Negative thermodynamic density of states and charge-density-wave instability in the lowest Landau level

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We derive the thermodynamic density of states (TDOS) and the charge-density-wave (CDW) instability condition by including both electron-electron interactions and electron-impurity interactions. We calculate the TDOS in two different ways. Our first approach involves obtaining the long-wavelength static polarizability, whereas the second one involves calculating the density derivative of the chemical potential. We present our results for GaAs-$Al_xGa_{1-x}As$ heterostructures in the lowest Landau level and ignore spin effects. We find that the TDOS becomes negative at high mobilities due to the renormalization effects of exchange and correlation. We also show, for the Hartree-Fock case, that when these renormalization effects are sufficiently strong, a CDW instability can occur. We derive an analytic expression for the CDW instability condition by considering the poles of the static-charge susceptibility.

I. INTRODUCTION

The discovery that the Hall resistance has plateaus at odd denominator filling factors in high mobility samples has shown that electron-electron (EE) interaction effects have important consequences.[1] Laughlin[2] proposed a trial many-electron wave function which has been fairly successful in explaining several features of the fractional quantum Hall effect. It has also been demonstrated that at these special filling factors the thermodynamic density of states (TDOS) becomes zero.[3] Thus the many-body effects on the TDOS is of interest both theoretically and experimentally. In the past, magnetocapacitance measurements were performed to determine the TDOS.[4,5] Recently, a new technique was employed to determine both the magnitude and the sign of the TDOS.[6] Interestingly enough, negative values of the TDOS were reported by Kravchenko, Pudalov, and Semenchinsky[5] and by Eisenstein, Pfeiffer, and West.[6] On the theoretical side, EE interactions were recognized to be sufficiently important to even result in negative values of the TDOS.[7] However, a full-fledged many-body analysis of the TDOS that consistently takes into account the vertex corrections due to both EE interactions and electron-impurity (EI) interactions has not been presented so far.

It has been presumed that at low filling factors or low densities, similar to the case of a zero magnetic field, the electronic interactions become sufficiently strong to result in the formation of a crystal.[8] Different investigators have predicted different values for the critical filling factor below which crystallization occurs.[9] Earlier on, within a Hartree-Fock approach, it was found that a charge-density-wave (CDW) transition occurs at a high melting temperature[10] and that the CDW can be regarded as a Wigner crystal with the same periodicity.[11] At low densities, the importance of including higher harmonic terms in the triangular CDW state was pointed out by Gerhardts.[12] Recently, Manolescu[13] has studied the CDW instability as a second-order transition by examining the temperature dependence of the charge susceptibility. Also, within a "parquet" approximation, Bychkov[14] has found that a CDW instability may occur at low filling factors when the wave vector is small. However, it is important to note that none of the previous works considers the effects of the EI interactions. On the experimental side, anomalies such as large diagonal resistivity, radio-frequency absorption resonances, sharp threshold conduction fields, and an additional luminescence line below a critical filling factor have been considered to be indicative of a pinned Wigner crystal.[15] Thus, all in all it is of interest to know the conditions for the formation of CDW's when both EE interactions and EI interactions are considered.

In this paper we obtain the TDOS and the CDW instability condition by including for the first time both EE interactions and EI interactions. We calculate the TDOS using two different methods. The first method involves consistently taking into account the vertex corrections due to EE interactions and EI interactions and obtaining the long-wavelength static polarizability. The second approach involves evaluating the TDOS from the derivative of the electronic density with respect to the chemical potential. We find that at 0 K when the typical EE interaction energy is much larger than the broadening of the single-electron density of states (SDOS), the renormalization effects due to exchange and correlation become large enough to significantly alter the shape of the TDOS and can even make the TDOS assume negative values. These renormalization effects are strongest somewhere between the center and the tails of the Landau level (LL). Our calculated curves for the negative TDOS are in qualitative agreement with the experimental results of Kravchenko, Pudalov, and Semenchinsky[5] and Eisenstein, Pfeiffer, and West.[6] Although negative TDOS does not imply an instability; nevertheless, we find that the renormalization effects that make the TDOS nega-

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tive could also lead to a divergence in the static charge susceptibility at some nonzero values of the momentum and thus lead to a CDW instability. We present an analytic expression for the CDW instability condition for a strictly two-dimensional Hartree-Fock (HF) case by taking into account the effects due to both EI interactions and EE interactions. We consider only the lowest Landau level (LL) because the typical EE interaction energy is the largest for this level and thus the above effects are most likely to occur here. For simplicity, we ignore spin effects.

The rest of the paper is organized as follows. In Sec. II we model the modulation-doped heterostructure as a capacitor and obtain the total compressibility in terms of the TDOS and the energy stored in the capacitor. In Sec. III we obtain the TDOS in terms of the long-wavelength static polarizability by taking into account the vertex corrections due to both EE interactions and EI interactions. Next in Sec. IV we derive the TDOS from the density derivative of the chemical potential again by including both EE interactions and EI interactions. Then in Sec. V we show that a CDW instability could result due to the same renormalization effects that produce negative TDOS. Here we also give the HF criterion for the CDW phase transition. Lastly, in Sec. VI we give our conclusions and present possible future problems.

II. BACKGROUND

We begin by considering the following model for the modulation-doped heterostructure. The two-dimensional electron gas (2DEG) and the donors that contribute electrons to the 2DEG are separated by a spacer. These donors are regarded as long range or remote scatterers. In addition to these, there are some impurities due to unintentional doping in the region of the 2DEG and these can be treated as short-range scatterers. All the impurities are assumed to be randomly distributed. The system of the remote donors and the electron gas can be regarded as a capacitor with stored potential energy due to the separation of opposite charges. Thus, unlike the case of a jellium model, we have a positive contribution to the Hartree term in modulation-doped heterostructures.

The thermodynamic density of states D_T is defined by

$$\frac{1}{D_T} \equiv \frac{d\mu}{dn}, \tag{1}$$

where μ is the chemical potential and is given by the shift of the Fermi energy (E_F) with respect to the noninteracting value of the bottom of the subband energy (E_0)

$$\mu = E_F - E_0. \tag{2}$$

The compressibility of the total system is related to the above quantities through

$$\frac{1}{\kappa} = \frac{n^2}{L}\frac{d^2E_{\rm tot}}{dn^2} = \frac{n^2}{L}\frac{dE_F}{dn} = \frac{n^2}{L}\left[\frac{dE_0}{dn} + \frac{1}{D_T}\right], \tag{3}$$

where $E_{\rm tot}$ is the total energy per unit area of the system and L is the length of the system in the third dimension. Strictly speaking, one should also include the rigidity of the background in the above expression for κ. Only if the background is totally flexible (i.e., offers no resistance to deformation) will the compressibility be given by Eq. (3). Now, it is well known that the system as a whole becomes unstable when the total compressibility becomes negative. Thus negative TDOS does not necessarily imply an instability because the compressibility could still be positive due to the rigidity of the background as well as due to the energy stored in the capacitor [i.e., the dE_0/dn term in Eq. (3)]. However, later on we will show, within the HF approximation, that when the renormalization effects, which make the TDOS negative, are sufficiently large a CDW instability will result in the uniform electron gas.

III. LONG-WAVELENGTH STATIC POLARIZABILITY

We will now show that at 0 K and high mobilities the TDOS becomes negative. The TDOS can be derived in two different ways. In this section we will present the first approach involving obtaining the long-wavelength zero-frequency limit of the polarizability $\chi(q,\omega)$. The dielectric function $\epsilon(q,\omega)$ is in general given in terms of the polarizability as follows:

$$\epsilon(q,\omega) \equiv 1 - V_q\chi(q,\omega), \tag{4}$$

where

$$\chi(q,\omega) \equiv \frac{\chi_0(q,\omega)}{1 + G_+(q,\omega)V_q\chi_0(q,\omega)}. \tag{5}$$

In the above equation V_q is the Fourier transform of the Coulombic interaction, χ_0 is the Lindhard polarizability of a noninteracting electron gas, and G_+ is the many-body local field which takes into account exchange and correlation effects.[16] Earlier on, the SDOS was derived within the framework of the self-consistent Born approximation (SCBA).[17] To obtain the TDOS, we will now proceed to derive the static long-wavelength value of the polarizability χ within the framework of the SCBA. The vertex corrections due to the effective Coulombic EE interactions are treated on the same footing as the vertex corrections due to the EI interactions as depicted in Fig. 1. We consider only the lowest LL and ignore spin effects in our analysis. The effective EE interaction is assumed to be a statically screened Coulombic interaction and thus corresponds to the screened Hartree-Fock (SHF) self-energy.[18] Next, we note that the vertex corrections due to the EE interactions and the EI interactions in the polarizability can be rearranged to obtain a series of random-phase-approximation-like bubbles with the bubbles containing only the vertex corrections due to the EI interactions and with the bubbles connected by the EE interactions. By inspection one can see that there is a one-to-one correspondence between the series of bubbles and the ladder diagrams corresponding to Fig. 1(b). We then sum up all the resulting bubbles to arrive at the following expression for the TDOS (Ref. 19):

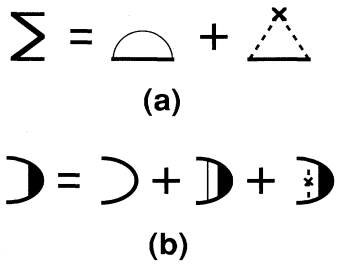

FIG. 1. (a) The self-energy Σ as the sum of self-energies due to the EE interactions and the EI interactions; (b) vertex corrections consistent with the self-energy. The thick solid line corresponds to the interacting electron Green's function, the thin solid to the screened Coulombic interaction, and the dashed line to the EI interaction.

$$D_T \equiv -\chi(q \to 0, 0) = \frac{D_{\rm imp}(\mu)}{1 - \sum_{\mathbf{k}} \frac{V_k J_{00}^2(kl)}{\epsilon(k,0)} \pi l^2 D_{\rm imp}(\mu)}, \quad (6)$$

where $D_{\rm imp}(\mu)$ is the SDOS at the Fermi surface and $J_{00}(kl)$ [$= \exp(-k^2l^2/4)$] is proportional to the probability amplitude of scattering an electron in the lowest LL back to the same LL. Furthermore, the degeneracy of the LL is taken to be $1/\pi l^2$ with l being the cyclotron radius. For the strictly two-dimensional HF situation, Eq. (6) simplifies to the following form:

$$D_T = \frac{D_{\rm imp}(\mu)}{1 - \frac{e^2}{\epsilon_S l}\sqrt{\frac{\pi}{2}} \pi l^2 D_{\rm imp}(\mu)}, \quad (7)$$

where ϵ_S is the dielectric constant. As the mobility of the sample is increased, the denominator on the right-hand side of Eq. (7) first becomes zero in the center of the LL leading to a divergence of the TDOS. Thus, for the HF case the negative values of the TDOS are most likely to occur in the center of the LL and within the SCBA the condition for this is $\Gamma\epsilon_S l/e^2 \leq \sqrt{\frac{2}{\pi}}$ where Γ is the broadening parameter.

In Fig. 2 we have plotted the TDOS as a function of the filling factor ν for the SHF case using Eq. (6) and the random-phase-approximation value of $\epsilon(k,0)$ [$= 1 - V_k\chi_0(k,0)$]. Here the Lindhard polarizability within the SCBA, on neglecting coupling between adjacent LL's, is given by[17]

$$\chi_0(q,0) = \frac{k_B T}{\pi l^2} \sum_{i\epsilon} \frac{G_0(i\epsilon) G_0(i\epsilon) J_{00}^2(ql)}{1 - \frac{\Gamma^2}{4} G_0(i\epsilon) G_0(i\epsilon) J_{00}^2(ql)}, \quad (8)$$

where G_0 is the interacting electron Green's function in the lowest LL. Furthermore, the chemical potential is obtained from the approximation

$$n = \int_{-\infty}^{\mu} dE\, D_{\rm imp}\left[\alpha(E)\right], \quad (9)$$

where $\alpha(E) = (E - 0.5\hbar\omega_C - \Sigma_{\rm EE})/\Gamma$ with ω_C being the noninteracting cyclotron energy and $\Sigma_{\rm EE}$ being the self-energy due to EE interactions. Our calculations were performed for GaAs-$Al_xGa_{1-x}As$ heterostructures. The quasi-two-dimensional nature of the electron gas has been taken into account by using a Fang-Howard type of wave function[20] with an average extent of 100 Å in the third dimension. In these heterostructures, we have taken the bare band mass of the electron to be $0.067m_e$ and the dielectric constant ϵ_S to be 12.4. In all our calculations, the broadening of the LL's was assumed to be only due to short-range scatterers and is taken to be given by the SCBA value $(\frac{2}{\pi}\hbar\omega_c\frac{\hbar}{\tau})^{1/2}$, where τ is the relaxation time in the absence of any magnetic fields.[17]

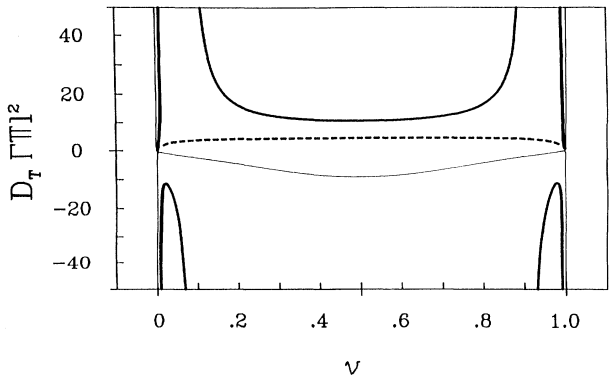

FIG. 2. Dimensionless TDOS $D_T\Gamma\pi l^2$ vs ν at 0 K obtained from the SCBA static long-wavelength polarizability given in Eq. (6). The curves were calculated at 10 T for the broadening $\Gamma = 2\Gamma_{\rm SCBA}$ and the following mobilities: (a) 1×10^6 cm^2/V s (thick dashed curve); (b) 2×10^6 cm^2/V s (thick solid curve); (c) 5×10^6 cm^2/V s (thin solid curve).

The curves in Fig. 2 were obtained at 0 K and for various values of the mobility. As the value of the mobility increases (i.e., as Γ decreases), the EE interaction term becomes more important in comparison to Γ and the TDOS, as shown in Fig. 2, differs more from its semielliptic shape of the SDOS. Furthermore, unlike the HF case, the divergence in the TDOS is most likely to occur away from the center of the LL on account of the screening effects. As the mobility increases from 1×10^6 cm^2/V s to 2×10^6 cm^2/V s the TDOS changes from being positive in the entire LL to a case with positive values in the center and the tails of the LL and negative values in between these two regions. In the latter case, depicted by the thick solid curve in Fig. 2, the TDOS diverges in the lower (upper) half of the LL around $\nu = 0.1$ (0.9) and $\nu = 0$ (1). The thick solid curve is in qualitative agreement with the experimental observations reported in Ref. 5. At an even higher mobility of 5×10^6 cm^2/V s the TDOS is negative over the entire LL except in the extreme tail regions where it diverges. This can be seen in the thin solid curve in Fig. 2 which is in qualitative agreement with the experimental results of Kravchenko, Pudalov, and Semenchinsky[5] and Eisenstein, Pfeiffer, and West.[6] However, unlike our curves which are symmetric about the center of the LL, these two groups observe larger negative values for the inverse of the TDOS around even filling factors as compared to around odd filling factors. This is probably due to weaker renormalization effects around odd filling factors arising due to the larger

screening effects produced by the overlap of LL's of the opposite spin. Since we ignore spin effects we do not take this effect into account around $\nu = 1$. In the next section we will discuss the divergence of the TDOS in the extreme tail regions of the LL.

Our calculations were done at a fixed magnetic field and varying density. Alternately, one could fix the density and vary the magnetic field. However, varying the magnetic field does not change the ratio $e^2/\epsilon_S l\Gamma$ for short-range scatterers. The only change is in the form of the Coulombic interaction which becomes more quasi-two-dimensional as the magnetic field increases. This is because when there is an increase in the magnetic field there is a corresponding decrease in the ratio of the cyclotron radius with respect to the average extent of the electronic wave function.

IV. DENSITY DERIVATIVE OF THE CHEMICAL POTENTIAL

We will now obtain the TDOS by an alternate approach involving use of its definition [see Eq. (1)]. In this approach, we include the Coulomb hole term in the electronic self-energy and also account for the change in the dielectric function as the density is varied. The chemical potential of the modulation-doped heterostructure at 0 K is given by

$$\mu = \tfrac{1}{2}\hbar\omega_C + \Sigma_{\rm EI} + \Sigma_{\rm EE}, \tag{10}$$

with $\Sigma_{\rm EI}$ being the self-energy due to the EI interactions. It is understood that the chemical potential in the above equation does not contain the Hartree contribution. Then from Eqs. (1) and (10) we obtain the following expression for the TDOS:

$$\frac{1}{D_T} = \frac{d\Sigma_{\rm EI}}{dn} + \frac{d\Sigma_{\rm EE}}{dn} = \frac{1}{D_{\rm imp}(\mu)} + \frac{d\Sigma_{\rm EE}}{dn}. \tag{11}$$

To evaluate the derivative of the self-energy due to the EE interactions we use the following quasistatic approximation (see Ref. 21):

$$\Sigma_{\rm EE} = -\sum_{\mathbf{q}} \frac{V_q J_{00}^2(ql)\nu}{\epsilon(q,0)} + \tfrac{1}{2}\sum_{\mathbf{q}} V_q J_{00}^2(ql)\left[\frac{1}{\epsilon(q,0)} - 1\right], \tag{12}$$

where the last term is due to the Coulomb hole contribution. To perform calculations, the dielectric function is assumed to be of the Thomas-Fermi type,

$$\epsilon(q,0) = 1 + V_q J_{00}^2(ql) D_{\rm imp}(\mu). \tag{13}$$

Then assuming that the SDOS $D_{\rm imp}(E)$ is a Gaussian[22] we obtain

$$\frac{d\Sigma_{\rm EE}}{dn} = -\sum_{\mathbf{q}} \frac{V_q J_{00}^2(ql)\pi l^2}{\epsilon(q,0)} - \sum_{\mathbf{q}} \frac{V_q^2 J_{00}^4(ql)}{\epsilon(q,0)^2}(\nu - \tfrac{1}{2})\frac{\alpha(\mu)}{\Gamma}. \tag{14}$$

Compared to the TDOS given by Eq. (6), the TDOS obtained from Eqs. (11) and (14) contain an additional term, namely the second term on the right-hand side of Eq. (14). The renormalization term of the TDOS, i.e., $d\Sigma_{\rm EE}/dn$ given in Eq. (14), has particle-hole symmetry, as it should, because both the SDOS and the TDOS have this symmetry. Furthermore, if the Coulomb hole term in Eq. (12) was not included the particle-hole symmetry is violated. Also, the SCBA SDOS would not be appropriate for the above method because its derivative diverges at the edges where it has sharp cutoffs.[17]

For the case where D_T is calculated using the formulas in Eqs. (11), (13), and (14), the TDOS values are plotted in Fig. 3 at 0 K. Similar to Fig. 2, here too as the parameter $e^2/\epsilon_S l\Gamma$ increases the deviation from the Gaussian shape increases for the TDOS. Furthermore, as before, at all filling factors away from the tails of the LL there is a critical value of the mobility at which the TDOS becomes negative and this critical value is the least somewhere between the center and the tails of the LL. In Fig. 3, for all the mobilities larger than 2.5×10^4 cm^2/V s, the TDOS diverges in the extreme tail regions of the LL. The corresponding curves are again qualitatively in agreement with the values of the TDOS observed by the authors of Refs. 5 and 6. Although Figs. 2 and 3 are qualitatively similar, nevertheless quantitatively they differ because of the different forms of the SDOS, the different screening effects, and also because additional Coulomb hole effects are included in Fig. 3.

Here we would like to point out that if we were to consider the effect of remote scatterers on the broadening, as we approach the tails of the LL, the contribution of the long-range scatterers to the broadening increases due to a decrease in screening. However, close to the center of the LL, the screening of the long-range scatterers would be most effective and the contribution of these scatterers to the broadening would be at its minimum.

In Figs. 2 and 3, as we approach the tail regions of the

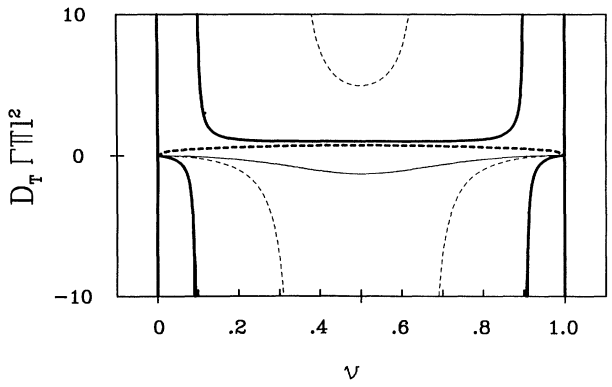

FIG. 3. Plot of the dimensionless TDOS vs ν at 0 K obtained using Eqs. (11), (13), and (14) and a Gaussian SDOS. The curves were calculated at 10 T for $\Gamma = \Gamma_{\rm SCBA}$ and the following values of the mobility: (a) 2.5×10^4 cm^2/V s (thick dashed curve); (b) 1×10^5 cm^2/V s (thick solid curve); (c) 1×10^6 cm^2/V s (thin dashed curve); (d) 1×10^7 cm^2/V s (thin solid curve).

LL from the center, the TDOS, if negative, diverges and changes sign. This is because the TDOS assumes positive values in the extreme tail regions due to vanishing values of the SDOS. It should, however, be noted that the linear screening picture we assume does not actually hold in these regions.[23] In the tails of the LL, the electron liquid breaks apart into puddles and can no longer screen the remote donors. The electron then sees fully the long-range fluctuations in the remote donor charge distribution. Furthermore, approximating the SDOS by a semiellipse or by a Gaussian is no longer valid in these regions.

V. CHARGE-DENSITY-WAVE INSTABILITY

In this section we will show that when the renormalization effects that make the TDOS negative are sufficiently large a CDW instability could result. The renormalization effects are accounted for through the many-body local field $G_+(q,\omega)$ defined in Eq. (5). We will now study the charge susceptibility χ_C for possible CDW instability when the many-body local field is large enough, i.e., G_+ is of the order unity. The charge susceptibility, on using Eqs. (4) and (5), is given by

$$\chi_C(q,\omega) \equiv \frac{1}{V_q}\left(\frac{1}{\epsilon(q,\omega)} - 1\right) = \frac{\chi_0(q,\omega)}{1-[1-G_+(q,\omega)]V_q\chi_0(q,\omega)}. \tag{15}$$

Then, for a strictly two-dimensional system, the Hartree-Fock SCBA condition for the denominator of the above equation to change sign is given by (see Fig. 1)

$$\frac{\Gamma\epsilon_S l}{e^2} = \left[\frac{4}{\pi q l} - \sqrt{\frac{2}{\pi}}\frac{I_0(q^2l^2/4)}{J_{00}(ql)}\right]\chi_0(q,0)\frac{\pi^2 l^2\Gamma}{2}, \tag{16}$$

where $I_0(y)$ is the modified Bessel function of order zero and the static Lindhard polarizability [see Eq. (8)] can be expressed as a compact formula as follows:

$$\begin{aligned}\chi_0(q,0) &= -\mathrm{Im}\frac{4}{\pi^2 l^2\Gamma^2}\int_{-\infty}^{\infty} d\epsilon\, n_F(\epsilon)\left\{\frac{1}{1-\frac{\Gamma^2}{4}G_0(\epsilon+i\delta)G_0(\epsilon+i\delta)J_{00}^2(ql)}\right\} \\ &= -\mathrm{Im}\frac{4}{\pi^2 l^2\Gamma^2}\int_{-\infty}^{\infty} d\epsilon\, n_F(\epsilon)\left\{\frac{1}{1-J_{00}^2(ql)\exp[-2i\cos^{-1}\alpha(\epsilon)]}\right\} \\ &= -\frac{2\sqrt{1-\alpha^2(\mu)}}{\pi^2 l^2\Gamma}\left[1-\frac{\arctan R(ql)}{R(ql)}\right],\end{aligned} \tag{17}$$

with $R = [2J_{00}\sqrt{1-\alpha^2(\mu)}]/(1-J_{00}^2)$. We now observe that $\chi_0(q,0)$ in the above equation is independent of the sign of $\alpha(\mu)$ and thus ensures particle-hole symmetry. In Fig. 4 we plot the instability condition given by Eq. (16). The minimum mobility at which the CDW occurs increases as we go from the center to the edge of the LL with the dimensionless CDW wave vector ql monotonically decreasing from 2.12 to 1.52. It is also of interest to note that the broadening at which the TDOS turns negative in the HF approximation [i.e., $\Gamma = e^2/(\epsilon_S l)\sqrt{2/\pi}\sqrt{1-\alpha^2(\mu)}$] is always higher than that leading to a CDW instability. However, the cause for the negative TDOS and the CDW instability are the renormalization effects given through the many-body local field $G_+(q,\omega)$.

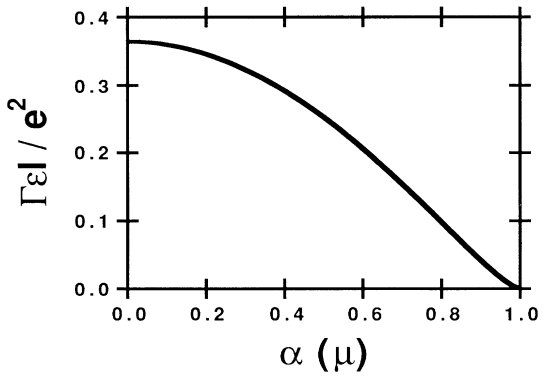

FIG. 4. Plot of the dimensionless broadening $\Gamma\epsilon_S l/e^2$ vs the dimensionless chemical potential shift $\alpha(\mu)$ expressing the CDW instability condition for a two-dimensional Hartree-Fock SCBA case.

Now the above analysis, within the HF approximation, predicts a second-order CDW phase transition when the conditions given by Eq. (16) are met. However, this does not imply that a state with a lower energy other than the predicted CDW state is not possible. In fact, in high mobility samples, we know that at odd denominator filling factors the lowest-energy state has value zero for the TDOS due to special correlations among the electrons and these states cannot be derived within a ladder-diagram type of approach. Furthermore, it is not clear whether the transition to a Wigner crystal (which is a superposition of CDW states) can be understood in terms of a topological long-range order[24] or in terms of the charge susceptibility. Thus although the phase transition could be different from the predicted second-order one, nevertheless the uniform electron-gas state is no longer stable and the CDW state given by Eq. (16) is one of the possible candidates for the lowest energy.

We will now remark on the consequences of including

additional effects on the CDW instability. First, we note that taking screening effects into account would shift the most likely position, at which CDW instability occurs, from the center of the LL to some point between the center and the edge of the LL. Second, including the effects due to the extent of the electronic wave function in the third dimension would decrease the strength of the electronic interactions and thus require higher mobilities for the occurrence of the CDW instability. Last, considering finite-temperature effects is similar to considering lower mobilities.[25]

VI. CONCLUSIONS

In conclusion, we say that taking the EE interactions into account can substantially alter the shape of the TDOS from the SDOS obtained by including only the EI interaction effects. At 0 K and high mobilities the renormalization effects due to exchange and correlation make the TDOS negative. We show this by calculating the TDOS from the static long-wavelength polarizability as well as from the density derivative of the chemical potential. The two approaches yield qualitatively similar results which are also qualitatively corroborated by the experimental results of Kravchenko, Pudalov, and Semenchinsky[5] and Eisenstein, Pfeiffer, and West.[6] Furthermore, for sufficiently strong renormalization effects there could be a CDW instability in the electron gas. We obtain an analytic expression for the CDW transition within a HF treatment by considering the poles of the static charge susceptibility. We find that for the HF case the CDW instability always sets in after the TDOS turns negative.

In the future, to calculate the TDOS, one could include the effects of long-range scatterers, the effects of dynamic screening, and the effects of nonlinear screening. As for the CDW transition, one could include screening effects and obtain the corresponding criterion for the instability. Figuring out the combination of CDW's that would minimize the total energy is also a problem for possible future studies.

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