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Probing the wave function of quantum confined states by resonant magnetotunneling

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We have measured the low-temperature (4.2 K) current-voltage characteristics $I(V)$ of a GaAs/(AlGa)As double-barrier resonant tunneling diode in which the quantum well is intentionally δ doped with Si donors. A peak in $I(V)$ at low voltage is observed and attributed to resonant tunneling of electrons from two-dimensional free-electron-like states into the fully localized bound states of the shallow donors. The magnetic-field dependence of this peak is fundamentally different from that of the main resonance. We show that the Fourier spectrum of the shallow-donor wave function may be deduced from the variation of the peak amplitude with magnetic field.

The quantum-mechanical solution for the states of a potential well is given by a set of energy eigenvalues together with corresponding wave functions. In general the quantized energy levels of a given potential may be probed experimentally using various spectroscopic techniques. However, direct measurement of the corresponding wave functions is much more difficult and typically requires the presence of a highly localized scattering center.[1,2] In this paper, we report an alternative technique for probing the wave function of a bound state by measuring the tunnel current through a potential barrier. The barrier separates the bound state under investigation from an initial plane-wave state with well-defined lateral momentum components in the plane of the tunnel barrier. The variation of tunnel current with transverse magnetic field (parallel to the tunnel barrier) allows us to determine the Fourier spectrum of the bound-state wave function. The experiments demonstrate clearly that subthreshold peaks in the current-voltage characteristics, $I(V)$, of resonant tunneling diodes can arise from tunneling into donor-bound states of the quantum well. The results are of relevance to the study of small area resonant tunneling devices[3] in which additional structure in $I(V)$ near threshold has been attributed to lateral quantization and Coulomb blockade.[4,5]

We employ a GaAs/(AlGa)As resonant tunneling diode[6] (RTD) in which the quantum well is lightly doped with Si donors. In conventional RTD's with no impurities in the well,[6] the presence of a transverse magnetic field leads to a characteristic voltage shift of the resonant tunneling peak.[7-9] This effect has recently been exploited as a different technique, magnetotunneling spectroscopy, to investigate the band structure of the confined states of electrons and holes in quantum wells.[10,11]

The double-barrier RTD's were grown by molecular-beam epitaxy (MBE) on n^+-type GaAs substrates at a temperature of 550 °C. They are of conventional form except that the center plane of the quantum well is δ doped with 4×10^9 cm^{-2} Si donors. Note that the mean separation of the donors, 160 nm, is much greater than the Bohr radius in GaAs, $a_B=9.9$ nm. The barrier and well thickness are $b=5.7$ nm and $w=9.0$ nm, respectively. The detailed composition, in order of growth, is as follows: 2-μm GaAs $n=2\times10^{18}$ cm^{-3}; 80.6-nm GaAs $n=2\times10^{17}$ cm^{-3}; 50.9-nm GaAs $n=2\times10^{16}$ cm^{-3}; 20.9-nm GaAs undoped; 5.7-nm $Al_{0.4}Ga_{0.6}As$ undoped; 4.5-nm GaAs undoped; δ-doped GaAs $n_a=4\times10^9$ cm^{-2}; 4.5-nm GaAs undoped; 5.7-nm $Al_{0.4}Ga_{0.6}As$ undoped; 20.9-mn GaAs undoped; 50.9-nm GaAs $n=2\times10^{16}$ cm^{-3}; 80.6-nm GaAs $n=2\times10^{17}$ cm^{-3}; 0.6-μm GaAs $n=2\times10^{18}$ cm^{-3}. The wafer was grown at low temperature and with wide spacer layers to avoid diffusion and segregation[12] of Si from the contact regions into the quantum well. Large area (> 100-μm diam) devices were fabricated by photolithography.

The low-temperature ($T=4.2$ K) $I(V)$ characteristics for a device fabricated from this wafer are shown in Fig. 1. For $B=0$ T, a shoulder in $I(V)$ is clearly observed at

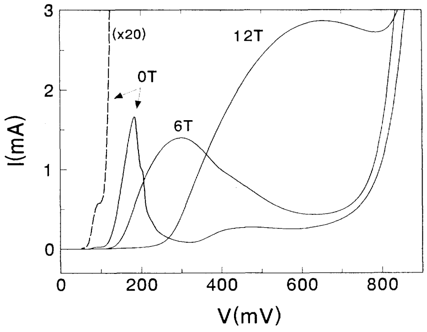

FIG. 1. Low-temperature ($T=4.2$ K) current-voltage characteristics, $I(V)$, for a resonant tunneling diode with a δ-doped quantum well for magnetic fields $B=0$, 6, 12 T applied perpendicular to the direction of current flow. A section of the 0-T curve is magnified by $\times20$ to show the feature due to donor-assisted resonant tunneling.

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a voltage $V \simeq 95$ mV. This shoulder is seen more clearly in Fig. 2, in which $I(V)$ close to the threshold for resonant tunneling is plotted for $B=0$ T. Also plotted in Fig. 2 is the differential conductance, dI/dV, in which the low voltage feature shows up clearly as a peak. We have established that this peak is due to the presence of the donors in the quantum well by making a comparison with the $I(V)$ of a control sample (curve *b* in Fig. 2) with no doping in the quantum well but an otherwise identical layer composition. No shoulder is observed for the sample with no dopants in the quantum well. In addition, a RTD with areal doping density of 8×10^9 cm^{-2} shows a peak of approximately twice the strength.

The mechanism giving rise to the donor-assisted resonant tunneling is shown in the inset to Fig. 2. The bound state of a shallow donor at the center of the well has a binding energy $E_d=12.8$ meV (from calculations by Greene and Bajaj[13]) and is *lower* in energy than the edge of the lowest two-dimensional (2D) subband of the quantum well. Electrons in the emitter therefore become aligned in energy with these donor bound states at a lower voltage than the threshold voltage, V_{th}, for tunneling into the 2D subband. The threshold voltage for tunneling into donor bound states, V_b, may be calculated from simple electrostatic considerations and is found to be $V_b \simeq 66$ mV, significantly lower than the calculated value for the 2D subband threshold voltage $V_{\text{th}}=136$ mV. These values have been calculated using the Fang-Howard approach[14] to treat the two-dimensional accumulation layer formed at the emitter barrier. The calculated values of V_b and V_{th} are close to the experimentally observed values of 67 and 125 mV, respectively, confirming the validity of the calculation. At the threshold for resonant tunneling V_{th}, the electron density in the accumulation layer is $n_s=1.06\times10^{11}$ cm^{-2} with a corresponding value of $k_F=8.16\times10^5$ cm^{-1} for the Fermi wave vector.

A similar conduction mechanism involving tunneling through a single donor has been invoked to account for a series of sharp peaks observed in the subthreshold $I(V)$ of a gated RTD with submicrometer lateral dimensions[3] and also to explain the magnetocapacitance of a quantum dot.[15] The peak we observe for the devices discussed in this paper is much broader. This is because Si segregation which occurs during the MBE growth causes the nominal δ-doping profile to be slightly smeared across the quantum well. Since the binding energy of a given donor is determined by its position within the well, the required voltage for donor-assisted tunneling varies from donor to donor, resulting in a broad peak. From the data in Refs. 12 and 13 we estimate that the variation in donor binding energy due to the smearing of the δ doping is ~10%.

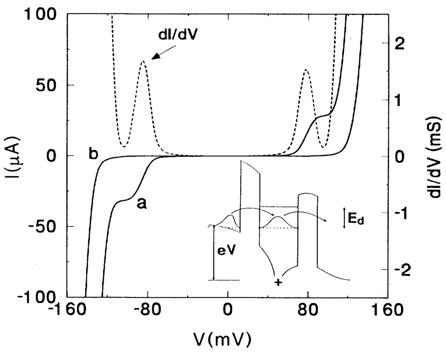

FIG. 2. Comparison of $I(V)$ of resonant tunneling diodes with (i) δ-doped quantum well (curve *a*) and (ii) an undoped quantum well, but otherwise identical layer composition (curve *b*). The shoulder corresponding to donor-assisted resonant tunneling is only observed in curve *a*. The dotted line shows the differential conductance, dI/dV, for the δ-doped sample. Inset: schematic diagram of the conduction-band profile of our device in the vicinity of a donor impurity in the quantum well.

Figure 3 shows the $I(V)$ of a device measured in a transverse magnetic field varying from $B=0$ to 12 T. These data emphasize the fact that the low voltage peak in dI/dV arises from a mechanism which is quite distinct from the main resonance. As B is progressively increased the shoulder in $I(V)$ develops into a clearly defined peak. The position of this peak is only weakly affected by a transverse magnetic field. In contrast, as B is progressively increased, the threshold of the main resonance shifts to higher voltage (this is seen most clearly in Fig. 1), as expected for a standard RTD with no impurities in the quantum well.[7-9] These trends are summarized in Fig. 4, in which the voltage position of the donor-assisted (filled triangles) and 2D subband (filled circles) peaks in $I(V)$ together with the experimental (open circles) and theoretical (solid curve) values of the threshold voltage, V_{th}, are plotted against magnetic field. We obtain V_{th} by extrapolating back to the voltage axis the near-linear rise in $I(V)$ which occurs at biases between threshold and the peak of the main resonance. Note that the low voltage peak is shifted in voltage from $V \simeq 95$ mV at $B=0$ T to $V \simeq 73$ mV at $B=12$ T. This small voltage shift is due to a difference in the diamagnetic shift of the bound states in 2D emitter and the quantum well.[16]

Consider first the variation of the threshold of the main resonance with transverse magnetic field. Under an ap-

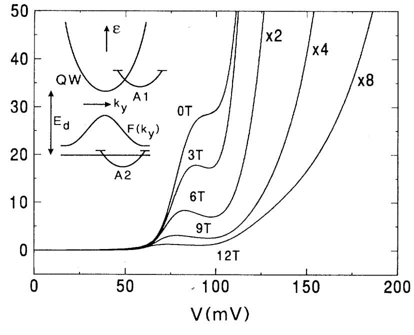

FIG. 3. Low-temperature ($T=4.2$ K) $I(V)$ for a δ-doped resonant tunneling diode in the presence of a magnetic field applied parallel to the barriers. $B=0$, 3, 6, 9, 12 T, respectively, from top to bottom. Inset: The inset shows the $\varepsilon(k_y)$ dispersion curve of the quantum well (QW) and the filled accumulation layer states ($A1$ and $A2$). The Fourier transform $F(k_y)$ of the donor state is also shown. Case 1 ($A1$) shows the position of the accumulation layer for tunneling in the quantum well. Case 2 shows the accumulation layer ($A2$) at lower voltage for tunneling into the donor state.

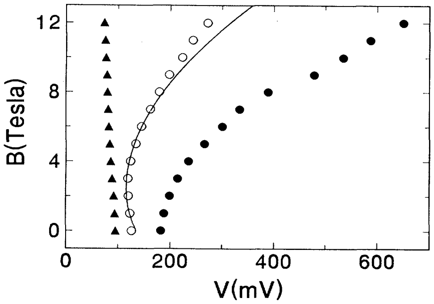

FIG. 4. Voltage position of the donor-assisted resonant tunneling peak (filled triangles), the threshold for resonant tunneling through the lowest 2D subband states (open circles: experiment; solid curve: theory), and the peak of the main resonance (filled circles) for a range of magnetic fields applied parallel to the tunnel barriers.

plied bias, a two-dimensional accumulation layer is formed close to the emitter barrier. We use the Landau gauge, $\mathbf{A}=(0,Bx,0)$ with the magnetic field along the z axis and the growth direction along the x axis. Electrons tunnel from an emitter state in the accumulation layer of lateral energy $\hbar^2k^2/2m^*$ to a state in the quantum well (QW) of lateral energy $\hbar^2(k+k_0)^2/2m^*$, with conservation of canonical momentum, $\hbar\mathbf{k}$.[7-9] Here $k_0=eB\Delta s/\hbar$, where Δs is the separation of the initial and final states. These displaced energy-momentum parabolas are shown schematically in Fig. 3. The voltage drop across the emitter which is required for resonant tunneling therefore depends on magnetic field, and the threshold voltage for the main resonance is expected to decrease for field for which $k_0<k_F$, and then increase for higher fields[7-9] in agreement with our data shown in Fig. 4. The solid curve is a fit to these data using the method described in Refs. 8 and 9.

The presence of the Si donor impurities breaks the translational symmetry in the plane of the well. Electrons tunnel from plane-wave emitter states to the zero-dimensional laterally bound shallow donor states in the well. The transition rate for this process is determined by the overlap of the plane-wave states in the emitter with the bound state in the well. For an electron tunneling from a state with wave vector $\mathbf{k}$ the overlap integral is given by $\Phi(\mathbf{k})$, the Fourier transform of the localized wave function, $\phi(y,z)$, of the shallow donor. The resonant current depends on the square of the Fourier components integrated over the occupied $\mathbf{k}$ states in the emitter. As the magnetic field is increased, the range of occupied $\mathbf{k}$ states in the emitter is shifted along the k_y axis. The amplitude of the donor-assisted peak is therefore determined by Fourier components with progressively higher wave vector. The Fourier transform of a localized wave function generally decreases with increasing wave vector so that the amplitude of the low voltage peak is expected to fall. This corresponds qualitatively with our observations which are shown in Fig. 5. At low magnetic field we find that $\ln(I)$ has a parabolic dependence on B, but at higher fields $\ln(I)$ has a linear dependence on B.

To analyze the amplitude dependence of the low voltage peak quantitatively, we have calculated the resonant tunneling current from two-dimensional plane-wave states into shallow-donor bound states using the Bardeen approach.[17] To simplify this calculation we have (i) assumed a separable donor bound-state wave function, $\psi(x,y,z)=g(x)\chi(y)\xi(z)$; (ii) modeled the two-dimensional emitter using the Fang-Howard wave function.

The current density $J(V,B)$ as a function of applied voltage and magnetic field applied in the z direction is given by

$$J(V,B)=\frac{-2em^*}{\pi\hbar^3}\int_{k_0-k_c}^{k_0+k_c}\frac{|M(k_y,k_v)|^2}{k_v}dk_y\,, \qquad (1)$$

where m^* is the effective mass and $M(k_y,k_z)$, proportional to the tunneling transition rate, is determined primarily by the overlap integrals between the lateral part of the initial- and final-state wave functions. Within our approximations, $M(k_y,k_z)\propto F(k_y)G(k_z)$, where $F(k_y)$ and $G(k_z)$ are the Fourier transforms of $\chi(y)$ and $\xi(z)$, respectively. The value of k_c is a function of the applied voltage and is determined by energy conservation so that $\hbar^2k_c^2/2m^*$ is the initial in-plane kinetic energy of the tunneling electron. Thus k_c varies from k_F at the voltage threshold for resonant tunneling via the shallow donor to $k_c=0$ at the maximum voltage for this mechanism to be permitted energetically. k_v is the value of k_z determined by energy conservation for a given value of k_y and is given by $k_v^2=k_c^2-(k_y-k_0)^2$.

In calculating Eq. (1) we have assumed that the amplitude of the initial- and final-state wave function within the barrier is independent of magnetic field. This is valid for our device for which the magnetic potential makes a small contribution to the total barrier height (for a discussion see Ref. 9). The magnetic-field dependence of the low voltage peak is therefore dominated by changes in the lateral overlap integrals and is given by

$$J(V_2,B)/J(V_2,0)=|F(k_0)|^2/|F(0)|^2\,, \qquad (2)$$

where V_2 is the voltage for which the $k=0$ state in the emitter is aligned with the bound-state energy. Hence the magnetic-field dependence gives a direct probe of the

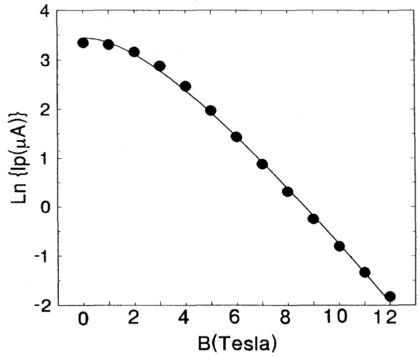

FIG. 5. Amplitude dependence of the donor-assisted peak current, I_p, vs magnetic field. The natural logarithm of I_p is plotted on the y axis. Also shown is a plot of the function given in Eq. (4).

Fourier spectrum of the donor wave function. Note that this result is valid for tunneling from a two-dimensional emitter state into an arbitrary zero-dimensional state. In Ref. 17 it is shown that the maximum donor-assisted resonant tunneling current occurs when $V=V_2$, and we therefore use the experimental dependence of the peak amplitude, I_p, on magnetic field to deduce the Fourier spectrum $F(k_y)$ of the donor wave function.

The comparison of theory and data is complicated by the fact that the wave function of the shallow-donor state is modified by a strong magnetic field. The Fourier spectrum of the donor wave function for $B=0$ may be deduced from the low-field dependence of the peak current. However, at high fields the donor-assisted peak amplitude is determined by the value of the Fourier transform at $k=k_0$ of a field-dependent wave function, i.e., both the value of k_0 and the wave function itself depend on B. We discuss below the field at which a crossover from the low- to the high-field regime occurs for our experiment.

The full analytic solution of Schrödinger's equation for a hydrogeniclike potential in a square well for arbitrary magnetic fields is not available, but it is possible to obtain a good fit to our data by assuming the following simple form for the wave function:

$$f(y)=A\exp[-y^2(\lambda^{-2}+ql_B^{-2})]\ , \tag{3}$$

where λ is the decay length of the wave function, $l_B=(\hbar/eB)^{1/2}$, is the magnetic length, q is a dimensionless parameter, and A is a (field-dependent) normalization factor. This function is a Gaussian with a magnetic-field-dependent decay length. In the limit of low B, $f(y)\rightarrow\exp[-y^2/\lambda^2]$ and for large B, $f(y)\rightarrow\exp[-qy^2/l_B^2]$. The high-field limit is the expected form for an eigenstate of a parabolic potential $V(y)=2q^2B^2y^2/m$. Assuming this form for the donor wave function gives

$$\ln[I_p(B)/I_p(0)]=\frac{-a^2B^2}{1+\gamma B}-\frac{\ln(1+\gamma B)}{2}\ , \tag{4}$$

where $a=e^2\Delta s^2\lambda^2/2\hbar^2$ and $\gamma=eq\lambda^2/\hbar$. We have fitted a function of this form to our data. The fitted curve is plotted in Fig. 5 and is in excellent agreement with experiment. The fitting procedure gives values for the constants $a=0.087\pm0.008\ \mathrm{T}^{-2}$ and $\gamma=0.11\pm0.02\ \mathrm{T}^{-1}$. From these parameters we deduce a value $\lambda=9.4\pm1$ nm, using a value $\Delta s=29$ nm calculated using the Fang-Howard wave function. This value for λ is very close to the value for the Bohr radius in GaAs, $a_B=9.9$ nm, which is the characteristic length for the donor wave function. In addition, we deduce a value of $q=0.8\pm0.2$. The inequality $\gamma B\ll1$ defines the low-field regime. Thus for $B\ll9$ T the experimental field dependence of the peak amplitude gives a measure of the Fourier spectrum of the unperturbed (i.e., $B=0$) donor wave function.

In conclusion, we have shown that donor-assisted resonant tunneling gives rise to an additional peak in the $I(V)$ of resonant tunneling diodes. From the dependence of the amplitude of this peak on transverse magnetic field, we are able to deduce the Fourier spectrum of the wave function of shallow donors in a quantum well.

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[1] J. Y. Marzin and J. M. Gerard, Phys. Rev. Lett. **62**, 2172 (1989).

[2] J. G. Williamson, C. E. Timmering, C. J. P. M. Harmans, J. J. Harris, and C. T. Foxon, Phys. Rev. B **42**, 7675 (1990).

[3] M. W. Dellow, P. H. Beton, C. J. G. M. Langerak, T. J. Foster, P. C. Main, L. Eaves, M. Henini, S. P. Beaumont, and C. D. W. Wilkinson, Phys. Rev. Lett. **68**, 1754 (1992).

[4] P. Gueret, N. Blanc, R. Germann, and H. Rothuizen, Phys. Rev. Lett. **68**, 1896 (1992).

[5] P. H. Beton, L. Eaves, and P. C. Main, Phys. Rev. Lett. **69**, 2995 (1992).

[6] L. L. Chang, L. Esaki, and R. Tsu, Appl. Phys. Lett. **24**, 593 (1974).

[7] R. A. Davies, D. J. Newson, T. G. Powell, M. J. Kelly, and H. W. Myron, Semicond. Sci. Technol. **2**, 61 (1987).

[8] M. L. Leadbeater, L. Eaves, P. E. Simmonds, G. A. Toombs, F. W. Sheard, P. A. Claxton, G. Hill, and M. A. Pate, Solid State Electron. **31**, 707 (1988).

[9] T. M. Fromhold, F. W. Sheard, and G. A. Toombs, in *Proceedings of the 20th International Conference on the Physics of Semiconductors, Thessaloniki* (World Scientific, Singapore, 1990), p. 1250.

[10] R. K. Hayden, D. K. Maude, L. Eaves, E. C. Valadares, M. Henini, F. W. Sheard, O. H. Hughes, J. C. Portal, and L. Cury, Phys. Rev. Lett. **66**, 1749 (1992).

[11] O. H. Hughes, M. Henini, E. S. Alves, M. L. Leadbeater, L. Eaves, M. Davies, and M. Heath, J. Vac. Sci. Technol. B **7**, 1041 (1989).

[12] J. J. Harris, J. B. Clegg, R. B. Beall, J. Castagne, K. Woodbridge, and C. Roberts, J. Cryst Growth **111**, 239 (1991); see also E. F. Schubert, H. S. Luftman, R. F. Kopf, R. L. Headrick, and J. M. Kuo, Appl. Phys. Lett. **57**, 1799 (1990), for a discussion of diffusion.

[13] R. L. Greene and K. K. Bajaj, Solid State Commun. **45**, 825 (1983).

[14] F. F. Fang and W. E. Howard, Phys. Rev. Lett. **46**, 767 (1966); L. Eaves, G. A. Toombs, F. W. Sheard, C. A. Payling, M. L. Leadbeater, E. S. Alves, T. J. Foster, P. E. Simmonds, M. Henini, O. H. Hughes, J. C. Portal, G. Hill, and M. A. Pate, Appl. Phys. Lett. **52**, 212 (1988).

[15] R. C. Ashoori, H. L. Stormer, J. S. Weiner, L. N. Pfeiffer, S. J. Pearton, K. W. Baldwin, and K. W. West, Phys. Rev. Lett. **68**, 3088 (1992).

[16] W. Demmerle, J. Smoliner, G. Berthold, E. Gornik, G. Weimann, and W. Schlapp, Phys. Rev. B **44**, 3090 (1991).

[17] T. M. Fromhold, F. W. Sheard, and L. Eaves, Acta Phys. Pol. A **82**, 737 (1992).