Magnetostriction of a granular superconductor

S. Sergeenkov* and M. Ausloos
S.U.P.R.A.S., B5,
Institute of Physics, University of Liège, B-4000 Liège, Belgium
(Received 22 July 1992; revised manuscript received 21 December 1992)

An exactly solvable model for the magnetostriction (MS) of a granular superconductor is presented. Both the "bulk-modulus-driven magnetostriction" (the change of the sample's volume in the magnetic free energy upon the applied stress) and the "change-of-phase magnetostriction" (due to the stress dependence of the weak-links-induced magnetization) are considered. It is found that the former contribution to MS dominates at small magnetic fields (in a Meissner phase of granular superconductor) while the latter one prevails at higher fields when the motion of Josephson vortices over grain boundaries is established. A useful link between MS and grain-boundary-pinning ability is obtained within the model. From magnetostriction measurements, this allows one to get rather unique information on the temperature and magnetic-field dependence of the weak-link pinning-force density and its field derivative. Analogy with the behavior of MS in the mixed state of type-II superconductors is stressed, and comparison of the model predictions with some experimental data on high-T_c ceramics is discussed.

As compared to the magnetoelastic behavior of low-T_c conventional superconductors (see the comprehensive review paper by Brändli[1]), that of high-T_c superconducting (HTS) ceramics is found to be more complicated due both to their perplexing vortex-lattice structure and to the important role of numerous defects in these materials.[2–6] Furthermore, the vast majority of experimental results on HTS has been obtained on ceramics samples, i.e., on granular superconductors. On the other hand, to our knowledge, the problem of weak-links-induced magnetostriction (MS) in granular HTS (and even in conventional superconductors) remains unsolved. In the present paper, we consider a rather simple model of MS for a granular superconductor which admits an exact treatment, and leads to quite reasonable conclusions as to the role of Josephson weak links in magnetoelastic properties of HTS ceramics.

The model is based on the well-known Hamiltonian[7] of a granular superconductor which in the so-called pseudospin representation[8] has the form

$$\mathcal{H}=-\frac{1}{2}\sum_{ij} J_{ij}S_i^+S_j^- +\mathrm{H.c.}\ , \tag{1}$$

where

$$J_{ij}(T,H)=J(T)\exp[iA_{ij}(H)]\ ,\quad S_i^+=\exp(+i\Phi_i)\ ,$$
$$A_{ij}(H)=\frac{\pi}{\Phi_0}(\mathbf{H}\times\mathbf{R}_{ij})\cdot\mathbf{r}_{ij}\ ,\quad \mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j\ , \tag{2}$$
$$\mathbf{R}_{ij}=(\mathbf{r}_i+\mathbf{r}_j)/2\ .$$

The above model describes the interaction between superconducting grains [with phases $\Phi_1(t)$] arranged in a three-dimensional lattice with coordinates $\mathbf{r}_i$ via Josephson (or proximity) coupling with energy $J(T)$. The system is under the influence of a frustrating applied magnetic field $\mathbf{H}$. For the sake of simplicity, we consider only a strain component U parallel to the c axis and to the magnetic field (thus we follow the usual geometry in HTS measurements of MS, see, e.g., Ref. 2).

As is well known,[9] the change of the free energy of a superconductor in the presence of an external magnetic field H is

$$\Delta G(H)=G(0)-G(H)=V\int_0^{H_i}dH\,M(T,H)\ , \tag{3}$$

where $M(T,H)$ is the magnetization of the sample, V its volume, and the internal field H_i is related to the applied field H via the demagnetization coefficient D, namely $H_i=H/(1-D)$. Recall[9] that $D=0$ for an infinite cylinder with its axis parallel to H, while for a sphere $D=1/3$. When the superconductor is under the influence of an external (homogeneous) stress σ, the magnetic energy (3) results in the associated strain component[1]

$$U=\frac{1}{V}\left[\frac{\partial\Delta G}{\partial\sigma}\right]\ . \tag{4}$$

Neglecting a possible slight change of the demagnetization coefficient D with the stress, Eqs. (3) and (4) give rise to the following two main contributions to the magnetostrictive strains, namely,[1] (a) the "bulk-modulus-driven MS" due to the change in the magnetic free energy arising from the stress dependence of sample volume:

$$U_{\mathrm{BMD}}=\left[\frac{1}{V}\frac{\partial\Delta G}{\partial\sigma}\right]_M=\left[\frac{1}{V}\frac{\partial V}{\partial\sigma}\right]\int_0^{H_i}dH\,M(T,H)\ ; \tag{5}$$

(b) the "change-of-phase MS" due to the stress dependence of the magnetization via the Josephson junction effective surface (see below):

$$U_{PH}=\left[\frac{1}{V}\frac{\partial\Delta G}{\partial\sigma}\right]_V=\int_0^{H_i}dH\frac{\partial M}{\partial\sigma}\ . \tag{6}$$

0163-1829/93/48(1)/604(3)/$06.00

As it was found experimentally[2-6] in HTS, the main contribution to MS at high fields arises from the irreversible motion of Abrikosov vortices; that is, the phase MS dominates in the mixed state of type-II superconductors. As we shall see below, a closely similar situation is realized on essentially a smaller field region in ceramics where the irreversible motion of the Josephson vortices dominates.

Let us recall that the magnetization with the Josephson junction array model [Eq. (1)] is defined via Josephson currents-induced diamagnetic moment $\boldsymbol{\mu}$,[7,8]

$$M(T,H)\equiv\overline{\langle\mu_z\rangle}/V\ , \tag{7}$$

where

$$\boldsymbol{\mu}=-\frac{\partial\mathcal{H}}{\partial\mathbf{H}}\ . \tag{8}$$

The bar over $\langle\mu\rangle$ in Eq. (7) denotes the configurational averaging over the randomly distributed grain coordinates $\mathbf{r}_i$, namely for any configuration-dependent quantity B_{ij},

$$\overline{B_{ij}}=\int_{-\infty}^{\infty}d\mathbf{r}_i d\mathbf{r}_j P(\mathbf{r}_i,\mathbf{r}_j)B_{ij}(\mathbf{r}_i,\mathbf{r}_j)\ . \tag{9}$$

Since according to Eq. (2),

$$\frac{\partial J_{ij}}{\partial\mathbf{H}}=-\frac{i\pi}{\Phi_0}(\mathbf{r}_{ij}\times\mathbf{R}_{ij})J_{ij}\ , \tag{10}$$

the diamagnetic moment [Eq. (8)] reads

$$\boldsymbol{\mu}=-\frac{i\pi}{2\Phi_0}\sum_{ij}J_{ij}S_i^+S_j^-(\mathbf{r}_{ij}\times\mathbf{R}_{ij})+\text{H.c.} \tag{11}$$

Thus, as it follows from Eqs. (2) and (7)–(11), the functional form of the magnetization $M(T,H)$ within the model essentially depends on the form of the distribution function $P(r_i,r_j)$. Assuming for simplicity, a site-type disorder allowing weak displacements of the grain sites at their positions of the original simple cubic lattice, i.e., within a radius $r_g=\sqrt{S/\pi}$, the new position is chosen randomly according to the normalized separable Gauss-like distribution function $P(r_i,r_j)=P(r_i)P(r_j)$, where

$$P(\mathbf{r}_i)=\frac{1}{\sqrt{2\pi S}}\exp\left[-\frac{\mathbf{r}_i^2}{2S}\right]\ , \tag{12}$$

we get from Eqs. (2), (9) and (12) for the averaged Josephson energy,

$$\overline{J_{ij}(T,H)}=\frac{J(T)}{\sqrt{1+H^2/H_0^2}}\ . \tag{13}$$

Finally, in view of Eqs. (2) and (7)–(13), the equilibrium magnetization of a granular superconductor reads (see Ref. 8 for more details)

$$M(T,H)=-H_0\chi(T)f(H/H_0)\ , \tag{14}$$

where

$$f(x)=\frac{x}{(1+x^2)^{3/2}}\ ,\quad \chi(T)=\frac{4J(T)SN}{r_g\Phi_0^2}\ ,\quad H_0=\frac{\Phi_0}{2S}\ , \tag{15}$$

where N is the number of grains. It is worthwhile to mention that as in the case of homogeneous type-II superconductors,[1] the equilibrium magnetization of a granular superconductor also has the scaling-law form as a function of the magnetic field (with H_0 here corresponding to the thermodynamic critical field H_c). According to Eqs. (5), (14), and (15) we get for the "bulk-modulus-drive MS",

$$U_{\mathrm{BMD}}(H_i,T)=-U_0(T)f_{\mathrm{BMD}}(H_i/H_0)\ , \tag{16}$$

where

$$f_{\mathrm{BMD}}(x)=1-\frac{1}{\sqrt{1+x^2}}\ ,$$
$$U_0(T)\equiv U_{\mathrm{BMD}}(\infty,T)=\kappa\chi(T)H_0^2,\quad \kappa\equiv-\frac{\partial\ln V}{\partial\sigma}\ . \tag{17}$$

Stankowski *et al.*[10] have studied experimentally the effect of pressure on the grain-boundary weak links of HTS ceramics using the microwave absorption method. In particular, they measured the Josephson current losses in granular superconductors and observed the shift of the characteristic Josephson field H_0 with the pressure. From the H_0 shift they extracted the change of the average (for their sample) effective surface of internal (grain boundary) Josephson junctions $S(\sigma)$ [recall that $H_0=\Phi_0/2S$, cf. Eq. (15)]. Namely, they found that the effective surface is decreasing with the applied stress, i.e., $\partial S/\partial\sigma<0$, leading to an essential improvement of grain-boundary weak links due to mechanical treatment. Thus, taking into account the following chain of the evident relations,

$$\frac{\partial M}{\partial\sigma}=\frac{\partial M}{\partial H_0}\frac{\partial H_0}{\partial\sigma}\ ,\quad \frac{\partial H_0}{\partial\sigma}=\frac{\partial H_0}{\partial S}\frac{\partial S}{\partial\sigma}\ ,$$

and assuming that the sample volume V and the projected area S are related in a usual way, $S\approx V^{2/3}$, we obtain from Eqs. (6) and (14)–(17) for the "change-of-phase magnetostriction,"

$$U_{PH}(H_i,T)=(4/3)U_0(T)f_{PH}(H_i/H_0)\ , \tag{18}$$

where

$$f_{PH}(x)=f_{\mathrm{BMD}}(x)-\left[\frac{x}{2}\right]\frac{\partial f_{\mathrm{BMD}}}{\partial x}\ . \tag{19}$$

Figure 1 shows the temperature dependence [which is via the Josephson energy $J(T)$] of the phase (dashed line), bulk-modulus (dotted line), and total, $U_{\mathrm{BMD}}+U_{PH}$ (solid line) reduced magnetostriction $U(H_i,T)/U_0(0)$ for a reduced magnetic field $H_i/H_0=0.5$. Such behavior qualitatively correlates with the experimental observations.[2-6] More interestingly, however, is the magnetic-field dependence of these two contributions to MS. Figure 2 presents the behavior of phase (dashed line), bulk-modulus (dotted line), and again total (solid line) reduced magnetostriction $U(H_i,T)/U_0(0)$ versus magnetic field (in reduced units H_i/H_0) for a reduced temperature $T/T_c=0.5$. As seen in Fig. 2, up to fields $H_i/H_0\approx2.5$,

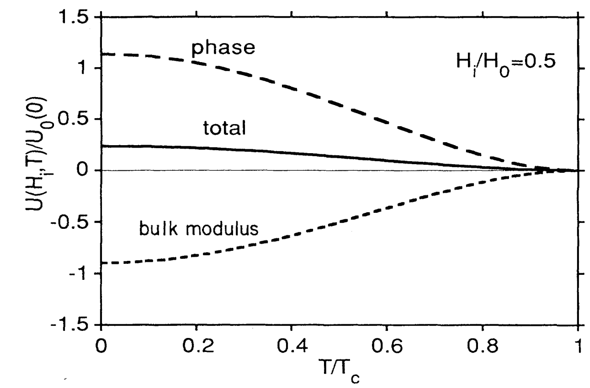

FIG. 1. Phase (dashed line), bulk-modulus (dotted line), and total (solid line) reduced magnetostriction $U(H_i,T)/U_0(0)$ vs reduced temperature T/T_c at the reduced magnetic field $H_i/H_0=0.5$.

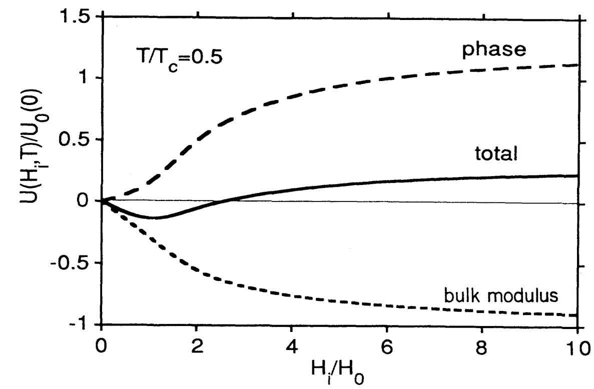

FIG. 2. Phase (dashed line), bulk-modulus (dotted line), and total (solid line) reduced magetostriction $U(H_i,T)/U_0(0)$ vs reduced magnetic field H_i/H_0 at reduced temperature $T/T_c=0.5$.

the MS of a granular superconductor is dominated by its bulk-modulus part [Eq. (16)]. Moreover, for small fields (below $H_i/H_0 \approx 1$) the total MS resembles the linear (Meissner-like) decrease of the magnetization, while the rising part of the curve (above $H_i/H_0 \approx 1$) is due to the Josephson vortex motion between grains. Thus the field $H_i/H_0 \approx 2.5$, where the total MS disappears is nothing but a decoupling field for the Josephson weak-link network. Above this field, magnetic flux moves freely through the percolative paths of the ceramics, and MS increases due to the increase of its phase part. Such a picture remarkably reproduces a similar behavior of MS expected to be realized in the mixed state of homogeneous type-II superconductors where the main contribution to the magnetostriction properties comes from the vortex motion.

Turning to the comparison of the model predictions with the experimental data, it is worth noting that a dip-like structure of MS, as seen in Fig. 2, has been observed[3] for longitudinal MS in Y-Ba-Cu-O ceramics at small fields ($B \approx 0.05T$) for $T/T_c \approx 0.5$. Using the typical values for ceramics parameters[10] such as the projected area $S \approx 1\ \mu\text{m}^2$, susceptibility $\chi \approx 1/4\pi$, critical field $H_0=\Phi_0/2S \approx 50$ G and compressibility $\kappa=-\frac{1}{2}\partial \ln S/\partial\sigma \approx -0.9\ (\text{G Pa})^{-1}$, for an applied field $B \approx 0.05$ T we get, in view of Eqs. (14)–(19), for a rough estimate of this dip, $U(H_i,T) \approx -10^{-7}$ which is very close to the value experimentally found in Ref. 3.

Another important result found in Ref. 3 concerns the hysteretic behavior of the MS resulting from the difference between field-cooled and zero-field-cooled magnetization regimes. It is well known that hysteresis in type-II superconductors is strictly related to the pinning-force densities of the material, $F_p(T,H)=J_c(H,T)H$, where $J_c(H,T)$ is the critical-current density. On the other hand, this hysteresis can be described within the model under consideration as the difference between dc and ac susceptibilities.[7,8] Furthermore, in view of Eqs. (7)–(14), we find that bulk-modulus-driven MS, Eqs. (16) and (17), can be presented via a dc form of the pinning-force density, i.e., $U_{\text{BMD}}(H_i,T)/U_0(0)+1 \approx F_p(H_i,T)/H_i$, while the phase MS, Eqs. (18) and (19), is a combination of dc and ac contributions, namely

$$U_{PH}(H_i,T)/U_0(0)-4/3 \approx (2/3)(-3F_p(H_i,T)/H_i+\partial F_p(H_i,T)/\partial H_i)\ .$$

So, we argue that the measurement of magnetostriction can provide rather unique information concerning the temperature and magnetic-field behavior of the grain-boundary pinning-force density, and its field derivative, in a granular superconductor. However, to make more definite conclusions as to the observability of the above predicted behavior of MS, more experimental data, especially in the small field region, are required.

Part of this work has been financed through the Incentive Program on High-Temperature Superconductors supported by the Belgian State-Prime Minister's Service-Science Policy Office (SU/02/13)

*Permanent address: Frank Laboratory of Neutron Physics, Joint Institute for Nuclear Research, Dubna, Russia.

[1] G. Brändli, Phys. Kondens. Mater. **11**, 93 (1970).
[2] E. du Tremolet de Lacheisserie *et al.*, J. Magn. Magn. Mater. **71**, L125 (1988).
[3] M. Braden *et al.*, Z. Phys. B **79**, 173 (1990).
[4] A. del Moral *et al.*, Physica **161C**, 48 (1989).
[5] A. Kadomsteva *et al.*, Physica **162-164C**, 1361 (1989).
[6] N. F. Oliveira *et al.*, Phys. Rev. B **39**, 2898 (1989).
[7] C. Ebner and D. Stroud, Phys. Rev. B **31**, 165 (1985).
[8] S. A. Sergeenkov, Physica **159C**, 43 (1989).
[9] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).
[10] J. Stankowski *et al.*, Physica **160C**, 170 (1989).