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Scattering of electrons off fractons in the normal state of high-T_c superconductors

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This paper presents a calculation of the temperature dependence of resistivity due to the interaction between the conduction electrons and fractons existing in the normal state of high-T_c superconductors. The results show that the resistivity from fracton scattering is nearly linear in temperature over a wide temperature range and $\rho(T)$ varies as $T^{3\bar{d}/D-1}$ for fracton scattering at low temperature. The effect of fractons on strongly coupled superconductors deserves to be further investigated.

The temperature-dependent resistivity in the normal state of metallic copper oxide superconductors has received considerable attention because of its ubiquitous nearly linear behavior over a wide temperature range for the optimum composition for superconductivity.[1–4] Some transport mechanisms have been proposed to account for the unusual properties.[5–14] Among these mechanisms, much effort has been devoted to researching the effect of the interaction between electrons and phonons. However, there is no consensus about the mechanism of high-T_c superconductivity. One of the reasons of inapplicability of conventional BCS theory is that it predicts the critical temperature T_c much lower than the observed values as T_c is proportional to the Debye frequency ω_D. However, as Rammal[15] showed, if the structure is of fractal nature, the cutoff frequency ω_{fD} in a fractal lattice can be much higher than the Debye frequency ω_D in a nonfractal structure. For example, on a Sierpinski structure $\omega_{fD}=2(d+1)\omega_D$, where d is the Euclidean dimension. Buttner and Blumen[16] suggested a possible explanation for the superconducting 240 K phase in the Y-Ba-Cu-O system based upon the experimental observation that the superconducting state may be of percolative nature,[17,18] possibly existing only on phase boundaries.[19,20] They proposed that high-T_c superconductivity may be envisaged as a fractal structure whose localized vibrations are responsible both for an effective electron-electron coupling mediated by the localized vibrational states (fractons) and a high critical temperature T_c. Tewari and Gumber[21] also showed that, under certain conditions, the effect of fractons in strongly coupled superconductors leads to a substantial increase in T_c in the fractal superconductor compared with that in the corresponding crystalline superconductor.

Excitations on a fractal structure differ from that in conventional crystals with periodicity. As is demonstrated by Alexander *et al.*, vibrational excitations in a percolating network have a crossover from phonons to fractons.[22] The former is extended with frequencies less than ω_c, the crossover frequency, the latter with frequencies larger than ω_c is localized, having a special wave function. Existence of fractons essentially affects the physical properties of condensed matter.[23,24] As Orbach *et al.* demonstrated, they can account for the thermal conductivity anomalies of amorphous solids at low temperatures.[25,26] In our previous papers, we calculated the interaction between conduction electrons and fractons in metallic glasses[27] and the temperature-dependent resistivity arising from the scattering of electrons off magnetic fractons and magnons in dilute two-dimensional Heisenberg antiferromagnets.[11] The former result presented a possible explanation for the resistivity minimum phenomena observed in metallic glasses at low temperatures and the later showed that the existence of magnetic fractons will lead to a linear temperature dependence of the resistivity over a wide temperature range while the magnon scattering will contribute a resistivity varying as $T^{3/5}$ almost over the whole temperature region.

One peculiarity of cuprate superconductors is that superconductivity is closely related to the oxygen deficiency in a limited range of concentration; beyond this range the ground state is either conventional paramagnetic metal or semiconductors. Only in the limited range of oxygen-deficiency concentration, superconductivity can come about below T_c, while for $T>T_c$ the system becomes strongly correlated metals.[28]

In this paper, we shall investigate the effect of vibrational fractons on the scattering of electrons aiming to see whether the fractal structure as a result of disorder introduced by critical doping of defects or by whatever kind of origin can explain the extraordinary linear dependence of resistivity of cuprates in the normal state.

We consider a model percolation network formed by the deficiency of oxygen. It has been proposed that[26] the percolation network appears to be homogeneous at length scales L larger than the percolation correlation

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length ξ and exhibits fractal geometry for the L less than ξ. The vibrational excitations on such a structure consist of phonons below ω_c and fractons over ω_c.

As is well understood, the electronic states that can contribute to the electrical resistivity are near the Fermi surface where the electrons are still in an extended state.[29] So the electron wave function can be regarded as a plane wave. We further assume that the electron states involved belong to a single band. Thus the model Hamiltonian may be written as[27]

$$H_I=\sum_{\mathbf{k}\mathbf{k}'\lambda} g_{\mathbf{k}\mathbf{k}'\lambda}(b_\lambda+b_\lambda^\dagger)C_{\mathbf{k}'}^\dagger C_{\mathbf{k}} , \tag{1}$$

where $C_{\mathbf{k}}^\dagger$ ($C_{\mathbf{k}}$), $b_\lambda^\dagger$ (b_λ) are creation (annihilation) operators for the electron with wave vector $\mathbf{k}$ and for the λth mode of fracton, respectively. The electron-fracton scattering matrix element $g_{\mathbf{k}\mathbf{k}'\lambda}$ is given by

$$g_{\mathbf{k}\mathbf{k}'\lambda}=\left[\frac{\hbar}{2B\omega_\lambda^{\mathbf{k}-\mathbf{k}'}}\right]^{1/2}\mathbf{e}_\lambda\cdot\mathbf{J}I(\mathbf{k}-\mathbf{k}') \tag{2}$$

in which B is the average bulk mass density, $\omega_\lambda^{\mathbf{k}-\mathbf{k}'}$ is the frequency of fractons, and $\mathbf{J}=-iV(\mathbf{k}-\mathbf{k}')/G(\mathbf{k}-\mathbf{k}')$, where G is the volume of the system and $V(\mathbf{k}-\mathbf{k}')$ is the Fourier component of the electron potential energy at the ion field. $\mathbf{e}$ is the polarization vector and $I(\mathbf{k}-\mathbf{k}')$ is the spatial Fourier transform of the fracton wave function

$$I(\mathbf{k}-\mathbf{k}')=\sum_\mu \Phi_\lambda(\mathbf{R}_\mu)e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_\mu} . \tag{3}$$

Here, $\Phi_\lambda(\mathbf{R}_\mu)$ was proposed by Alexander, Entin-Wohlman, and Orbach.[30] It is

$$\Phi_\lambda(\mathbf{R}_\mu)=C|\mathbf{R}_\mu|^{-(d-D)/2}(l_{\omega_\lambda})^{-D/2}\exp\left[-\frac{1}{2}\left[\frac{R_\mu}{l_{\omega_\lambda}}\right]^{d_\phi}\right] , \tag{4}$$

where C is a constant, l_{ω_λ} is the fracton localization length with frequency ω_λ, D is the fractal dimensionality, d_ϕ is the superlocalization exponent, and d is the Euclidean dimensionality. The absolute square of $\mathbf{I}(\mathbf{k}-\mathbf{k}')$ has also been treated carefully by Alexander *et al.* and the superlocalization of fractons leads to[30]

$$|I(\mathbf{k}-\mathbf{k}')|^2=|\mathbf{k}-\mathbf{k}'|^{-D} . \tag{5}$$

Since there is a factor $\mathbf{e}_\lambda\cdot(\mathbf{k}-\mathbf{k}')$ in the Hamiltonian (1), we need only consider the longitudinal components of fractons. Inserting (5) into Eq. (2), we obtain the square of $g_{\mathbf{k}\mathbf{k}'\lambda}$:

$$|g_{\mathbf{k}\mathbf{k}'\lambda}|^2=\frac{\hbar\lambda_0^2}{2B}\frac{|\mathbf{k}-\mathbf{k}'|^{2-D}}{\omega_\lambda^{\mathbf{k}-\mathbf{k}'}} , \tag{6}$$

where $\lambda_0=V(\mathbf{k}-\mathbf{k}')/G$ is the electron-fracton coupling constant.

For calculating the electrical resistivity in cuprate oxides due to fracton scattering, we will deal with a Boltzmann equation to obtain the inverse relaxation time $1/\tau_f$ as Allen derived it for electron-phonon interaction.[31,32] By solving the Boltzmann equation variationally to the first-order approximation and assuming the bandwidth is much larger than k_BT and fracton frequency $\omega_\lambda^{\mathbf{k}-\mathbf{k}'}$, the relaxation time of the electrons due to scattering by fracton is given by

$$\begin{gathered}\frac{1}{\tau_f}=4\pi k_BT\int_{\omega_c}^{\omega_{fD}}d\omega\frac{\alpha_{\rm tr}^2F(\omega)}{\omega}P\left[\frac{\hbar\omega}{2k_BT}\right] ,\\ P(x)=\left[\frac{x}{\sinh x}\right]^2 ,\end{gathered} \tag{7}$$

where the integral extends over all frequencies for which the function $\alpha_{\rm tr}^2F(\omega)$ is nonzero and $\alpha_{\rm tr}^2F(\omega)$ is called the transport spectral functional defined by[32]

$$\alpha_{\rm tr}^2F(\omega)=N(E_F)\sum_{\mathbf{k}\mathbf{k}'\lambda}(\mathbf{k}-\mathbf{k}')_\parallel^2|g_{\mathbf{k}\mathbf{k}'\lambda}|^2\times\delta(\omega-\omega_\lambda^{\mathbf{k}-\mathbf{k}'})/2k_F^2 , \tag{8}$$

where $N(E_F)$ is the density of states for one spin direction at Fermi energy and k_F is the Fermi wave number. The electron-fracton scattering matrix element $g_{\mathbf{k}\mathbf{k}'\lambda}$ is described in Eqs. (2) and (6). The transport spectral function $\alpha_{\rm tr}^2F(\omega)$ is analogous to the function $\alpha^2F(\omega)$ used in Eliashberg theory of superconductivity except for the extra factors of momentum $(\mathbf{k}-\mathbf{k}')_\parallel^2/2k_F^2$. Furthermore, we assume that the Fermi surface does not change significantly within ω_{fD} and may replace the complicated $\mathbf{k},\mathbf{k}'$ summation with the Fermi surface average in Eq. (8). Since the quantities in Eq. (8) only depend on the change of the electronic wave vector, $\mathbf{q}=\mathbf{k}-\mathbf{k}'$, which is also the momentum of the fracton, the double summation $\sum_{\mathbf{k}\mathbf{k}'}$ can be transformed into a single one $\sum_{\mathbf{q}}$. As pointed out by Kresin and Wolf,[33] to the first approximation, one can suppose the Fermi surface for the high-T_c oxides to be cylindrically shaped, which corresponds to neglecting the interlayer transitions although the interlayer transitions lead to small derivations from the cylindrical shape. Considering this approximation, we could convert the summation over the Fermi surface into an integral about $\mathbf{q}$ according to the following rule:[29]

$$\sum_{\mathbf{q}}=\frac{K_{d-1}}{(2\pi)^d}\int q^{d-2}dq\int_0^\pi(\sin\theta)^{d-2}d\theta \tag{9}$$

for phonon excitation, and similarly

$$\sum_{\mathbf{q}}=\frac{K_{D-1}}{(2\pi)^D}\int q^{D-2}dq\int_0^\pi(\sin\theta)^{D-2}d\theta \tag{10}$$

for fracton excitation with $K_x=\pi^{x/2}/\Gamma(x/2)$. The integrals in Eqs. (9) and (10) are understood to be over the symmetrical Fermi surface—from one point $\mathbf{k}$ to all other point $\mathbf{k}'$, where $\mathbf{k}$ and $\mathbf{k}'$ both have magnitude k_F.[34]

The equivalent of a dispersion relation for a fracton, at least for low frequencies, is given by[35]

$$\omega^{\mathbf{q}}=\omega_c[(\tfrac{1}{2}\xi)q]^{D/\tilde{d}} . \tag{11}$$

Here, ω_c is the crossover frequency which is a characteristic vibrational frequency separating a high-frequency (fracton) regime from a low-energy (phonon) regime, ξ is the percolation correlation length, and $\tilde{d}$ is the fracton dimensionality. For two- and three-dimensional percolation networks $\tilde{d}=4/3$, $D=1.9$ and $\tilde{d}=1.42$, $D=2.5$, respectively.[22] Inserting Eqs. (10) and (11) into Eq. (8) brings us to the spectral function

$$\alpha_{\mathrm{tr}}^2 F(\omega)=C_f^{\mathrm{tr}}\omega^{3\tilde{d}/D-2} \tag{12}$$

for fracton excitation with the coefficient

$$C_f^{\mathrm{tr}}=N(E_F)\frac{K_{D-1}}{(2\pi)^D}\frac{\hbar\lambda_0^2}{2B}\frac{1}{2k_F^2}\frac{\tilde{d}}{D}\left[\frac{2}{\xi\omega_c^{\tilde{d}/D}}\right]^3 .$$

Using the low-frequency phonon dispersion law and noticing that $|I(\mathbf{q})|^2$ in Eq. (5) is unity for phonons, we can easily obtain

$$\alpha_{\mathrm{tr}}^2 F(\omega)=C_{\mathrm{ph}}^{\mathrm{tr}}\omega^{d+1} \tag{13}$$

for phonon excitation with

$$C_{\mathrm{ph}}^{\mathrm{tr}}=N(E_F)\frac{K_{d-1}}{(2\pi)^d}\frac{\hbar\lambda_0^2}{2B}\frac{1}{2k_F^2}\frac{1}{v_s^{d+3}} .$$

It is now simple to compute the scattering rate in formula (7). Especially at low temperature ($\hbar\omega_{fD}\gg 2k_BT$, but keep $2k_BT\gg\hbar\omega_c$), $1/\tau_f$ behaves as $T^{3\tilde{d}/D-1}$, while at high temperature Eq. (7) becomes proportional to T as $P(x)$ approaches rapidly to unity.

Generally, we may express the resistivity from fractons as

$$\rho(T)=\frac{m^*}{ne^2}\frac{1}{\tau_f} , \tag{14}$$

where the electronic effective mass m^* is a constant related to the electron band structure and τ_f is the relaxation time of the electrons due to scattering by fractons discussed above. For simplicity, the coefficient C_f^{tr} is chosen so that the $\alpha_{\mathrm{tr}}^2F(\omega)$ function is normalized, i.e.,

$$\lambda_{\mathrm{tr}}=2\int_{\omega_c}^{\omega_{fD}} d\omega\frac{\alpha_{\mathrm{tr}}^2F(\omega)}{\omega}=1 .$$

The numerical results of the resistivity $\rho(T)$ as a function of temperature in two- and three-dimensional percolation networks with different parameter ω_{fD} (but same crossover frequency $\omega_c=10$ K) are shown in Figs. 1 and 2, respectively. The resistivity $\rho(T)$ in Figs. 1 and 2 appears to be essentially linear over wide temperature ranges and consistent with the measurements for most cuprates which show the optimum superconductivity.[2–4,36] For confirming the linear relation between the resistivity and temperature, we further calculate the differential of resistivity about temperature and the results are also shown in Figs. 1 and 2. The slope of resistivity is indeed nearly constant above 100 K (it is just the T_c value for high-T_c superconductors), except for the case $C(c)$ ($\omega_{fD}=3000$ K), where ω_{fD} may be too high to satisfy the condition of the dispersion law for a fracton in formula (11).

Very recently, Takagi *et al.*[4] reported the observed results on high-quality samples of $La_{2-x}Sr_xCuO_4$ showing that only in the narrow composition region ($x=0.15$ or so), which associated with optimal superconductivity, the T-linear in-plane resistivity is observed and in the overdoped range ($x>0.2$) the resistivity follows a novel power-law dependence $\rho\sim T^{1.5}$ over the wide temperature range up to 1000 K. The newly observed resistivity-temperature relation in the underdoped and overdoped range requires further theoretical investigation.

It is interesting to compare our result with that of Entin-Wohlman, Alexander, and Orbach.[29] They calculated the temperature dependence of the inelastic scattering rate for degenerate electron gas which is proportional

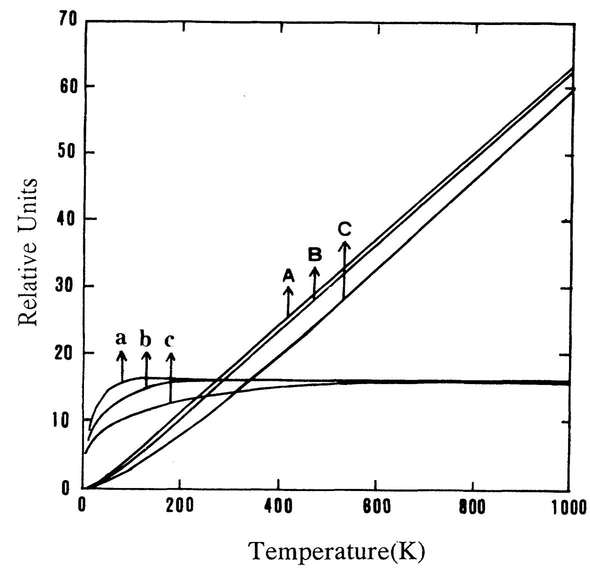

FIG. 1. Temperature-dependent resistivity $\rho(T)$ (curves A, B, C) and its slope $d\rho(T)/dT$ (curves a, b, c) calculated from the fracton scattering in the two-dimensional percolation network (both in relative unites). Curves A(a), B(b), and C(c) correspond to $\omega_{fD}=500$ K, $\omega_{fD}=1000$ K, and $\omega_{fD}=3000$ K, respectively.

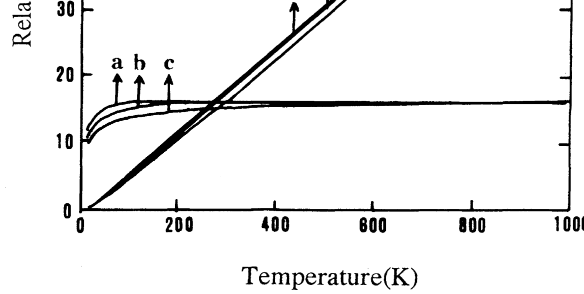

FIG. 2. Results as in Fig. 1 but for the three-dimensional percolation network.

to $T^{8/7}$ for fractons in a two-dimensional percolation network with scalar forces and to $T^{5/19}$ with central and bending forces. If we consider an infinite cluster in percolation, the fracton dimension is $\tilde{d}=2D/(2+\theta)$ and then at low temperature the resistivity of our result is also proportional to $T^{8/7}$ for scalar forces with $\theta=4/5$ and to $T^{5/19}$ for central and bending forces with $\theta=11/4$. The fact that we obtain the same result of Entin-Wohlman *et al.* reflects the common nature of the interaction between the extended electrons and fractons.

Furthermore, Tewari and Gumber[21] indicate that within the frame of a strongly coupled superconductor, just by simple introduction of fractons (the substitution of fracton density of states in place of phonon density of states) the critical temperature T_c cannot be increased. Only under certain conditions, T_c may be increased substantially. The so-called "certain conditions" is to assume arbitrarily that the factor $\alpha^2(\omega)$ in the superconductivity function $\alpha^2F(\omega)$ is equal to k/ω^n, and their numerical results show that n should be equal to or greater than 1.65. In fact, in an analogous way for calculating the function $\alpha_{\rm tr}^2F(\omega)$ we can also easily derive that the superconductivity function $\alpha^2F(\omega)$ behaves as $\omega^{\tilde{d}/D-2}$ for fractons and ω^{d-1} for phonons under Debye approximation. Therefore the factor $\alpha^2(\omega)$ automatically varies as $\omega^{\tilde{d}/D-\tilde{d}-1}$, which gives $\omega^{-1.63}$ in the two-dimensional percolation network and $\omega^{-1.85}$ in the three-dimensional network, respectively, rather than being a constant like for phonons. So we may conclude that the presence of fractons in a strongly coupled superconductor results in an increase in its T_c if McMillan's equation and Kresin's equation in Ref. 21 are adopted to evaluate the T_c.

In summary, the fracton dispersion law, together with its superlocalization nature, results in a nearly linear temperature dependence for the electrical resistivity, especially at low temperature where it varies as $T^{3\tilde{d}/D-1}$. This prediction agrees well with known experimental measurements for high-T_c oxides in normal states. Furthermore, the effect of fractons on strongly coupled superconductors may result in an increase in T_c in fractal superconductors over its value in the corresponding crystalline superconductor.

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