k_\perp^2}[G(\mathbf{k})-1] \; . \tag{A17}$$

Following the standard scheme [see, e.g., (Ref. 17)] we derive the elastic energy for the vortex lattice

$$E_{\mathrm{el}}=\frac{1}{2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}W_{\alpha\beta}(\mathbf{k})u_\alpha(\mathbf{k})u_\beta(-\mathbf{k}) \; , \tag{A18}$$

with dynamic matrix $W_{\alpha\beta}(\mathbf{k})$ given by

$$W_{\alpha\beta}(\mathbf{k})=\left[\frac{B}{d\Phi_0}\right]^2\sum_{\mathbf{Q}}[E_{\mathrm{int}}(\mathbf{k}_\perp+\mathbf{Q},k_z)(k_{\perp\alpha}+Q_\alpha)(k_{\perp\beta}+Q_\beta)-E_{\mathrm{int}}(\mathbf{Q},0)Q_\alpha Q_\beta] \; . \tag{A19}$$

Here $u_\alpha(\mathbf{k})$ is the elastic displacement, $\mathbf{Q}$ are the reciprocal-lattice vectors and the integration over $\mathbf{k}$ is performed over the first Brillouin zone of the lattice. With the use of Eq. (A17) the dynamic matrix takes the form

$$W_{\alpha\beta}(\mathbf{k})=\frac{B^2}{4\pi}\sum_{\mathbf{Q}}[G(\mathbf{k}_\perp+\mathbf{Q},k_z)-1]\frac{(k_{\perp\alpha}+Q_\alpha)(k_{\perp\beta}+Q_\beta)}{\lambda^2(\mathbf{k}_\perp+\mathbf{Q})^2}-[G(\mathbf{Q},0)-1]\frac{Q_\alpha Q_\beta}{\lambda^2\mathbf{Q}^2}\ . \tag{A20}$$

The pinning properties are determined by shear deformations $\mathbf{u}_t(\mathbf{k}_\perp,k_z)$, e.g., a twist of a vortex bundle. The shear energy $E_{\rm el}^{(t)}$ describing such deformations is obtained from (A20) by dropping the $\mathbf{Q}=0$ term. In the limit $\lambda K_0 \gg 1$, $k_\perp \ll K_0$ ($K_0=\sqrt{4\pi B/\Phi_0}$), this energy can be represented as

$$E_{\rm el}^{(t)}=\frac{1}{2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}[C_{44}(k_z)\tilde{k}_z^2+C_{66}k_\perp^2]|\mathbf{u}_t(\mathbf{k})|^2\ , \tag{A21}$$

with $\tilde{k}_z=(2/s)\sin(sk_z/2)$.

The shear modulus C_{66} and the tilt modulus C_{44} are determined by the following expressions:

$$C_{66}=\frac{B\Phi_0}{(8\pi\lambda)^2}\ , \tag{A22}$$

$$C_{44}(k_z)\tilde{k}_z^2=\frac{B^2}{8\pi\lambda^2}\sum_{\mathbf{Q}}[G(\mathbf{Q},0)-G(\mathbf{Q},k_z)]\ . \tag{A23}$$

Substituting in the latter equation the expression for $G(\mathbf{k})$ (A14) we finally obtain

$$C_{44}(k_z)=\frac{B^2s^2}{16\pi\lambda^4}\sum_{\mathbf{Q}}\frac{\sinh(\chi s)}{[\cosh(\chi s)-1][\cosh(\chi s)-\cos(k_z s)]}\frac{1}{1+\lambda_s^2Q^2}\left[1-\frac{\sinh(Q_sd_s)-\exp(\alpha)[\cosh(Q_sd_s)-1]}{Q_sd_s}\right]\ . \tag{A24}$$

The sum is performed over reciprocal vectors $\mathbf{Q}$ of the triangular vortex lattice,

$$Q=Q_0\sqrt{(m_1+m_2/2)^2+3m_2^2/4}$$

with $Q_0=2\pi(2B/\sqrt{3}\Phi_0)^{1/2}$. The parameters $\mathbf{Q}_s$, χ, and α are defined by the relations

$$Q_s=\sqrt{Q^2+\lambda_s^{-2}}\ ,$$

$$\cosh\chi s=\cosh Qd_i\cosh Q_sd_s+\frac{1}{2}\left[\frac{Q_s}{Q}+\frac{Q}{Q_s}\right]\sinh Qd_i\sinh Q_sd_s\ ,$$

$$\sinh\alpha=\frac{1}{2}\left[\frac{Q_s}{Q}+\frac{Q}{Q_s}\right]\frac{\sinh Qd_i}{\sinh\chi s}\ .$$

2. Region B close to H_{c2}

As follows from the result (A24) of the preceding section, the tilt modulus does not depend on the transversal wave vector in contrast to the case of a 3D superconductor.[17] It means that in order to estimate the tilt modulus it is sufficient to calculate the energy change $\delta E_{\rm tilt}(\mathbf{u})$ under homogeneous tilt deformation $\mathbf{u}(z_n)$. This simplifies the problem because in this case only the magnetic part E_m of the energy changes. For the case $a \ll s$ which will be considered in this section this change can be represented as

$$\delta E_{\rm tilt}(\mathbf{u})=E_m(\mathbf{u})-E_m(0)\ , \tag{A25}$$

with

$$E_m(\mathbf{u})=-\int d^2\mathbf{r}\sum_n\frac{1}{c}\mathbf{A}_1(z_n)\mathbf{j}_n+\int d^3\mathbf{r}\left[\frac{\mathbf{A}_1^2}{8\pi\lambda_{\rm eff}^2}+\frac{\mathbf{B}_1^2}{8\pi}\right]\ . \tag{A26}$$

Here $\mathbf{A}_1$ and $\mathbf{B}_1=\nabla\times\mathbf{A}_1$ are the oscillating components of the vector potential and the magnetic field,

$$\lambda_{\rm eff}^{-2}=\langle f^2\rangle\lambda^{-2}\ ,$$

$f(\mathbf{r}_\perp)$ being the equilibrium order parameter normalized to its value at $B=0$. The 2D current density in the nth layer is given by

$$\mathbf{j}_n(\mathbf{r}_\perp)=\mathbf{j}_0[\mathbf{r}_\perp-\mathbf{u}(z_n)]\ ,$$

$\mathbf{j}_0(\mathbf{r}_\perp)$ being the equilibrium 2D current density,

$$\mathbf{j}_0(\mathbf{r}_\perp)=\frac{csf^2(\mathbf{r}_\perp)}{4\pi\lambda^2}\left[\frac{\Phi_0}{2\pi}\nabla\phi-\mathbf{A}\right]\ , \tag{A27}$$

which can be expressed as a Fourier series

$$\mathbf{j}_0(\mathbf{r}_\perp)=\sum_{\mathbf{Q}}\mathbf{j}_Q\exp(i\mathbf{Q}\mathbf{r}_\perp)\ .$$

Variation of the Eq. (A26) with respect to $\mathbf{A}_1$ leads to the following equation:

$$\Delta\mathbf{A}_1-\lambda_{\rm eff}^{-2}\mathbf{A}_1=-\frac{4\pi}{c}\sum_n\mathbf{j}_n(\mathbf{r}_\perp)\delta(z-z_n)\ . \tag{A28}$$

The solution is given by

$$\mathbf{A}_1(\mathbf{r})=\sum_{n,\mathbf{Q}}\frac{2\pi\mathbf{j}_{\mathbf{Q}}}{c\sqrt{\mathbf{Q}^2+\lambda_{\rm eff}^{-2}}}\exp[-\sqrt{\mathbf{Q}^2+\lambda_{\rm eff}^{-2}}|z-z_n|+i\mathbf{Q}(\mathbf{r}_\perp-\mathbf{u}_n)]\ . \tag{A29}$$

Substituting this expression into the Eq. (A25) and expanding with respect to

$$\mathbf{u}(z_n)=\int(dk_z/2\pi)\mathbf{u}(k_z)\exp(ik_z z_n)\ ,$$

we obtain the following expression for the tilt energy valid in the whole region of $\mathbf{B}$ under the conditions $s \ll Q_0^{-1} \ll \lambda_{\rm eff}$ and

$$\delta E_{\rm tilt}(\mathbf{u})=\int \frac{dk_z}{2\pi}\frac{C_{44}(k_z)}{2}|\mathbf{u}(k_z)|^2\ , \tag{A30}$$

with

$$C_{44}(k_z)=\sum_{\mathbf{Q}}\frac{2\pi|\mathbf{j}_Q|^2}{c^2s^2(\tilde{k}_z^2+\mathbf{Q}^2)}\ . \tag{A31}$$

This expression is quite general and can be used in the whole range $H_{c1} \ll B < H_{c2}$. Near H_{c2} the 2D current density is given by[26]

$$\mathbf{j}_0(\mathbf{r}_\perp)=\frac{cs\Phi_0}{(4\pi\lambda)^2}\mathbf{n}_z\times\nabla f^2(\mathbf{r}_\perp)\ . \tag{A32}$$

Fourier transform of $f^2(\mathbf{r}_\perp)$ can be found in the review[17]

$$f^2(\mathbf{r}_\perp)=\langle f^2\rangle\sum_{\mathbf{Q}}\exp[i\phi(\mathbf{Q})-(\xi Q)^2/4+i\mathbf{Q}\mathbf{r}_\perp]\ . \tag{A33}$$

Using Eqs. (A32) and (A33) we obtain

$$|\mathbf{j}_Q|=\frac{(1-b)cs\Phi_0 Q}{(4\pi\lambda)^2\beta_A}\exp[-(\xi Q)^2/4]\ , \tag{A34}$$

$b=B/H_{c2}$, $B_A=1.16$. With very good accuracy one can keep in the sum over $\mathbf{Q}$ in Eq. (A31) only the six terms corresponding to the minimum reciprocal-lattice vector $Q_0=4\pi/(\sqrt{3}a)$. In the case $sQ_0 \ll 1$ this gives Eq. (8).

APPENDIX B: PARAMETERS CHARACTERIZING STRENGTH OF DISORDER

Vortex lines in layered superconductors are composed of weakly interacting pancake vortices located in superconducting layers. Imperfections in the crystalline lattice generate a random contribution to the energy of a single pancake vortex $\delta\epsilon_v(\mathbf{r})$. The statistical properties of this random potential are described by the correlation function

$$\langle\delta\epsilon_v(\mathbf{r})\delta\epsilon_v(\mathbf{r}')\rangle=\gamma_p r_p^4 f(\mathbf{r}-\mathbf{r}')\ . \tag{B1}$$

The function $f(\mathbf{r})$ decreases over distances $\mathbf{r}\sim r_p$ and is normalized by the condition $\int d^2\mathbf{r} f(\mathbf{r})=1$, e.g., $f(\mathbf{r})=(1/\pi r_p^2)\exp[-(r/r_p)^2]$. The mean-square random force γ_p gives a natural measure of the strength of disorder in the 2D planes. The typical pinning energy U_p is connected with γ_p by the relation

$$U_p=\sqrt{\gamma_p}r_p\ . \tag{B2}$$

The maximum possible value of γ_p, $\gamma_{\max}$, is realized when at distances of the order of the coherence length ξ the energy variations are of the order of the vortex core condensation energy $\epsilon_c=s\Phi_0^2/(4\pi\lambda)^2$, i.e.,

$$\gamma_{\max}\approx(\epsilon_c/\xi)^2\ . \tag{B3}$$

The value of γ_p can be estimated from the maximum value of the critical current j_{c0} which occurs at small temperatures and magnetic fields

$$\gamma_p=\left[\frac{\Phi_0 s}{c}j_{c0}\right]^2\ . \tag{B4}$$

In the magnetically coupled system there is a typical value of γ_p, γ_{p0}, above which the interaction with the random potential becomes stronger than the interaction between the pancake vortices composing a vortex line. This value is determined by the condition $U_p \sim U_{\rm tilt}$ and marks the crossover between regimes 1 and 2 discussed in Sec. III:

$$\gamma_{p0}\approx\left[\frac{\Phi_0^2 r_p s}{(4\pi)^2\lambda^4}\right]^2\approx\frac{r_p^2\xi^2}{\lambda^4}\gamma_{\max}\ . \tag{B5}$$

The theory of collective pinning[6] uses the mean-square value of the random force W which determines the typical value of the random force F_r acting on a volume V of the vortex lattice by $F_r \approx \sqrt{WV}$. In the limit $B \ll H_{c2}$ the parameter W is connected with the parameter γ_p by the relation

$$W=\frac{n_v\gamma_p}{s}\ . \tag{B6}$$

Here $n_v=B/\Phi_0$ is the density of vortices.

To obtain the field dependence of the critical current it is necessary to know the field dependence of the parameter W in the whole field range $B \le H_{c2}$. For disorder of the "random transition temperature"-type (δT_c pinning[23]) the parameter W vanishes as $(1-b)^2$ as B approaches H_{c2} ($b=B/H_{c2}$). The natural interpolation for $W(b)$ is

$$W=W_0 b(1-b)^2\ . \tag{B7}$$

This kind of interpolation has been used in studies of collective pinning.[1] The field independent parameter W_0 is connected with γ_p by the relation

$$W_0=\frac{\gamma_p}{2\pi\xi^2 s}\ . \tag{B8}$$

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