Frequency and wave-vector dependence of the fluctuation electromagnetic response near the superconducting transition

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Expressions for the fluctuation components of frequency- and wave-vector-dependent transverse and longitudinal dielectric functions are obtained for a normal massive isotropic metal near T_c. The characteristic radius of nonlocality of the fluctuation response is shown to be of the order of the temperature-dependent superconducting coherence length $\xi(T)$. Certain consequences of the nonlocal nature of the static-fluctuation diamagnetic response above and below T_c are discussed. Frequency and wave-vector dependences of the fluctuation electromagnetic response are found to determine the characteristic value of the differential cross section for small-angle magnetic neutron scattering near T_c. This value appears to be 10^3–10^5 times greater in high-temperature superconductors than in low-temperature superconductors with large zero-temperature coherence lengths.

I. INTRODUCTION

The contribution to the electromagnetic response, originating from superconducting fluctuations, is of importance near the superconducting transition temperature T_c. The appropriate contribution to the conductivity and diamagnetic response[1–5] has a marked temperature dependence and can be extracted from experimental observations (see, e.g., Ref. 6). Usually considerations near T_c are restricted to the case of the local dependence on the applied electromagnetic field. However, nonlocality (spatial dispersion) appears to be essential for a range of problems, as we will show below. The objective of this work is to obtain the spatial dispersion of the fluctuation electromagnetic response near T_c and to examine some of the effects resulting from this nonlocality. More familiar temporal (frequency) dispersion of the response will also be considered. Fluctuation effects are more noticeable in high-temperature superconductors (HTSC), than in low-temperature ones due to a greater magnitude of Ginzburg number.[7] This fact can benefit experimental observation of discussed effects.

In Sec. II, we derive the general expressions for the fluctuation parts of the transverse and longitudinal electromagnetic response above T_c with characteristic frequencies $\Omega \ll T_c$ and wave vectors $Q \ll \xi_0^{-1}$, where ξ_0 is the zero-temperature coherence length. Both Aslamazov-Larkin and Maki-Thompson terms are discussed in this context. The spatial dispersion of the Aslamazov-Larkin term is shown to be characterized by a large scale $\xi(T)$, whereas much smaller scales ξ_0, l are characteristic for the Maki-Thompson term (where l is the mean free path). So the nonlocality of the latter term can be ignored. The large macroscopic value of a radius of superconducting correlations $\xi(T)$ near T_c means that the nonlocality of the Aslamazov-Larkin term is of importance even for a macroscopically varying field.

The static limit of fluctuation response near T_c is discussed in detail in Sec. III. The fluctuation diamagnetic response in homogeneous metal and near the vacuum-metal boundary is considered in Sec. III A. The relation of the fluctuation current to magnetic field is found to be governed exactly by the phenomenological Pippard kernel, which was proposed by Pippard to describe the relation between the superconducting current and vector potential **A**. The profile of fluctuation magnetization near the vacuum-metal boundary for the parallel magnetic field is substantially modified at distances less than or of the order $\xi(T)$ from the boundary. The problem of diamagnetic response in a slab geometry is studied in Sec. III B. The magnetization in a slab with width $d \lesssim \xi(T)$ in a parallel field is essentially suppressed, as compared to the magnetization in the interior of a thick sample. Fluctuation corrections to the penetration depth below T_c are obtained in Sec. III C. The fluctuation correction to the penetration depth without an account of nonlocality was considered earlier in Ref. 8. The nonlocality of the fluctuation response turns out to be more important for the type-I superconductors than for the type-II ones.

Magnetic neutron scattering on the superconducting fluctuations above T_c is examined in Sec. IV. The obtained temperature dependence of the corresponding cross section describes the original growth of this cross section approaching T_c and its subsequent saturation (the appearance of the plateau) due to the joint influence of spatial and frequency dispersion of the fluctuation response. The conditions are found under which either frequency or spatial dispersion dominates in forming the plateau. Recent experiment on HTSC,[9] where a plateau of such type was observed is discussed. In this case it is shown that the spatial dispersion of the fluctuation response is to be taken account of.

Finally, the fluctuation contribution to the reflectivity of plane metal surface above T_c is discussed briefly in Sec. V.

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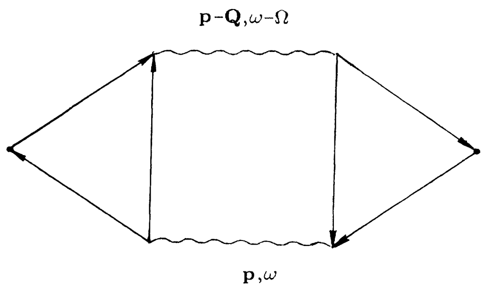

FIG. 1. Aslamazov-Larkin diagram. Solid lines correspond to electron propagators, wavy lines correspond to fluctuation propagators.

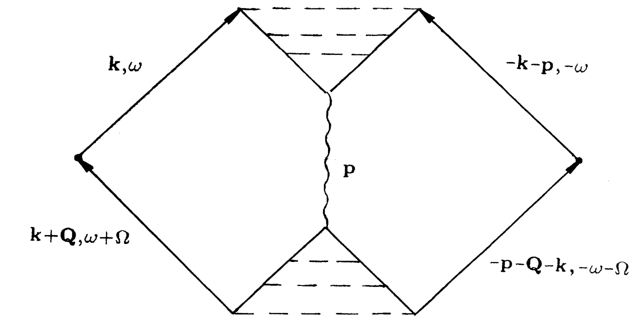

FIG. 2. Maki-Thompson term. Dashed lines denote impurity scattering.

II. FLUCTUATION COMPONENT OF ELECTROMAGNETIC RESPONSE

In a normal metal near T_c and out of the critical region, the basic contribution to the fluctuation linear electromagnetic response is provided, as is known, by the Aslamazov-Larkin and Maki-Thompson terms[1,4,5] (see Figs. 1 and 2). Let us first consider the former term, which can be described within the framework of a time-dependent Ginzburg-Landau equation with a Langevin source (see, e.g., Ref. 10):

$$\gamma\frac{\partial\Psi}{\partial t}+(a+2i\gamma e\phi)\Psi-\frac{1}{4m}\left[\nabla-\frac{2ie}{c}\mathbf{A}\right]^2\Psi=g(\mathbf{r},t)\ . \tag{1}$$

Here $a=\alpha(T-T_c)$, $\langle|g(\mathbf{Q},\Omega)|^2\rangle=2\gamma T$, and the validity of inequalities $\Omega\ll T_c$ and $Q\ll\xi_0^{-1}$ is implied. Expanding Ψ over the set of plane waves $\Psi(\mathbf{r},t)=\sum_{\omega,\mathbf{p}}c_{\omega,\mathbf{p}}e^{i\mathbf{p}\cdot\mathbf{r}-i\omega t}$ and performing the Fourier transformation in Eq. (1), one can get the following expression for the Fourier component of the superconducting current:

$$\mathbf{J}(\Omega,\mathbf{Q})=\frac{e}{m}\sum_{\omega,\mathbf{p}}\left[\mathbf{p}+\frac{\mathbf{Q}}{2}\right](c^{(0)*}_{\omega,\mathbf{p}}c^{(1)}_{\omega+\Omega,\mathbf{p}+\mathbf{Q}}+c^{(1)*}_{\omega,\mathbf{p}}c^{(0)}_{\omega+\Omega,\mathbf{p}+\mathbf{Q}})-\frac{2e^2}{mc}\sum_{\omega,\mathbf{p}}\sum_{\omega',\mathbf{p}'}c^{(0)*}_{\omega,\mathbf{p}}c^{(0)}_{\omega',\mathbf{p}'}\mathbf{A}(\Omega+\omega-\omega',\mathbf{Q}+\mathbf{p}-\mathbf{p}')\ . \tag{2}$$

The applied transverse field (div$\mathbf{E}=0$) is regarded here as a perturbation and the quantities $c^{(0)},c^{(1)}$ are of zero and first order in $\mathbf{A}$, respectively. On averaging Eq. (2) over superconducting fluctuations of the order parameter, the kernel, relating $\mathbf{j}=\langle\mathbf{J}\rangle$ to $\mathbf{A}$, is obtained. For $T>T_c$ this kernel must be zero in the limit $\Omega\to0$, $Q\to0$ due to gauge invariance. However, in calculating this kernel, the Ginzburg-Landau theory results in an integration over momenta, which is divergent at the upper limit. It is associated with the invalidity of the Ginzburg-Landau theory for large momenta $Q\gtrsim\xi_0^{-1}$. In order to obtain the finite result within the framework of the Ginzburg-Landau theory, it is sufficient to subtract the integrand, corresponding to the limit $\Omega\to0$, $Q\to0$.

Following this procedure we get in the linear approximation in $\mathbf{A}$ the relation $\mathbf{j}(\Omega,\mathbf{Q})=(-i\Omega/4\pi)\varepsilon^f_{\rm tr}(\Omega,Q)\mathbf{E}(\Omega,\mathbf{Q})$, where

$$\varepsilon^f_{\rm tr}(\Omega,Q)=\frac{128\pi e^2T_c}{\Omega^2}\int\frac{d^3p}{(2\pi)^3}p_y^2\left[\frac{1}{[2\xi^{-2}(T)+p^2+(\mathbf{p}-\mathbf{Q})^2-4im\gamma\Omega][\xi^{-2}(T)+p^2]}-\frac{1}{2[\xi^{-2}(T)+p^2]^2}\right]\ . \tag{3}$$

Here $\xi(T)=1/2(ma)^{1/2}$. It is convenient to introduce the dimensionless frequency $\omega=2m\gamma\xi^2(T)\Omega$ and wave vector $\mathbf{q}=\mathbf{Q}\xi(T)/2$. Then, performing an integration in Eq. (3), one finds the final result for $\varepsilon^f_{\rm tr}(\Omega,Q)$:

$$\varepsilon^f_{\rm tr}(\Omega,Q)=\frac{16\gamma^2m^2e^2T_c\xi^3(T)}{\omega^2}\left\{1+\frac{i\omega}{2q^2}[1-(1+q^2-i\omega)^{1/2}]-\frac{1}{q}\left[1+q^2\left[1-\frac{i\omega}{2q^2}\right]^2\right]\left[\arctan\left[q-\frac{i\omega}{2q}\right]+\arctan\left[\frac{i\omega}{2q(1+q^2-i\omega)^{1/2}}\right]\right]\right\}\ . \tag{4}$$

Here the function arctan$z$ is defined in a complex plane z with cuts $(i,i\infty)$, $(-i,-i\infty)$. Radicals have cuts along the real negative axis.

One can also obtain Eq. (4) from the Aslamazov-Larkin diagram, represented in Fig. 1. The Aslamazov-Larkin diagram is generally considered for the external frequency $\Omega\ll T_c$ in the limiting case $Q\to0$, where Q is the wave vector of the external electromagnetic field. Suppose, that the external momentum can be comparable to or even exceed the inverse radius of superconducting correlations $\xi^{-1}(T)$, remaining much less than ξ_0^{-1}. In the calculation of the diagram of Fig. 1, this circumstance leads to the essential dependence of fluctuation propagators on Q, while electron loops may be described as before by expressions, corresponding to the $Q \to 0$ limit. The thing is that the characteristic momentum for fluctuation propagators $K^{R,A}(p,\omega)=1/(a+p^2/4m \mp i\gamma\omega)$ is $\xi^{-1}(T)$ and the one for normal electron Green functions is either ξ_0^{-1} or l^{-1} for clean and dirty limits, respectively.

The appropriate integral over momenta, giving response at $Q=0$, $\Omega=0$ appears to be divergent as earlier. However, according to Ref. 1 the account of the other diagrams, which are important at $Q=0$, $\Omega=0$, results in the total zero value of response in this case. So the subtraction procedure stated above is justified.

For small wave vectors $q^2 \ll \omega$, expression (4) gives

$$\varepsilon_{\rm tr}^f(\Omega,Q)=\varepsilon^f(\Omega)+i\frac{256\gamma^2m^2e^2T_c\xi^3(T)}{15\omega^5}q^2[1-\tfrac{5}{2}i\omega-(1-i\omega)^{5/2}-\tfrac{15}{8}\omega^2(1-i\omega)^{1/2}] , \tag{5}$$

where the fluctuation dielectric function, in which only frequency dependence is taken into account,

$$\varepsilon^f(\Omega)=\frac{32\gamma^2m^2e^2T_c\xi^3(T)}{\omega^2}\left[1+\frac{2i}{3\omega}[1-(1-i\omega)^{3/2}]\right] , \tag{6}$$

is in agreement with the results of Refs. 1 and 2.

For relatively large wave vectors $q^2 \gg \omega$, we have, from Eq. (4),

$$\varepsilon_{\rm tr}^f(\Omega,Q)=\frac{4\pi c^2Q^2}{\Omega^2}\chi^f(Q)+\frac{4\pi i}{\Omega}\sigma^f(Q) . \tag{7}$$

Here $\chi^f(Q)$ and $\sigma^f(Q)$ are the wave-vector-dependent fluctuation diamagnetic susceptibility and transverse conductivity of a normal metal near T_c. The expression for $\chi^f(Q)$ is

$$\chi^f(Q)=\chi_0 f(q)=\frac{3\chi_0}{2q}\left[\left[1+\frac{1}{q^2}\right]\arctan q-\frac{1}{q}\right] , \tag{8}$$

where $\chi_0=-e^2T_c\xi(T)/6\pi c^2$ is the well-known expression for the fluctuation diamagnetic susceptibility of a massive sample in a uniform field. The spatial dispersion of the static transverse fluctuation conductivity is given by

$$\sigma_{\rm tr}^f(Q)=\frac{2\sigma_{\rm AL}}{q}\left[\arctan q+\frac{1}{q}[1-(1+q^2)^{1/2}]\right] , \tag{9}$$

where $\sigma_{\rm AL}=e^2\gamma m T_c\xi(T)/\pi$ is the fluctuation Aslamazov-Larkin conductivity. It follows from Eq. (9) that the spatial dispersion of the fluctuation conductivity of metal becomes essential even for relatively small values of the wave vector $Q \sim \xi^{-1}(T) \ll \xi_0^{-1}$, whereas the wave-vector dependence of other terms in conductivity may be ignored under such conditions.

The expression for the longitudinal fluctuation dielectric function can be obtained in a way similar to that outlined above, after removing the restriction $\mathrm{div}\mathbf{E}=0$. Then we have

$$j_i(\Omega,\mathbf{Q})=-\frac{i\Omega}{4\pi}\left[\varepsilon_{\rm tr}^f(\Omega,Q)\left[\delta_{im}-\frac{Q_iQ_m}{Q^2}\right]+\varepsilon_l^f(\Omega,Q)\frac{Q_iQ_m}{Q^2}\right]E_m(\Omega,\mathbf{Q}) , \tag{10}$$

where the fluctuation component of the longitudinal dielectric function is given by

$$\mathbf{Q}\varepsilon_l^f(\Omega,Q)=\frac{64\pi i\gamma m e^2T_c}{\Omega}\int\frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}+\mathbf{Q}/2}{\xi^{-2}(T)+Q^2/4+(\mathbf{p}+\mathbf{Q}/2)^2-2im\gamma\Omega}\left[\frac{1}{\xi^{-2}(T)+p^2}-\frac{1}{\xi^{-2}(T)+(\mathbf{p}+\mathbf{Q})^2}\right] . \tag{11}$$

On carrying out the integration in Eq. (11) one obtains

$$\varepsilon_l^f(\Omega,Q)=i\frac{16\gamma^2m^2e^2T_c\xi^3(T)}{\omega q^2}\left\{-1+(1+q^2-i\omega)^{1/2}+\frac{i\omega}{2q}\left[\arctan\left[q-\frac{i\omega}{2q}\right]+\arctan\left[\frac{i\omega}{2q(1+q^2-i\omega)^{1/2}}\right]\right]\right\} . \tag{12}$$

For small wave vectors $q^2 \ll \omega$ expression (12) gives

$$\varepsilon_l^f(\Omega,Q)=\varepsilon^f(\Omega)+i\frac{256\gamma^2m^2e^2T_c\xi^3(T)}{5\omega^5}q^2[1-\tfrac{5}{2}i\omega-\tfrac{5}{4}\omega^2-(1-i\omega)^{5/2}-\tfrac{5}{8}\omega^2(1-i\omega)^{1/2}] , \tag{13}$$

where $\varepsilon^f(\Omega)$ is defined in Eq. (6).

For relatively large wave vectors $q^2 \gg \omega$ we have $\varepsilon_l^f(\Omega,Q)=4\pi i\sigma_l^f(Q)/\Omega$ and the longitudinal fluctuation conductivity $\sigma_l^f(Q)$ is as follows:

$$\sigma_l^f(Q)=\frac{2\sigma_{\rm AL}}{q^2}[(1+q^2)^{1/2}-1] . \tag{14}$$

Temperature dependences of $\varepsilon^f(\Omega)$, $\chi^f(Q)$, $\sigma_{\rm tr}^f(Q)$ and $\sigma_l^f(Q)$ for small ω or q are entirely governed by a factor $\xi(T)$, being proportional to $(T-T_c)^{-1/2}$ as follows from Eqs. (6), (8), (9), and (14). In approaching T_c, the values of ω and q may become large enough ($\gtrsim 1$). Then temperature dependences of $\varepsilon^f(\Omega)$, $\chi^f(Q)$, $\sigma_{\rm tr}^f(Q)$, and $\sigma_l^f(Q)$ are saturated. This results in the appearance of the corresponding plateau. Such behavior is quite common. For instance, the usual correlation function of an order parameter in the Landau theory of the second-order phase transition increases with temperature for small momenta and becomes independent of temperature with approaching to T_c due to momentum dependence.

Contrary to the Aslamazov-Larkin diagram, the Maki-Thompson diagram,[4,5] represented in Fig. 2, allows the parametrization with the external momentum passing only through the electron Green functions. The external momentum is completely excluded from the fluctuation propagator (as can be seen in Fig. 2). The qualitative difference in the behavior of the spatial dispersion for the Aslamazov-Larkin and Maki-Thompson terms stems from this fact. If $Q \ll \xi_0^{-1}$, the spatial dispersion of the Maki-Thompson correction $\sigma_{\rm MT}$ to the conductivity of metal can be neglected. In the dirty limit, for instance, the nonlocal correction to the Maki-Thompson term is of order $(Ql)^2$. This correction originates from the momentum dependence of electron Green functions. The impurity vertex does not result in dependence on the external momentum Q because it is associated only with the sum of the incoming momenta of electron propagators.

Straightforward calculations yield a frequency dispersion of this term resulting from the diffusive frequency dependence of the impurity vertex in the dirty limit:

$$\sigma_{\rm MT}(\Omega) =\frac{e^2}{4\pi\xi_0^2}\int_0^\infty \frac{p^2dp}{(p^2-i\Omega/D+1/\tau_\phi D)[p^2+\xi^{-2}(T)]} . \tag{15}$$

Here D is the diffusion constant, τ_ϕ is the phase relaxation time. Contributions to τ_ϕ may be of different physical origin.[11] This parameter may be extracted from experimental observations, e.g., magnetoresistance data.[12] Taking into account the relation $\xi_0^2=\pi D/8T_c$, we obtain, after integrating Eq. (15),

$$\sigma_{\rm MT}(\Omega)=\frac{1}{8}\frac{e^2}{\xi_0[t^{1/2}+(\delta-\pi i\Omega/8T_c)^{1/2}]} , \tag{16}$$

where $t=|T-T_c|/T_c$, $\delta=\pi/8T_c\tau_\phi$. In the limit of small frequencies we recover from Eq. (16) a familiar result for $\sigma_{\rm MT}$:

$$\sigma_{\rm MT}(\Omega)=\frac{1}{8}\frac{e^2}{\xi_0(t^{1/2}+\delta^{1/2})} . \tag{17}$$

If $\delta\sim 1$ that takes place for HTSC,[12] we may discard both the frequency and wave vector dependences of $\sigma_{\rm MT}$ under assumed conditions $\Omega \ll T_c$, $Q \ll \xi_0^{-1}$.

III. STATIC FLUCTUATION RESPONSE

A. Fluctuation diamagnetic response of a massive sample

The nonlocal kernel, relating fluctuation magnetization to magnetic field **H**, according to Eq. (8) exactly coincides with the phenomenological Pippard kernel[13] (see also Ref. 14), relating **j** to **A** in superconductors. In the coordinate space it is given by

$$\mathbf{M}(\mathbf{r})=\frac{3\chi_0}{2\pi\xi(T)}\int \frac{\mathbf{R}(\mathbf{R}\mathbf{H}(\mathbf{r}'))}{R^4}e^{-2R/\xi(T)}d\mathbf{r}' ,\qquad \mathbf{R}=\mathbf{r}-\mathbf{r}' . \tag{18}$$

Here $\mathbf{M}(\mathbf{r})=[\mathbf{B}(\mathbf{r})-\mathbf{H}]/4\pi$. It is essential that the characteristic radius of nonlocality in Eq. (18) is $\xi(T)$.

The external magnetic field **H**, parallel to the plane boundary of massive normal metal, induces magnetization $\mathbf{M}_0=\chi_0\mathbf{H}$ in its interior. In this problem the magnetic field **H** is completely uniform over the whole space. A nonuniform profile of magnetization $\mathbf{M}(\mathbf{r})=[\int\chi(\mathbf{r},\mathbf{r}')d\mathbf{r}']\mathbf{H}$ appears within the metal near the surface owing to the inhomogeneity of diamagnetic susceptibility. This inhomogeneity is, in turn, associated with the spatial dispersion of the fluctuation diamagnetic response (8). The profile of magnetization turns out to have characteristic length $\sim\xi(T)$. M is varying from the zero at the boundary to M_0 in the depth of the sample.

Under the specular reflection condition the half-space problem is known to reduce to the effective whole space problem, in which **A** is continued symmetrically across the boundary or, equivalently, **H** is continued antisymmetrically. Then, if the sample occupies half-space $x>0$, the magnetization is given by

$$M(x)=\frac{2}{\pi}H\int_0^\infty \frac{\sin(Qx)}{Q}\chi^f(Q)dQ . \tag{19}$$

On calculating this integral with an account of Eq. (8), we obtain

$$M(x)=M_0\left\{1-e^{-2x/\xi(T)}+\frac{x}{2\xi(T)}\left[\left[\frac{2x}{\xi(T)}-1\right]e^{-2x/\xi(T)}+2\left[\frac{2x^2}{\xi^2(T)}-3\right]Ei\left[-\frac{2x}{\xi(T)}\right]\right]\right\} . \tag{20}$$

Here $Ei(-x)=\int_{-\infty}^{-x} dt\, e^t/t$. For $x \ll \xi(T)$ we have

$$M(x)=\frac{3M_0 x}{\xi(T)} \ln\left[\frac{\xi(T)}{2.16x}\right]. \tag{21}$$

And in the region $x \gg \xi(T)$, Eq. (20) gives

$$M(x)=M_0\left[1-\frac{3\xi^2(T)}{4x^2}e^{-2x/\xi(T)}\right]. \tag{22}$$

It can be seen from Eqs. (20)–(22) that experimental investigation of magnetization near the surface on the scale of $\xi(T)$ can yield the data on the properties of nonlocal fluctuation response. The dependence of $M(x)/M(0)$ on the dimensionless coordinate $2x/\xi(T)$ is represented in Fig. 3.

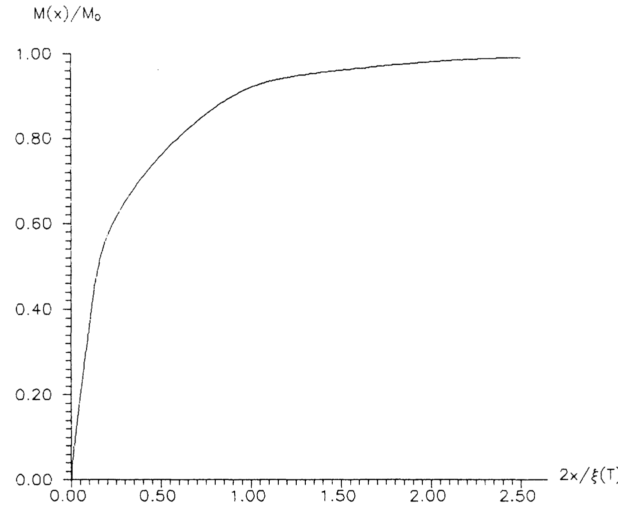

FIG. 3. Fluctuation magnetization $M(x)/M_0$ near the vacuum-metal boundary.

B. Static response in a slab geometry

In this subsection we will consider the fluctuation diamagnetic response to the static magnetic field in a slab geometry. Let the normal to the slab coincide with the x direction and the plane boundaries have coordinates $x=0$ and d. If $\mathbf{H}=(H\cos\alpha, H\sin\alpha, 0)$ we can choose a vector potential such as $A_z=H(y\cos\alpha - x\sin\alpha)$. Then the condition of the absence of the current normal to the surface takes the form $\nabla\Psi|_\perp=\mathbf{0}$. Substitute an expansion of Ψ over the set of functions $\cos(n\pi x/d)\exp(i\mathbf{k}\cdot\mathbf{r})$ into the Ginzburg-Landau effective Hamiltonian in the Gaussian approximation:

$$F=\int\left\{a|\Psi|^2+\frac{1}{4m}\left|\left[\nabla-\frac{2ie}{c}\mathbf{A}\right]\Psi\right|^2\right\}dV. \tag{23}$$

The parameter $n=0,1,2,\ldots$; $\mathbf{k},\mathbf{r}$ are two dimensional and correspond to y,z. Further calculations are analogous to those presented above in Sec. II if the integration over p_x is replaced by a summation over n. The components of current or magnetic field may also be parametrized by these numbers. In order to take into account that for $n=0$ and $k\to 0$, the kernel relating current to the vector potential has to vanish, the subtraction procedure, analogously stated above, must be used.

It is convenient to use magnetization $\mathbf{M}$ instead of the current $\mathbf{j}=c\,\mathrm{curl}\mathbf{M}$. For the averaged over x magnetization $\mathbf{M}$ we shall get $\mathbf{M}=\chi_\perp\mathbf{H}_\perp+\chi_\parallel\mathbf{H}_\parallel$, where indices $\perp$ and $\parallel$ denote components perpendicular and parallel to the slab.

On the integrating over the components p_y, p_z, susceptibility $\chi_\parallel$ is given by

$$\chi_\parallel=\frac{48}{\pi^3}\chi_0\tau^{1/2}\sum_{\nu=2k+1>0}\frac{1}{\nu^4}\sum_{n=-\infty}^{+\infty}\left[\frac{\tau+[(n+\nu)^2+n^2]/2}{(n+\nu)^2-n^2}\ln\frac{\tau+(n+\nu)^2}{\tau+n^2}-1\right], \tag{24}$$

where $\tau=[d/\pi\xi(T)]^2$ is the dimensionless temperature. If $\tau \gg 1$,

$$\chi_\parallel=\chi_0\left[1-\frac{3}{8\pi\tau^{1/2}}\right]. \tag{25}$$

Naturally, this result follows from Eq. (20), when the decrease of the average fluctuation response due to the distribution (20) is calculated. In the opposite limit $\tau \ll 1$,

$$\chi_\parallel=\frac{\pi}{2}\chi_0\tau^{1/2}\ln\left[\frac{0.28}{\tau}\right]. \tag{26}$$

As was noted in Ref. 6 on the basis of qualitative considerations, susceptibility $\chi_\parallel$ should not practically depend on temperature for $\tau \ll 1$. According to the quantitative result (26) this susceptibility reveals logarithmic temperature dependence.

The result for perpendicular susceptibility appears to be much simpler:

$$\chi_\perp=\frac{2}{\pi}\chi_0\tau^{1/2}\sum_{n=0}^{\infty}\frac{1}{\tau+n^2}. \tag{27}$$

For small τ it recovers the well-known expression $\chi_\perp=2\chi_0/\pi\tau^{1/2}$ for a thin film, while for large τ

$$\chi_\perp=\chi_0\left[1+\frac{1}{\pi\tau^{1/2}}\right]. \tag{28}$$

In Fig. 4 numerical results for $\chi_\perp$ and $\chi_\parallel$ measured in units of $\chi'=-Te^2d/6c^2$ are represented as functions of dimensionless temperature τ. For example, according to the calculations, the relation $\chi_\perp=2\chi_\parallel$ is fulfilled at $d\approx 1.7\xi(T)$.

C. Fluctuation corrections to the penetration depth

The spatial nonlocality of fluctuation response can also manifest itself below T_c. In the superconductor at $T<T_c$ the length scale of the nonlocal coupling of the main (nonfluctuation) part of the current with the field is known to be determined over the entire temperature range by the zero-temperature coherence length ξ_0. Near T_c we have $\xi_0 \ll \xi(T)$ and according to approximations, under which we can use the Ginzburg-Landau theory, we ignore the nonlocality with the length scale ξ_0. In the Gaussian approximation the fluctuation contribution to the screening current amounts to only a small correction to the main superconducting current. However, one can hope that this correction may be extracted from experiments.

Under the gauge condition div $\mathbf{A}=0$, the expression for the average current in the approximation linear in the field can be represented as $\mathbf{j}(\mathbf{Q})=-L(Q)\mathbf{A}(\mathbf{Q})$. The fluctuation correction to the value of $L(0)$ was found in Ref. 8.

In the equilibrium states the averaging of supercurrent $\mathbf{J}(\mathbf{Q})$ can be carried out with the help of a standard Ginzburg-Landau effective Hamiltonian. As compared to the case $T>T_c$ [see Eq. (23)], the quartic term $b|\Psi|^4/2$ does contribute to the Gaussian fluctuations here. Owing to this fact averaged quantities like $\langle\Psi\Psi\rangle$ and $\langle\Psi^*\Psi^*\rangle$ appear to be nonzero apart from common ones $\langle\Psi^*\Psi\rangle$ type. This procedure leads to the following expression for the average current in the linear approximation:

$$j_i(\mathbf{Q})=\frac{16e^2T_c}{c}\int\frac{d^3p}{(2\pi)^3}\left[\frac{(p_i+Q_i/2)(p_l+Q_l/2)}{(\mathbf{p}+\mathbf{Q})^2[p^2+2\xi^{-2}(T)]}-\frac{1}{4}\left[\frac{1}{p^2}+\frac{1}{p^2+2\xi^{-2}(T)}\right]\delta_{il}\right]A_l(\mathbf{Q})\ . \tag{29}$$

In integrating Eq. (29) over momenta we obtain linearly divergent at the upper limit terms. Regularization of the integral is associated exclusively with the limit $Q=0$ and is fulfilled analogously to Ref. 8. As a result the dependence of the fluctuation response on the wave vector in the Gaussian approximation is given by

$$L(Q)=\frac{c}{4\pi}\lambda_L^{-2}\left[1+\frac{1}{3}\left[\frac{Gi}{t}\right]^{1/2}\left[1+\frac{Q^2\xi^2(T)}{2}\right]f\left[\frac{Q\xi(T)}{\sqrt{2}}\right]\right]\ . \tag{30}$$

Here $\lambda_L=(mc^2b/8\pi e^2|a|)^{1/2}$ is the London penetration depth. The Ginzburg number is defined as $Gi=2T_cm^3b^2/\pi^2\alpha$. The function $f(x)$ was introduced in Eq. (8).

In the case $Q\xi(T)\ll 1$, which corresponds to large values of the Ginzburg-Landau parameter $\kappa\gg 1$, we find from Eq. (30)

$$L(Q)=\frac{c}{4\pi}\lambda_L^{-2}\left[1+\frac{1}{3}\left[\frac{Gi}{t}\right]^{1/2}[1+\tfrac{2}{5}Q^2\xi^2(T)]\right]\ . \tag{31}$$

It follows from here that the fluctuation correction to the penetration depth is

$$\lambda^{-2}=\lambda_L^{-2}\left[1+\frac{1}{3}\left[\frac{Gi}{t}\right]^{1/2}\left[1+\frac{2}{5\kappa^2}\right]\right],\quad \kappa\gg 1\ . \tag{32}$$

Expression (32) differs from the corresponding result in Ref. 8 in that it also incorporates the small correction ($\propto\kappa^{-2}$) due to the nonlocal nature of response.

If the condition $Q\xi(T)\gg 1$ is satisfied, that takes place for type-I superconductors with $\kappa\ll 1$, Eq. (30) leads to

$$L(Q)=\frac{c}{4\pi}\lambda_L^{-2}\left[1+\frac{\pi Q\xi(T)}{4\sqrt{2}}\left[\frac{Gi}{t}\right]^{1/2}\left[1-\frac{4\sqrt{2}}{\pi Q\xi(T)}\right]\right]\ . \tag{33}$$

For the penetration depth determined from the formula $\lambda=\int_0^\infty B(x)dx/H$, we find from Eq. (33) the following result for the case of specular reflection at the boundary:

$$\lambda^{-2}=\lambda_L^{-2}\left[1+\frac{1}{2\sqrt{2}\kappa}\left[\frac{Gi}{t}\right]^{1/2}[1-2\sqrt{2}\kappa]\right],\quad \kappa\ll 1\ . \tag{34}$$

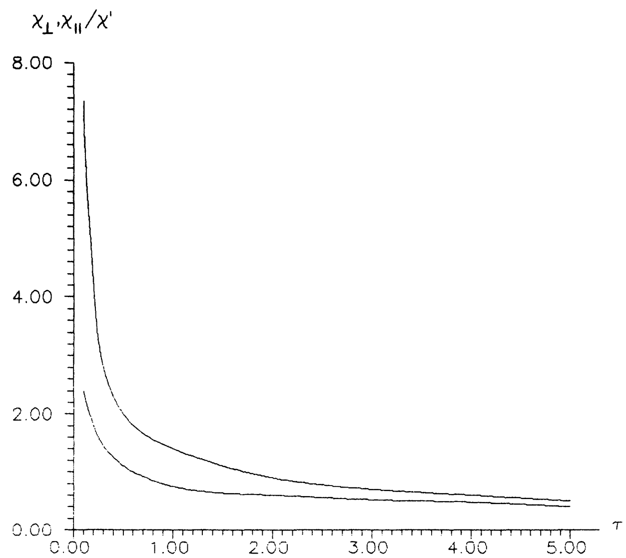

FIG. 4. Fluctuation diamagnetic susceptibility χ/χ' of a slab as a function of dimensionless temperature $\tau=[d/\pi\xi(T)]^2$. The upper curve corresponds to the perpendicular susceptibility $\chi_\perp$, the lower curve corresponds to $\chi_\parallel$.

Here, contrary to Eq. (32), the nonlocal nature of the diamagnetic response is important for the first fluctuation correction to the penetration depth.

The range of the applicability of the results derived in this section is determined by the applicability of the Ginzburg-Landau theory, the assumption that the fluctuations in the order parameter are small, and the validity of ignoring the fluctuations of the magnetic field. These conditions impose the requirements

$$Q \ll \xi_0^{-1}, \quad \max\{Gi, \kappa^{-6}Gi\} \ll t \ll 1 \ . \tag{35}$$

The condition $t \gg Gi/\kappa^6$ is important for a type-I superconductor with $\kappa < 1$. Because of this condition, in describing small fluctuations in the order parameter one can ignore the circumstance that the fluctuations in the magnetic field make the superconducting transition a first-order transition.[15] In particular, we find $(Gi/t)^{1/2}\kappa^{-1} \ll \kappa^2$ from this condition. Consequently, the second term in Eq. (34) is always small in comparison with the first, as it should be.

IV. FLUCTUATION CONTRIBUTION TO THE MAGNETIC NEUTRON SCATTERING ABOVE T_c

Specific dependences of dielectric function on the temperature, wave vector, and frequency may be revealed in the differential magnetic neutron-scattering cross section in normal metal near T_c. Behavior of this type was observed recently in experiments on HTSC polycrystal samples.[9] In the absence of the magnetic ordering, the magnetic neutron scattering is caused by their interaction with an equilibrium fluctuating magnetic field. The corresponding differential scattering cross section of an unpolarized beam into the solid-angle region dO and the interval of transferred energy $d\Omega$ with energy increase Ω for the massive homogeneous isotropic sample is as follows:

$$\frac{d^2S}{d\Omega dO} = \frac{g^2e^2V}{32\pi^3c^2}\frac{p'}{p}\langle B^2\rangle_{\Omega,Q} \ . \tag{36}$$

Here $\mathbf{Q}$ is the transferred momentum, $\mathbf{p}=m_N\mathbf{v}_N$, and $\mathbf{p}'=\mathbf{p}+\mathbf{Q}$ are the initial and final neutron momenta, $g=1.91$ is the neutron gyroscopic factor. According to the fluctuation-dissipation theorem applied to the electromagnetic field, the spectral component of the bilinear correlator of the magnetic field is described by the expression

$$\langle B_iB_j\rangle_{\Omega,Q} = \frac{8\pi}{1-e^{-\Omega/T}}\left[\delta_{ij}-\frac{Q_iQ_j}{Q^2}\right] \times \operatorname{Im}\frac{1}{1-(\Omega^2/c^2Q^2)\varepsilon_{tr}(\Omega,Q)} \ . \tag{37}$$

Substitution of Eq. (37) into Eq. (36) under the condition $\Omega \ll T_c$ leads to

$$\frac{d^2S}{d\Omega\, dO} = \frac{g^2e^2VT_c}{2\pi^2c^2}\frac{p'}{p}\frac{1}{\Omega}\operatorname{Im}\frac{1}{1-(\Omega^2/c^2Q^2)\varepsilon_{tr}(\Omega,Q)} \ . \tag{38}$$

In Ref. 9 the total scattering cross section dS/dO in a given direction was measured for low-energy neutrons and small scattering angles near the forward direction. In order to calculate this quantity, one has to integrate Eq. (38) over energy Ω at a fixed scattering angle θ, or equivalently at fixed transferred momentum $K=2p\sin(\theta/2)$, corresponding to quasielastic scattering at angle θ. In integrating Eq. (38) we have $\Omega/v_N \sim K \ll p$ for the small scattering angles. Thus, we can set approximately $\mathbf{Q}=\mathbf{K}+\mathbf{n}'\Omega/v_N$, $Q^2=K^2+\Omega^2/v_N^2$, where $\mathbf{n}'=\mathbf{p}'/p'$. In the discussed region of frequencies and wave vectors we can represent the dielectric function in the form $\varepsilon_{tr}(\Omega,Q)=4\pi i\sigma/\Omega+\varepsilon_{tr}^f(\Omega,Q)$, where σ is the nonfluctuation part of the static conductivity of normal metal and $\varepsilon_{tr}^f(\Omega,Q)$ is defined in Eq. (4). The spin contribution to $\varepsilon_{tr}(\Omega,Q)$ in the given region of Ω and Q is assumed to have no specific temperature dependence near T_c and therefore it is ignored.

Estimates based upon the experimental conditions of Ref. 9 show that the inequality $c^2Q^2 \gg \Omega^2\varepsilon_{tr}(\Omega,Q)$ holds with sufficient accuracy. Taking into account this inequality we get explicit analytic expressions for the contributions of superconducting fluctuations in the cross section dS^f/dO in the angle intervals $K \ll m\gamma v_N/\hbar^2$ and $K \gg m\gamma v_N/\hbar^2$:

$$\frac{dS^f}{dO} = \frac{16g^2e^4VT_c^2}{\pi c^4\hbar^8}(m\gamma v_N)^2\xi^3(T)\left[\frac{2}{3\tilde{k}^3}[(1+\tilde{k})^{3/2}-1]-\frac{1}{\tilde{k}^2}\right], \quad K \ll m\gamma v_N/\hbar^2 \ , \tag{39}$$

$$\frac{dS^f}{dO} = \frac{g^2e^4VT_c^2\gamma v_N}{2\pi c^4\hbar^4 a}\left[\frac{1}{k^2}\left[1+\frac{k}{2}-(1+k^2)^{1/2}\right]+\frac{2\chi^f(K)}{3\pi\chi_0}\right], \quad K \gg m\gamma v_N/\hbar^2 \ . \tag{40}$$

Dimensionless units $k=K\xi(T)/2$, $\tilde{k}=2m\gamma v_N\xi^2(T)K/\hbar^2$ have been introduced here and the Planck constant is no longer set to be a unit up to the end of this section. Conditions $K \ll m\gamma v_N/\hbar^2$ and $K \gg m\gamma v_N/\hbar^2$ correspond to the limiting cases $q^2 \ll \omega$ and $q^2 \gg \omega$ (if $\Omega \sim Kv_N$). Thus, the dependence of the cross section dS^f/dO on momentum $\hbar K$ in the interval of angles $K \ll m\gamma v_N/\hbar^2$ is associated with the dependence of the dielectric function (6) on the frequency Ω. In this case the influence of spatial dispersion may be neglected. At the same time the dependence of dS^f/dO on momentum $\hbar K$ in the angle interval $K \gg m\gamma v_N/\hbar^2$ [see Eq. (40)] is directly connected with the dependence of fluctuation conductivity (9) on the wave vector. Hence the account of spatial dispersion becomes important for $K \gtrsim m\gamma v_N/\hbar^2$, $\xi^{-1}(T)$. According to the estimate, wave vectors of such an order of magnitude were present in measurements.[9]

The fact that a number of terms on the right-hand side of Eq. (40) combine into $\chi^f(K)$ does not seem to be occasional. Indeed, according to the sum rule that stems from the Leontovich dispersion relationships,[16] we have

$$\mu(K)=1+4\pi\chi(K)=\frac{1}{\pi}\mathrm{vp}\int_{-\infty}^{+\infty}\frac{d\Omega}{\Omega}\,\mathrm{Im}\frac{1}{1-\{\Omega^2/c^2[\mathbf{K}+\mathbf{u}(\Omega/c)]^2\}\varepsilon_{\mathrm{tr}}[\Omega,\mathbf{K}+\mathbf{u}(\Omega/c)]}\ . \tag{41}$$

Here the inequality $u^2\le 1$ must be satisfied and relations

$$\lim_{\Omega\to 0}\frac{1}{1-(\Omega^2/c^2K^2)\varepsilon_{\mathrm{tr}}(\Omega,K)}=\mu(K)\ ,\qquad \lim_{\Omega\to 0}\Omega^2\varepsilon_l(\Omega,K)=0 \tag{42}$$

are supposed to be valid. The integral with which we deal in the magnetic scattering problem is very close to Eq. (41), but for the magnitude of appropriate parameter u. Instead of the wave-vector argument $\mathbf{K}+\mathbf{u}\Omega/c$ as in Eq. (41), the quantity $\mathbf{K}+\mathbf{n}'\Omega/v_N$ appears. So the effective value of parameter u^2 is $c^2/v_N^2>1$ and the sum rule (41) cannot be applied in our case. The presence of $\chi^f(K)$ in Eq. (40) may be regarded as a reminiscence of the broken sum rule.

Under the condition $K\ll m\gamma v_N/\hbar^2$ any of the inequalities $\tilde{k}\ll 1$ and $\tilde{k}\gg 1$ may, generally speaking, hold. Analogously, the inequality $K\gg m\gamma v_N/\hbar^2$ does not exclude both limits $k\ll 1$ and $k\gg 1$. In the limiting cases $\tilde{k}\ll 1$ and $k\ll 1$, expressions (39) and (40) coincide, as it must be:

$$\frac{dS^f}{dO}=\frac{g^2e^4VT_c^2\gamma v_N m^{1/2}}{\pi c^4\hbar^5[\alpha(T-T_c)]^{1/2}K}=\frac{2g^2e^2VT_c\sigma_{\mathrm{AL}}v_N}{c^4\hbar^2K}\ . \tag{43}$$

In this situation the scattering cross section increases as $(T-T_c)^{-1/2}$ with temperature approaching T_c and is proportional to $1/K$. If $\tilde{k}\gg 1$ in Eq. (39) or $k\gg 1$ in Eq. (40), the cross section no longer increases with temperature approaching T_c:

$$\frac{dS^f}{dO}=\frac{8g^2e^4VT_c^2[2m\gamma v_N]^{1/2}}{3\pi c^4\hbar^5K^{3/2}}\ ,\qquad \frac{\hbar^2}{2m\gamma v_N\xi^2(T)}\ll K\ll m\gamma v_N/\hbar^2 \tag{44}$$

$$\frac{dS^f}{dO}=\frac{16}{\pi^2}\left[\frac{\pi}{2}-1\right]\frac{g^2e^4VT_c^2m\gamma v_N}{c^4\hbar^6K^2}\ ,\qquad K\gg m\gamma v_N/\hbar^2,\ \frac{1}{\xi(T)}\ . \tag{45}$$

In fact, any term $\Delta\sigma$ in the conductivity of metal leads to expression (43) for scattering angles, where both spatial and frequency dispersion of $\Delta\sigma$ can be disregarded [under the condition $c^2K^2\gg\Omega^2\varepsilon_{\mathrm{tr}}(\Omega,K)$ all the contributions to the cross section from different terms in $\varepsilon_{\mathrm{tr}}(\Omega,K)$ are additive to a first approximation].

The contribution of the Maki-Thompson term (16) is obtained in an analogous way on integrating Eq. (38):

$$\frac{dS^f_{\mathrm{MT}}}{dO}=\frac{g^2e^4VT_cv_N}{4c^4\hbar^3\xi_0K\{t^{1/2}+[\delta+(\pi Kv_N/8T_c)]^{1/2}\}}\ . \tag{46}$$

Due to the short phase relaxation time in the HTSC, it is possible to ignore the term $\pi Kv_N/8T_c$ as compared to δ in Eq. (46). It corresponds to the possibility of ignoring the frequency dispersion of the Maki-Thompson term in Eq. (16). Then we have

$$\frac{dS^f_{\mathrm{MT}}}{dO}=\frac{2g^2e^2VT_c\sigma_{\mathrm{MT}}v_N}{c^4\hbar^2K} \tag{47}$$

in agreement with Eq. (43). This term can be disregarded because the ratio of the contribution (47) to the cross section given by Eq. (45) is characterized by a small parameter $K\xi_0$.

Since the superconducting fluctuations are supposed to be Gaussian, the plateau height is correctly described by Eqs. (44) and (45) for $T>T_{Gi}$, where T_{Gi} is the boundary between the regions of Gaussian and strong fluctuations. In particular, the validity of Eq. (45) demands $[K\xi(T_{Gi})]^2\gg 1$. Under the experimental conditions of Ref. 9, the quantity $K\xi(T_{Gi})$ is likely to be equal to several units, hence we can use Eq. (45) at least for qualitative estimates.

Let us consider the cross section dS^f/dO as a function of temperature at a given value of the transferred momentum, satisfying the inequalities $K\xi_0\ll 1$ and $\hbar K\ll p$. With decreasing of temperature and approaching T_c, the value dS^f/dO primarily increases according to Eq. (43). Then, due to the growth of $\xi(T)$, one of the inequalities (44) or (45) becomes valid, so the plateau appears in the graph of dS^f/dO. Such a behavior was observed in Ref. 9.

According to our results, (44) and (45), the plateau height decreases with the increase of the transferred momentum. It is proportional to $K^{-3/2}$ if the effects of frequency dispersion in contrast to spatial nonlocality are important ($K\ll m\gamma v_N/\hbar^2$). It is also proportional to K^{-2} if the account of the spatial dispersion is indispensable and frequency dispersion can be ignored ($K\gg m\gamma v_N/\hbar^2$). The numerical calculation gives the dependence of the plateau height for intermediate values of transferred momenta. Let us represent $dS^f/dO=f_1(K)f_2(K)$, where $f_1(K)$ is given by Eq. (45). The graph of $f_2(K)$ is shown in Fig. 5.

One can see that experimental data[9] of the plateau height is better described by K^{-2} dependence than by $K^{-3/2}$. This circumstance together with other abovementioned estimates leads us to the conclusion, that the account of spatial dispersion is needed for the analysis of the results of measurements.[9]

The height of the plateau is proportional to $T_c^{3/2}/\xi_0$ for the scattering angles (44) and to T_c/ξ_0^2 for the angles (45) [here we set $\xi_0\sim\hbar/(m\alpha T_c)^{1/2}$ and $\gamma\propto\alpha$]. Hence, for the HTSC the plateau height, i.e., the characteristic value of the contribution of superconducting fluctuations, appears to be 10^3-10^5 times greater than for ordinary low-temperature superconductors (with sufficiently large coherence length).

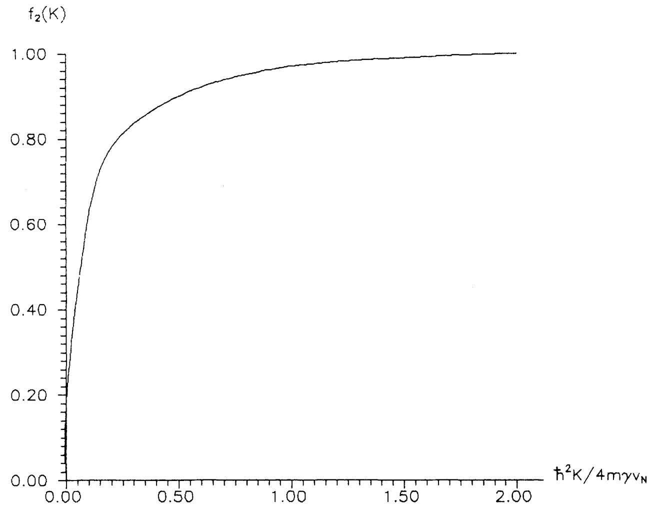

FIG. 5. Function $f_2(K)$, giving the plateau height of the magnetic neutron-scattering cross section $dS^f/dO = f_1(K)f_2(K)$, where $f_1(K)$ is defined in Eq. (45).

We have assumed above that the sample is isotropic and homogeneous. Naturally, for the quantitative analysis of the effects in the HTSC, the generalization of the results for the case of anisotropic superconductors and, probably, the polycrystal structure of the samples is needed.

V. FLUCTUATION CONTRIBUTION TO THE REFLECTIVITY OF PLANE METAL SURFACE

Let us finally return to the frequency dependence of fluctuation conductivity $\sigma^f(\Omega)=\Omega\varepsilon^f(\Omega)/4\pi i$, where $\varepsilon^f(\Omega)$ is defined in Eq. (6). This quantity contributes, for example, to the surface impedance of plane metal boundary under normal skin effect conditions:

$$Z(\Omega)=(1-i)\left[\frac{\Omega}{8\pi[\sigma+\sigma^f(\Omega)]}\right]^{1/2}. \tag{48}$$

The HTSC dimensionless frequency $\omega=2m\gamma\xi^2(T)\Omega$ that governs the frequency dependence of $\sigma^f(\Omega)$ may be much greater than a unit, in particular, for the millimeter wave region near T_c, whereas $\Omega \ll T_c$. So the high-frequency limit

$$\sigma^f(\Omega)=4(1+i)e^2T_c(m\gamma)^{1/2}/3\pi\Omega^{1/2}$$

must be used for sufficiently low-frequency waves. While this contribution may be noticeable, corrections due to nonlocality turn out to be small here and are characterized by a small parameter $\sigma/m\gamma c^2$.

VI. CONCLUSION

A fluctuation contribution to the dielectric function with an account of spatial and temporal dispersion has been obtained near T_c. Both the Aslamazov-Larkin, (4) and (12) and Maki-Thompson (16) terms were considered for wave vectors of the external field $Q \ll \xi_0^{-1}$ and frequencies $\Omega \ll T_c$. The possibility of ignoring the spatial dispersion in the Maki-Thompson term for $Q \ll \xi_0^{-1}$ was substantiated. The Aslamazov-Larkin term has the radius of nonlocality $\sim\xi^{-1}(T)$. The nonlocality of fluctuation diamagnetism in a homogeneous massive sample of the normal metal was examined. It governs the profile of fluctuation magnetization near the metal-vacuum boundary (20). The static diamagnetic response in a slab geometry was also calculated, Eqs. (24) and (27). Nonlocal fluctuation corrections to the penetration depth below T_c were found in Sec. III C. The wave-vector dependence of the response essentially modifies the behavior of the fluctuation contribution to magnetic neutron scattering near T_c (45). The frequency dispersion of fluctuation conductivity appears to be very important for the surface impedance of the normal phase of the HTSC, in particular, for the far infrared and millimeter electromagnetic waves. Results obtained here concern homogeneous isotropic metals near T_c. Consideration of the influence of anisotropy is in process.

Note added in proof. Recently, an article appeared by L. I. Glazman, F. W. J. Hekking, and A. Zyuzin [Phys. Rev. B **46**, 9074 (1992)], where similar general aspects of the spatial dispersion of the Aslamazov-Larkin term and the Maki-Thompson term were observed for a different particular problem of fluctuation conductivity of a hollow cylinder with a magnetic flux.

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[1]L. G. Aslamazov and A. I. Larkin, Fiz. Tverd. Tela (Leningrad) **10**, 1104 (1968).

[2]H. Schmidt, Z. Phys. **216**, 336 (1968).

[3]A. Schmid, Phys. Rev. **180**, 527 (1969).

[4]K. Maki, Prog. Theor. Phys. **39**, 897 (1968).

[5]R. S. Thompson, Phys. Rev. B **1**, 327 (1970).

[6]W. J. Skockpol and M. Tinkham, Rep. Prog. Phys. **38**, 1049 (1975).

[7]L. N. Bulaevskii, V. L. Ginzburg, and A. A. Sobyanin, Zh. Eksp. Teor. Fiz. **94**, 355 (1988).

[8]A. Buzdin and B. Vuyichit, Mod. Phys. Lett. B **4**, 485 (1990).

[9]N. R. Bernhoeft, P. J. Allen, D. McK. Paul, S. M. Hayden, P. A. Timmins, and G. G. Lonzarich, Nature **350**, 690 (1991).

[10]M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York 1975).

[11]W. Brenig, M. Chang, E. Abrahams, and P. Wölfle, Phys. Rev. B **31**, 7001 (1985).

[12]J. B. Bieri, K. Maki, and R. S. Thompson, Phys. Rev. B **44**, 4709 (1991).

[13]A. B. Pippard, Proc. R. Soc. London Ser. A **216**, 547 (1953).

[14]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[15]B. I. Halperin, T. C. Lubensky, and S. K. Ma, Phys. Rev. Lett. **32**, 292 (1974).

[16]M. A. Leontovich, Zh. Eksp. Teor. Fiz. **40**, 907 (1961).