Time scales of the flux creep in superconductors

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(Received 22 April 1993)

We have studied both theoretically and experimentally flux-creep dynamics in superconductors. A theoretical analysis of nonlinear flux diffusion shows that the relaxation of the electric field proves to be similar for different models of thermally activated flux creep, whereas the long-time decay of the magnetic moment $M(t)$ can be essentially model dependent. A proposed scaling analysis indicates that the short-time decay of $M(t)$ in the subcritical region $j<j_c$ is universal and consists of two stages. The initial nonlogarithmic stage is due to a transient redistribution of magnetic flux over the sample cross section, the duration of this stage τ_0 being entirely determined by macroscopic quantities, such as sample sizes, flux creep rate $M_1(T,B)=dM/d\ln t$, and magnetic ramp rate $\dot{B}_e=dB/dt$. The second stage corresponds to the approximately logarithmic relaxation $M(t)=M_c-M_1\ln(t/t_0)$, with t_0 being a macroscopic time constant that also depends on sample sizes, $M_1(T,B)$, and the voltage criterion E_c at which the critical current density j_c is defined. We consider different models of flux dynamics with nonlinear flux-creep-activation barriers $U(j)$ and obtain explicit formulas for τ_0 and t_0 for the exponential V-I curve and the vortex-glass model. We have also performed an experimental study of magnetic relaxation in grain-oriented $YBa_2Cu_3O_7$ in which the time constant τ_0 has been measured directly at different temperatures 4.2 K $<T<$ 88 K, magnetic fields $0<B<8$ T, and ramp rates 5 μT/s $<\dot{B}_e<$ 10 mT/s. We have observed the inverse dependence of τ_0 upon $\dot{B}_e$, with τ_0 ranging from 1 s to 10^3 s and reaching 5000 s at $\dot{B}_e=5$ μT/s, $B=6$ T, and $T=20$ K. It is shown that, in accordance with our model, the dependences of τ_0 upon T and B coincide with those for the flux-creep rate $M_1(T,B)=dM/d\ln t$ measured on the logarithmic stage of the flux creep. We have also measured the dependences of the initial magnetic moment $M(0)$ on T, B, and $\dot{B}_e$. Manifestations of the obtained results in magnetic and relaxation measurements on high-T_c superconductors are discussed.

I. INTRODUCTION

Significant relaxation of irreversible magnetization (flux creep) in high-temperature superconductors (HTS) is known to be a very important factor limiting the current-carrying capacity of these materials. This can manifest itself in a dependence of magnetization curves on eddy electric fields induced by ac magnetic fields $H_e(t)$, strong dependences of the critical current density $j_c(T,B)$ upon temperature T, magnetic induction B, and the voltage criterion E_c. As a result, j_c vanishes above the irreversibility field $B_*(T)$ which can be well below the upper critical field $B_{c2}(T)$. In addition, macroscopic electrodynamic properties of HTS become sensitive to the highly nonlinear part of the current-voltage (I-V) characteristic in the subcritical region ($j<j_c$) determined by microscopic mechanisms of flux dynamics and pinning (see, e.g., Refs. 1 and 2, and references cited therein). This situation differs from that of conventional low-T_c superconductors (LTS) which can be well described by the universal Bean model, regardless of particular mechanisms of resistivity at $j<j_c$.

Under these conditions flux-creep measurements become a very useful tool for studying the microscopic mechanisms of flux dynamics and pinning in HTS. This is due to the fast current relaxation in HTS, which enables one to measure a significant portion of the decay of initial magnetic moment $M(0)$ within a limited experimental time window $t_i<t<t_f$ (typically, $t_i\sim 1-10$ s, and $t_f\sim 10^5-10^6$ s) and thereby to reconstruct the I-V curve deep within the subcritical region $j<j_c$,[3–9] where the electric field $E(j)$ can be written in the form

$$E=E_c\exp\left[-\frac{U(j)}{kT}\right]. \quad (1)$$

Here the flux-creep potential barrier, $U(j)$, is a nonlinear function of j which vanishes at $j=j_c$, and E_c is a voltage criterion at which j_c is defined, with E_c being of the order of a crossover electric field between the flux-flow and flux-creep regimes. For instance, the essentially nonlogarithmic relaxation observed in HTS about the irreversibility line[5–9] has been interpreted in the literature in terms of vortex-glass[10] and collective creep[11,12] models. In addition, HTS exhibit unusual behavior of the flux-creep parameters, in particular, nonmonotonic dependences of the creep rate $s(T,B)=\partial\ln M/\partial\ln t$ on T and B.[13–16]

Flux creep occurs due to a finite resistivity at $j<j_c$ caused by a thermally activated hopping of fluxons between neighboring pinning positions. This leads to a

directional drift of magnetic flux under the action of the Lorentz force, which is accompanied by a dissipation, giving rise to a decay of induced magnetization currents. Essential features of the flux creep have been clarified by the Anderson-Kim model[17] which assumes a thermally activated uncorrelated hopping of pointlike vortex bundles in some effective pinning potential. In this model the energy barrier $U(j)=U_0(1-j/j_c)$ is linear, which gives rise to the logarithmic decay of $M(t)$ at $t \gg t_0$,

$$M(t)=M_c-M_1 \ln \left[\frac{t}{t_0}\right] , \qquad (2)$$

observed in both LTS and HTS materials (see, e.g., Refs. 1, 18, and 19). Here M_c is the unrelaxed value of M given by the Bean model, $M_1=kTM_c/U_a$, U_a is an apparent flux-creep activation energy, and t_0 is a time constant which was originally ascribed to an inverse attempt frequency of pinned fluxons (usually t_0 is assumed to be of order 10^{-10}–10^{-13} s).

Although Eq. (2) was first obtained within the framework of the simplified Anderson-Kim model, the logarithmic dependence $M(t)$ describes the initial stage of magnetic relaxation in more elaborated flux-creep models as well, for example, in vortex-glass[10] and collective creep[11,12] models which consider a thermally activated hopping of strongly interacting vortex lines in a random pinning potential. Such a similarity of the observed current relaxation for these qualitatively different models takes place over the significant region of the phase diagram well below the irreversibility line. Here the flux-creep rate $M_1=dM/d\ln t$ is small enough, therefore $M(t)$ decays slowly within the experimental time window $t_i<t<t_f$, and the local $j(\mathbf{r})$ remains close to j_c. On these time scales, this enables one to expand the flux-creep activation barrier $U(j)$ in Eq. (1) in a power series in j_c-j,[20]

$$U(j)=\frac{(j_c-j)}{j_1}+\cdots , \qquad (3)$$

and neglect higher-order terms in j_c-j. This results in the exponential E-j characteristic

$$E=E_c \exp\left[\frac{j-j_c}{j_1}\right] \qquad (4)$$

similar to that of the Anderson-Kim model in which $j_1=j_ckT/U_a$. Meanwhile, the above arguments do not assume any particular mechanism of resistivity and are based only on the thermally activated character of flux dynamics at $j<j_c$. In turn, Eq. (4) together with the Maxwell equations give the logarithmic decay of $j(t)$ which can be interpreted in terms of a macroscopic nonlinear diffusion of magnetic flux through the cross section of a superconductor.[18,19]

Such a universality implies that the initial stage of the flux creep can be described in terms of directly measured macroscopic quantities, regardless of specific mechanisms of flux dynamics and pinning. For instance, the value j_1 in Eq. (4) is related to the observed flux-creep rate $j_1(T,B)=-\partial j/\partial \ln t$. By contrast, the parameters j_c and E_c cannot be extracted separately from electromagnetic measurements.[21,22] Indeed, instead of j_c and E_c in Eq. (4), one can take another pair j_c' and E_c' related to j_c and E_c as follows:

$$j_c'=j_c-j_1\ln(E_c/E_c') . \qquad (5)$$

Such a transformation does not change the E-j characteristic (4), which implies that the parameters j_c and E_c are not independent, since only the combination $j_c-j_1\ln E_c$ has physical relevance. In other words, Eq. (5) links two critical current densities j_c and j_c' which are defined at different voltage criteria E_c and E_c'. Once E_c is fixed (say, $E_c=1$ μV/cm), the value j_c can be extracted from resistive or magnetization measurements, which allows one to express Eq. (4) only via observed macroscopic parameters.

Equation (2) becomes invalid at short times $t<\tau_0$ when the initial stage of the flux creep can be described phenomenologically as follows:[22–24]

$$M(t)=M(0)-M_1\ln\left[1+\frac{t}{\tau_0}\right] , \qquad (6)$$

where $M(0)$ is the initial value of $M(t)$ at $t=0$, and τ_0 is a time constant which determines a transient stage before the beginning of the logarithmic relaxation of $M(t)$. Such a transient regime is due to specific features of flux-creep measurements in which a superconductor is placed in an external magnetic field $B_e(t)$ which is increased with a constant ramp rate $\dot{B}_e=dB_e/dt$ until $t=0$ and then kept fixed. This induces the initial electric field $E\sim\dot{B}_er$ which then decays at $t>0$ owing to a finite resistivity in the subcritical region $j<j_c$. The sharp change of external conditions at $t=0$ causes the transient regime during the time $0<t<\tau_0$ needed for a nonlinear diffusion redistribution of magnetic flux over the sample cross section.[22] The conclusions about the macroscopic origin of t_0 and τ_0, as well as the dependence of τ_0 on the initial conditions have been made by several groups.[12,22,24–35] Here the time constants τ_0 and t_0 which correspond to the transient and the steady-state regimes of relaxation, respectively, turn out to be qualitatively different, since the value τ_0, unlike t_0, can be strongly affected by the ramp rate $\dot{B}_e$.

Recently the time constant τ_0 has been extracted from flux-creep measurements.[28,33–35] The results of these experiments indicate that τ_0 is indeed a macroscopic quantity which proves to be inversely proportional to the ramp rate $\dot{B}_e$. For instance, in grain-oriented $YBa_2Cu_3O_{7-x}$ the value of τ_0 ranges from 1 to 10^4 s when changing $\dot{B}_e$ from 10^{-2} to 10^{-6} T/s.[28] Hence it follows that at small $\dot{B}_e$ the transient nonlogarithmic regime can take a considerable time $\tau_0(\dot{B}_e)$ which can even exceed the time window $t_i<t<t_f$. At the same time, measurements of t_0 performed by Sun *et al.*[16] on proton-irradiated Y-Ba-Cu-O single crystals gave $t_0\sim10^{-1}$ -10^{-4} s which is much smaller than τ_0.

Therefore, the time constants τ_0 and t_0 can be expressed in terms of directly measured macroscopic quantities [sample sizes, flux-creep rate $j_1(t,B)$, ramp rate $\dot{B}_e$, voltage criterion E_c, etc.] without invoking such uncertain microscopic parameters as an attempt frequency, vortex bundle size, mean hopping distance, etc. This is due to the fact that the initial stage of the flux creep turns out to be insensitive to specific mechanisms of flux dynamics and can be described by Eqs. (2) and (6) in different flux-creep models. Such a universality is virtually due to a slow decay of $M(t)$ well below the irreversibility field $B_*(T)$, unlike the situation at higher T and B for which $M(t)$ can substantially change in the time window $t_i < t < t_f$. In the latter case the nonlinearity of $U(j)$ at larger deviations $j_c - j(t)$ manifests itself in a nonlogarithmic relaxation of $M(t)$ which indeed has been observed in HTS in long-time flux-creep measurements.[6-9,24,31,33,34]

A nonlogarithmic decay of $M(t)$ can occur in short-time flux-creep measurements as well if a superconductor is placed in pulse magnetic fields which cause the initial eddy electric fields $E(r) \sim \dot{B}_e r$ much higher than E_c. This enables one to study the supercritical region of the *I-V* curve $j > j_c$, and, in particular, to trace a crossover between the flux-creep and flux-flow regimes.[5,35] For instance, Huang *et al.*[5] reported a nonlogarithmic relaxation in $YBa_2Cu_3O_7$ on ms time scales observed with the use of a pulse technique which gives $\dot{B}_e \sim 10^2 - 10^3$ T/s. In this case the induced electric fields $E \sim 10^2 - 10^3$ μV/cm are much larger than the conventional voltage criterion $E_c = 1$ μV/cm for j_c.

In this paper we focus on the subcritical region $j < j_c$ and present theoretical and experimental studies of the initial stages of the magnetic relaxation in the framework of the approach which has been proposed in our previous communication.[28] The aim of this work is to demonstrate the universality of the time constants t_0 and τ_0 and to study their dependences on T, B, and $\dot{B}_e$ by using reduced ramp rates $\dot{B}_e$. The paper is organized as follows. In Sec. II, we consider qualitative features of the nonlinear flux diffusion in superconductors, in particular, the case of the exponential *V-I* curve (4) which pertains to the initial stages of the flux creep. It is shown that the time constant τ_0 can be calculated by a dimensional analysis and proves to be similar for different flux-creep models over a wide region of the parameters. Explicit formulas which describe the relaxation of $M(t)$ for the exponential $E(j)$ and the vortex-glass model are obtained. In Sec. III, we present detailed experimental results of the flux-creep measurements done on grain-oriented $YBa_2Cu_3O_{7-x}$ at different sweep rates $10^{-2} < \dot{B}_e < 10^{-6}$ T/s, temperatures $4.2 < T < 77$ K, and magnetic inductions $0 < B_e < 8$ T. Our experimental data are fully consistent with both the theoretical analysis given in the Sec. II and our previous results.[28] For instance, we have observed the inverse dependence of τ_0 on $\dot{B}_e$ with τ_0 reaching 5×10^3 s at $\dot{B}_e = 5 \times 10^{-6}$ T/s. Moreover, the dependences of the time constant $\tau_0(T,B)$ on T and B are shown to coincide with those of $M_1(T,B)$. Section IV contains discussions of the results obtained and their manifestations in resistive and magnetic properties of HTS.

II. NONLINEAR FLUX DIFFUSION IN SUPERCONDUCTORS

Flux creep in superconductors can be formulated in terms of a nonlinear diffusion of magnetic flux through a sample.[18] This process is described by the Maxwell equations

$$\frac{\partial \mathbf{B}}{\partial t} = -\text{curl}\mathbf{E} , \tag{7}$$

$$\text{curl}\mathbf{H} = j(E)\frac{\mathbf{E}}{E} , \tag{8}$$

where the dependence $j(E)$ is determined by particular mechanisms of resistivity. We neglect here the effect of anisotropy, assuming that $\mathbf{j}$ is always parallel to $\mathbf{E}$ (in layered HTS this corresponds to $\mathbf{H}$ parallel to the c axis when magnetization currents flow in the nearly isotropic ab plane). Furthermore, we confine ourselves to the field region $H_{cl} \ll H \ll H_{c2}$, where one can put $B = \mu_0 H$. Expressing $\mathbf{E}$ via $\mathbf{j} = \text{curl}\mathbf{H}$ by means of Eq. (1), one can get a nonlinear equation for $\mathbf{H}$, which describes the evolution of vortex density.[18] However, for our aims it is more convenient to exclude $\mathbf{H}$ and present Eqs. (7) and (8) as a single equation for $\mathbf{E}$. We consider here a slab of thickness $2a$ along the x axis and infinite in the yz plane with the external magnetic field $\mathbf{H}_e$ parallel to the z axis (Fig. 1). Let $H_e(t)$ increase with a constant ramp rate $\dot{H}_e$ until $t=0$, and then remain fixed. This induces an initial eddy electric field $E(x) = x\dot{B}_e$ which then decays at $t > 0$ due to the flux creep. For that planar case the electric field $\mathbf{E} = \mathbf{y}E$ has only the y component, and Eqs. (7) and (8) reduce to

$$\frac{\partial^2 E}{\partial x^2} = g(E)\frac{\partial E}{\partial t} , \tag{9}$$

$$g(E) = \mu_0 \frac{\partial j}{\partial E} . \tag{10}$$

Substituting Eq. (1) into Eq. (10), one can present the function $g(E)$ in the form

$$g(\mathbf{E}) = -\frac{\mu_0 kT}{E}\left[\frac{\partial U}{\partial j}\right]^{-1} . \tag{11}$$

The relaxation of $E(x,t)$ is described by the solution of Eq. (9) which satisfies the initial condition $E(x,0) = \dot{B}_e x$ and the boundary conditions $\partial E/\partial x = \partial B_e/\partial t = 0$ at

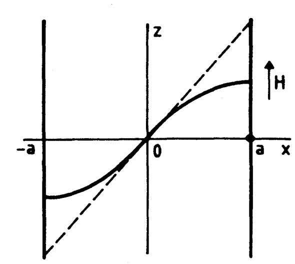

FIG. 1. Infinite plate of thickness $2a$ in a parallel magnetic field H_e. The solid and dashed curves show the electric field profile $E_y(x,t)$ described by Eq. (21) and the initial distribution $E(x,0) = \dot{B}_e x$, respectively.

$x = \pm a$. As follows from Eqs. (9) and (10), the relaxation of $E(x,t)$ is described by a diffusionlike nonlinear partial differential equation which is analogous to the equation for a nonlinear heat diffusion in a medium with the "thermal conductivity" $\kappa=1$ and "heat capacity" $g(E)$. Nonlinear flux dynamics described by Eq. (9) or a similar equation for $H(x,t)$ have been studied in the literature for different flux-creep models[17,18,30,36–38] (see also Ref. 39). For instance, there has been obtained some partial analytical solution of Eq. (9) which describes a relaxation of $E(x,t)$.[17,36] Vinokur, Feigelman, and Geshkenbein[37] have obtained exact solutions which describe a dynamics of magnetic flux penetration for the logarithmic potential barrier $U(j)=(U_0/kT)\ln(j_c/j)$. Extensive numerical studies of various dynamic regimes have been performed by van der Beek *et al.*[38] and Gough *et al.*[35]

Qualitative features of the flux creep in different models can be obtained by a dimensional analysis.[28] We consider here three characteristic examples, namely, the exponential E-j curve (4), power dependence $E=E_c(j/j_c)^m$,[40] and the E-j curve of the form

$$E(j)=E_c \exp[1-(j_c/j)^\beta]U_0/kT$$

predicted by vortex-glass[10] and collective creep[11,12] models. Substituting these formulas for $E(j)$ into Eq. (11), we obtain

$$g(E)=\frac{\mu_0 j_1}{E}\ , \tag{12}$$

$$g(E)=\frac{\mu_0 j_c}{mE}\left[\frac{E_c}{E}\right]^{1/(1+m)}\ , \tag{13}$$

$$g(E)=\frac{\mu_0 kTj_c}{U_0\beta E}\left[1+k\frac{T}{U_0}\ln\frac{E_c}{E}\right]^{-1-1/\beta}\ , \tag{14}$$

for the exponential, power, and vortex glass $E(j)$, respectively. Despite the differences of physical mechanisms behind these models, the function $g(E)$ proves to be close to $1/E$ dependence in all three cases, if one neglects slowly varying logarithmic factors or takes account of the fact that $m \gg 1$ well below the irreversibility line.[40] Therefore, these models lead to similar time evolutions of $E(x,t)$ which are close to that of the Anderson-Kim scenario for the exponential $E(j)$. As follows from Eq. (11) such a universality results from the thermally activated flux dynamics in the subcritical region $j<j_c$ and takes place for any power dependence of the energy barrier $U(j)$ in Eq. (1). However, the time decays of $j(x,t)$ and $M(t)$ are much more sensitive to the particular dependence $U(j)$ and turn out to be essentially different in the above models.

Therefore, qualitative features of the time evolution of $E(x,t)$ are described by Eqs. (9) and (12), where j_1 is some effective model-dependent parameter. For instance, in the vortex-glass model the value j_1 can be found by comparison of Eqs. (12) and (14), which yields

$$j_1=\frac{kTj_c}{\beta U_0}\left[1+\frac{kT}{U_0}\ln\frac{E_c}{E_{\rm eff}}\right]^{-1-1/\beta}\ , \tag{15}$$

where $E_{\rm eff}$ is a characteristic electric field in a sample. Owing to the weak logarithmic dependence of j_1 on $E_{\rm eff}$, the exact value of $E_{\rm eff}$ in the time interval $t_i<t<t_f$ is not that essential, especially at $B \ll B_*$, where $kT \ll U_0$. For instance, at the initial stage of the flux creep, $E_{\rm eff}$ can be estimated as a characteristic initial electric field $E_{\rm eff} \sim \dot{B}_e a$.[12]

Equation (9) and the corresponding boundary and initial conditions can be written in the following dimensionless form:

$$|\varepsilon|\varepsilon''=\dot{\varepsilon}\ , \tag{16}$$

$$\varepsilon'(\pm 1,\theta)=0,\quad \varepsilon(\eta,0)=\eta\ , \tag{17}$$

where the prime and overdot denote the differentiations with respect to the dimensionless coordinate $\eta=x/a$ and time $\theta=t/\tau$, respectively, and $\epsilon=E/\dot{B}_e a$. The time constant τ is given by

$$\tau=\frac{\mu_0 j_1 a}{\dot{B}_e}\ . \tag{18}$$

Since Eqs. (16) and (17) do not contain any characteristics of the sample geometry and the initial conditions, the electric field $E(x,t)$ can be presented in the following scaling form:

$$E(x,t)=a\dot{B}_e\varepsilon\left[\frac{x}{a},\frac{t}{\tau}\right]\ , \tag{19}$$

where $\epsilon(\eta,\theta)$ is a universal dimensionless function which obeys Eqs. (16) and (17).

The exact solution of the nonlinear partial differential equation (16) which satisfies conditions (17) is unknown, although some analytical solutions have been considered in the literature. For instance, there is a solution which obeys both Eq. (16) and the boundary condition $E'(\pm a,t)=0$:[17]

$$E(x,t)=\frac{\mu_0 j_1}{2t}(2ax-x^2{\rm sgn}x)\ . \tag{20}$$

This solution is independent of the initial conditions and thereby gives an exact long-time asymptotics of $E(x,t)$ at $t \gg \tau$. Although Eq. (20) was first obtained within the framework of the Anderson-Kim model,[17] formula (20) virtually has much wider range of applicability and describes with the logarithmic accuracy the time decay of $E(x,t)$ in any flux-creep model with power activation barrier $U(j)$. This enables one to calculate the relaxation of $M(t)$, including explicit calculations of the time constants, which will be done in the next section.

Now we consider the transient stage ($t<\tau$) which essentially depends upon the initial conditions.[28] In this case one can write another exact solution of Eq. (16),[28,36]

$$E(x,t)=\frac{\mu_0 j_1}{2(t+\tau_0)}(2ax-x^2{\rm sgn}x)\ , \tag{21}$$

which also obeys the boundary condition $E'(x,0)=0$, but, unlike Eq. (20), does not contain the singularity at $t=0$. Although Eq. (21) does not satisfy the initial condition $E(x,0)=\dot{B}_e x$, we employ Eq. (21) for a selfconsistent calculation of the magnetic moment $M(t)$, by choosing the time constant τ_0 such that Eq. (21) would result in the exact initial value of $M(t)$ at $t=0$. The calculation of τ_0 given below yields $\tau_0=C\tau$, where τ is the universal time constant (18), and C is a numerical coefficient of the order of unity which depends upon the sample geometry.

III. RELAXATION OF $M(t)$

A. Initial stage

We first consider the initial stage of magnetic relaxation of the subcritical state ($j<j_c$) for which one can use the I-V curve of form (4). Then $M(t)$ is given by

$$M(t)=\frac{1}{2}\int_{-a}^{a} xj(x,t)dx$$
$$=\int_0^a \left[j_c+j_1\ln\frac{E(x,t)}{E_c}\right]xdx \ . \qquad (22)$$

Substitution of Eq. (21) into Eq. (22) and integration over x yield

$$M(t)=\frac{a^2}{2}\left[j_c+j_1\ln\left[\frac{8\mu_0 j_1 a^2}{\tau_0 E_c}\right]-3j_1\right]$$
$$-\frac{j_1 a^2}{2}\ln\left[1+\frac{t}{\tau_0}\right] \ . \qquad (23)$$

Now we choose the time constant τ_0 such that Eq. (23) would give the correct initial value $M(0)$ at $t=0$. The quantity $M(0)$ can be calculated from Eq. (22) with $E(x,0)=\dot{B}_e x$, whence

$$M(0)=\frac{a^2}{2}\left[j_c+j_1\ln\left[\frac{a\dot{B}_e}{E_c}\right]-\frac{j_1}{2}\right] \ . \qquad (24)$$

By equation the expressions in the square brackets in Eqs. (23) and (24), we can calculate τ_0 and write $M(t)$ in the form

$$M(t)=M(0)-M_1\ln\left[1+\frac{t}{\tau_0}\right] , \qquad (25)$$

$$\tau_0=\frac{8\mu_0 j_1 a}{e^{5/2}\dot{B}_e}=0.657\tau, \quad M_1=\frac{a^2}{2}j_1 \ . \qquad (26)$$

Here $M(0)$ and τ are determined by Eqs. (24) and (18), respectively, and $e=2.718$. Therefore, this procedure gives the interpolation formulas (24)–(26) which provide the correct asymptotics of $M(t)$ at $t=0$ and $t\gg\tau_0$. Equations similar to Eqs. (25) and (26) have also been obtained in Refs. 12, 22, 24, and 26–32 with an accuracy to numerical coefficients.

The function $M(t)$ described by Eq. (25) is shown in Fig. 2. Here the time constant τ_0 determines the duration of the initial nonlogarithmic relaxation caused by the transient diffusion redistribution of magnetic flux over the sample cross section after the stepwise change of $\dot{B}_e$ at $t=0$. If plotted as a function of the variable $u=\ln t$, the dependence $M(u)$ exhibits a plateau at small $\ln t$. As

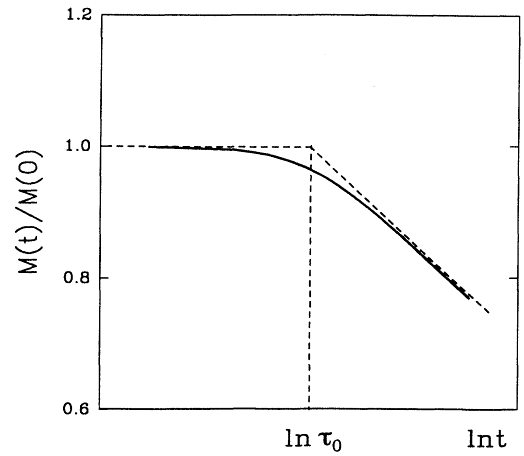

FIG. 2. Time relaxation of $M(t)$ described by Eq. (25). The dashed lines show the asymptotics of $M(\ln t)$ for $t\ll\tau_0$ and $t\gg\tau_0$, respectively.

follows from Eq. (25), the value of $\ln\tau_0$ is determined by the intersection of two straight lines $M(u)=M(0)$ and $M(u)=[M(0)+M_1\ln\tau_0]-M_1 u$, which are the short- and long-time asymptotics of $M(u)$, respectively. It should be emphasized that in the real-time scale formula (25) does not give any plateau in $M(t)$ at short t, since the plateau in $M(\ln t)$ actually results from a logarithmic "compression" of the t axis upon the transformation $u=\ln t$. However, the plotting of M versus $\ln t$ is a convenient mathematical procedure which enables one to reveal the nonlogarithmic stage when analyzing experimental data (see below).

B. Long-time relaxation

At $t\gg\tau_0$ the value $\dot{B}_e$ in Eqs. (24)–(26) cancels, thereby $M(t)$ becomes independent of the initial conditions, and formula (25) reduces to the well-known Anderson-Kim result (2). Notice that the constants M_c and t_0 in Eq. (2) can be chosen arbitrarily, provided that the observed quantity $M_c+\ln t_0$ remains constant. In other words, instead of particular M_c and t_0, one can take another pair M_c' and t_0' such that

$$M_c'=M_c+M_1\ln\frac{t_0}{t_0'} \ . \qquad (27)$$

It is convenient to define M_c by analogy to the Bean model, $M_c=a^2 j_c/2$,[19] where j_c corresponds to the particular voltage criterion E_c. In this case Eqs. (24)–(26) allow one to express the time constant t_0 only via observed parameters a, j_1, and E_c as follows:[28]

$$t_0=\frac{8a^2\mu_0 j_1}{e^3 E_c} \ . \qquad (28)$$

Therefore, unlike the time constant τ_0 which is fixed unambiguously by the initial conditions and the macroscopic parameters of a superconductor, the value t_0 essentially depends on the voltage criterion E_c and the definition of M_c. For this reason, there are many different definitions of t_0 in the literature (see, e.g., Refs. 12, 22, and 24–32, although they virtually give similar descriptions of the observed relaxation of $M(t)$. In any case, however, the flux-creep time constant t_0 is determined by the macroscopic nonlinear flux diffusion and has nothing to do with the inverse microscopic "attempt" frequency of pinned fluxons usually assumed to be of order 10^{-10}–10^{-13} s. Indeed, taking, for example, the conventional criterion $E_c=1\ \mu$V/cm, we find for the sample with $a=0.1$ mm, $j_1=10^4$ A/cm^2,[15] that $t_0\sim 0.01$ s. Similar conclusion on the macroscopic origin of t_0 has been made in Refs. 12, 22, and 24–32 within the framework of different flux-creep models. Another qualitative estimation of t_0 and E_c could be done if one considers a linear flux-flow part of $E(j)$, for which $E(j)=(j-j_c)\rho_f$, where ρ_f is the flux-flow resistivity.[28] The flux-flow regime occurs at $j-j_c \gg j_1$, since the parameter j_1 determines the smearing of the *I-V* curve due to the flux creep [see Eq. (4)]. Hence, it follows that the crossover electric field $E_c\sim\rho_f j_1$ can essentially depend on T and B. Taking $\rho_f=\rho_n B/B_{c2}=0.1\ \mu\Omega$ cm, with $B/B_{c2}=0.01$ and $\rho_n\sim 10\ \mu\Omega$ cm, the normal-state resistivity extrapolated down to $T\sim 10$ K, we obtain that $E_c\sim 1$ mV/cm, and $t_0\sim 10^{-5}$ s. The difference between these two estimations indicates that the value of t_0 can be fairly sensitive to the definition of t_0 discussed above.

Formulas (25) and (26) are valid as long as the decay of $M(t)$ during the flux-creep measurements $t_i<t<t_f$ is small compared with $M(t_i)$, which enables one to use the exponential approximation (4) of $E(j)$. However, the long-time relaxation of $M(t)$ becomes sensitive to the particular form of the flux-creep barrier $U(j)$ and generally is not described by Eqs. (25) and (26). For example, in the vortex-glass model $M(t)$ at $t\gg\tau_0$ can be presented in the form[12]

$$M(t)=\frac{M_c}{[1+(kT/U_0)\ln(t/t_0)]^{1/\beta}} . \tag{29}$$

Here the values j_1 and t_0 are given by Eqs. (15) and (28), respectively. Notice that the interpolation formula (29) was obtained with a logarithmic accuracy because of a weak logarithmic dependence of j_1 on $E_{\rm eff}$ in Eqs. (15) and (28). In other respects, the time constant t_0 in the vortex-glass model displays the same qualitative dependence on the sample size and the voltage criterion E_c as that for the exponential $E(j)$ [see Eq. (28)]. This is essentially due to the universality of the relaxation of $E(t)$ discussed above.

The term $\ln(E_c/E_{\rm eff})$ in Eq. (15) brings an uncertain numerical factor in t_0 which is determined by a characteristic change of electric fields during the flux-creep measurements $t_i<t<t_f$. Since $E(t)$ mostly decays as $1/t$, the term $\ln(E_c/E_{\rm eff})$ gives rise to a weak time dependence of t_0 in Eq. (29). The character of this dependence can be estimated with a logarithmic accuracy by substituting $E_{\rm eff}\sim\mu_0 j_c kTa^2/\beta U_0 t$ which follows from Eqs. (15) and (20) into Eq. (15). Hence,

$$t_0=\frac{8a^2\mu_0 j_c kT}{e^3 E_c\beta U_0}\left[1+\frac{kT}{U_0}\ln\frac{t}{t_*}\right]^{-1-1/\beta} , \tag{30}$$

$$t_*\simeq\frac{8\mu_0 a^2 j_c kT}{e^3\beta U_0 E_c} . \tag{31}$$

Therefore, the time constant t_0 in Eq. (29) actually logarithmically decreases with t, which may affect the interpretation of long-time flux-creep experiments. For instance, at $t_i\simeq 10$ s and $t_f\simeq 10^5$–10^6 s we get $\ln(t_f/t_i)\simeq 10$–12. Hence, it follows that at $(kT/\beta U_0)\ln(t_f/t_i)>1$ the value t_0 can noticeably vary within the time window $t_i<t<t_f$, since the exponent β can be quite small in certain regions of T and B [for instance, the collective creep model predicts that $\frac{1}{7}<\beta<\frac{3}{2}$ (Refs. 11 and 12).

Unlike t_0, the transient time constant τ_0 essentially depends upon $\dot{B}_e$ and can be measured directly. For instance, if $a=1$ mm, and $j_1=2\times 10^4$ A/cm^2, we obtain from Eq. (26) that $\tau_0=16$ s at $\dot{B}_e=10^{-2}$ T/s, and $\tau_0=1.6\times 10^4$ s at $\dot{B}_e=10^{-5}$ T/s. Therefore, the initial nonlogarithmic stage of magnetic relaxation can take considerable time, which may even exceed a time window in flux-creep experiments. In addition, the value $\tau_0(T,B)$ depends on T and B, since $\tau_0(T,B)$ is proportional to the value $j_1(T,B)$ which determines the slope of the curves $M(\ln t)$ at $t>\tau_0$.

IV. MEASUREMENTS OF THE INITIAL STAGE OF THE FLUX CREEP

A. Experimental

In this section we present results of flux-creep measurements performed on grain-oriented $YBa_2Cu_3O_{7-x}$ with 5% of Ag. The samples prepared by a liquid-phase processing technique have a plateletlike shape with typical grain size about 1 cm in the *ab* plane and the thickness of the grains along the *c* axis between 20 and 30 μm. The specimen of thickness 0.5 mm was cut with the *c* axis parallel to the macroscopic slab surface of 3.3×2.5 mm. The magnetic field was parallel to the *c* axis such that the screening magnetization currents flow in the *ab* plane. Such a geometry has been chosen in order to minimize the effect of anisotropy on the stability of current configurations.[41]

All measurements of $M(t)$ were performed by means of a vibrating sample magnetometer (Oxford Instruments, Model 3001). At a constant temperature, T, we apply a sufficiently high initial field B_e which then is reduced with the desired sweep rate $\dot{B}_e$ to the field B at which the magnetic moment $M(t)$ is measured. The value B ranged from 2 to 10 T which ensured the complete flux penetration and full critical state in the sample. Since we were interested in the relaxation of the irreversible magnetization, the reversible part $M_{\rm eq}$ was subtracted from the measured $M(t)$ by taking the mean value of two branches of the magnetization curve which correspond to the increasing and decreasing $B_e(t)$ (Fig. 3). In our measurements no asymmetry of the relaxation was observed,

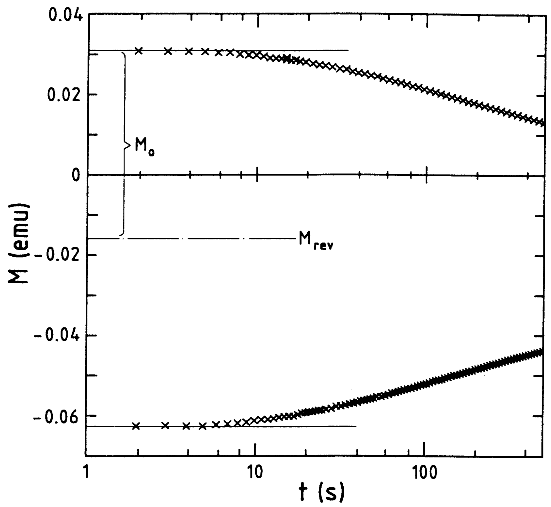

FIG. 3. An example of relaxation curves $M(\ln t)$ for two branches of M which correspond to the increasing and decreasing $B_e(t)$ (upper and lower curves, respectively) at 77 K and 3 T.

which indicates that the influence of the surface barrier[42] is negligible. The time t_i between the beginning of the magnetic relaxation and the first measurement is determined by an integration constant of the signal amplifier and was about 2 s.

B. Results

We have measured the relaxation of $M(t)$ for the ramp rates $\dot{B}_e$ ranging from 5 μT/s to 10 mT/s at different temperatures 4.2 K$<T<$80 K and magnetic fields $0<B<8$ T. As an illustration, Figs. 4(a) and 4(b) show typical relaxation curves $M(\ln t)$ for $T=77$ K, $B=3$ T and $T=20$ K, $B=6$ T at different ramp rates $\dot{B}_e$. As seen from Fig. 4(b), the character of $M(\ln t)$ at $\ln t<100$ essentially depends upon $\dot{B}_e$, whereas at large times the influence of the initial conditions on $M(t)$ becomes much less pronounced. For instance, at $\dot{B}_e>0.2$ mT/s the curves $M(\ln t)$ have a downward curvature, the relaxation rate $dM/d\ln t$ increasing with $\dot{B}_e$. As $\dot{B}_e$ decreases, the curvature of $M(\ln t)$ changes the sign at small $\ln t$, which gives rise to the developing of a plateau in $M(\ln t)$ which increases as $\dot{B}_e$ decreases.

We suppose that these features of the observed $M(\ln t)$ can be interpreted in terms of the nonlinear diffusion motion of magnetic flux in which initial velocity is proportional to the ramp rate $\dot{B}_e$. For instance, the faster initial relaxation of $M(\ln t)$ at higher $\dot{B}_e$ shown in Fig. 4(b) implies a larger differential resistivity dj/dE at the corresponding electric fields $E(0)\sim a\dot{B}_e$. As $\dot{B}_e$ decreases, both $E(0)$ and dj/dE decrease, thereby the relaxation rate $dM/d\ln t$ slows down. Simultaneously, this results in the increase of the transient time τ_0 needed for the beginning of a steady-state flux creep after the diffusion redistribution of the magnetic flux over the sample cross section due to the abrupt change of $\dot{B}_e$ at $t=0$. As the value τ_0 becomes larger than the integration time constant of the amplifier t_i, the curvature of $M(\ln t)$ changes the sign, giving rise to a quasiplateau in the relaxation curves at small $t<\tau_0$. However, as has been already mentioned, such a plateau in $M(\ln t)$ is just a manifestation of the transient nonlogarithmic stage which does not imply any plateau in $M(t)$ dependence in the linear time scale. Notice that in our experiments we have specially chosen the reduced ramp rate $\dot{B}_e$ in order to trace the appearance of the transient stage. Usually the value $\dot{B}_e$ is taken by several orders of magnitude larger than our lowest $\dot{B}_e$, which shifts this stage into the ms region.

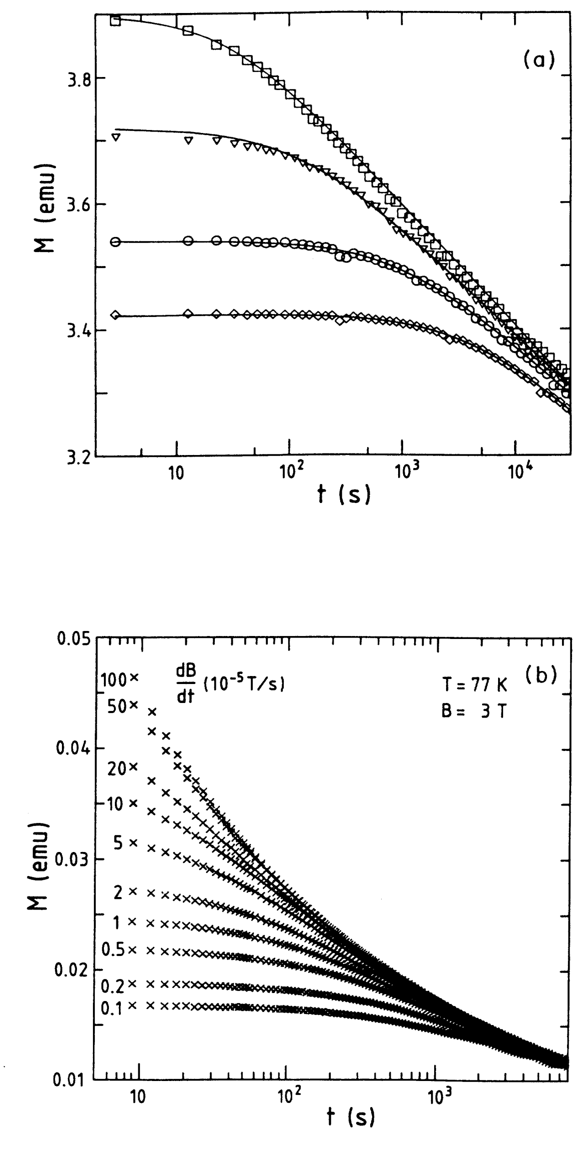

FIG. 4. Examples of the relaxation curves $M(\ln t)$ for $T=20$ K, $B=6$ T (a), and $T=77$ K, $B=3$ T (b). Solid curves in (a) correspond to Eq. (6) with fit parameters $M(0)$, M_1, and τ_0 at various $\dot{B}_e$: 1 mT/s (□), 0.13 mT/s (▽), 20 μT/s (○), and 5 μT/s (◇).

As an illustration, we consider the case $T=20$ K and $B=6$ T in more detail. Shown in Fig. 4(a), the relaxation curves $M(\ln t)$ do display a plateau corresponding to the transient nonlogarithmic stage, the plateau increasing as $\dot{B}_e$ decreases. As follows from Fig. 4(a), the observed curves $M(\ln t)$ can be well described by Eq. (6) when treating $M(0)$, M_1, and τ_0 as fit parameters. This confirms the above analysis and enables one to extract the quantities $M(0)$, M_1 and τ_0 from the flux-creep measurements and then to compare their dependencies on T, B, and $\dot{B}_e$ to those given by the theoretical consideration.

We first examine the time constant $\tau_0(T,B,\dot{B}_e)$ written

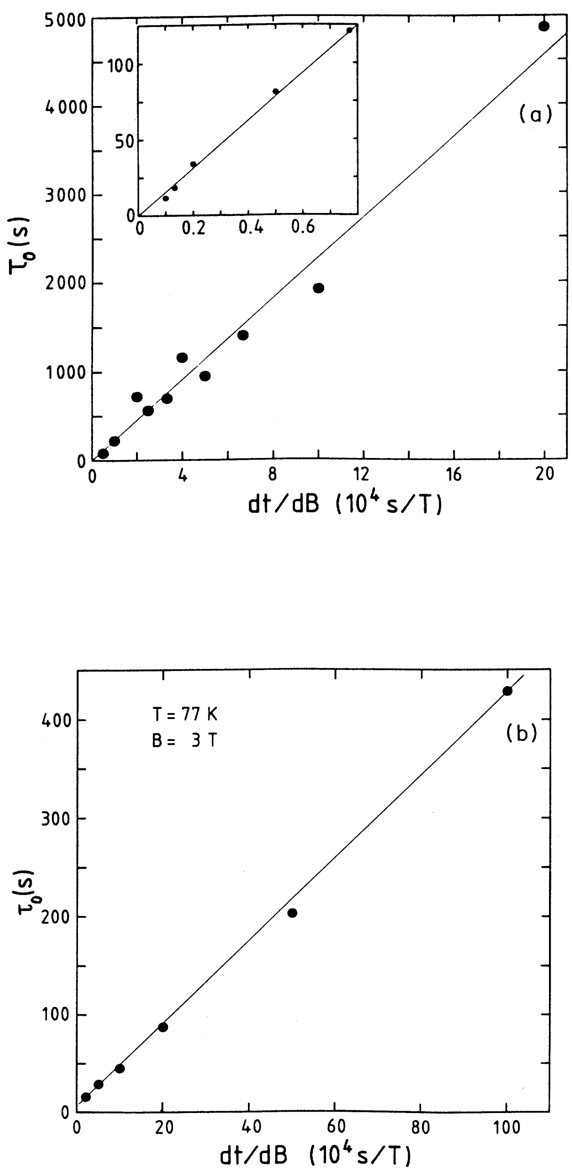

FIG. 5. Dependence of the time constant τ_0 on $1/\dot{B}_e$ extracted from the relaxation curves in Fig. 4. Inset in Fig. 4(a) shows the data for higher $\dot{B}_e$. The lines give the best fits; their slopes in Fig. 4(a) and the inset differ by $\simeq 20\%$.

in the form which is convenient for direct comparison of Eq. (26) with experimental data:

$$\tau_0=G\left|\frac{dM}{d\ln t}\right|\frac{1}{\dot{B}_e}\ . \tag{32}$$

Here the factor G contains all parameters which characterize the sample geometry, the value G being independent of T, B, and $\dot{B}_e$. The second factor $dM/d\ln t$ is the observed flux-creep rate $M_1(T,B)$ in the regime of the logarithmic relaxation. The function $M_1(T,B)$ essentially depends on T and B,[13–16] and can be directly extracted from the slope of the relaxation curves at $t \gg \tau_0$. The third term $1/\dot{B}_e$ results from the effect of initial conditions which allow one to vary significantly the duration of the transient stage upon changing the ramp rate $\dot{B}_e$. Therefore, Eq. (32) predicts the linear proportionality of τ_0 to M_1 and the inverse dependence of τ_0 from $\dot{B}_e$, which has been confirmed by our flux-creep measurements.

Figure 5 shows the dependence of τ_0 extracted from the relaxation curves $M(\ln t)$ versus the inverse ramp rate $1/\dot{B}_e$. As follows from Fig. 5, the time constant $\tau_0(\dot{B}_e)$ proves to be inversely proportional to $\dot{B}_e$ over the region of $\dot{B}_e$ examined. We have observed, however, some deviations from the inverse dependence $\tau_0=C/\dot{B}_e$. For instance, at $T=20$ K and $B=6$ T the slope C at $5<\dot{B}_e<100$ μT/s turns out to be about 20% smaller than the value of C at $0.1<\dot{B}_e<1.2$ mT/s [Fig. 5(a)]. We believe that this may be due to nonexponential $E(j)$ at $E_1<E<E_2$, where $E_{1,2}=a\dot{B}_e/2$ is a mean-induced electric field in the superconductor for the minimum and maximum ramp rates (for our sample $E_1=1.25\times10^{-5}$ μV/cm at $\dot{B}_e=5$ μT/s and $E_2=2.5\times10^{-3}$ μV/cm at $\dot{B}_e=1$ mT/s, respectively). In particular, for the vortex-glass model this effect manifests itself in a weak logarithmic dependence of the parameter j_1 on $\dot{B}_e$, or in a nonzero value of $1/m$ for the power I-V curve [see Eq. (13)]. Anyway, the time constant τ_0 essentially increases upon reducing $\dot{B}_e$ and can even exceed the time window at small $\dot{B}_e$. For instance, at 20 K and 6 T we have observed the increase of τ_0 from 11 s at $\dot{B}_e=1$ mT/s to 5000 s at $\dot{B}_e=5$ μT/s.

Shown in Fig. 6, are the temperature and field dependences of τ_0 at fixed $\dot{B}_e=10$ μT/s together with the dependences of the relaxation rate $dM/d\ln t$ on T and B extracted from the slopes of the relaxation curves $M(\ln t)$ at $t\gg\tau_0$. The time constant τ_0 strongly depends both on T and B, the temperature dependence being nonmonotonic. However, as seen from Fig. 6, the values τ_0 and $dM/d\ln t$ exhibit remarkably similar dependencies on T and B,[34] which implies that both quantities are indeed proportional over the whole domains of T and B studied. This result enables one to obtain the geometrical constant $G\simeq0.2$ T/emu and, using Eq. (32), to calculate τ_0 for arbitrary values of $\dot{B}_e$. Therefore, the dependencies of τ_0 upon $\dot{B}_e$, T, and B predicted by Eq. (32) are in agreement with our experimental data. Recently a similar behavior of $\tau_0(T,B)$ was observed by Brawner, Ong, and Wang[33] who found that the temperature and field dependences of τ_0 are close to those of the critical current density

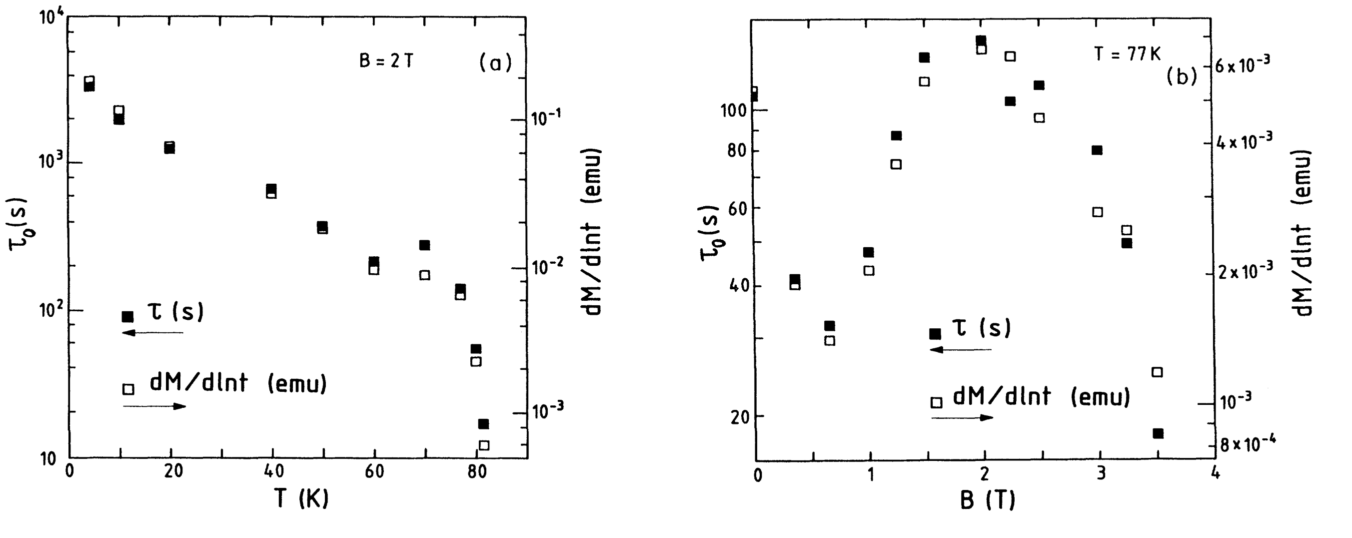

FIG. 6. The time constant $\tau_0(T,B)$ and the creep rate $M_1(T,B)$ plotted in the logarithmic scale vs T (a) and B (b), respectively. The value M_1 is obtained from the slope of $M(\ln t)$ at $t \gg \tau_0$ at $\dot{B}_e = 10\ \mu$T/s.

$j_c(T,B)$. However, as follows from our results, the time constant $\tau_0(T,B)$ scales as the flux-creep rate $j_1(T,B) = -dj/d\ln t$ on the logarithmic stage of magnetic relaxation. The quantity j_1 is related to j_c as follows: $j_1(T,B) = s(T,B)j_c(T,B)$, where $s(T,B) = -d\ln M/d\ln t$ is a dimensionless flux-creep rate. Figure 7 shows an example of the field dependence of $s(B)$ for our sample at 77 K (see also Refs. 13–16). Such a strong nonmonotonic behavior of $s(B)$ can manifest itself in different dependences of j_1 and j_c upon T and B, which should be taken into account when analyzing the time constant $\tau_0(T,B)$.

Another quantity which turns out to be dependent on $\dot{B}_e$ is the initial magnetic moment $M(0)$. As follows from

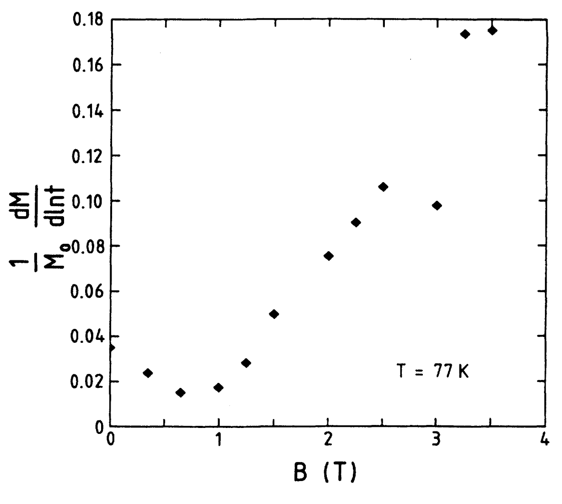

FIG. 7. Dimensionless flux-creep rate $s = d\ln M/d\ln t$ as a function of B at 77 K.

Eq. (24), the value $M(0)$ logarithmically increases with $\dot{B}_e$ in the case of the exponential I-V curve which results in the logarithmic relaxation at $t \gg \tau_0$. Our measurements have shown that such a regime does occur at low T. For instance, Fig. 8(a) shows the linear dependence of $M(0)$ upon $\ln\dot{B}_e$ at $T = 20$ K and $B = 6$ T extracted from the relaxation curves presented in Fig. 4. However, at higher T and B the flux-creep dynamics can be essentially nonlogarithmic, especially about the irreversibility field $B_*(T)$,[5–9] where the dependence of $M(0)$ on $\ln\dot{B}_e$ becomes nonlinear. An example of such a behavior at $T = 77$ K and $B = 3$ T is presented in Fig. 8(b), where the nonlinear dependence of $M(0)$ on $\ln\dot{B}_e$ correlates with the nonlogarithmic decay of $M(t)$ shown in the inset.

We have also measured the temperature dependence of $M(0)$ at a constant $\dot{B}_e = 0.01$ mT/s and $B = 2$ T. The results shown in Fig. 9 indicate approximately exponential dependence of $M(t_i)$ on T below 60 K, where the moment $M(t_i)$, taken as usual at $t \simeq 10$–100 s, only slightly differs from $M(0)$. However, above 60 K, the value $M(t_i)$ becomes much smaller than $M(0)$ due to a considerable increase of the flux-creep rate when approaching the irreversibility line.

V. CONCLUDING REMARKS

The above experimental and theoretical results indicate that the initial stage of the flux creep in the subcritical state $j < j_c$ is universal and is determined by a nonlinear flux diffusion, regardless of particular microscopic mechanisms of resistivity. We have shown that there are two characteristic time scales τ_0 and t_0 of short- and long-time magnetic relaxation, respectively. Both τ_0 and t_0 are macroscopic quantities which can be expressed in terms of directly measured parameters, such as a, j_c, j_1,

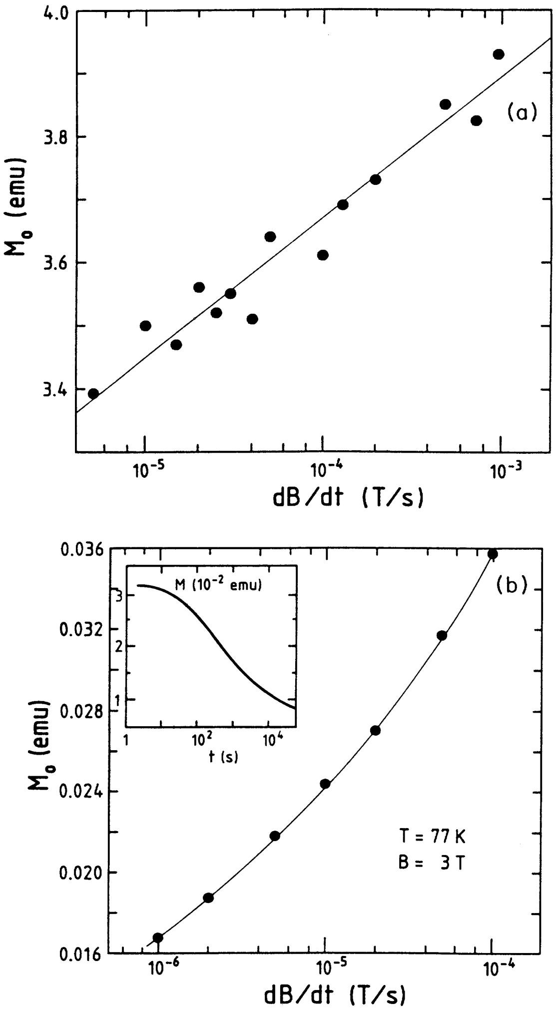

FIG. 8. Dependences of $M(0)$ at $T=20$ K, $B=6$ (a) and $T=77$ K, $B=3$ T (8b), respectively. The nonlinear dependence of $M(0)$ upon $\ln\dot{B}_e$ in Fig. 7(b) correlates with the nonlogarithmic relaxation of $M(t)$ shown in the inset.

$\dot{B}_e$, and E_c. It should be emphasized that τ_0 turns out to be inversely proportional to the sweep rate $\dot{B}_e$, which enables one to substantially increase τ_0 by decreasing $\dot{B}_e$. By contrast, the value t_0 does not depend on $\dot{B}_e$, but depends upon the voltage criterion E_c. As follows from our data, the temperature and field dependences of τ_0 are entirely determined by the observed creep rate $M_1(T,B)$ in the regime of the steady-state logarithmic flux creep.

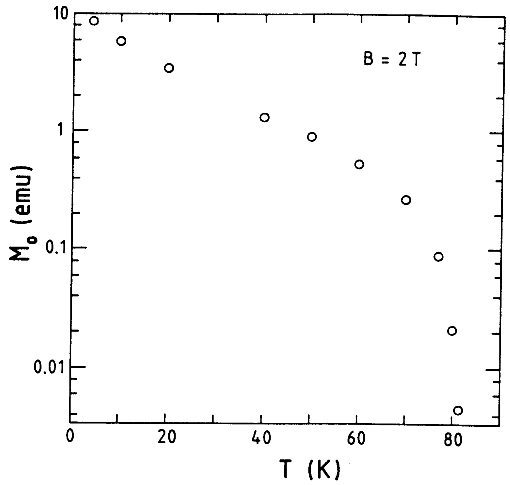

FIG. 9. Temperature dependence of $M(0)$ at $B=2$ T and $\dot{B}_e=10\ \mu$T/s.

The features of magnetic relaxation discussed in this paper result from the essential nonlinearity of the *I-V* curves at $j<j_c$, and could be observed on both HTS and LTS materials. However, in HTS they seem to be more pronounced due to much stronger flux creep,[1] which, for example, can manifest itself in a significant dependence of magnetization curves upon the sweep rate.[32,35,43] Here the time constant τ_0 determines the minimum time needed for measurements of stationary magnetic or electric characteristics of superconductors.[28] In particular, in our experiments the value τ_0 reached about 5000 s.

ACKNOWLEDGMENTS

The authors thank K. Salama, D. Lee, and V. Selvamanickam from the Texas Center of Superconductivity at University of Houston for the preparation of the specimens and B. Runtsch and A. Will for technical assistance.

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