RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 48, NUMBER 7 15 AUGUST 1993-I

Critical conductivity exponent of Si:P in a magnetic field

Peihua Dai, Youzhu Zhang, Snezana Bogdanovich, and M. P. Sarachik
Physics Department, City College of the City University of New York, New York, New York 10031
(Received 14 May 1993)

The critical conductivity exponent of Si:P changes from near $\frac{1}{2}$ in zero field to 0.86 ± 0.15 in a magnetic field of 8 T, consistent with the theoretical expectation of 1. According to recent theory, similar behavior found earlier in Si:B, where spin-orbit scattering is strong, corresponds to the universality class for magnetic impurities. These measurements in Si:P thus constitute a clear determination of the critical conductivity exponent near the metal-insulator transition in the universality class for high magnetic field.

Based largely on the elegant stress-tuning experiments of Paalanen *et al.*,[1] the metal-insulator transition that takes place in doped semiconductors as a function of dopant concentration is thought to be a continuous, second-order phase transition.[2] The critical behavior near the transition is governed by the dominant process, be it spin-orbit scattering, magnetic impurities, or magnetic field, that determines the symmetry of the system and its universality class. The critical exponent μ, which characterizes the approach to the transition of the zero-temperature conductivity $\sigma=\sigma_0[(n/n_c)-1]^\mu$, is generally assumed equal to the critical exponent ν for the correlation length, an assumption that is strictly valid only in the critical region. Although most theoretical work yields a critical exponent of 1 under most circumstances, that result is not well established, particularly in the generic class[3] where the spin-flip and spin-orbit scattering rates and the Zeeman splitting are all small compared to the temperature. Based on work of Finkelshtein,[4] various theoretical studies[5-7] have indicated that the exponent should be close to 1 in the magnetic-field (MF) universality class and for the cases of magnetic impurities (MI) and spin-orbit (SO) scattering.

Experiments have shown that μ is generally close to 1 in the absence of a magnetic field in doped semiconductors as well as in the amorphous metal-semiconductor mixtures.[8] Since the exponent is expected to also be 1 in the MF universality class, the effect of applying an external magnetic field cannot be tested in such systems. Thus, the finding of Ootuka, Matsuoka, and Kobayashi[9] that the exponent changes in Ge:Sb from 0.9 at $B=0$ to 1 in $B=4$ T is suggestive but not conclusive. On the other hand, it has been known for some years that the critical conductivity exponent[10] μ is between $\frac{1}{2}$ and $\frac{2}{3}$ in the silicon-based *n*-type doped semiconductors Si:P,[11] Si:As,[12,13] Si:B,[8] Si:P,As,[14] and probably Si:Sb,[15] as well as amorphous Ga:Ar.[16] It is only in these materials that one can determine whether or not the exponent changes to the expected value of 1 in a magnetic field. Indeed, magnetic tuning measurements by Shafarman *et al.*[17] indicate a possible change in the critical conductivity exponent of Si:As in a magnetic field.

In an earlier publication[18] we reported that the critical exponent in Si:B, where spin-orbit effects are strong, changes from 0.65 in zero field to approximately 1 in a magnetic field of 7.5 T. We attributed these results to the magnetic-field universality class. It has been noted,[3,6,19] however, that for a material that is in the SO universality class in the absence of a magnetic field, the application of a large magnetic field places it instead in the universality class corresponding to MI even when there are no magnetic impurities present in the system; in other words, SO+MF=MI. This implies that if spin-orbit effects are indeed strong in Si:B, our earlier finding of $\mu=1$ in this material in a large magnetic field corresponds to the MI universality class rather than the MF class. The MF universality class can only be realized in systems that belong to the generic class in zero field, and the critical exponent for the MF class can be determined only from high-field measurements of a material in which the spin-flip and spin-orbit scattering rates are small compared to the temperature.[3]

In this paper we report measurements of the critical conductivity exponent of Si:P, a material where the spin-orbit and spin-flip scattering are unimportant, and which is in the generic class down to temperatures on the order of 3 mK in the absence of a magnetic field.[3] We find that the conductivity exponent is 0.58 in zero field, in agreement with earlier results of other investigators,[1,11] and changes to near 1 in a magnetic field of 8 T. This is a clear determination of the critical conductivity exponent at the metal-insulator transition for the universality class corresponding to a strong magnetic field.

Czochralski-grown Si:P was obtained from Crysteco; Table I lists room-temperature resistivities, the resistance ratios $R(4.2\text{ K})/R(300\text{ K})$ and the dopant concentrations based on the Thurber[20] scale. Data were taken between 0.06 and 1.2 K in magnetic fields to 8 T using standard techniques described elsewhere.[8,18]

The conductivities of the nine samples of Si:P, with different phosphorus concentrations as labeled, are plotted as a function $T^{1/2}$ in zero field in Fig. 1. The conductivities of seven of the samples are plotted in Fig. 2 in a magnetic field of 8 T. As was found also for Si:B, while the slopes of the conductivity curves change numerical sign as the transition is approached in zero field, they all

 ©1993 The American Physical Society

TABLE I. For the Si:P samples used, the table lists room-temperature resistivities, resistance ratios, and dopant concentrations derived from the scale of Thurber *et al.* (Ref. 20).

ρ(300 K) (10^{-3} Ω cm)	ρ(4.2 K)/ρ(300 K)	n (10^{18} cm^{-3})
9.84	0.838	4.65
10.18	0.936	4.43
10.73	1.202	4.05
11.22	1.666	3.74
11.30	1.764	3.70
11.53	2.220	3.58
11.63	2.483	3.53
11.78	2.991	3.46
11.98	3.812	3.35

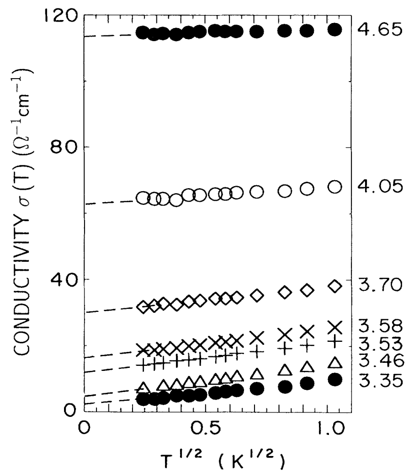

FIG. 2. Conductivity vs $T^{1/2}$ in a magnetic field of 8 T for Si:P. Dopant concentrations are indicated next to each curve in units of 10^{18} cm^{-3}. The dashed lines represent linear-regression fits to the data.

become positive when a large magnetic field is applied. Zero-temperature extrapolations were obtained by fitting the data to $\sigma(T)=\sigma(0)+m(n)T^{1/2}$, where the temperature-dependent term is associated with electron-electron interactions.[21] The intercepts $\sigma(0)$ deduced from these linear-regression fits are plotted in Fig. 3 as open circles in zero field and closed circles at $B=8$ T.

It is clear in Figs. 1 and 2 that progressively lower temperatures are required for a reliable determination of $\sigma(0)$ as the transition is approached. Measurements by Paalanen *et al.*[1] and by Rosenbaum *et al.*[11] down to very low temperatures on the order of 1–5 mK show that the conductivity changes rapidly below 60 mK for samples very near the transition. Detailed comparison of our data with the results obtained by these investigators in Si:P samples with equivalent conductivities and dopant concentrations indicates that extrapolations from above 60 mK (the lowest temperature available in our current experiments) yield substantial overestimates of $\sigma(0)$ near the transition, and that our lowest concentration sample is actually on the insulating side. As found in other similar studies,[12,14] this give rise near the transition to the "rounding" shown in Fig. 3; we exclude the data for the two lowest concentration samples from the fitting procedure described below.

The critical behavior of the conductivity of Si:P shown in Fig. 3 in a magnetic field is qualitatively different from that in zero field: while the zero-field approach is clearly sublinear, the behavior in a magnetic field is consistent with an exponent near 1. Nonlinear least-squares fits by $\sigma(T\rightarrow 0)=\sigma_0[(n/nc)-1]^{\mu}$ yield $\sigma_0=260\pm 20$ Ω^{-1}cm^{-1}, $n_c=(3.46\pm 0.06)\times 10^{18}$ cm^{-3}, and $\mu=0.58$ ± 0.08 in the absence of a magnetic field,[22] in agreement with earlier determinations.[1,11] In a magnetic field of 8

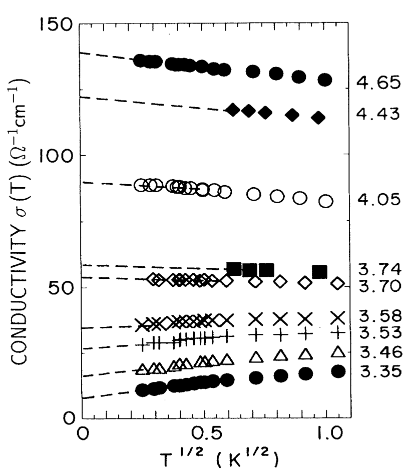

FIG. 1. Conductivity in zero field plotted as a function of $T^{1/2}$ for Si:P. Dopant concentrations are indicated next to each curve in units of 10^{18} cm^{-3}. The dashed lines represent linear-regression fits to the data.

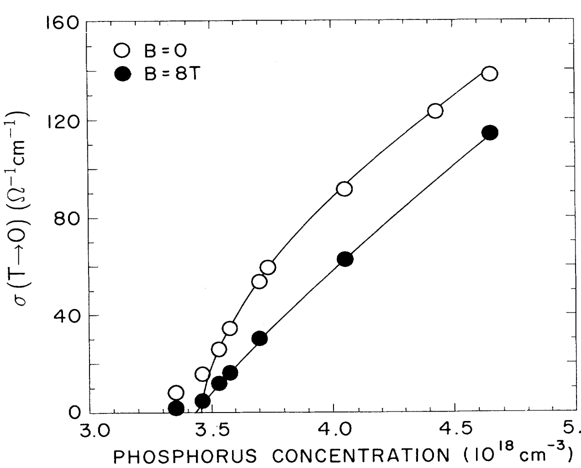

FIG. 3. Zero temperature conductivity vs dopant concentration. The open and closed circles refer to data in zero field and in a magnetic field of 8 T, respectively. The lines are best fits by $\sigma(T\rightarrow 0)=\sigma_0[(n/n_c)-1]^{\mu}$ with $\mu=0.58$ in zero field and $\mu=0.86$ at 8 T.

T, the prefactor $\sigma_0 = 280 \pm 30\ \Omega^{-1}\,\mathrm{cm}^{-1}$ and the critical concentration $n_c = (3.45 \pm 0.06) \times 10^{18}\,\mathrm{cm}^{-3}$ remain substantially the same, while the critical conductivity exponent increases to $\mu = 0.86 \pm 0.15$.

To summarize, the critical conductivity exponent of Si:P is 0.58 in zero field, in agreement with earlier results of other investigators,[1,11] and changes to near 1 in a magnetic field of 8 T, in agreement with theoretical expectations. This is a determination of the critical conductivity exponent at the metal-insulator transition for the universality class corresponding to a strong magnetic field.

We thank D. Belitz and T. Kirkpatrick for very useful discussions and communicating their results prior to publication. We thank C. Di Castro and J. C. Phillips for calling our attention to several relevant references. We are grateful to V. Dobrosavljevic for illuminating explanations of the theory, and for his guidance in understanding some of the published literature. This work was supported by the U.S. Department of Energy under Grant No. DE-FG02-84-ER45153. Support for Y.Z. and S.B. was provided by the U.S. Air Force Office of Scientific Research Grant No. F49620-92-J-0190.

---

[1]M. A. Paalanen, T. F. Rosenbaum, G. A. Thomas, and R. N. Bhatt, Phys. Rev. Lett. **48**, 1284 (1982).

[2]For arguments in favor of a first-order transition see, for example, A. Mobius, Phys. Rev. B **40**, 4194 (1989); J. Phys. C **18**, 4639 (1985); A. Mobius, D. Elefant, A. Heinrich, R. Muller, J. Schumann, H. Vinzelberg, and G. Zies, *ibid.* **16**, 6491 (1983); A. Mobius, H. Vinzelberg, C. Gladun, A. Henrich, D. Elefant, J. Schumann, and G. Zies, *ibid.* **18**, 3337 (1985).

[3]D. Belitz and T. R. Kirkpatrick (unpublished).

[4]A. M. Finkelshtein, Zh. Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys. JETP **57**, 97 (1983)]; Z. Phys. B **56**, 189 (1984); Zh. Eksp. Teor. Fiz. **86**, 367 (1984) [Sov. Phys. JETP **59**, 212 (1984)]; Pis'ma Zh. Eksp. Teor. Fiz. **40**, 63 (1984) [JETP Lett. **40**, 796 (1984)].

[5]See C. Castellani, G. Kotliar, and P. A. Lee, Phys. Rev. Lett. **59**, 323 (1987), and references therein.

[6]C. Castellani, C. Di Castro, P. A. Lee, and M. Ma, Phys. Rev. B **30**, 527 (1984).

[7]C. Castellani, C. Di Castro, G. Forgacs, and S. Sorella, Solid State Commun. **52**, 261 (1984).

[8]See P. Dai, Y. Zhang, and M. P. Sarachik, Phys. Rev. Lett. **66**, 1914 (1991), and references therein.

[9]Y. Ootuka, H. Matsuoka, and S. Kobayashi, in *Anderson Localization,* edited by T. Ando and H. Fukuyama (Springer-Verlag, Berlin, 1988), p. 40.

[10]The critical conductivity exponent $\mu < 1$ is generally considered an interesting unresolved puzzle. We note that for uncompensated material in the absence of a magnetic field, however, an exponent $\mu = \frac{1}{2}$ has been obtained using set theoretic methods by J. C. Phillips, Phys. Rev. B **43**, 8679 (1991); **45**, 5863 (1992).

[11]T. F. Rosenbaum, K. Andres, G. A. Thomas, and R. N. Bhatt, Phys. Rev. Lett. **45**, 1723 (1980); T. F. Rosenbaum, R. F. Milligan, M. A. Paalanen, G. A. Thomas, R. N. Bhatt, and W. Lin, Phys. Rev. B **27**, 7509 (1983).

[12]P. F. Newman and D. F. Holcomb, Phys. Rev. B **28**, 638 (1983).

[13]W. N. Shafarman, D. W. Koon, and T. G. Castner, Phys. Rev. B **40**, 1216 (1989).

[14]P. F. Newman and D. F. Holcomb, Phys. Rev. Lett. **51**, 2144 (1983).

[15]A. P. Long and M. Pepper, J. Phys. C **17**, L425 (1984); Solid State Electron. **28**, 61 (1985).

[16]Th. Zint, M. Rohde, and H. Micklitz, Phys. Rev. B **41**, 4831 (1990).

[17]W. N. Shafarman, T. G. Castner, J. S. Brooks, K. P. Martin, and M. J. Naughton, Phys. Rev. Lett. **56**, 980 (1986).

[18]P. Dai, Y. Zhang, and M. P. Sarachik, Phys. Rev. Lett. **67**, 136 (1991).

[19]C. Di Castro, in *Anderson Localization* (Ref. 9), p. 96; C. Castellani, C. Di Castro, and G. Strinati, in *Fluctuations and Stochastic Phenomena in Condensed Matter,* edited by L. Garrido (Springer-Verlag, Berlin, 1987), p. 175.

[20]W. R. Thurber, R. L. Mattis, Y. M. Liu, and J. J. Filliben, J. Electrochem. Soc. **127**, 1807 (1980).

[21]B. L. Al'tshuler and A. G. Aronov, Zh. Eksp. Teor. Fiz. **77**, 2028 (1979) [Sov. Phys. JETP **50**, 968 (1979)]; Pis'ma Zh. Eksp. Teor. Fiz. **30**, 514 (1979) [JETP Lett. **30**, 482 (1979)].

[22]Using the scale of F. Mousty, P. Ostoja, and L. Passari, J. Appl. Phys. **45**, 4576 (1974), this corresponds to $n_c = (3.71 \pm 0.06) \times 10^{18}\,\mathrm{cm}^{-3}$ in zero magnetic field, consistent with $n_c = 3.75 \times 10^{18}\,\mathrm{cm}^{-3}$ found by Paalanen *et al.* (Ref. 1) and Rosenbaum *et al.* (Ref. 11).