Dynamics of two-dimensional excitons in semiconductor heterostructures

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Two different processes of quasi-two-dimensional exciton dynamics involving acoustic phonons are studied for semiconductor quantum wells. The first process involves the scattering of an exciton from an initial state to a final state through three different channels and the second process involves the decay of an exciton into a free-electron-hole pair by absorption of an acoustic phonon. Rates for both processes are calculated using a variational wave function for the two-dimensional exciton. The calculated results agree quite well with recent experimental results.

I. INTRODUCTION

One of the most active topics in current semiconductor research for both fundamental and practical investigations is the study of the optoelectronic properties of excitons in semiconductor quantum wells.[1–4] The confinement of a quasi-two-dimensional (2D) exciton between two parallel potential barriers is responsible for physical properties with importance in the fabrication of photonic devices such as quantum-well lasers and fast electro-optic switches.[4–6] Among quantum-well systems, GaAs/AlGaAs is the simplest and most extensively studied. In this system GaAs acts as a quantum well (QW) and AlGaAs as barrier between quantum wells. In such wells, excitons get compressed perpendicularly towards the quantum-well layers, therefore the binding energy[7,8] and oscillator strength[9] of excitons in quantum wells are larger than those of excitons in bulk. This enables one to observe the optical-absorption spectra of excitons in quantum wells rather easily even at room temperature.[10,11]

Much work has been done in the area of dynamics of excitons in bulk,[12] and some in quantum-well structures[13–15] as well. Takagahara[15] has considered the scattering in the localized to weakly delocalized region to study the excitonic relaxation processes in alloyed quantum-well structures. However, in this paper we investigate the decay and scattering of free (delocalized) excitons due to acoustic phonons in quantum wells, which are dominant at low temperatures. In general, however, exciton scattering in semiconductors is caused by imperfections in the periodic lattice potential. These imperfections can arise due to lattice vibrations, external impurities (both ionized and neutral), and the presence of excess carriers. The kind of scattering we have considered here is essentially the scattering of an exciton from an initial state of wave vector $\mathbf{K}$ to another state of wave vector $\mathbf{K}'$ by emitting or absorbing an acoustic phonon. Such scattering of excitons due to phonons provide an important transfer mechanism of both energy and momentum in condensed matters. Following our earlier work[16] we have also calculated the rate of decay of an exciton into an electron-hole pair due to interaction with acoustic phonons.

In semiconductor quantum wells, due to the reduction in symmetry along the axis of growth of the quantum-well structure and the presence of band discontinuities at the interfaces, there exist two kinds of excitons: the heavy-hole (HH) and the light-hole (LH) excitons.[17] Here we investigate the mechanism of decay and scattering for both types of excitons in quantum wells. The rates of both decay and scattering of excitons obtained here agree quite well with recent experimental results.

II. THEORY

A. Wave functions of initial and final exciton states in quantum wells

The exciton wave functions in quantum-well systems differ from those in the bulk because of the confinement within the potential well. Here we use a wave function for an exciton in a single QW obtained within the two-band approximation, i.e., only the lowest electron subband and highest hole subband are included. We neglect any coupling between the light and heavy holes. The exciton state vector denoted by $|\alpha,\mathbf{K}\rangle$ can then be written as[18]

$$|\alpha,\mathbf{K}\rangle=\frac{v_0}{L}\sum_{\mathbf{r}_e,\mathbf{r}_h}\exp(i\mathbf{K}\cdot\mathbf{R})f_\alpha(\mathbf{r}_e-\mathbf{r}_h,z_e,z_h)\times a^\dagger_{1,\mathbf{r}_e}a_{0,\mathbf{r}_h}|0,n\rangle \quad (1)$$

where v_0 is the volume of the unit cell and L is the quantum well width, $a^\dagger_{1,\mathbf{r}_e}$ is the creation operator of an electron in the conduction band, denoted by 1, and $a_{1,\mathbf{r}_h}$ is the annihilation operator of an electron in the valence band, denoted by 0, at positions r_e and r_h, respectively. In Eq. (1), $|0,n\rangle$ is given by

$$|0,n\rangle=|0\rangle|n\rangle \quad (2)$$

where $|0\rangle$ is the electronic vacuum state of the system representing completely filled valence and empty conduction bands and

$$|n\rangle=|n_1,n_2,\ldots,n_q\rangle$$

where $n_{\mathbf{q}}$ is the occupation number of phonons with wave vector $\mathbf{q}$. $\mathbf{R}$ is the coordinate of the center of mass of an exciton defined by

$$R=\frac{m_e\mathbf{r}_e+m_h\mathbf{r}_h}{m_e+m_h} \tag{3}$$

where m_e (m_h) is the effective mass of electron (hole) and $f_\alpha(\mathbf{r}_e-\mathbf{r}_h,z_e,z_h)$ is the envelope function for an exciton in the αth state in a quantum well. Here we consider the lowest ($1s$) exciton state for which we assume a variational envelope function[19,20] as

$$f_{1s}(\mathbf{r}_e-\mathbf{r}_h,z_e,z_h)=A\exp\left[-\frac{br}{2}\right]\cos\frac{\pi z_e}{L}\cos\frac{\pi z_h}{L} \tag{4}$$

where A is the normalization constant and b is a variational parameter. The form of the wave function (4) is obtained by assuming that the QW is of an infinite barrier height and by neglecting the relative motion of electrons and holes along the z direction. Writing the operators of (1) in the Bloch representation and converting the summation over $\mathbf{r}_e$ and $\mathbf{r}_h$ into corresponding integrations, one gets

$$|\alpha=1s,\mathbf{K}\rangle=\sum_{\mathbf{r}_e,\mathbf{r}_h}F_{1s}(\mathbf{k},\mathbf{k}',\mathbf{K})\delta_{\mathbf{k}-\mathbf{k}',\mathbf{K}}a^\dagger_{1,\mathbf{k}}a_{0,\mathbf{k}'}|0,n\rangle \tag{5}$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the wave vectors of the electron and hole, respectively, and

$$F_{1s}(\mathbf{k},\mathbf{k}',\mathbf{K})=\frac{1}{L}\int d^2r\int dz_e\int dz_h f_{1s}(\mathbf{r}_e-\mathbf{r}_h,z_e,z_h)\times\exp[i(\alpha_e\mathbf{K}-\mathbf{k})\cdot\mathbf{r}-ik_zz_e+ik_z'z_h]\,, \tag{6}$$

where

$$\alpha_e=\frac{m_e}{(m_e+m_h)}\,,\quad \alpha_h=\frac{m_h}{(m_e+m_h)}\,.$$

In Eqs. (4)–(6) all position and wave vectors have been resolved into components parallel and perpendicular to the quantum-well interfaces as $(\mathbf{r},z)$ and $(\mathbf{k},k_z)$. For the calculation of both rates of scattering and decay of excitons, we will consider the form of the initial-state wave function of an exciton as given in (5).

For calculating the rate of scattering, we consider that an exciton gets scattered from its initial state of wave vector $\mathbf{K}$ to a final state through the following three channels.

(i) Exciton ($\mathbf{K}$)$\rightarrow$ exciton ($\mathbf{K}'$)+ virtual phonons.

(ii) Exciton ($\mathbf{K}$)$\rightarrow$ exciton ($\mathbf{k}$)+ acoustic phonon emitted ($\mathbf{K}-\mathbf{k}$).

(iii) Exciton ($\mathbf{K}$)$\rightarrow$ exciton ($\mathbf{k}$)+ acoustic phonon absorbed ($\mathbf{k}-\mathbf{K}$).

The corresponding wave functions of the three types of final states described above can then be written as

$$|f_0\rangle=|1s,\mathbf{K}'\rangle\,, \tag{7a}$$

$$|f_+\rangle=\sum_{\mathbf{k}}b^\dagger_{\mathbf{K}-\mathbf{k}}|1s,\mathbf{k}\rangle\,, \tag{7b}$$

$$|f_-\rangle=\sum_{\mathbf{k}}b_{\mathbf{k}-\mathbf{K}}|1s,\mathbf{k}\rangle \tag{7c}$$

where $b^\dagger_{\mathbf{k}}$ ($b_{\mathbf{k}}$) represents a phonon creation (annihilation) operator of wave vector $\mathbf{k}$.

For the decay process, we consider that initially an exciton with wave vector $\mathbf{K}$ decays into an electron-hole pair by absorbing an acoustic phonon. Thus the wave function of the final state of the decay process can be written as

$$|f_d\rangle=\sum_{\mathbf{k}}a^\dagger_{1,\mathbf{k}}a_{0,\mathbf{k}}b_{\mathbf{k}-\mathbf{K}}|0,n\rangle\,. \tag{8}$$

B. Exciton-phonon interaction in quantum wells

Having defined the eigenvectors of initial and final states we assume that the transition from initial to final states takes place due to a quasi-2D exciton-phonon interaction operator. Using the exciton state vectors and variational wave function in (3), the interaction operator can be obtained as[18]

$$H^{DF(2\,D)}_{\text{ex-ph}}=4\sum_{\mathbf{K},\mathbf{K}',q_z}\left[\frac{\hbar[(\mathbf{K}-\mathbf{K}')^2+q_z^2]^{1/2}}{2\rho uV}\right]^{1/2}\left[\frac{D_cG(q_zL)}{[1+(\beta_h/2)^2]^{3/2}}-\frac{D_vG(q_zL)}{[1+(\beta_e/2)^2]^{3/2}}\right]\times|1s,\mathbf{K}'\rangle\langle\mathbf{K},1s|(b_{\mathbf{q}}+b^\dagger_{-\mathbf{q}})\delta_{\mathbf{K}-\mathbf{K}',\mathbf{q}} \tag{9}$$

where D_c and D_v are the deformation potentials for the conduction and valence bands, ρ is the mass density, and u is the sound velocity in the material, $\beta_h=(q\alpha_h)/b$ and $\beta_e=(q\alpha_e)/b$. The δ function conserves the momentum of the scattered exciton and phonon along the x-y plane. The function $G(x)$ is derived as

$$G(x)=\frac{1}{4}\left[\frac{\sin(x+2\pi)}{x+2\pi}+\frac{\sin(x-2\pi)}{x-2\pi}+\frac{\sin x}{x}\right]\,. \tag{10}$$

It is to be noted that $G(x)$ differs from the function evaluated in Ref. 18 due to the use of a different variational envelope function.

C. Matrix element for exciton scattering

Using the final state given in (7a), the matrix element for the transition of an excitonic state $|1s,\mathbf{K}\rangle$ (5), to another state $|1s,\mathbf{K}'\rangle$ (7a), involving virtual phonons, is obtained as

$$H_{\text{ex-ph}}^{DF(2\text{ D})}=4\sum_{q_z}\left[\frac{\hbar[(\mathbf{K}-\mathbf{K}')^2+q_z^2]^{1/2}}{2\rho uV}\right]^{1/2}\left[\frac{D_cG(q_zL)}{[1+(\beta_h/2)^2]^{3/2}}-\frac{D_vG(q_zL)}{[1+(\beta_e/2)^2]^{3/2}}\right](n_q+\tfrac{1}{2}\pm\tfrac{1}{2})^{1/2}\delta_{\mathbf{K}-\mathbf{K}',\mathbf{q}}\ , \tag{11}$$

where the plus and minus correspond to a phonon emission and absorption process, respectively, and for simplification, we have used $G(x)=\frac{1}{4}$, which is valid for small values of x. $\bar{n}_q$ is the thermalized average number of phonons given by

$$\bar{n}_q=\frac{1}{\exp[\hbar\omega(q)/K_BT]-1}\ , \tag{12}$$

where $\hbar\omega(q)$ is the energy of an acoustic phonon with wave vector $\mathbf{q}$. The acoustic-phonon energy $\hbar\omega(q)$ involved here is smaller than the thermal energy K_BT and hence $K_BT/\hbar\omega(q)\gg1$. Using this inequality in Eq. (12), we can approximate

$$\bar{n}_q+1\simeq\bar{n}_q\simeq\frac{K_BT}{\hbar\omega(q)}=\frac{K_BT}{\hbar u|q|}\ . \tag{13}$$

The matrix elements for the second and third scattering processes described by (7b) and (7c) can be derived in a similar way using (5), (7b), (7c), and (9) as

$$\langle f_\pm|H_{\text{ex-ph}}^{DF(2\text{ D})}|1s,\mathbf{K}\rangle=4\sum_{q_z}\left[\frac{\hbar[(\mathbf{K}-\mathbf{k})^2+q_z^2]^{1/2}}{2\rho uV}\right]^{1/2}\left[\frac{D_cG(q_zL)}{[1+(\beta_h/2)^2]^{3/2}}-\frac{D_vG(q_zL)}{[1+(\beta_e/2)^2]^{3/2}}\right]\xi_q^{\pm}\delta_{\mathbf{K}-\mathbf{k},\mathbf{q}}\ , \tag{14}$$

where $\xi_q^+=\bar{n}_q+1$ corresponds to the scattering of excitons involving emission of a phonon (7b) and $\xi_q^-=\bar{n}_q$ corresponds to that involving absorption of a phonon (7c). It is to be noted that the expression of the matrix element in (14) depends linearly on the phonon number $\bar{n}_q$ whereas that in (11) depends on the square root of the phonon number.

D. Matrix element for exciton decay

The matrix element for the exciton decay process of transition from an initial state of a free exciton to a final state of an electron-hole pair by absorbing an acoustic phonon can be obtained using (5), (6), (8), and (9) as

$$\langle f_d|H_{\text{ex-ph}}^{DF(2\text{ D})}|1s,\mathbf{K}=0\rangle=\left[\frac{8\pi}{S}\right]^{1/2}\sum_q\left[\frac{\hbar q}{2\rho uV}\right]^{1/2}[D_c-D_v]\times\frac{b^2}{(b^2+q^2)^{3/2}}n_q^{1/2} \tag{15}$$

where S is the surface area of quantum well. Also the exciton wave vector in the initial state is assumed to be at $K=0$. Using (13) in (15), the latter can be further simplified as

$$\langle f_d|H_{\text{ex-ph}}^{DF(2\text{ D})}|1s,\mathbf{K}=0\rangle=\left[\frac{8\pi}{S}\right]^{1/2}\left[\frac{K_BT}{2\rho u^2V}\right]^{1/2}[D_c-D_v]\sum_q\frac{b^2}{(b^2+q^2)^{3/2}}\ . \tag{16}$$

The summation over q in (16) gives

$$\sum_q\frac{b^2}{(b^2+q^2)^{3/2}}=\frac{Sb}{2\pi}\ . \tag{17}$$

Using (17) in (16) we get

$$\langle f_d|H_{\text{ex-ph}}^{DF(2\text{ D})}|1s,\mathbf{K}=\mathbf{0}\rangle=\left[\frac{K_BT}{\pi\rho u^2L}\right]^{1/2}[D_c-D_v]b\ . \tag{18}$$

It is to be noted that the elements in (18) are inversely proportional to the quantum-well width L.

E. Transition rates of exciton scattering

Calculations of a transition rate can be done using the Fermi golden rule given by

$$W=\frac{2\pi}{\hbar}\sum_f|\langle f|H^I|i\rangle|^2\delta(E_i-E_f)\ , \tag{19}$$

where $|i\rangle$ and $|f\rangle$ represent the initial and final states with energy E_i and E_f, respectively, and H^I is the interaction operator. Here we first consider the case of exciton scattering from an initial state of exciton with wave vector $\mathbf{K}$ to a final exciton state with wave vector $\mathbf{K}'$ involving virtual phonons. The energy of initial and final states are used as

$$E_i=E_{\text{ex}}+\frac{\hbar^2K^2}{2M^*}+\frac{\hbar^2\pi^2}{2M^*L^2}\ , \tag{20a}$$

$$E_f=E_{\text{ex}}+\frac{\hbar^2K'^2}{2M^*}+\frac{\hbar^2\pi^2}{2M^*L^2}\ , \tag{20b}$$

where E_{ex} is the exciton energy and $M^*=m_e^*+m_h^*$. The

last terms of both (20a) and (20b) are due to exciton confinement in quantum wells. Using (11) and (20) in (19), we get

$$W_s(K)=\frac{\pi b^6 K_B T}{\hbar\rho u^2 V}\sum_{q_z,\mathbf{K}'}\left[\frac{D_c}{[1+(\beta_h/2)^2]^{3/2}}-\frac{D_v}{[1+(\beta_e/2)^2]^{3/2}}\right]^2\times\delta(E_{\mathbf{K}}-E_{\mathbf{K}'}) \quad (21)$$

and converting the summation over K' and q into integrals we obtain an expression for the rate of exciton scattering as

$$W_s(K)=\frac{3b^6 K_B T M^*}{4\pi\hbar^3\rho u^2 L}\int_0^{2\pi}\left[\frac{D_c}{[1+(\beta_h/2)^2]^{3/2}}-\frac{D_v}{[1+(\beta_e/2)^2]^{3/2}}\right]^2\times(1-\cos\theta)d\theta\ , \quad (22)$$

where $q^2=4K^2\sin(\theta/2)$ and θ is the angle between $\mathbf{K}$ and $\mathbf{K}'$. To obtain (22), we have considered the exciton scattering to be elastic during the transition, so that initially an exciton with wave vector $\mathbf{K}$ emits a virtual phonon of wave vector $\mathbf{q}$ and then reabsorbs it to get to the final state of wave vector $\mathbf{K}'$. As β_h and β_e depend on q, which in turn depends on θ, it is difficult to solve the integral in (22) analytically. It can, however, be evaluated using the approximation $b\geq\beta_e,\beta_h$, which gives

$$W_s(K)\simeq W(0)-\frac{27K_B T K^2}{2\hbar^3\rho u^2 L b^2 M^*}\times[D_v^2 m_h^2+D_c^2 m_c^2-D_c D_v(m_e^2+m_h^2)] \quad (23)$$

where

$$W_s(0)=\frac{3K_B T M^*}{2\hbar^3\rho u^2 L}[D_c-D_v]^2\ . \quad (24)$$

It is to be noted that $W_s(0)$, which represents the rate of scattering at the bottom of the exciton band ($K=0$), is independent of the variational parameter b.

Next we consider the case of exciton scattering from an initial state of exciton with wave vector $\mathbf{K}$ to a final exciton state with emission or absorption of an acoustic phonon. The energy of the initial exciton state would be the same as in (18a) whereas that of the final state can be written as

$$E_f=E_{\rm ex}+\frac{\hbar^2k^2}{2M^*}+\frac{\hbar^2\pi^2}{2M^*L^2}\pm\hbar\omega(q) \quad (25)$$

where the phonon energies $+\hbar\omega$ and $-\hbar\omega$ correspond to emission and absorption of an acoustic phonon, respectively. Thus using (14), (20b), and (25) in (19), the rate is obtained as

$$W_s^{\pm}(K)=\frac{9\omega_D f^{\pm}(T)M^*}{\hbar^2\rho u^2 L}[D_c-D_v]^2 \quad (26)$$

where

$$f^-(T)=\int_0^1 t^{2.5}[\exp(\hbar\omega_D t/K_B T)-1]^{-1}dt \quad (27a)$$

for the scattering of an exciton with absorption of an acoustic phonon and

$$f^+(T)=\int_0^1 t^{2.5}\{[\exp(\hbar\omega_D t/K_B T)-1]^{-1}+1\}dt \quad (27b)$$

for the scattering of an exciton with emission of an acoustic phonon. The integrals in (27a) and (27b) are later evaluated numerically to obtain values for the scattering rates.

F. Transition rate of exciton decay

To evaluate the exciton decay rate, we replace $\delta(E_i-E_f)$ in (19) by $\delta(\hbar\omega-E_b)$ where E_b is the binding energy of the exciton and $\hbar\omega$ is the energy of an emitted acoustic phonon.[16] Using (18) and (19), the exciton decay rate can be obtained as

$$W_D=\frac{6b^2[D_c-D_v]^2 K_B T E_b^2}{\hbar L\rho u^2(\hbar\omega_D)^3}\ . \quad (28)$$

In order to derive (28), we have converted the summation over q into an integration in energy space where the Debye model for the density of acoustic phonon states given by

$$\rho[\hbar\omega(q)]=\frac{3N(\hbar\omega(q))^2}{(\hbar\omega_D)^3} \quad (29)$$

is used. The use of a three-dimensional density of phonon states for GaAs/GaAlAs is justified because of the small difference in lattice constants of GaAs and GaAlAs. Hence it may not be possible to confine acoustic phonons only to the well region.

III. RESULTS AND DISCUSSION

We have presented here a method of calculating the rates of 2D exciton scattering and decay of 2D excitons in quantum wells through various channels. We will give the calculated result for the case of HH exciton in GaAs-$Al_xGa_{1-x}As$ quantum wells, but our calculations can easily be extended to the case of the LH exciton as well. The rates are calculated using (23), (26), and (28) and values[18] for $D_c=-6.5$ eV, $D_v=3.1$ eV, $u=4.8\times10^5$ cm/s, $\hbar\omega_D=40$ cm^{-1}, $\rho=5.3$ g cm^3, and $b=10^8$ m^{-1}.

Figure 1 shows the rate of the HH exciton scattering involving virtual phonons as a function of the quantum-well width calculated for three different temperatures of 10, 50, and 100 K and for two selected exciton wave vectors of $|\mathbf{K}|=0$ and $|\mathbf{K}|=3\times10^7$ m^{-1} obtained using (23) and (24). As can be seen from Fig. 1, the rates of scattering for exciton wave vectors $|\mathbf{K}|=0$ and $|\mathbf{K}|\neq0$ do not differ from each other appreciably. This, in conjunction with Eq. (23), suggests that the rate of scattering at $|\mathbf{K}|=0$ is dominant. The rate of scattering through channel 1 increases linearly with temperature as can be seen from Eq. (23).

Our calculated rates of HH exciton scattering are ob-

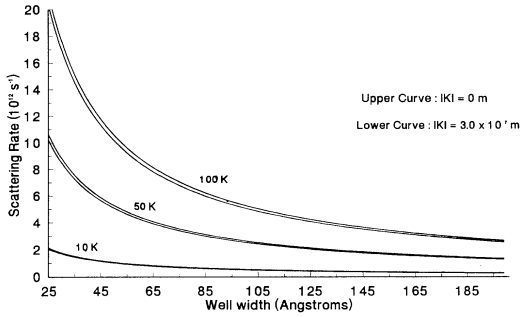

FIG. 1. Rates of exciton scattering by virtual phonons as a function of well width at 10, 50, and 100 K.

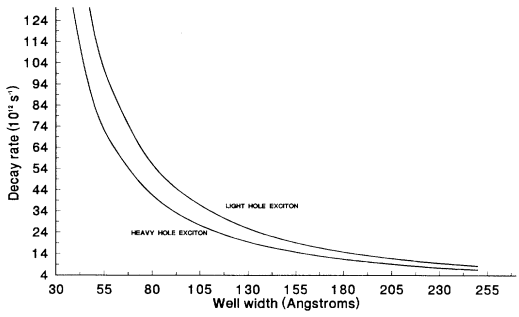

FIG. 3. Rates of exciton decay as a function of well width at 5 K.

tained in the range of about 2×10^{11} to 20×10^{12} s^{-1} at temperatures ranging from 10 to 100 K and for well widths varying from 25 to 200 Å. These results compare well with the experimental rates[21] of 3×10^{11} to 3×10^{12} s^{-1} obtained for the HH exciton at temperatures ranging from 5 to 100 K and for well widths varying from 40 to 140 Å, taking into account the difference in temperature as well as the well width range.

Experimental results[21] also indicate that at a given temperature, excitonic linewidths from wider wells are smaller than those for narrower wells. This also agrees very well with the scattering rate expressions in (23) and (24). The numerical calculations using (23) show that the exciton scattering of the HH exciton is sometimes greater (of the order of M_{HH}/M_{LH}) than the LH exciton. It is thus possible to use this result to calculate the distribution of HH and LH excitons in semiconductor QW's. It is, however, to be noted that experimental results indicate slightly higher scattering rates for LH excitons in comparison with that for HH excitons, but our calculations show that it is the other way around. This may be due to the fact that scattering by the effect of interface roughness, which is not taken into account here, can be expected to influence the LH exciton more than the HH exciton.

Figure 2 shows the variation of the rates of HH exciton scattering with emission and absorption of a single acoustic phonon with temperature. The rates are calculated at a quantum-well width of 100 Å. The variation of rates

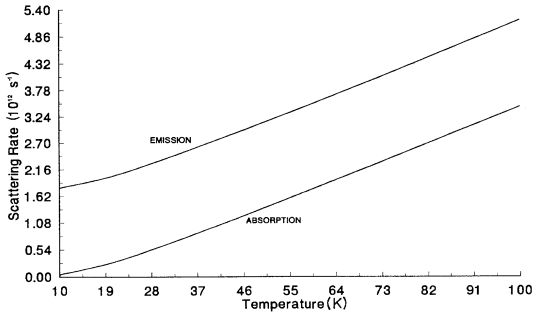

FIG. 2. Rates of exciton scattering with emission/absorption of a single acoustic phonon as a function of temperature at a well width of 100 Å.

with the quantum-well width is not shown here, as it is found to be the same as that shown in Fig. 1 for exciton scattering involving virtual phonons. Both rates of exciton scattering with emission and absorption of acoustic phonons increase nearly linearly with temperature (Fig. 2). From Figs. 1 and 2, it can be seen that the rate of exciton scattering through channel 1 (involving virtual phonons) is higher than that for the other two channels (involving emission and absorption of an acoustic phonon). One can therefore conclude that exciton scattering through channel 1 is more efficient in comparison with that through channels 2 and 3.

It may be desirable to compare the results obtained here with those obtained by Takagahara[15] for localized quasi-2D excitons in InGaAs/InP alloy quantum wells. In the localized regime, the rates of phonon-assisted migration of excitons among localized sites within a quantum-well layer is found to be sensitive to the exciton energy. It may be attributed to the fact that an exciton of higher energy is more likely to migrate than that of lower energy. However, for delocalized excitons, the rate of scattering by acoustic phonons is independent of the exciton energy as can be seen from Eqs. (23) and (26). Our calculated rate of migration of a delocalized exciton is found to be an order of magnitude higher than that calculated by Takagahara. As a localized exciton is less likely to be scattered than a delocalized exciton, a higher rate of migration for delocalized exciton is fully justified.

For the case of delocalized excitons, Takagahara has found that the rate of scattering of delocalized excitons by a single phonon depends on temperature linearly. In our case, such a linear dependence is found only for exciton scattering through channel 1 (involving virtual phonons). However, for channels (ii) and (iii) involving emission and absorption of an acoustic phonon, we have found a nonlinear dependence of the exciton scattering rate on temperature in the low-temperature region (≤ 30 K). This can be justified on the basis of the expression for the average number of phonons given in Eq. (12), which is then used in Eq. (15). As $\bar{n}_q$ (12), the average number of phonons used in the transition matrix element (15), depends nonlinearly on temperature, such a nonlinear behavior in the transition rate of scattering can be expected.

Nash and Mowbray[14] have discussed exciton scattering

by optical phonons where there are more than the three specific channels considered here for the case of acoustic phonons. Exciton scattering by optical phonons, which is dominant at higher temperatures, also includes exciton transitions to higher subbands. This explains the higher rates ($\sim 1.0\times 10^{13}$ s^{-1}) obtained[14] for exciton scattering by optical phonons for well widths ranging up to 300 Å.

In Fig. 3, we have shown rates of both HH and LH excitons decay as a function of the quantum-well width. The exciton binding energy E_b and the corresponding quantum-well width values used here are obtained using methods in earlier works.[22–24] At small widths, the rate of LH exciton decay seems to be larger than that of HH decay, but the difference nearly diminishes at large well widths. This is expected as the decay rate in (28) depends on the exciton mass through the exciton binding energy. Since the binding energy of the LH exciton is generally higher than that of the HH exciton, the decay rate of the LH exciton is $(E_b^{\rm LH}/E_b^{\rm HH})^2$ times higher than that of the HH exciton. This is consistent with recent experimental measurements[25] which show that the decay time for the light excitons is twice as much as in the case of the HH exciton.

The rate of exciton decay into a free-electron-hole pair calculated using (28) is found to be 1.5×10^{14} s^{-1} for the HH exciton at 300 K and well width of 100 Å, which agrees reasonably well with the inverse of the ionization time[26] of 300 fs measured for excitons in 65 periods of GaAs quantum wells 96 Å thick, alternated with $Al_{0.3}Ga_{0.7}As$ barrier layers 98 Å thick at 300 K. Experimental results[27] also indicate that the exciton lifetime decreases significantly with temperature and increases with well width, which agrees well with the decay rate expression in (28).

It is to be noted that for exciton scattering and decay processes, we have considered only the exciton interaction with acoustic phonons due to deformation potential. The piezoelectric coupling which arises due to electrostatic interaction between the acoustic phonons and the polarization field has been ignored here as its strength is weaker[28,29] compared to the interaction considered here. The inclusion of scattering via piezoelectric coupling is, however, expected to change the calculated rates by some factors which will not affect the orders of their magnitudes.

Although we have considered a process whereby energy is conserved by emitting a single acoustic phonon, the number of phonons emitted would obviously depend upon the internal energy state of the exciton, initial density of electron-hole pairs, the surrounding temperature, and wave vector associated with the phonon modes. Multiphonon processes[30,31] have not been considered here as they involve higher-order terms in the perturbing Hamiltonian.

In conclusion, we have presented in this paper the scattering and decay rates of excitons due to their interaction with virtual and acoustic phonons for various quantum-well widths and temperatures. It is expected that our results may have importance in the quantitative understanding of future experimental work involving semiconductor optical devices.

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